Universidade Estadual de Campinas

Propagador: Oscilador Harmônico Simples (OHS)

FI001 - Mecânica Quântica I
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Vimos que podemos definir o propagador como,

$$K\left(\mathbf{x}'', t; \mathbf{x}', t_0\right) = \sum_{a'} \left\langle x''|a'\right\rangle \left\langle a'|x'\right\rangle \exp\left[\frac{-iE_{a'}\left(t - t_0\right)}{\hbar}\right] \tag{1}$$

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$$a_n(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega x^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}} x\right), \quad n = 0, 1, 2, \ldots \tag{2}$$

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(H_n são polinômios de Hermite). Por conseguinte:

$$\left\langle x'' | a' \right\rangle = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega x''^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x''\right) \tag{3}$$

$$\left\langle a' | x' \right\rangle = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega x'^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x'\right) \tag{4}$$

Substituindo,

$$K(\mathbf{x}'',t;\mathbf{x}',t_0) = \sum_{a'} \langle x''|a'\rangle \langle a'|x'\rangle \exp\left[\frac{-iE_{a'}(t-t_0)}{\hbar}\right] =$$
$$= \sum_{a'} \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega x''^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x''\right) \cdot$$
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Os autovalores de energia para o OHS são $E_n = (n + \frac{1}{2})\hbar\omega$, portanto,

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$$K(\mathbf{x}'', t; \mathbf{x}', t_0) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{2}} e^{-\frac{m\omega\left(x''^2 + x'^2\right)}{2\hbar}} \sum_{n=0} \frac{1}{2^n n!} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x'\right) H_n\left(\sqrt{\frac{m\omega}{\hbar}}x''\right) \cdot$$
$$\cdot \exp[-i\omega(n+1/2)(t-t_0)] \tag{6}$$

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$$
\begin{aligned}
K(\mathbf{x}'', t; \mathbf{x}', t_0) = {} & \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} e^{-\frac{m\omega\left(x''^2+x'^2\right)}{2\hbar}} e^{-\frac{i\omega}{2}(t-t_0)} \sum_{n=0} \frac{1}{2^n n!} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x'\right) \cdot \\
& \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x''\right) \left(e^{-i\omega(t-t_0)}\right)^n = \\
= {} & \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} e^{\frac{m\omega\left(x''^2+x'^2\right)}{2\hbar}} e^{-\frac{i\omega}{2}(t-t_0)} e^{-\frac{m\omega\left(x''^2+x'^2\right)}{\hbar}} \cdot \\
& \cdot \sum_{n=0} \frac{1}{2^n n!} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x'\right) H_n\left(\sqrt{\frac{m\omega}{\hbar}}x''\right) \left(e^{-i\omega(t-t_0)}\right)^n
\end{aligned}
\tag{7}
$$

$$K(\mathbf{x}'', t; \mathbf{x}', t_0) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} e^{-\frac{m\omega\left(x''^2+x'^2\right)}{2\hbar}} e^{-\frac{i\omega}{2}(t-t_0)} \sum_{n=0} \frac{1}{2^n n!} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x'\right) \cdot$$
$$\cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x''\right)\left(e^{-i\omega(t-t_0)}\right)^n =$$
$$= \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} e^{\frac{m\omega\left(x''^2+x'^2\right)}{2\hbar}} e^{-\frac{i\omega}{2}(t-t_0)} e^{-\frac{m\omega\left(x''^2+x'^2\right)}{\hbar}} \cdot$$
$$\cdot \sum_{n=0} \frac{1}{2^n n!} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x'\right) H_n\left(\sqrt{\frac{m\omega}{\hbar}}x''\right)\left(e^{-i\omega(t-t_0)}\right)^n \tag{7}$$

Usando a relação (Seção 2.6 Sakurai):

$$\left(\frac{1}{\sqrt{1-\zeta^2}}\right)\exp\left[\frac{-(\varepsilon^2+\eta^2-2\varepsilon\eta\zeta)}{(1-\zeta^2)}\right] = \exp[-(\varepsilon^2+\eta^2)]\sum_{n=0}\left(\frac{\zeta^n}{2^n n!}\right)H_n(\varepsilon)H_n(\eta) \tag{8}$$

$$
\begin{aligned}
K(\mathbf{x}'',t;\mathbf{x}',t_0) &= \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} e^{\frac{m\omega\left(x''^2+x'^2\right)}{2\hbar}} e^{-\frac{i\omega}{2}(t-t_0)} \left[\frac{1}{\sqrt{1-e^{-2i\omega(t-t_0)}}}\right] \cdot \\
&\cdot \exp\left(-\frac{m\omega}{\hbar}\left[\frac{x'^2+x''^2-2x''x'e^{-i\omega(t-t_0)}}{1-e^{-2i\omega(t-t_0)}}\right]\right) = \\
&= \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} e^{\frac{m\omega\left(x'^2+x''^2\right)}{2\hbar}} \left[\frac{1}{e^{\frac{i\omega}{2}(t-t_0)}\sqrt{1-e^{-2i\omega(t-t_0)}}}\right] \cdot \\
&\cdot \exp\left(-\frac{m\omega}{\hbar}\left[\frac{\left(x'^2+x''^2\right)-2x''x'e^{-i\omega(t-t_0)}}{e^{-i\omega(t-t_0)}\left[e^{i\omega(t-t_0)}-e^{-i\omega(t-t_0)}\right]}\right]\right) = \\
&= \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} e^{\frac{m\omega\left(x'^2+x''^2\right)}{2\hbar}} \left[\frac{1}{\sqrt{e^{i\omega(t-t_0)}-e^{-i\omega(t-t_0)}}}\right] \cdot \\
&\cdot \exp\left(-\frac{m\omega}{\hbar}\left[\frac{(x'^2+x''^2)e^{i\omega(t-t_0)}-2x''x'}{e^{i\omega(t-t_0)}-e^{-i\omega(t-t_0)}}\right]\right)
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
K(\mathbf{x}'', t; \mathbf{x}', t_0) =& \sqrt{\frac{m\omega}{2\pi i\hbar \sin[\omega(t-t_0)]}} e^{\frac{m\omega\left(x''^2+x'^2\right)}{2\hbar}} \cdot \\
& \cdot \exp\left(-\frac{m\omega}{\hbar}\left[\frac{\left(x'^2+x''^2\right)e^{i\omega(t-t_0)}-2x''x'}{2i\sin\left[\omega\left(t-t_0\right)\right]}\right]\right) = \\
=& \sqrt{\frac{m\omega}{2\pi i\hbar \sin[\omega(t-t_0)]}} \cdot \\
& \cdot \exp\left(-\frac{m\omega}{2\hbar}\left[\frac{\left(x'^2+x''^2\right)e^{i\omega(t-t_0)}-2x''x'}{i\sin\left[\omega(t-t_0)\right]}-\left(x'^2+x''^2\right)\right]\right)
\end{aligned} \tag{10}
$$

$$K(\mathbf{x}'', t; \mathbf{x}', t_0) = \sqrt{\frac{m\omega}{2\pi i\hbar \sin[\omega(t-t_0)]}} \cdot \\ \cdot \exp\left[\frac{im\omega}{2\hbar \sin[\omega(t-t_0)]}[(x'^2 + x''^2)\cos[\omega(t-t_0)] - 2x''x']\right] \tag{11}$$

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Obrigada!