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# Adição de Momento Angular

Instituto de Física "Gleb Wataghin"

## Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

FI – 001 – Mecânica Quântica 1

Régis Schiavon de Oliveira

Junho 2020

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Vamos começar a diagonalização com $S_z$:

Formulário de Apoio:

$$S_x = \frac{\hbar}{2}\sigma_x \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{2}\sigma_y \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad S_z = \frac{\hbar}{2}\sigma_z \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A \otimes B = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \otimes \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}B & a_{12}B \\ a_{21}B & a_{22}B \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

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Temos então $S_z$ diagonal e seus Autovalores 1, 0, 0, -1

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$$\boldsymbol{S}^2 = (\boldsymbol{S}_1 + \boldsymbol{S}_2)^2$$

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O termo $\boldsymbol{S}_1.\boldsymbol{S}_2 = \boldsymbol{S}_{1x}\,\boldsymbol{S}_{2x} + \boldsymbol{S}_{1y}\,\boldsymbol{S}_{2y} + \boldsymbol{S}_{1z}\,\boldsymbol{S}_{2z} = \frac{\hbar^2}{4}(\sigma_{1x} \otimes \sigma_{2x} + \sigma_{1y} \otimes \sigma_{2y} + \sigma_{1z} \otimes \sigma_{2z})$

$$\sigma_{1x} \otimes \sigma_{2x} = \begin{pmatrix} \mathbf{0}.\sigma_{2x} & \mathbf{1}.\sigma_{2x} \\ \mathbf{1}.\sigma_{2x} & \mathbf{0}.\sigma_{2x} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1.0 & 1.1 \\ 0 & 0 & 1.1 & 1.0 \\ 1.0 & 1.1 & 0 & 0 \\ 1.1 & 1.0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\sigma_{1y} \otimes \sigma_{2y} = \begin{pmatrix} \mathbf{0}.\sigma_{2y} & -\boldsymbol{i}.\sigma_{2y} \\ \boldsymbol{i}.\sigma_{2y} & \mathbf{0}.\sigma_{2y} \end{pmatrix} = \begin{pmatrix} 0 & 0 & -i.0 & -i.-i \\ 0 & 0 & -i.i & -i.0 \\ i.0 & i.-i & 0 & 0 \\ i.i & i.0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\sigma_{1z} \otimes \sigma_{2z} = \begin{pmatrix} \mathbf{1}.\sigma_{2z} & \mathbf{0}.\sigma_{2z} \\ \mathbf{0}.\sigma_{2z} & -\mathbf{1}.\sigma_{2z} \end{pmatrix} = \begin{pmatrix} 1.1 & 1.0 & 0 & 0 \\ 1.0 & 1.-1 & 0 & 0 \\ 0 & 0 & -1.1 & -1.0 \\ 0 & 0 & -1.0 & -1.-1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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# Diagonalização $S^2$e $S_z$ na base $\{m_1, m_2\}$

Instituto de Física "Gleb Wataghin"

Portanto o termo $\boldsymbol{S_1.S_2}$ fica:

$$\boldsymbol{S_1.S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}\right]$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Portanto o termo $\boldsymbol{S_1}.\boldsymbol{S_2}$ fica:

$$\boldsymbol{S_1}.\boldsymbol{S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \Longrightarrow \boldsymbol{S_1}.\boldsymbol{S_2} = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Portanto o termo $\boldsymbol{S_1 . S_2}$ fica:

$$\boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \Rightarrow \boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\,\boldsymbol{S_1 . S_2}$ , ainda nos resta os termos $\boldsymbol{S_1^2}$ $\boldsymbol{e}$ $\boldsymbol{S_2^2}$

