PHYSICAL REVIEW B **69**, 140402(R) (2004)

Domain-wall scattering in an interacting one-dimensional electron gas

R. G. Pereira and E. Miranda
Instituto de Fisica Gleb Wataghin, Unicamp, Caixa Postal 6165, 13083-970 Campinas, SP, Brazil
(Received 24 September 2003; published 6 April 2004)

We study the transport in a Luttinger liquid coupled to a magnetic chain containing a Bloch domain wall. We compute the leading correction to the adiabatic limit of a long domain wall, which causes no scattering. We show that the problem is reminiscent of an impurity in a Luttinger liquid, but with a different dependence on the interaction parameters due to spin-flip scattering. For repulsive interactions, we find that the domain-wall resistance diverges with decreasing temperature. This may be relevant for the design of one-dimensional systems with large magnetoresistance at low temperatures.

DOI: 10.1103/PhysRevB.69.140402 PACS number(s): 75.47.Jn, 85.75.−d, 73.63.Nm

The large magnetoresistance associated with the nucleation of domain walls in magnetic wires and nanocontacts[1–7] has potential applications in the design of high-density magnetic memories and sensors. The negative magnetoresistance observed in these systems was originally explained by the mistracking of carrier spins when the local magnetization rotates in a distance comparable to the Fermi wavelength.[8] Strictly speaking, none of the available experiments has reached the extreme one-dimensional (1D) limit. It would be interesting to look for effects specific to 1D systems. These systems fall into the universality class of Luttinger Liquids (LL),[9] which are distinguished by the absence of stable quasiparticle excitations and unique transport properties, such as a power-law temperature dependence of the conductance through a nonmagnetic impurity.[10–12] At $T=0$, a vanishingly small barrier is able to produce perfect reflection if the carriers interact repulsively.

The effect of nonmagnetic impurities suggests a similar phenomenon in the case of a magnetic inhomogeneity. In this article we show that a magnetic domain wall behaves as a spin-flip impurity in a LL. We analyze the backscattering term of the domain wall in the limit of weak scattering. It is governed by an anomalous dimension given primarily by $(K_c+K_s^{-1})/2$, where K_c and K_s are the LL interaction parameters. There is also a correction due to the asymmetry between up- and down-spin electrons introduced by the exchange field. In the case of local repulsive interactions, this should lead to an anomalously large and temperature-dependent magnetoresistance in one-dimensional systems.

We consider interacting electrons coupled to a magnetic domain wall as described by the Hamiltonian

$$H=\sum_{k\sigma} \epsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} - J_K \sum_j \mathbf{S}_j \cdot \mathbf{s}_j + H_{int}, \tag{1}$$

where $c_{k\sigma}$ destroys a conduction electron with momentum k and spin projection σ, $\epsilon_k=k^2/2m$ for quadratic dispersion, J_K is the Kondo coupling constant between conduction electrons and localized spins $\mathbf{S}_j$, and $\mathbf{s}_j=\frac{1}{2}\Sigma_{\alpha\beta} c_{j\alpha}^{\dagger}\sigma_{\alpha\beta}c_{j\beta}$ is the conduction electron-spin density at site j. We assume a *static*, *pinned* magnetic domain wall described in the continuum limit by setting $\mathbf{S}(x)=S\cos\theta(x)\hat{\mathbf{z}}+S\sin\theta(x)\hat{\mathbf{y}}$. For a Bloch domain wall, we take $\cos\theta(x)=-\tanh(x/\lambda)$, with λ being the wall width. The term H_{int} accounts for electron-electron interactions. In a uniformly magnetized system, the spin polarization gives rise to different interaction constants $g_\uparrow$ and $g_\downarrow$ between electrons with the same spin and $g_\perp$ between electrons with opposite spins, due to the absence of SU(2) symmetry.[13] In a system containing a domain wall, the local magnetization acts as an effective magnetic field on the conduction electrons. The first-order approach is to assume that the interaction constants are different for the spin densities in the direction fixed by the spin background

$$H_{int}=\int dx\left\{\frac{g_\uparrow}{2}\rho_\wedge^2+\frac{g_\downarrow}{2}\rho_\vee^2+g_\perp\rho_\wedge\rho_\vee\right\},$$

where

$$\rho_{\wedge,\vee}(x)=\psi^{\dagger}(x)\,\frac{1\pm\sigma\cdot\mathbf{e}(x)}{2}\,\psi(x),$$

and $\mathbf{e}(x)=\cos\theta(x)\hat{\mathbf{z}}+\sin\theta(x)\hat{\mathbf{y}}$. This expression should be exact in the limit of long domain walls. For low polarizations $(J_K\to 0)$, we recover spin degeneracy and all the interaction constants must be equal $(g_\uparrow=g_\downarrow=g_\perp)$.

