Partícula carregada sujeita à um campo eletromagnético

Fluxo de probabilidade

FI001 - Mecânica Quântica I

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Schrödinger:

$$\langle x'|H|\alpha, t_0; t\rangle = i\hbar \frac{\partial}{\partial t}\langle x'|\alpha, t_0; t\rangle \quad \textbf{(1)}$$

Hamiltoniana em termos de A e φ:

$$H = \frac{1}{2m}\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right) + e\phi = \frac{\boldsymbol{\Pi}^2}{2m} + e\phi \quad \textbf{(2)}$$

Continuidade:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\boldsymbol{j} = \mathbf{0} \quad \textbf{(3)}$$

Veja aula 10; j=fluxo de probabilidade

$$\rho = \psi^*\psi \quad \textbf{(4)}$$

$$\psi = \langle x'|\alpha, t_0; t\rangle \quad \textbf{(5)}$$

$$\frac{\partial\rho}{\partial t} = \frac{\partial\psi^*}{\partial t}\psi + \psi^*\frac{\partial\psi}{\partial t} \quad \textbf{(6)}$$

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$$i\hbar\frac{\partial}{\partial t}\langle x'|\alpha,t_0;t\rangle = \frac{1}{2m}\langle x'\left|\left(\boldsymbol{p}-\frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)^2\right|\alpha,t_0;t\rangle + e\phi(x')\langle x'|\alpha,t_0;t\rangle$$

$$= \frac{1}{2m}\left(-i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\langle x'\left|\left(\boldsymbol{p}-\frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\right|\alpha,t_0;t\rangle + e\phi(x')\langle x'|\alpha,t_0;t\rangle$$

$$= \frac{1}{2m}\left(-i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\left(-i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\langle x'|\alpha,t_0;t\rangle + e\phi(x')\langle x'|\alpha,t_0;t\rangle$$

Veja Slide 15 da aula

$$i\hbar\frac{\partial}{\partial t}\langle x'|\alpha,t_0;t\rangle = \frac{1}{2m}\left(-i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\left(-i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\langle x'|\alpha,t_0;t\rangle + e\phi(x')\langle x'|\alpha,t_0;t\rangle$$

$$-i\hbar\frac{\partial}{\partial t}\langle x'|\alpha,t_0;t\rangle^* = \frac{1}{2m}\left(i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\left(i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\langle x'|\alpha,t_0;t\rangle^* + e\phi^*(x')\langle x'|\alpha,t_0;t\rangle^*$$

Fazendo: vermelho.Ψ – Ψ*.verde

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$$\frac{\partial \rho}{\partial t} = -\frac{1}{ih}\frac{1}{2m}\left(i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\left(i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\langle x'|\alpha, t_0; t\rangle^*\langle x'|\alpha, t_0; t\rangle$$

$$-e\phi^*(x')\langle x'|\alpha, t_0; t\rangle^*\langle x'|\alpha, t_0; t\rangle$$

$$+\frac{1}{2m}\langle x'|\alpha, t_0; t\rangle^*\left(-i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\left(-i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\langle x'|\alpha, t_0; t\rangle$$

$$+\langle x'|\alpha, t_0; t\rangle^* e\phi(x')\langle x'|\alpha, t_0; t\rangle$$

$\phi(x') = \phi^*(x')$ -> Termo azul se cancela

$$\frac{\partial \rho}{\partial t} = \frac{1}{ih}\left[-\langle x'|\frac{\boldsymbol{\Pi}^{*2}}{2m}|\alpha, t_0; t\rangle^*\langle x'|\alpha, t_0; t\rangle + \langle x'|\alpha, t_0; t\rangle^*\langle x'|\frac{\boldsymbol{\Pi}^2}{2m}|\alpha, t_0; t\rangle\right]$$

Eq. no Slide 16 da aula

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$$\mathbf{\Pi}^2 = \left(-i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\left(-i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)$$

Operador atuando sobre vetor potencia A e função de onda

$$= -\hbar^2\nabla'^2 + \frac{e}{c}i\hbar\nabla'\cdot\boldsymbol{A}(\boldsymbol{x}') + \frac{2e}{c}i\hbar A(x')\cdot\nabla' + \frac{e^2}{c^2}A^2(x')$$

$$\nabla'\cdot A(x') = 0 \; (Gauge\ de\ Coulomb)$$

$$\mathbf{\Pi}^2 = -\hbar^2\nabla'^2 + \frac{2e}{c}i\hbar A(x')\cdot\nabla' + \frac{e^2}{c^2}A^2(x')$$

$$\mathbf{\Pi}^{*2} = \left(i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)\left(i\hbar\nabla' - \frac{e}{c}\boldsymbol{A}(\boldsymbol{x}')\right)$$

$$= -\hbar^2\nabla'^2 - \frac{e}{c}i\hbar\nabla'\cdot\boldsymbol{A}(\boldsymbol{x}') - \frac{2e}{c}i\hbar A(x')\cdot\nabla' + \frac{e^2}{c^2}A^2(x')$$

$$\mathbf{\Pi}^{*2} = -\hbar^2\nabla'^2 - \frac{2e}{c}i\hbar A(x')\cdot\nabla' + \frac{e^2}{c^2}A^2(x')$$

