Magnetically controlled impurities in quantum wires with strong Rashba coupling

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We investigate the effect of strong spin-orbit interaction on the electronic transport through nonmagnetic impurities in one-dimensional systems. When a perpendicular magnetic field is applied, the electron spin polarization becomes momentum-dependent and spin-flip scattering appears, to first order in the applied field, in addition to the usual potential scattering. We analyze a situation in which, by tuning the Fermi level and the Rashba coupling, the magnetic field can suppress the potential scattering. This mechanism should give rise to a significant magnetoresistance in the limit of large barriers.

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The spin-orbit interaction in low-dimensional systems is a powerful tool in the development of spintronics, the electronics that intends to explore the electron spin to store and transport information.[1] In semiconductor heterostructures, the spin-orbit interaction arises intrinsically from the asymmetry of the potential that confines the two-dimensional electron gas (2DEG). In this context, it is usually referred to as Rashba coupling.[2] In addition to this, there is also the Dresselhaus spin-orbit coupling, which is a result of the lack of inversion-symmetry in the bulk.[3] Both terms introduce an effective magnetic field that depends on the in-plane momentum of the electron. The spin precession associated with the Rashba coupling led Datta and Das to propose a spin field-effect transistor (spin-FET).[4] The interest in this device is furthered a great deal by the demonstrated possibility of tuning the spin-orbit coupling by means of applied gate voltages.[5] However, the presence of impurities poses an obstacle to the spin-FET because they scatter electrons between states with different momentum and, consequently, randomize the spin direction. This is known as the Elliot-Yafet mechanism of spin relaxation.[6,7] More recently, it has been pointed out that a nonballistic spin-FET, which would be robust against impurity scattering, can be realized by matching the Rashba and Dresselhaus parameters.[8]

Quantum wires are created when the propagation in the 2DEG is further confined in one of its directions. If the wire width is comparable to the Fermi wavelength, the transverse sub-bands of the quasi-one-dimensional system are quantized. In the strict one-dimensional (1D) limit of vanishing width, one can neglect the mixing between the sub-bands, which is proportional to the transverse momentum, and recover a well-defined spin-polarization axis.[9] The dependence of the electron spinors on the momentum is restored and conveniently controlled by applying a magnetic field perpendicular to the spin-polarization axis.[10] Therefore, although impurity scattering is enhanced in 1D systems due to interaction effects,[11–13] a nonmagnetic impurity is able to cause spin relaxation only in the presence of an external magnetic field.

An effective theory of 1D conductors which includes spin-orbit interaction as well as Zeeman splitting has been developed.[14,15] In this work, we address the question of how the spin splitting characteristic of spin-orbit interaction affects the scattering off nonmagnetic impurities in 1D systems. For zero magnetic field, we obtain the usual potential scattering without spin flip. The application of a perpendicular magnetic field opens a gap in the vicinity of $k=0$, where the spin subbands are degenerate, and introduces a spin-flip scattering term. We analyze the effect of this term in the limit of strong Rashba coupling, i.e., when the band splitting is comparable to the Fermi energy. In Ref. 16, it was argued that electron scattering at a potential step in the interface between a metallic gate and the semiconductor wire can work as a spin filter when the Fermi level coincides with the position of the gap. We will show that, by making the same choice for the Fermi level, the spin-flip scattering which we consider becomes the most relevant process at low temperatures and the scattering off the impurity can be controlled by the magnetic field. The possibility of observing this effect in a realistic experimental situation is also discussed.

The Hamiltonian for one electron moving on the xy plane subjected to the Rashba spin-orbit interaction is

$$H_0 = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{\alpha}{\hbar}(\sigma_x p_y - \sigma_y p_x) + V(x), \tag{1}$$

where $\mathbf{p}$ is the momentum operator, α is the Rashba coupling constant, $\boldsymbol{\sigma}$ is the vector of Pauli matrices, and $V(x)$ is a potential that confines the electron in the x direction. The latter is usually taken as the potential of a harmonic oscillator $V(x)=m\omega_0^2 x^2/2$, and the 1D limit is achieved by taking $\hbar\omega_0 \gg \epsilon_F$, with ϵ_F the Fermi energy. This means that the wire width w must be small enough that $w \ll \lambda_F$, where λ_F is the Fermi wavelength. In this limit, the transverse degrees of freedom are frozen at low temperatures and we can simply write