It is now convenient to perform a spin gauge transformation that aligns the spin of the conduction electrons with the local magnetization.[2] This amounts to rotating the spin-density operator $\mathbf{s}(x)$ by the angle $\theta(x)$ around the x axis, which is accomplished by the operator

$$U=\exp\left\{\frac{i}{2}\int dx\ \theta(x)(\psi_\downarrow^{\dagger}\psi_\uparrow+\psi_\uparrow^{\dagger}\psi_\downarrow)\right\},$$

where $\psi_\sigma(x)$ is the field operator for conduction electrons. The rotation of H through U yields

$$\tilde{H}=U^{\dagger}HU=\sum_{k\sigma}\epsilon_{k\sigma}c_{k\sigma}^{\dagger}c_{k\sigma}+\tilde{H}_{int}+H_w. \tag{2}$$

Here, $\epsilon_{k\sigma}=\epsilon_k-\sigma J_K S/2$ expresses the fact that the effective magnetic field of the local moments breaks the spin degeneracy of the electron gas. $\tilde{H}_{int}$ is obtained from H_{int} by changing $\rho_{\wedge,\vee}\to\rho_{\uparrow,\downarrow}$. The transformation also makes explicit the scattering term due to the presence of the domain wall

$$H_w = -\frac{i}{4m}\int dx\,\partial_x\theta\,\psi^\dagger\sigma_x\partial_x\psi + \text{H.c.} + O(\lambda^{-2}), \quad (3)$$

where we intend to carry out the calculations to leading order in $1/\lambda$. This corresponds to the first correction to the adiabatic limit of a very long domain wall, which produces no scattering.

We now focus on the long-wavelength limit of the conduction electrons. In this limit, we can linearize the dispersion around the Fermi points. Since each of the two spin branches has a different Fermi wave vector $k_{F\sigma}$, we must have two Fermi velocities $v_{F\sigma}=v_F(1+\sigma\zeta)$, with v_F the mean Fermi velocity and ζ the velocity mismatch

$$\zeta = \frac{v_{F\uparrow}-v_{F\downarrow}}{v_{F\uparrow}+v_{F\downarrow}}.$$

The linearized dispersion for spin σ reads $\epsilon_{k\sigma}=v_{F\sigma}(k \mp k_{F\sigma})$, where the minus (plus) sign applies to right (left) moving electrons. The field operator ψ_σ then naturally separates into right and left parts

$$\psi_\sigma(x) = e^{ik_{F\sigma}x}\psi_{+,\sigma}(x) + e^{-ik_{F\sigma}x}\psi_{-,\sigma}(x).$$

Bosonization enables one to build an effective theory by mapping the fermionic operators into associated bosonic fields.[9] In terms of these fields, the field operators are given by

$$\psi_{r,\sigma}(x) = \frac{1}{\sqrt{2\pi\alpha}}\exp\{-i\sqrt{\pi}[\theta_\sigma(x) - r\phi_\sigma(x)]\}, \quad (4)$$

where α^{-1} is a momentum cutoff and ϕ_σ and θ_σ are dual fields satisfying $[\phi_\sigma(x),\partial_x\theta_\sigma(x')] = i\delta(x-x')$. We further define the charge and spin bosons $\phi_{c,s}=(\phi_\uparrow \pm \phi_\downarrow)/\sqrt{2}$. Upon bosonizing the free part of the Hamiltonian (2), we get the LL Hamiltonian[9]

$$H_{LL} = \sum_{\nu=c,s}\frac{v_\nu}{2}\int dx\left\{K_\nu(\partial_x\theta_\nu)^2 + \frac{1}{K_\nu}(\partial_x\phi_\nu)^2\right\} + \int dx\{\zeta v_1\partial_x\theta_c\partial_x\theta_s + \zeta v_2\partial_x\phi_c\partial_x\phi_s\}, \quad (5)$$

where

$$v_1 = v_F + \frac{g_{4\uparrow}-g_{4\downarrow}+g_{2\uparrow}-g_{2\downarrow}}{2\pi\zeta},$$

$$v_2 = v_F + \frac{g_{4\uparrow}-g_{4\downarrow}-g_{2\uparrow}+g_{2\downarrow}}{2\pi\zeta},$$

where $g_{2\sigma}$ and $g_{4\sigma}$ are the interaction constants between electrons in different branches and in the same branch, respectively. For not very large ζ, we will take $g_{i\uparrow}-g_{i\downarrow}\propto\zeta$ $(i=2,4)$ (Ref. 13) so that $v_{1,2}$ are approximately independent of ζ. It is clear from Eq. (5) that the spin background introduces scattering between charge and spin excitations, which are no longer the normal modes of system.