# Diagonalização $S^2$e $S_z$ na base $\{m_1, m_2\}$

Portanto o termo $\boldsymbol{S_1.S_2}$ fica:

$$\boldsymbol{S_1.S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \Longrightarrow \boldsymbol{S_1.S_2} = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\,\boldsymbol{S_1.S_2}$ , ainda nos resta os termos $\boldsymbol{S_1^2}$ $\boldsymbol{e}$ $\boldsymbol{S_2^2}$

$$\boldsymbol{S_1^2} = \frac{3\hbar^2}{4}\boldsymbol{I_1} \otimes \boldsymbol{I_2} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.\boldsymbol{I_2} & 0.\boldsymbol{I_2} \\ 0.\boldsymbol{I_2} & 1.\boldsymbol{I_2} \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Portanto o termo $\boldsymbol{S_1 . S_2}$ fica:

$$\boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \Rightarrow \boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + \mathbf{2}\, \boldsymbol{S_1 . S_2}$ , ainda nos resta os termos $\boldsymbol{S_1^2}$ $\boldsymbol{e}$ $\boldsymbol{S_2^2}$

$$\boldsymbol{S_1^2} = \frac{3\hbar^2}{4} \boldsymbol{I_1} \otimes \boldsymbol{I_2} = \frac{3\hbar^2}{4}\begin{pmatrix} \mathbf{1.}\boldsymbol{I_2} & \mathbf{0.}\boldsymbol{I_2} \\ \mathbf{0.}\boldsymbol{I_2} & \mathbf{1.}\boldsymbol{I_2} \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.1 & 1.0 & 0 & 0 \\ 1.0 & 1.1 & 0 & 0 \\ 0 & 0 & 1.1 & 1.0 \\ 0 & 0 & 1.0 & 1.1 \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Portanto o termo $\boldsymbol{S_1 . S_2}$ fica:

$$\boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \longrightarrow \boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + \boldsymbol{2}\, \boldsymbol{S_1 . S_2}$ , ainda nos resta os termos $\boldsymbol{S_1^2}\ \boldsymbol{e}\ \boldsymbol{S_2^2}$

$$\boldsymbol{S_1^2} = \frac{3\hbar^2}{4} \boldsymbol{I_1} \otimes \boldsymbol{I_2} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.\boldsymbol{I_2} & 0.\boldsymbol{I_2} \\ 0.\boldsymbol{I_2} & 1.\boldsymbol{I_2} \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.1 & 1.0 & 0 & 0 \\ 1.0 & 1.1 & 0 & 0 \\ 0 & 0 & 1.1 & 1.0 \\ 0 & 0 & 1.0 & 1.1 \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Portanto o termo $\boldsymbol{S_1.S_2}$ fica:

$$\boldsymbol{S_1.S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \rightarrow \boldsymbol{S_1.S_2} = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\,\boldsymbol{S_1.S_2}$ , ainda nos resta os termos $\boldsymbol{S_1^2}$ $e$ $\boldsymbol{S_2^2}$

$$\boldsymbol{S_1^2} = \frac{3\hbar^2}{4}\boldsymbol{I_1} \otimes \boldsymbol{I_2} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.\boldsymbol{I_2} & 0.\boldsymbol{I_2} \\ 0.\boldsymbol{I_2} & 1.\boldsymbol{I_2} \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.1 & 1.0 & 0 & 0 \\ 1.0 & 1.1 & 0 & 0 \\ 0 & 0 & 1.1 & 1.0 \\ 0 & 0 & 1.0 & 1.1 \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Analogamente, $\boldsymbol{S_2^2} = \boldsymbol{S_1^2}$ , portanto, temos: $\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\,\boldsymbol{S_1.S_2}$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Portanto o termo $\boldsymbol{S_1 . S_2}$ fica:

$$\boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \Rightarrow \boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\,\boldsymbol{S_1 . S_2}$ , ainda nos resta os termos $\boldsymbol{S_1^2}$ *e* $\boldsymbol{S_2^2}$

$$\boldsymbol{S_1^2} = \frac{3\hbar^2}{4}\boldsymbol{I_1} \otimes \boldsymbol{I_2} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.\boldsymbol{I_2} & 0.\boldsymbol{I_2} \\ 0.\boldsymbol{I_2} & 1.\boldsymbol{I_2} \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.1 & 1.0 & 0 & 0 \\ 1.0 & 1.1 & 0 & 0 \\ 0 & 0 & 1.1 & 1.0 \\ 0 & 0 & 1.0 & 1.1 \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Analogamente, $\boldsymbol{S_2^2} = \boldsymbol{S_1^2}$ , portanto, temos: $\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\,\boldsymbol{S_1 . S_2}$

$$\boldsymbol{S^2} = \frac{3\hbar^2}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \frac{\hbar^2}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Diagonalização $S^2$e $S_z$ na base $\{m_1, m_2\}$