$$H_0 \approx \frac{p^2}{2m} + \frac{\alpha}{\hbar}\sigma_x p + \frac{\hbar\omega_0}{2}, \tag{2}$$

where $p \equiv p_y$ is the component of the momentum in the direction of the wire and $\hbar\omega_0/2$ is the zero point energy (lowest subband) of the oscillator in the transverse direction. We also include in the model the effect of a magnetic field applied along the z-direction

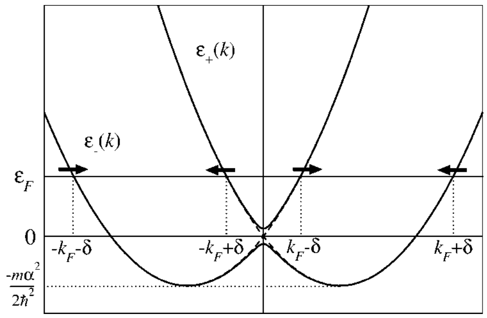

FIG. 1. Electron dispersion in the presence of spin-orbit interaction, for $B=0$ (dashed line) and $B\neq 0$ (solid line). If $\alpha k_F \gg g\mu_B B$, the electron states at the Fermi surface have spins approximately parallel to $+x$ ($\rightarrow$) or $-x$ ($\leftarrow$).

$$H_0 \approx \frac{p^2}{2m} + \frac{\alpha}{\hbar}\sigma_x p - \frac{g\mu_B B}{2}\sigma_z + \frac{\hbar\omega_0}{2}. \tag{3}$$

Here, g is the effective electron g-factor, μ_B is the Bohr magneton, and B is the applied magnetic field. As usual, we ignore the orbital effect of the magnetic field in the 1D limit. Actually, what really matters for our purposes is that the external field is perpendicular to the spin direction set by the spin-orbit coupling. Thus, a field applied along the y-direction (which causes no orbital effect) would serve as well.

The Hamiltonian H_0 in Eq. (3) is promptly diagonalized in momentum space. For $B=0$, the eigenfunctions are plane waves with eigenvalues (omitting the zero point energy) $\epsilon_\sigma = \hbar^2k^2/2m + \sigma\alpha k$, where $\sigma=+$ ($\sigma=-$) refers to spins parallel to $+x$ ($-x$). As a result, the spin bands are "horizontally" split even at $B=0$. The energy minima are shifted to $k=\pm m\alpha/\hbar^2 \equiv \pm\delta$ and the bands are degenerate at $k=0$ (Fig. 1).

However, for $B\neq 0$, the eigenfunctions take the form $\Psi_\lambda(x)=e^{ikx}\chi_\lambda(k)$, $\lambda=\pm$, where the momentum-dependent spinors are written in the basis of eigenstates of σ_z as

$$\chi_+(k) = \begin{pmatrix} \sin\frac{\theta(k)}{2} \\ \cos\frac{\theta(k)}{2} \end{pmatrix}, \quad \chi_-(p) = \begin{pmatrix} \cos\frac{\theta(k)}{2} \\ -\sin\frac{\theta(k)}{2} \end{pmatrix}. \tag{4}$$

Here, we have introduced the polar angle $\theta(k)$ that specifies the polarization direction in the xz plane, given by

$$\theta(k) = \tan^{-1}\frac{2\alpha k}{g\mu_B B}. \tag{5}$$

The eigenvalues of H_0 are

$$\epsilon_\pm(k) = \frac{\hbar^2k^2}{2m} \pm \sqrt{(\alpha k)^2 + \left(\frac{g\mu_B B}{2}\right)^2}. \tag{6}$$