The bosonized form of the scattering terms H_w can be obtained easily by using the relation (4). We retain only the backscattering term, which scatters electrons from right to left moving states (and vice versa) and is important for the departure from perfect conductance.[10–12] It can be written in terms of charge and spin fields as

$$H_w^{(b)} = \frac{\zeta k_F A_{2k_F}}{m\pi\alpha}\sin[\sqrt{2\pi}\theta_s(0)]\sin[\sqrt{2\pi}\phi_c(0)], \quad (6)$$

where $k_F=(k_{F\uparrow}+k_{F\downarrow})/2$ and $A_q=\int dx e^{-iqx}\partial_x\theta(x)$ is real for symmetric walls. We note that the $2k_F$ mode of the domain wall cancels the oscillation of the backscattering term. Moreover, the scattering amplitude increases with growing ζ and thinner walls.

The free Hamiltonian H_{LL} as given by Eq. (5) is not in diagonal form. However, it is still quadratic in the bosonic fields and can be diagonalized by means of a canonical transformation to new fields $\theta'_{c,s}$ and $\phi'_{c,s}$. We define the bosonic field vectors

$$\theta = \begin{pmatrix}\theta_c\\ \theta_s\end{pmatrix}, \quad \phi = \begin{pmatrix}\phi_c\\ \phi_s\end{pmatrix},$$

so that H_{LL} can be rewritten as

$$H_{LL} = \frac{1}{2}\int dx\{\partial_x\theta A\partial_x\theta + \partial_x\phi B\partial_x\phi\},$$

where we have introduced the matrices

$$A = \begin{pmatrix} v_cK_c & \zeta v_1\\ \zeta v_1 & v_sK_s\end{pmatrix}, \quad B = \begin{pmatrix} v_c/K_c & \zeta v_2\\ \zeta v_2 & v_s/K_s\end{pmatrix}.$$

Our aim is to diagonalize A and B simultaneously. In order for the LL to be stable, the corresponding eigenvalues (the velocities of the natural excitations) must be positive; this limits the validity of our solution to the interval

$$\frac{\zeta^2 v_1^2}{v_c v_s} < K_c K_s < \frac{v_c v_s}{\zeta^2 v_2^2}. \quad (7)$$

Outside this interval, the polarization is large enough to make one of the velocities vanish and the spinonlike excitation becomes gapped. We start the diagonalization by rotating A and B through an angle φ, as expressed by the matrix

$$R = \begin{pmatrix}\cos\varphi & \sin\varphi\\ -\sin\varphi & \cos\varphi\end{pmatrix}.$$

We choose the angle φ in such a way that, applying next the rescaling

$$\Lambda = \begin{pmatrix}\sqrt{\kappa} & 0\\ 0 & \sqrt{\mu}\end{pmatrix},$$

we shall have $\Lambda^{-1}R^tAR\Lambda^{-1} = \Lambda R^tBR\Lambda$. This condition requires

$$\kappa = \sqrt{\frac{v_cK_c\cos^2\varphi - \zeta v_1\sin 2\varphi + v_sK_s\sin^2\varphi}{\frac{v_c}{K_c}\cos^2\varphi - \zeta v_2\sin 2\varphi + \frac{v_s}{K_s}\sin^2\varphi}}, \quad (8a)$$

$$\mu=\sqrt{\frac{v_sK_s\cos^2\varphi+\zeta v_1\sin 2\varphi+v_cK_c\sin^2\varphi}{\frac{v_s}{K_s}\cos^2\varphi+\zeta v_2\sin 2\varphi+\frac{v_c}{K_c}\sin^2\varphi}}, \tag{8b}$$

$$\kappa\mu=\frac{2\zeta v_1\cos 2\varphi+(v_cK_c-v_sK_s)\sin 2\varphi}{2\zeta v_2\cos 2\varphi+\left(\frac{v_c}{K_c}-\frac{v_s}{K_s}\right)\sin 2\varphi}. \tag{8c}$$

The restriction (7) assures that κ and μ are both real. We then determine φ in the interval $[-\pi/4,\pi/4]$ for arbitrary ζ by imposing that the three expressions (8) are solved simultaneously. Being equal, the two transformed matrices can be made diagonal by performing a second rotation S. As a result, the Hamiltonian (5) assumes the form

$$H_{LL}=\sum_{\nu=c,s}\frac{v'_\nu}{2}\int dx\{(\partial_x\theta'_\nu)^2+(\partial_x\phi'_\nu)^2\}, \tag{9}$$

where $v'_{c,s}$ are the eigenvalues of the final matrix. The original bosonic field vectors are written in terms of the new ones as

$$\theta=T^\theta\theta' \qquad \phi=T^\phi\phi',$$

where $T^\theta=R\Lambda^{-1}S$ and $T^\phi=R\Lambda S=[(T^\theta)^{-1}]^t$.

