Non-Gaussian Spatial Correlations Dramatically Weaken Localization

H. Javan Mard,[1] E. C. Andrade,[2] E. Miranda,[3] and V. Dobrosavljević[1]
[1]Department of Physics and National High Magnetic Field Laboratory, Florida State University, Tallahassee, Florida 32306, USA
[2]Institut für Theoretische Physik, Technische Universität Dresden, 01062 Dresden, Germany
[3]Instituto de Física Gleb Wataghin, Unicamp, R. Sérgio Buarque de Holanda, 777, Campinas, SP 13083-859, Brazil
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We perform variational studies of the interaction-localization problem to describe the interaction-induced renormalizations of the effective (screened) random potential seen by quasiparticles. Here we present results of careful finite-size scaling studies for the conductance of disordered Hubbard chains at half-filling and zero temperature. While our results indicate that quasiparticle wave functions remain exponentially localized even in the presence of moderate to strong repulsive interactions, we show that interactions produce a strong decrease of the characteristic conductance scale g^* signaling the crossover to strong localization. This effect, which cannot be captured by a simple renormalization of the disorder strength, instead reflects a peculiar non-Gaussian form of the spatial correlations of the screened disordered potential, a hitherto neglected mechanism to dramatically reduce the impact of Anderson localization (interference) effects.

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According to the scaling theory of localization [1], any amount of disorder suffices to localize all (noninteracting) electrons at $T = 0$ in dimension $d \leq 2$. In the presence of electron-electron interactions, however, no such general statement exists, and the transport behavior of disordered interacting electrons remains an outstanding open problem [2]. Since the relevant analytical results are available only in some limiting cases [3–5], complementary computational methods play a crucial role in providing insight and information. Several numerical approaches have been recently utilized to investigate transport properties of these systems, including variational Hartree-Fock (HF) [6–8] and slave boson (Gutzwiller approximation) [9] methods, as well as (numerically exact) quantum Monte Carlo techniques [10–12].

These studies provided evidence that repulsive electron-electron interactions generally increase the conductance in small systems, with the suppression of electronic localization being tracked down to partial screening of the disorder potential. In principle, interactions could modify either the amplitude or the form of spatial correlations [6] of the renormalized disorder potential. The former mechanism is known to be significantly enhanced by strong correlation effects [9] and to survive even in high dimensions, while the latter is more pronounced [13] in the weak-coupling regime and in low dimensions [4].

Despite this progress, several important questions remained unanswered: (1) What is the dominant physical mechanism for disorder screening, and can it qualitatively modify the noninteracting picture? (2) Can the interaction effects overcome Anderson localization and stabilize the metallic phase in low dimensions? The task to carefully and precisely answer these important questions in a model calculation is the main goal of this Letter. To do this, we utilize two different variational methods to describe the statistics of the renormalized disorder potential in an idealized dirty Fermi liquid. In contrast to most previous attempts, here we perform a careful finite size scaling analysis of the conductance, which allows us to reach conclusive results for the transport properties of the model we consider.

Model and method.—We study the paramagnetic phase of a disordered Hubbard model

$$H = -t\sum_{i,j,\sigma}(c^{\dagger}_{i\sigma}c_{j\sigma} + \text{H.c.}) + \sum_{i,\sigma}\varepsilon_i n_{i\sigma} + U\sum_i n_{i\uparrow}n_{i\downarrow}, \tag{1}$$

where t is the hopping amplitude between nearest-neighbor sites, $c^{\dagger}_{i\sigma}(c_{i\sigma})$ are the creation (annihilation) operators of an electron with spin $\sigma =$ at site i, U is the on-site Hubbard repulsion, and $n_{i\sigma} = c^{\dagger}_{i\sigma}c_{i\sigma}$. The spatially uncorrelated random site energies ε_i are drawn from a uniform distribution of zero mean and width W. We work at half-filling, in units such that $t = a = e^2/h = 1$, where a is the lattice spacing, h is Planck's constant, and e is the electron charge. To be able to carry out the large scale computations needed for conclusive finite-size scaling of the conductance, we focus our attention on a one-dimensional model. Within the variational description of a dirty Fermi liquid we consider, we expect the main trends to persist in higher dimensions.