Portanto o termo $\boldsymbol{S_1.S_2}$ fica:

$$\boldsymbol{S_1.S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \longrightarrow \boldsymbol{S_1.S_2} = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\,\boldsymbol{S_1.S_2}$ , ainda nos resta os termos $\boldsymbol{S_1^2}$ *e* $\boldsymbol{S_2^2}$

$$\boldsymbol{S_1^2} = \frac{3\hbar^2}{4}\boldsymbol{I_1} \otimes \boldsymbol{I_2} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.\boldsymbol{I_2} & 0.\boldsymbol{I_2} \\ 0.\boldsymbol{I_2} & 1.\boldsymbol{I_2} \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.1 & 1.0 & 0 & 0 \\ 1.0 & 1.1 & 0 & 0 \\ 0 & 0 & 1.1 & 1.0 \\ 0 & 0 & 1.0 & 1.1 \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Analogamente, $\boldsymbol{S_2^2} = \boldsymbol{S_1^2}$ , portanto, temos: $\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\,\boldsymbol{S_1.S_2}$

$$\boldsymbol{S^2} = \frac{3\hbar^2}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \frac{\hbar^2}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \boldsymbol{S^2} = \frac{\hbar^2}{1}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Portanto o termo $\boldsymbol{S_1 . S_2}$ fica:

$$\boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\left[\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right] \longrightarrow \boldsymbol{S_1 . S_2} = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\, \boldsymbol{S_1 . S_2}$ , ainda nos resta os termos $\boldsymbol{S_1^2}$ *e* $\boldsymbol{S_2^2}$

$$\boldsymbol{S_1^2} = \frac{3\hbar^2}{4} \boldsymbol{I_1} \otimes \boldsymbol{I_2} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.\boldsymbol{I_2} & 0.\boldsymbol{I_2} \\ 0.\boldsymbol{I_2} & 1.\boldsymbol{I_2} \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1.1 & 1.0 & 0 & 0 \\ 1.0 & 1.1 & 0 & 0 \\ 0 & 0 & 1.1 & 1.0 \\ 0 & 0 & 1.0 & 1.1 \end{pmatrix} = \frac{3\hbar^2}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Analogamente, $\boldsymbol{S_2^2} = \boldsymbol{S_1^2}$ , portanto, temos: $\boldsymbol{S^2} = (\boldsymbol{S_1} + \boldsymbol{S_2})^2 = \boldsymbol{S_1^2} + \boldsymbol{S_2^2} + 2\, \boldsymbol{S_1 . S_2}$

$$\boldsymbol{S^2} = \frac{3\hbar^2}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \frac{\hbar^2}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \boldsymbol{S^2} = \frac{\hbar^2}{1}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

Temos que na base $\{m_1, m_2\}$ $\boldsymbol{S^2}$ não é diagonal, porém, na base {s, ms} gerada pela base {m1,m2} é diagonal!
Devemos então determinar os autovalores e autovetores de $\boldsymbol{S^2}$ e então aplicarmos à equação: $\boldsymbol{S^2_{diagonal}} = \boldsymbol{U^{-1}}\, \boldsymbol{S^2}\, \boldsymbol{U^1}$
Onde $\boldsymbol{U^1}$ é construída com os autovetores de $\boldsymbol{S^2}$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det\begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix}$$

# Diagonalização $S^2$e $S_z$ na base $\{m_1, m_2\}$

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Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det\begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

Cujas raízes são 0 e 2 (triplamente degenerado)

Os autovalores 0 e 2 podem ser escritos como $S(S+1)\,\hbar^2$ (S=0 ou 1)

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det\begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

Cujas raízes são 0 e 2 (triplamente degenerado)

Os autovalores podem ser escritos como S(S+1) $\hbar^2$ (S=0 ou 1)