The dispersion for $B\neq 0$ is shown in Fig. 1. We note that the mixing between $\sigma=\pm$ eigenstates opens a gap $\Delta=g\mu_B B$ at $k=0$. Furthermore, we can no longer associate a fixed spin direction with the bands $\lambda=\pm$. Consider, for example, the limit of strong Rashba coupling $\alpha k_F \gg g\mu_B B$, where $k_F = n\pi/2$, with n the average electron density, stands for the Fermi momentum when $\alpha=0$. For $\epsilon_F>0$ (as in Fig. 1), the electron states with $\lambda=-$ on the Fermi surface have spin approximately parallel to $+x$ at $k=-k_F-\delta$ and $-x$ at $k=+k_F+\delta$. The opposite happens for $\lambda=+$. As one moves from one Fermi point to the other along each band, the spin polarization changes continuously from $\pm x$ to $\mp x$, passing through $\pm z$ in the vicinity of the gap, where $|k| \ll g\mu_B B/\alpha$.

We consider now the scattering of electrons described by the spin states in Eq. (4) off nonmagnetic impurities. The free Hamiltonian of the electron gas can be written in second-quantized form

$$H_0 = \sum_{k,\lambda} \epsilon_\lambda(k) c^\dagger_{k\lambda} c_{k\lambda}, \tag{7}$$

where $\epsilon_\lambda(k)$ are the dispersions in Eq. (6) and $c_{k\lambda}$ destroys an electron in the eigenstate $|k\lambda\rangle$ with momentum k in the band λ. The nonmagnetic impurity corresponds to the perturbation

$$V = \sum_{kp\lambda\mu} \langle k\lambda|V(x)|p\mu\rangle c^\dagger_{k\lambda} c_{p\mu}, \tag{8}$$

where $V(x)$ is a localized potential. Using the spinors in Eq. (4), we can calculate the matrix elements in Eq. (8) to find

$$V = \sum_{kp\lambda} \frac{V_{k-p}}{N}\cos\left[\frac{\theta(k)-\theta(p)}{2}\right] c^\dagger_{k\lambda} c_{p\lambda} - \sum_{kp\lambda} \lambda \frac{V_{k-p}}{N}\sin\left[\frac{\theta(k)-\theta(p)}{2}\right] c^\dagger_{k\lambda} c_{p,-\lambda}, \tag{9}$$

where V_k is the the Fourier transform of $V(x)$ and N is the number of sites in the wire. The first term in Eq. (9) describes scattering between states in the same band whereas the second one describes scattering between states in different bands. That these two terms exist in general is a feature of the spin-orbit interaction, which makes $\theta(k)\neq\theta(p)$ if $k\neq p$. Since a given band does not have a fixed spin direction, spin-flip scattering terms can emerge from both terms. In order to make these terms explicit, we focus on the limit of $g\mu_B B \ll \alpha k$ for k around the Fermi surface in Fig. 1 and expand $\theta(k) \approx \mathrm{sgn}(k)\pi/2 - g\mu_B B/2\alpha k$. To zeroth order in the magnetic field, we find

$$V^{(0)} = \sum_{kp\lambda} \frac{V_{k-p}}{N}\left[\frac{1+\mathrm{sgn}(kp)}{2} c^\dagger_{k\lambda} c_{p\lambda} - \lambda \frac{\mathrm{sgn}(k)-\mathrm{sgn}(p)}{2} c^\dagger_{k\lambda} c_{p,-\lambda}\right]. \tag{10}$$

It is easy to verify that the two terms in Eq. (10) correspond to the usual potential scattering in the limit $B\rightarrow 0$. It is possible to scatter between states in the same band and with momenta of the same sign or states in different bands and with momenta of opposite signs. According to Fig. 1, only the latter case can occur if $\epsilon_F>0$ and it consists of a non spin-flip process that connects $k_F\pm\delta$ with $-k_F\pm\delta$. True spin-flip processes appear at first order in B

$$V^{(1)} = \frac{g\mu_B B}{4\alpha}\sum_{kp\lambda}\frac{V_{k-p}}{N}\left(\frac{1}{k}-\frac{1}{p}\right)\times\left[\frac{\mathrm{sgn}(k)-\mathrm{sgn}(p)}{2}c^{\dagger}_{k\lambda}c_{p\lambda}-\lambda\frac{1+\mathrm{sgn}(kp)}{2}c^{\dagger}_{k\lambda}c_{p,-\lambda}\right]. \tag{11}$$

The terms in $V^{(1)}$ connect $k_F \pm \delta$ with $-(k_F \pm \delta)$, which are states with spins polarized in opposite directions ($\pm x$). The momentum transfer in the scattering is $\Delta k = 2(k_F \pm \delta)$.