In order to analyze the effect of the backscattering term (6), we work out an effective action for the free Hamiltonian that depends only on the fields at the origin.[10] In terms of the new bosonic fields, we have

$$H_w^{(b)}=\gamma\sin[\sqrt{2\pi}[T_{21}^\theta\theta'_c(0)+T_{22}^\theta\theta'_s(0)]]$$
$$\times\sin[\sqrt{2\pi}[T_{11}^\phi\phi'_c(0)+T_{12}^\phi\phi'_s(0)]],$$

where $\gamma=\zeta k_FA_{2k_F}/m\pi\alpha$. Thus, the effective action must depend on both conjugate fields. We start with the free partition function in imaginary time

$$Z_0=\int\prod_\nu D\phi'_\nu D\theta'_\nu\exp\left\{\int dxd\tau[i\partial_\tau\phi'_\nu\partial_x\theta'_\nu-\mathcal{H}(\phi'_\nu,\theta'_\nu)]\right\},$$

where $\mathcal{H}(\phi'_\nu,\theta'_\nu)$ is the Hamiltonian density in Eq. (9). Then we integrate out the degrees of freedom for $x\neq 0$ and find the effective action

$$S_0^{eff}[\phi_0,\theta_0]=\frac{1}{\beta}\sum_{\nu,n}|\omega_n|\phi'_{0\nu}(\omega_n)\phi'_{0\nu}(-\omega_n)$$
$$+\frac{1}{\beta}\sum_{\nu,n}|\omega_n|\theta'_{0\nu}(\omega_n)\theta'_{0\nu}(-\omega_n),$$

where ω_n are bosonic Matsubara frequencies. A renormalization group analysis gives the flow of the coupling constant γ at low energies ($\ell\to\infty$) (Refs. 10–12)

$$\frac{d\gamma}{d\ell}=(1-D)\gamma,$$

where D is the dimension of the backscattering operator, given by

$$D=\tfrac{1}{2}[(T_{21}^\theta)^2+(T_{22}^\theta)^2+(T_{11}^\phi)^2+(T_{12}^\phi)^2]. \tag{10}$$

We would like to express D in terms of the LL parameters. Remarkably, it does not depend on the matrix S and reduces to

$$D=\frac{1}{2}\left[\kappa\cos\varphi+\frac{1}{\kappa}\sin\varphi+\frac{1}{\mu}\cos\varphi+\mu\sin\varphi\right].$$

For small ζ, we get

$$D=\frac{1}{2}\left(K_c+\frac{1}{K_s}\right)+\frac{(K_cK_sv_2-v_1)[K_cK_s(K_cK_sv_2+v_1)v_s^2+2(K_c^2K_s^2v_2-v_1)v_cv_s-(K_cK_sv_2+v_1)v_c^2]\zeta^2}{4K_cK_s^2v_cv_s(v_c+v_s)^2}. \tag{11}$$

For nonmagnetic impurities, $D_{imp}=(K_c+K_s)/2$, which is different from the $\zeta\to 0$ limit of our result. This should be attributed to the spin-flip scattering explicit in the form (3), in contrast with the charge-only scattering by a nonmagnetic impurity.

The possible phases can be obtained similarly to Refs. 10–12. We first focus on the $\zeta\to 0$ case. For $D>1$ or $K_c+K_s^{-1}>2$, the scattering is irrelevant and the fixed point is a LL with perfect transmission of charge and spin. For $D<1$ or $K_c+K_s^{-1}<2$, which is favored for increasingly repulsive interactions (decreasing K_c), the scattering is relevant and the system flows to the strong-coupling limit. This limit corresponds to two semi-infinite LL's with spins polarized in opposite directions and coupled through a small hopping term that flips the electron spin in the tunneling process. This term has been analyzed in the context of a magnetic impurity in a LL,[14] where the hopping is found to be irrelevant for repulsive interactions. As a result, the fixed point is a spin-charge insulator at $T=0$. The straight line in Fig. 1 represents the marginal line $D=1$ in the limit $\zeta\to 0$.