Sabemos que $S^2$ e $S_z$ comutam, portanto, podemos determinar um conjunto de autovetores de $S_z$ que também são de $S^2$

# Diagonalização $S^2$e $S_z$ na base $\{m_1, m_2\}$

Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det\begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

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Sabemos que $S^2$e $S_z$ comutam, portanto, podemos determinar um conjunto de autovetores de $S_z$ que também são de $S^2$

Logo $S_z|\Psi\rangle = M_s\,\hbar^1|\Psi\rangle$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

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Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det\begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

Cujas raízes são 0 e 2 (triplamente degenerado)

Os autovalores podem ser escritos como $S(S+1)\,\hbar^2$ (S=0 ou 1)

Sabemos que $S^2$ e $S_z$ comutam, portanto, podemos determinar um conjunto de autovetores de $S_z$ que também são de $S^2$

Logo $S_z|\Psi\rangle = M_s\,\hbar^1|\Psi\rangle$

$$\Rightarrow \frac{\hbar}{1}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = M_s\frac{\hbar}{1}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det\begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

Cujas raízes são 0 e 2 (triplamente degenerado)

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$$\frac{\hbar}{1}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = M_s\frac{\hbar}{1}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \begin{aligned} (1 - M_s)x &= 0 \\ (0 - M_s)y &= 0 \\ (0 - M_s)z &= 0 \\ (-1 - M_s)w &= 0 \end{aligned}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

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P/ λ = 0 ou (S=0) e $M_s$ = 0

$$|S = 0,\ M_s = 0\rangle = \begin{pmatrix} 0 \\ y \\ -z \\ 0 \end{pmatrix}$$

# Diagonalização $S^2$e $S_z$ na base $\{m_1, m_2\}$

Resolvendo autovalores: $\boldsymbol{S^2}\ |\Psi\rangle = \lambda\, \hbar^2 |\Psi\rangle$

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det \begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

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P/ λ = 0 ou (S=0) e $M_s$ = 0

$$|S = 0,\ M_s = 0\rangle = \begin{pmatrix} 0 \\ y \\ -z \\ 0 \end{pmatrix}$$

$$|00\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

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P/ λ = 0 ou (S=0) e $M_s$ = 0

$$|S=0,\ M_s = 0\rangle = \begin{pmatrix} 0 \\ y \\ -z \\ 0 \end{pmatrix}$$

$$\Downarrow$$

$$|00\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

P/ λ = 2 ou (S=1) e $M_s$ = 1

$$|S=1,\ M_s = 1\rangle = \begin{pmatrix} x \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det\begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

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$$|S=0,\ M_s=0\rangle = \begin{pmatrix} 0 \\ y \\ -z \\ 0 \end{pmatrix}$$

⬇

$$|00\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

P/ λ = 2 ou (S=1) e $M_s$ = 1

$$|S=1,\ M_s=1\rangle = \begin{pmatrix} x \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

⬇

$$|11\rangle = \frac{1}{1}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

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$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det\begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

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P/ λ = 2 ou (S=1) e $M_s$ = 1

$$|S=1,\ M_s=1\rangle = \begin{pmatrix} x \\ 0 \\ 0 \\ 0 \end{pmatrix} \Downarrow \quad |11\rangle = \frac{1}{1}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

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P/ $\lambda$ = 2 ou (S=1) e $M_s$ = 1

$$|S=1,\ M_s=1\rangle = \begin{pmatrix} x \\ 0 \\ 0 \\ 0 \end{pmatrix} \downarrow \quad |11\rangle = \frac{1}{1}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

P/ $\lambda$ = 2 ou (S=1) e $M_s$ = 0

$$|S=1,\ M_s=0\rangle = \begin{pmatrix} 0 \\ y \\ z \\ 0 \end{pmatrix} \downarrow \quad |10\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

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Resolvendo autovalores: $\boldsymbol{S^2}\,|\Psi\rangle = \lambda\,\hbar^2|\Psi\rangle$

$$\begin{pmatrix} \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{2} \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \boldsymbol{w} \end{pmatrix} = \lambda \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \boldsymbol{w} \end{pmatrix} \Rightarrow \det \begin{pmatrix} (\mathbf{2}-\lambda) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (\mathbf{1}-\lambda) & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & (\mathbf{1}-\lambda) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & (\mathbf{2}-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