Our starting point is the nonmagnetic HF scheme [6], where the renormalized site energies v_i are given by

$$v_i = \varepsilon_i + \frac{U}{2}\langle n_i\rangle. \tag{2}$$

Here, $\langle n_i \rangle = \sum_\sigma \langle n_{i\sigma} \rangle$, the average site occupation, is determined self-consistently in the ground state, for each disorder realization. To cross-check our HF predictions within a theory that is able to capture strong correlation effects, we repeated the same calculations using the slave boson (SB) mean-field theory (i.e., the Gutzwiller approximation) of Kotliar and Ruckenstein [9,14], generalized to disordered systems [13]. The SB theory features two local variational parameters: the renormalized site energies v_i and the quasiparticle weight Z_i ($Z_i = 1$ within HF) [15–17]. We found that, for moderate interaction strength (not close to the Mott transition) and the low-dimensional situation we consider, both methods produce qualitatively the same behavior (see Fig. 2), dominated by a peculiar type of spatial correlation of the screened disorder potential. The strong correlation effects (corresponding to $Z_i \ll 1$) do not appear to play a significant role in this regime (in contrast to the situation explored in Refs. [13,16]). This makes it possible to search for the relevant screening mechanism within the simpler and physically very transparent HF scheme, which we focus on in presenting most of our results.

To study the nature of the ground state we focus on the dimensionless conductance g, which we obtain applying the standard Landauer approach to our quasiparticle Hamiltonian [17–20]. We numerically calculate g in a setup where we attach our system to two noninteracting metallic leads at its ends [21]. For simplicity, we consider the wide band limit, where the leads' self-energies are simply given by $\Sigma_{1(L)} = -i\eta/2$ [17], and in all our results we consider $\eta = 1.0t$ (we carefully checked that all our conclusions are independent of η [17]). Since we are working in one dimension, the conductance displays wide sample-to-sample fluctuations. We therefore focus on its typical value, as given by the geometrical average $g = g_{\rm typ} = \exp \overline{\ln g_s}$ [22,23]. In every case, we averaged our results over 2000 disorder realizations, which was sufficient to obtain very accurate results.

Conductance scaling.—In the noninteracting limit, the dependence of the conductance on disorder and system size can be expressed in a simple scaling function $g_0(x)$, with $x = L/\xi$, where $\xi = \xi(W)$ is the localization length [1,24]. Specifically, $g_0 \propto L^{d-1}$ for $g \gg g^*$ (Ohmic regime, $x \ll 1$) and $g_0 \propto \exp(-L/\xi)$ for $g \ll g^*$ (localized regime, $x \gg 1$), where g^* is the characteristic dimensionless conductance which marks the crossover between these two regimes. In particular, we use the expected exponential decay of the conductance to determine ξ for fixed values of W.

Using this scaling ansatz, we can collapse the system size dependence of the conductance onto a scaling curve $g(L/\xi)$ even in the presence of interactions, as shown in Fig. 1. Note that the error bars are approximately the size of the data symbols or even smaller. We find that the localization length increases considerably with U (see [17] for more details). This enhancement of the localization length with interactions has been often observed in studies of disordered interacting systems [7,12,25,26]. We should, nevertheless, stress that, despite the huge enhancement of ξ with U, there is always an exponential decrease for large L and we do not see any evidence of extended states.

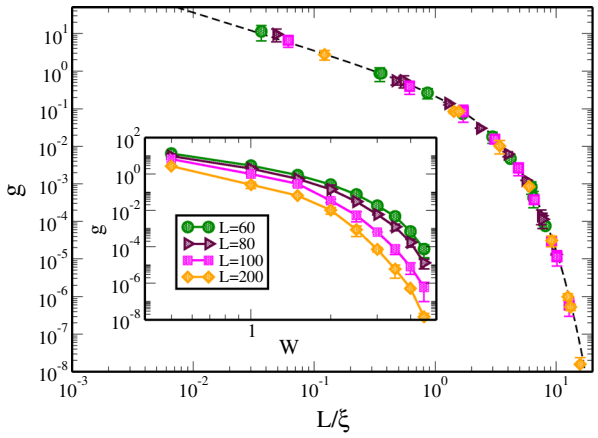

FIG. 1 (color online). The conductance scaling function for $U = 1.0t$ obtained with system sizes $L = 60, 80, 100$, and 200 at several disorder levels $W \leq 4.5t$. The dashed line is a modified version of the one-dimensional conductance scaling function proposed in Ref. [22] [see also Eqs. (3) and (5)]: $g = g^*/[\exp(x) - 1]$, with $x = L/\xi$ and $g^* = 0.366$. In the inset, the conductance as a function of the disorder level is shown for fixed system sizes.