The correction for finite ζ vanishes when $K_cK_s=v_1/v_2$. Actually, this cancellation happens to all orders in ζ because the Eqs. (8) are always satisfied for $\varphi=0$, $\kappa=K_c$, and $\mu=K_s=v_1/v_2K_c$. Consequently, the condition $K_cK_s=v_1/v_2$ defines a line in parameter space where the dimension of the scattering term is ζ invariant. In particular, the noninteracting point $K_c=K_s=1$ (and $v_1=v_2=v_F$) is always marginal. For $K_cK_s\neq v_1/v_2$, the dimension varies with ζ.

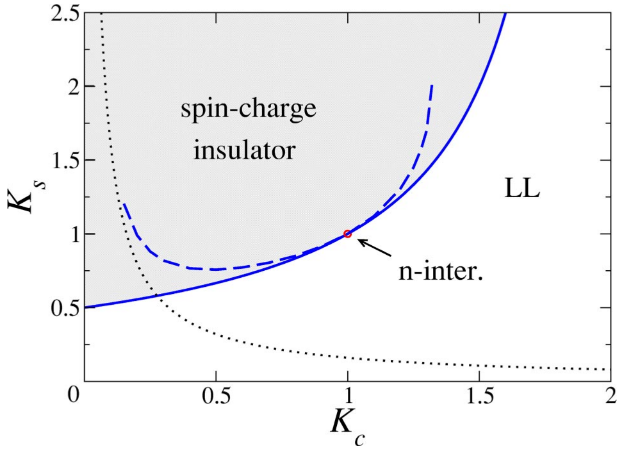

FIG. 1. Phase diagram for a Luttinger liquid coupled to a magnetic domain wall. The backscattering term $H_w^{(b)}$ is marginal on the straight line in the limit $\zeta \to 0$, and on the dashed one for $\zeta = 0.4$ (with all velocities equal). The dotted line corresponds to the lower bound of stability of the Luttinger liquid according to Eq. (7).

The dashed line in Fig. 1 shows how the marginal line is modified for $\zeta = 0.4$ and $v_c = v_s = v_1 = v_2 = v_F$.

The dimension D manifests itself in the exponent of the frequency-dependent domain-wall resistance. The resistivity associated with the backscattering off the wall at low frequencies is $\rho(\omega) \propto \omega^{2(D-1)}$. Likewise, the finite-temperature resistance turns out to be $\rho(T) \propto T^{2(D-1)}$. Therefore, the domain-wall scattering in a LL gives rise to a temperature-dependent resistance. For $D > 1$, the resistance vanishes as a power law when $T \to 0$; for $D < 1$, it diverges in the limit $T \to 0$. The LL behavior is cut off at a temperature $T^* \sim v_F/L$, below which the transport is dominated by the Fermi-liquid leads.[10] This can be understood as follows. The domain wall is known to induce long-ranged spin-density oscillations in the electron gas.[7] Similarly to what happens with charge-density oscillations created by nonmagnetic impurities,[12] the scattering by these spin-density oscillations diverges at low ω in one dimension. As a result, the electrons are totally reflected by the wall.

Finally, let us estimate the exponent in the particular case of the Hubbard model.[13] Due to the absence of SU(2) symmetry, we cannot take $K_s = 1$ as usual. Instead, the parameters K_c and K_s depend implicitly on ζ. To lowest order in ζ, $K_s \approx 1 + [2\ln(\zeta^{-1})]^{-1}$. Note that this correction has a lower order dependence on ζ than the explicit one (order ζ^2) in Eq. (11). Furthermore, a finite polarization makes $K_s > 1$ and so pushes the model into the insulating region of the phase diagram. For small U, $K_c \approx 1 - aU/2\pi v_F + O(\zeta)$, where U is the on-site repulsion and a is the lattice spacing. Then, $D \approx 1 - aU/4\pi v_F - [4\ln(\zeta^{-1})]^{-1}$. As an experimental test of this theory, one should look for the dependence of the resistance exponent on the polarization ζ of the underlying system of carriers.

In conclusion, we have shown that the domain-wall scattering in a Luttinger liquid is the magnetic analog of the Kane-Fisher problem. Just as a nonmagnetic impurity, a domain wall breaks the translation symmetry of the electron gas. The $2k_F$ mode of the wall gives rise to a spin-flip backscattering term which is relevant for repulsive interactions. In this case, the magnetoresistance diverges as a power law in the limit of zero temperature. By applying magnetic fields one can insert or remove a single domain wall and then switch between a spin-charge insulator and a Luttinger liquid with perfect conductance. This should be relevant in view of the quest for systems exhibiting large magnetoresistance.

This work was supported by Fapesp through Grants No. 01/12160-5 (R.G.P.) and 01/00719-8 (E.M.), and by CNPq through Grant No. 301222/97-5 (E.M.).

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