In the limit of strong Rashba coupling and low magnetic field, the potential scattering off the impurity overcomes the spin-flip scattering because the amplitude of the latter is proportional to $g\mu_B B/\alpha k_F \ll 1$. Hence, we do not expect a large variation of the resistance of the wire with the magnetic field. However, the band structure in Fig. 1 suggests a special case in which we can suppress the potential scattering. As first proposed in Ref. 16, it suffices to lower the Fermi energy until $\epsilon_F = 0$, so that the Fermi level lies exactly inside the gap. In this case, the states to which electrons would be scattered without spin flip become forbidden and this process will depend on thermal activation to states above the gap. Consequently, the spin-flip scattering will dominate at temperatures much lower than the gap, which is controlled by the external magnetic field. The condition $\epsilon_F = 0$ relies on a fine tuning between the Rashba coupling and the electronic density, and can be stated as $\alpha = \hbar v_F$, where $v_F = \hbar k_F/m$ is the Fermi velocity.

We calculate the resistance for this situation perturbatively by using the memory function formalism.[17] First, we note that even the "usual" scattering (in the sense that it reduces to the potential scattering in the limit $B \to 0$) from $k = \pm(k_F + \delta) \equiv \pm 2k_F$ to $k \approx 0$ involves a rotation of $\pm\pi/2$ in the spin direction. For $k_B T \ll \Delta \ll \alpha k_F$, the resistance due to this scattering is

$$R_1 = \frac{h}{e^2}\left(\frac{ma|V_{2k_F}|}{\pi\hbar^2 n}\right)^2\sqrt{\frac{\pi g\mu_B B}{k_B T}}\exp\left(-\frac{g\mu_B B}{2k_B T}\right), \tag{12}$$

where a is the lattice parameter. Likewise, the resistance due to the spin-flip scattering ($\Delta k = 4k_F$) reads

$$R_2 = \frac{h}{e^2}\left(\frac{ma|V_{4k_F}|}{\pi\hbar^2 n}\right)^2\left(\frac{g\mu_B B}{4\alpha k_F}\right)^2. \tag{13}$$

We see that R_1 decreases and R_2 increases as the magnetic field is increased. In principle, we should be able to observe a crossover to a regime in which the total resistance is due mostly to the spin-flip scattering (Fig. 2). Moreover, there must be a total resistance minimum for a given value of $B = B_0$. Let us estimate this minimum resistance $R_T^{\min}$. By putting $V_{2k_F} \approx V_{4k_F}$, $T = 1$ K, $k_F = 2 \times 10^6$ cm^{-1}, $m = 0.036\ m_0$ (for InAs quantum wells[5]), where m_0 is free electron mass, and $g = 13$ (Ref. 18), we get $B_0 \approx 3.2$ T and

$$\frac{R_T^{\min}}{R_T^0} \approx 3 \times 10^{-5},$$

where $R_T^0 = 2(h/e^2)(ma|V_{2k_F}|/\pi\hbar^2 n)^2$ is the resistance associated with the impurity scattering at $B = 0$. In practice, this

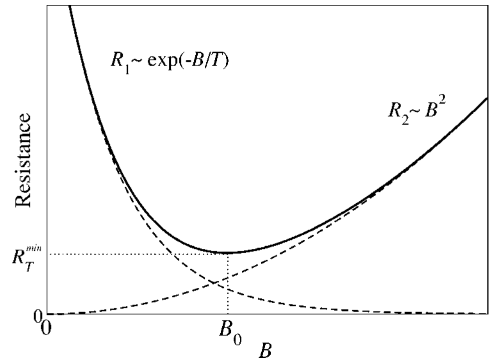

FIG. 2. Resistance of the quantum wire with Rashba coupling constant $\alpha = \hbar v_F$, for a fixed temperature and as a function of the magnetic field B.

mechanism removes the impurity scattering at low temperatures.