Interestingly, all the curves $g(L/\xi)$ for different interaction strengths can be made to collapse onto a single universal curve by a proper interaction-dependent rescaling of the conductance, see Fig. 2 [27]. We call the conductance rescaling factor $g^*(U)$, and stress that g^* is a function of U only. Its U dependence for both HF and SB approaches is shown in the inset of Fig. 2, where an exponential decrease with U fits well the data in both cases.

The above scaling implies that the full disorder and interaction dependence of the conductance can be written as

$$g = g^*(U) g_0[L/\xi(W, U)], \tag{3}$$

where $g_0(x)$ is the noninteracting scaling function, $g^*(U)$ sets the crossover conductance which separates the weak localization regime ($g \gg 1$) from the strongly localized one ($g \ll 1$), and we have explicitly shown all the W and U dependences.

The scaling function in Eq. (3) can then be used to generate the beta function $\beta(g) = d \ln g / d \ln L$. It follows immediately that the only effect introduced by interactions on $\beta(g)$, as compared to its noninteracting counterpart, is the rescaling of g by the characteristic conductance $g^*(U)$

$$\beta(g) = \beta_0[g/g^*(U)], \tag{4}$$

where $\beta_0(g)$ is the noninteracting beta function. In particular, if we use the form of $\beta_0(g)$ proposed in Ref. [22] we obtain

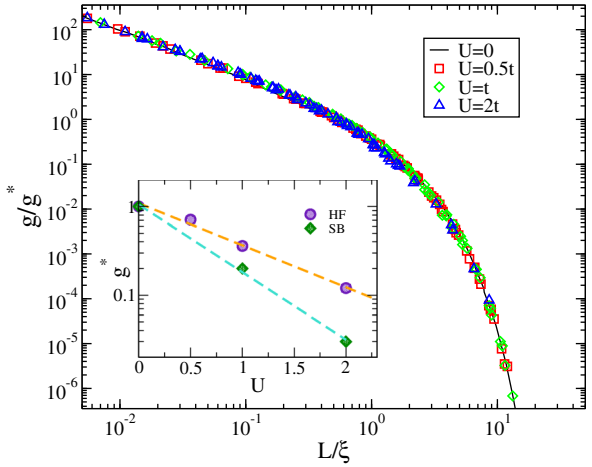

FIG. 2 (color online). Conductance scaling curves for different U values collapse onto a single universal curve after rescaling both L (by ξ) and g [by $g^*(U)$]. In the inset, the characteristic conductance g^* is plotted as a function of U for both HF and SB approaches. The behavior is well fitted by an exponential: $g^*(U) = 1.09(8)\exp[-1.09(7)U]$ (HF) and $g^*(U) = 1.05(2)\exp[-1.75(3)U]$ (SB).

$$\beta(g) = -\left[1 + \frac{g}{g^*(U)}\right] \ln\left[1 + \frac{g^*(U)}{g}\right]. \qquad (5)$$

The validity of Eq. (5) can be double-checked through a direct examination of the behavior of the beta function for different values of U, as shown in Fig. 3. We stress that interaction-induced renormalizations of the localization length alone are not capable of describing the results of Fig. 3, as they drop out of the beta function. Finally, using (5), we are able to give an operational definition of the characteristic conductance: $g = g^*(U)$ at $L = L^* = (\ln 2)\xi(W, U)$ [17].