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$$\Rightarrow \frac{\hbar}{1}\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{-1} \end{pmatrix}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \boldsymbol{w} \end{pmatrix} = M_s\frac{\hbar}{1}\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \boldsymbol{w} \end{pmatrix} \Rightarrow \begin{aligned} (1-M_s)x &= 0 \\ (0-M_s)y &= 0 \\ (0-M_s)z &= 0 \\ (-1-M_s)w &= 0 \end{aligned}$$

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P/ λ = 2 ou (S=1) e $M_s$ = 1

$$|S=1,\ M_s=1\rangle = \begin{pmatrix} \boldsymbol{x} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \;\Downarrow\; |11\rangle = \frac{\mathbf{1}}{\mathbf{1}}\begin{pmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

P/ λ = 2 ou (S=1) e $M_s$ = 0

$$|S=1,\ M_s=0\rangle = \begin{pmatrix} \mathbf{0} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \mathbf{0} \end{pmatrix} \;\Downarrow\; |10\rangle = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\begin{pmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{1} \\ \mathbf{0} \end{pmatrix}$$

P/ λ = 2 ou (S=1) e $M_s$ = -1

$$|S=1,\ M_s=-1\rangle = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{w} \end{pmatrix} \;\Downarrow$$

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# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

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$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Rightarrow \det\begin{pmatrix} (2-\lambda) & 0 & 0 & 0 \\ 0 & (1-\lambda) & 1 & 0 \\ 0 & 1 & (1-\lambda) & 0 \\ 0 & 0 & 0 & (2-\lambda) \end{pmatrix} \Rightarrow \lambda^4 - 6\lambda^3 + 12\lambda^2 - 8\lambda^1 = 0$$

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P/ λ = 2 ou (S=1) e $M_s$ = 0

$$|S=1,\ M_s=0\rangle = \begin{pmatrix} 0 \\ y \\ z \\ 0 \end{pmatrix} \ \Downarrow \ |10\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

P/ λ = 2 ou (S=1) e $M_s$ = -1

$$|S=1,\ M_s=-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ w \end{pmatrix} \ \Downarrow \ |1{-1}\rangle = \frac{1}{1}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

$|00\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$ $|11\rangle = \frac{1}{1}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $|10\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ $|1-1\rangle = \frac{1}{1}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

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# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

$\lfloor 00\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$ $\lfloor 11\rangle = \frac{1}{1}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $\lfloor 10\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ $\lfloor 1{-1}\rangle = \frac{1}{1}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ Com os autovetores ao lado construímos $\boldsymbol{U^1}$ de $\boldsymbol{S^2_{diagonal} = U^{-1}\, S^2\, U^1}$

$$\boldsymbol{U^1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{e} \quad \boldsymbol{U^{-1}} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

$|00\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$ $|11\rangle = \frac{1}{1}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $|10\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ $|1-1\rangle = \frac{1}{1}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ Com os autovetores ao lado construímos $\boldsymbol{U^1}$ de $\boldsymbol{S^2_{diagonal} = U^{-1}\, S^2\, U^1}$

$$\boldsymbol{U^1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{e} \quad \boldsymbol{U^{-1}} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Logo temos $\boldsymbol{S^2_{diagonal} = U^{-1}\, S^2\, U^1}$

# Diagonalização $S^2$ e $S_z$ na base $\{m_1, m_2\}$

$\lfloor 00\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$ $\lfloor 11\rangle = \frac{1}{1}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $\lfloor 10\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ $\lfloor 1{-1}\rangle = \frac{1}{1}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ Com os autovetores ao lado construímos $\boldsymbol{U^1}$ de $\boldsymbol{S^2_{diagonal} = U^{-1}\, S^2\, U^1}$

$$\boldsymbol{U^1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{e} \quad \boldsymbol{U^{-1}} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Logo temos $\boldsymbol{S^2_{diagonal} = U^{-1}\, S^2\, U^1}$ ➡

$$\frac{\hbar^2}{1}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} = \frac{\hbar^2}{1}\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} . \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$