We define the magnetoresistance as the relative difference between the resistance at $B = B_0$ and the resistance at zero magnetic field

$$\frac{\Delta R}{R} = \frac{(G_0^{-1} + R_T^{\min}) - \left(\frac{1}{2}G_0^{-1} + R_T^0\right)}{(G_0^{-1} + R_T^{\min})} \approx \frac{1}{2} - \frac{1}{2}\left(\frac{ma|V_{2k_F}|}{\hbar^2 k_F}\right)^2, \tag{14}$$

where $G_0 = e^2/h$ is the conductance quantum. In the limit of weak barriers ($|V_{2k_F}| \to 0$), the magnetoresistance is positive because the main effect of the magnetic field is to open the gap at $k = 0$ and reduce the number of electron channels at the Fermi level to the equivalent of a single band ($\lambda = -$). As a result, the ballistic conductance decreases, according to the Landauer formula,[19] from $2G_0$ to G_0 (Ref. 10). However, in the limit of large barriers ($R_T^0 \gg G_0^{-1}$), the magnetoresistance is negative because, upon effectively removing the impurity with the magnetic field, we increase the conductance from almost zero to G_0. Actually, this is the most relevant case in the 1D limit, since repulsive interactions renormalize upwards the scattering off an impurity in a Luttinger liquid.[11–13] Therefore, the scattering amplitude $|V_{2k_F}|$ in Eq. (14) should be replaced by the effective amplitude $|V_{2k_F}|(T/T_F)^{K-1}$, where $K < 1$ for repulsive interactions, leading to singular scattering at $T = 0$. For this reason, a large negative magnetoresistance should be observed. It must also be remarked that at $T = 0$ the spin-flip scattering is the only source of electron scattering. Since this process converts right-moving "spin-down" ($\leftarrow$) electrons into left-moving "spin-up" ($\rightarrow$) electrons, and vice versa, it conserves the persistent spin current $\langle J_s \rangle = \langle J_{\rightarrow} - J_{\leftarrow} \rangle \neq 0$ in a wire with spin-orbit interaction. On the other hand, it does not conserve spin density because the magnetic field restores the Elliot-Yafet mechanism of spin-relaxation.

Finally, we consider the possibility of observing this effect with the available experimental conditions. For a density as low as $k_F = 2 \times 10^6$ cm^{-1} and for $m = 0.036\ m_0$, we need $\alpha \approx 4 \times 10^{-10}$ eVm, which is one order of magnitude above the reported values of α (Ref. 5). However, the condition $\alpha = \hbar v_F$ could be achieved in principle by either further reducing the electron density or pursuing higher values of α, by means of applied electric fields or more asymmetric heterostructures. Besides, it has been predicted that interaction effects should enhance α at low densities.[20] The fine tuning between the Rashba splitting and the Fermi level (with an error smaller than the gap Δ) could also be guaranteed, since it is possible to control these two properties independently.[5]

We would like to end by mentioning an interesting recent analysis of the magnetoresistance of quantum wires in the presence of a domain wall and spin-orbit interaction.[21] Unlike us, however, that work focuses on the limit where the effective magnetic field "seen" by the conduction electrons (produced by the polarized spin background) is much greater than the spin-orbit scale: $g\mu_B B \gg \alpha k_F$. Furthermore, the tuning of the Fermi level to the gap is not considered. The role of the chemical potential when there is a gap due to a perpendicular magnetic field has also appeared in a related work.[22] It focuses on a combination of spin-orbit interaction and the time-dependent magnetic field provided by the coupling to a nonequilibrium nuclear spin polarization, which is predicted to induce fluctuations in the charge conductance.[22]

In conclusion, we have shown that the scattering off nonmagnetic impurities is strongly affected by a perpendicular magnetic field when the Fermi level crosses the degeneracy point of 1D bands split by spin-orbit interaction. The opening of a gap proportional to the Zeeman energy suppresses the potential scattering at low temperatures. This mechanism does not depend on the scattering amplitude. Therefore, one can effectively remove all the impurities in the wire with the magnetic field and observe a negative magnetoresistance.

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