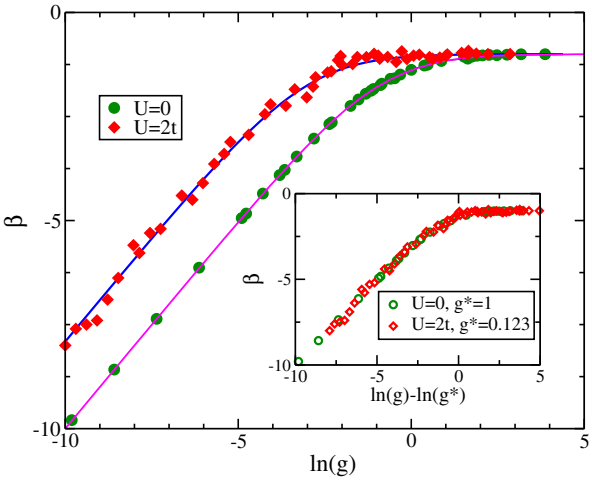

FIG. 3 (color online). The beta function for $U = 0$ and $U = 2t$. The numerical data are well described by Eq. (5) with $g^*(U = 0) = 1.0$ and $g^*(U = 2t) = 0.123$. Inset: The curve for $U = 2t$ collapses onto the noninteracting one with a shift of $\ln(g^*)$ along the horizontal axis.

Disorder screening and non-Gaussian spatial correlations.—A commonly invoked explanation for this conductance enhancement is the fact that interactions act to "screen" the one-body potential [6,9,16]. Within a mean-field picture, an electron moving in the one-body potential v_i "sees" site energies renormalized by the average interaction with the other electrons, as in Eq. (2). In the inset of Fig. 4, we compare the conductance in the full HF calculation for $W = 0.5t$ and $U = 1t$ with the one obtained in the noninteracting case with an effective disorder W_{eff} obtained from the width of the v_i distribution [17]. It is clear that the screening effect by itself is not enough to reproduce the conductance enhancement of the full HF calculation. This is further confirmed when, after obtaining the fully converged self-consistent HF values of v_i's, we then calculate the conductance of a noninteracting system whose site energies are a random permutation (RP) of the same v_i's. Not surprisingly, the conductance of the randomized system is essentially the same as the one for the noninteracting system with uncorrelated site energies distributed uniformly with strength W_{eff} (inset of Fig. 4). In the main panel of Fig. 4, we also show the beta function obtained from the RP of the HF results. As can be seen, it reduces to the noninteracting one. The effect of a RP of the renormalized site energies is to eliminate the spatial correlations between them. In the following, we argue that it is precisely these correlations which shift the crossover

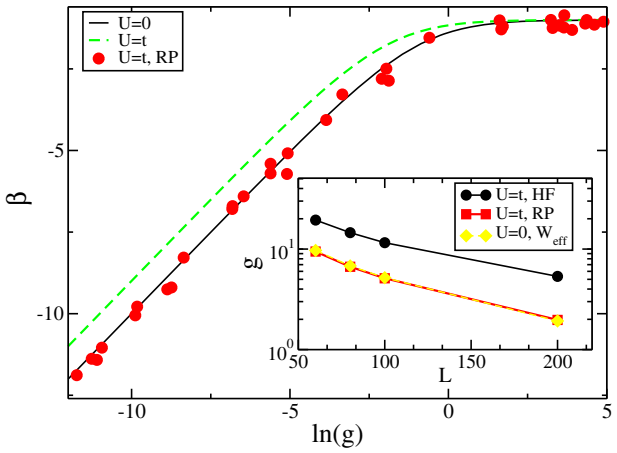

FIG. 4 (color online). Beta function for $U = 1t$ obtained after randomizing the self-consistently determined HF renormalized energies v_i's (red circles; see also the text for an explanation of the procedure). Black and green lines are plotted using Eq. (5). In the inset, the conductance g is shown as a function of L for: a system with $W = 0.5t$ and $U = 1t$ in the HF approximation (black dots), a noninteracting system whose site energies are a random permutation of the renormalized site energies v_i of the HF approximation (red squares), and a noninteracting system with $U = 0$ and $W = W_{\text{eff}} = 0.41t$ (gold diamonds).

scale $g^*(U)$ to much smaller values as compared to the $U = 0$ case.