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$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}\left[-\langle x'| \frac{-\hbar^2\nabla'^2 - \frac{2e}{c} i\hbar A(x')\cdot\nabla' + \frac{e^2}{c^2}A^2(x')}{2m} |\alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle + \langle x'|\alpha, t_0; t\rangle^* \langle x'| \frac{-\hbar^2\nabla'^2 + \frac{2e}{c} i\hbar A(x')\cdot\nabla' + \frac{e^2}{c^2}A^2(x')}{2m} |\alpha, t_0; t\rangle \right]$$

$$= \langle x' \left| \frac{\hbar\nabla'^2}{i2m} \right| \alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle + \langle x' \left| \frac{e}{mc} A(x')\nabla' \right| \alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle$$

$$- \langle x'| \frac{e^2}{2mc^2} A^2(x') |\alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle$$

$$- \langle x'|\alpha, t_0; t\rangle^* \langle x'| \frac{\hbar\nabla'^2}{i2m} |\alpha, t_0; t\rangle + \langle x'|\alpha, t_0; t\rangle^* \langle x'| \frac{e}{mc} A(x')\nabla' |\alpha, t_0; t\rangle$$

$$+ \langle x'|\alpha, t_0; t\rangle^* \langle x'| \frac{e^2}{2mc^2} A^2(x') |\alpha, t_0; t\rangle$$

$\frac{e^2}{2mc^2}A^2(x') \ldots real$ $\langle x'|\alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle = |\psi|^2$ -> Termo azul se cancela

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$$\frac{\partial\rho}{\partial t} = \langle x' \left|\frac{\hbar\nabla'^2}{i2m}\right| \alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle + \langle x' \left|\frac{e}{mc} A(x')\nabla'\right| \alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle$$

$$-\langle x'|\alpha, t_0; t\rangle^* \langle x'| \frac{\hbar\nabla'^2}{i2m} |\alpha, t_0; t\rangle + \langle x'|\alpha, t_0; t\rangle^* \langle x'| \frac{e}{mc} A(x')\nabla'|\alpha, t_0; t\rangle$$

$$-\frac{\partial\rho}{\partial t} = \nabla' \cdot \boldsymbol{j} = \frac{i\hbar}{2m}[\langle x'|\nabla'^2|\alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle + \langle x'|\alpha, t_0; t\rangle^* \langle x'|\nabla'^2|\alpha, t_0; t\rangle]$$

$$-\frac{e}{mc}[\langle x'|A(x')\nabla'|\alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle + \langle x'|\alpha, t_0; t\rangle^* \langle x'|A(x')\nabla'|\alpha, t_0; t\rangle]$$

$$\frac{i\hbar}{2m}(\langle x'|\nabla'^2|\alpha, t_0; t\rangle^* \langle x'|\alpha, t_0; t\rangle - \langle x'|\alpha, t_0; t\rangle^* \langle x'|\nabla'^2|\alpha, t_0; t\rangle)$$

$$= -\frac{i\hbar}{2m}(\psi^*\nabla'^2\psi - \psi\nabla'^2\psi^*)$$

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$$\nabla' \cdot \boldsymbol{j} = -\frac{i\hbar}{2m}\left(\psi^*\nabla'^2\psi - \psi\nabla'^2\psi^*\right) - \frac{e}{mc}\nabla'[(\psi^*\psi)\mathbf{A}]$$

$$= \nabla'\left[-\frac{i\hbar}{2m}\left(\psi^*\nabla'\psi - \psi\nabla'\psi^*\right) - \frac{e}{mc}A|\psi|^2\right]$$

$$\langle x'|\alpha, t_0; t\rangle^*\langle x'|\alpha, t_0; t\rangle = \psi^*\psi = |\psi|^2 \qquad A(x') \ldots real$$

$$\boldsymbol{j} = -\frac{i\hbar}{2m}\left(\psi^*\nabla'\psi - \psi\nabla'\psi^*\right) - \frac{e}{mc}\boldsymbol{A}|\psi|^2$$

$$z - z^* = (a + ib) - (a - ib) = 2ib = 2Im(z)$$

$$\boxed{\boldsymbol{j} = \frac{\hbar}{m}Im(\psi^*\nabla'\psi) - \frac{e}{mc}\boldsymbol{A}|\psi|^2}$$

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No ausência do campo j fica:

$$\boldsymbol{j} = \frac{\hbar}{m} Im(\psi^* \nabla' \psi) = \frac{\hbar}{2mi}(\psi^* \nabla' \psi - \psi \nabla' \psi^*)$$

$$= \frac{1}{m} \frac{\psi^*(-i\hbar\nabla\psi) + \psi(i\hbar\psi^*)}{2} = \frac{1}{m} \frac{\psi^*(-i\hbar\nabla\psi) + \psi(i\hbar\psi)^*}{2}$$

$$= \frac{1}{m} Re[\psi^*(-i\hbar\nabla\psi)] = \frac{1}{m} Re[\psi^* \boldsymbol{p} \psi]$$

Com campo, fazer a substituição: $\boldsymbol{p} \rightarrow \boldsymbol{p} - \frac{e}{c}\boldsymbol{A}$

$$\boldsymbol{j} = \frac{1}{m} Re\left[\psi^* \left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right)\psi\right] = \frac{\hbar}{m} Im(\psi^* \nabla' \psi) - \frac{e}{mc}\boldsymbol{A}|\psi|^2$$

$$\boxed{\boldsymbol{j} = \frac{\hbar}{m} Im(\psi^* \nabla' \psi) - \frac{e}{mc}\boldsymbol{A}|\psi|^2}$$