To further elucidate the pivotal role of spatial correlations, we start by looking at the limit of weak disorder $W \to 0$. A perturbative calculation shows that the correlations among the v_i's are given by $\langle v_i v_j \rangle \sim r_{ij}^{-1}$, for $r_{ij} \gg 1$, where $r_{ij} = |r_i - r_j|$ [13,17]. These long-ranged correlations of the effective disorder potential come from the usual Friedel oscillations. When properly tailored, a correlated disorder potential may drive a metal-insulator transition in $d = 1$ [28–32]. In order to go beyond weak disorder, we first generate numerically the two point correlation function $\langle v_i v_j \rangle$ from our HF results [17]. We then implement a standard procedure to generate random v_i's with Gaussian correlations of zero mean and covariance matrix $\langle v_i v_j \rangle$ (note that the generated data have no correlations beyond Gaussian). Finally, we calculate the conductance of a noninteracting system with the latter site energies. Essentially, we want to know if the Gaussian correlations contained in $\langle v_i v_j \rangle$ are sufficient to account for the g^* renormalization. Figure 5 displays the results of this numerical procedure [which we dubbed Gaussian spatial correlations (GSC)]. Although the conductance is enhanced as in the case of W_{eff} (see the inset of Fig. 5), the scaling curve coincides with the noninteracting one, implying there is no g^* renormalization from purely Gaussian correlations.

Taken together, these facts imply that there are significant non-Gaussian spatial correlations in the v_i's which considerably delay the crossover to the strongly localized regime. Such correlations introduce a very exciting new dimension to the physics of disordered systems, because much of the existing lore about Anderson localization focused on the effects of random potentials with simple Gaussian statistics—incorrectly assuming that higher-order correlations play only a secondary role. In the Supplemental Material [17] we further characterize these intersite correlations and show how their incorporation is essential for a $g^*(U) < 1$.

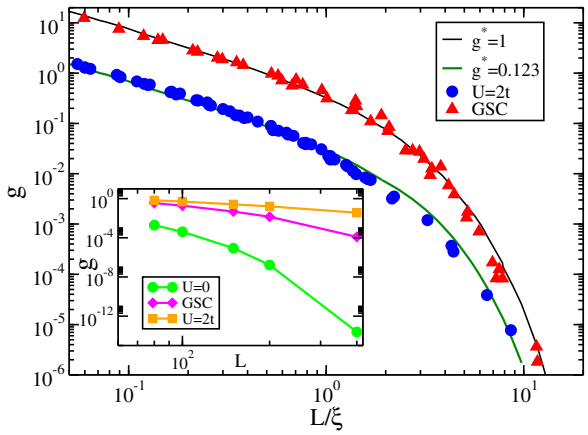

FIG. 5 (color online). Comparison of the conductance scaling function of a disordered chain with $U = 2t$ in the HF approximation and the corresponding noninteracting system with the same Gaussian correlated site energies (GSC) (the inset shows the unscaled data). Solid lines are drawn by using $g = g^*/[\exp(x) - 1]$, as in Fig. 1.

Extension to higher dimensions.—It is tempting to speculate on what would happen if our main conclusions persist in $d > 1$. If we follow the same phenomenological extension as in Shapiro's work [33], we can write $\tilde{\beta}_d(g) = \beta(g) + d - 1$, where $\beta(g)$ is given in Eqs. (4) or (5). Graphically, this corresponds to a vertical shift of $\beta(g)$ for $d = 2, 3$. In particular, for $d = 3$, $\tilde{\beta}_d(g)$ changes sign as expected [1,24]. By construction, $\tilde{\beta}_d$ has the correct asymptotic limits: $\tilde{\beta}_d \simeq d - 2$ for $g \gg g^*$ and $\tilde{\beta}_d \propto \ln(g/g^*)$ for $g \ll g^*$. Assuming, as we found, that the main effect of interactions is to rescale the crossover scale g^*, the net result would be to shift this crossover in $\tilde{\beta}_d$ to much smaller conductances. This implies a much more extended Ohmic region, even though at $T = 0$ all electronic states should still remain localized in $d = 2$ [12,26]. In addition, the proposed interaction-induced renormalization $g \to g/g^*$ should dramatically reduce the amplitude of the weak-localization correction; precisely such an effect was observed in $d = 2$ magnetoresistance experiments [34]. In practice, this would open the possibility that competing (e.g., Mott or Wigner-Mott) mechanisms for localization [35,36] could become dominant well before Anderson localization effects set in.

Conclusions.—Adding interactions to a disordered system gives rise to new effects that assist transport even if the single particle states are all Anderson localized. Our careful numerical studies show that the typical value of the scaled conductance follows the same noninteracting behavior but with a large decrease of the conductance scale $g^*(U)$ signaling the crossover to the strongly localized regime. Surprisingly, we find that this reduction is brought about by non-Gaussian intersite correlations, a mechanism overlooked in previous works. This opens an exciting new door to understanding the effects of interactions in disordered systems.

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