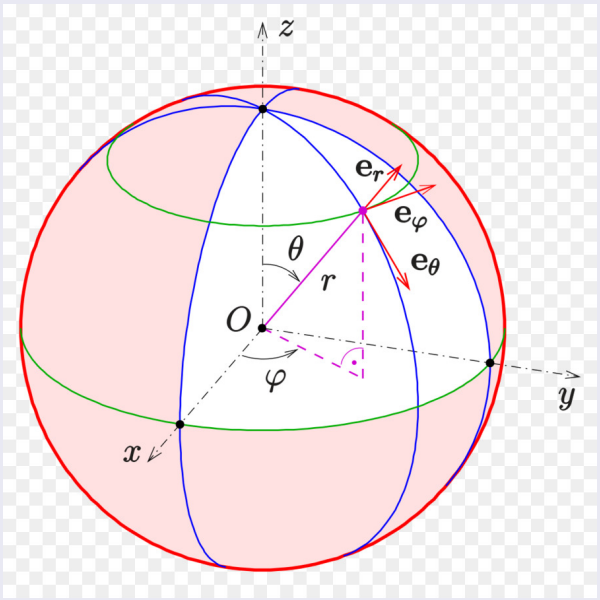
Operadores Tensoriais

Victor Sanchez

Campinas, 2020

Considering $\hat{n} = (n_x, n_y, n_z) = (\frac{x}{r}, \frac{y}{r}\frac{z}{r}) \Rightarrow (V_x, V_y, V_z)$ and spherical harmonics from Appendix B eq. (B.5.7) pg. 528.

Coordenadas esféricas



$$cos(\theta) = \frac{z}{r} \Rightarrow V_z$$
$$sen(\theta)sen(\phi) = \frac{y}{r} \Rightarrow V_y$$
$$sen(\theta)cos(\phi) = \frac{x}{r} \Rightarrow V_x$$

Harmônicos esféricos

$$Y_2^0 = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$$
$$Y_2^{\pm 1} = \mp\sqrt{\frac{15}{8\pi}}(\sin\theta\cos\theta)e^{\pm i\phi}$$
$$Y_2^{\pm 2} = \sqrt{\frac{15}{32\pi}}(\sin^2\theta)e^{\pm 2i\phi}$$

Exercicio 1

$$Y_2^0 = \sqrt{\frac{5}{16\pi}}(3cos^2\theta - 1)$$

$$T_{\pm 2}^{(0)} = \sqrt{\frac{5}{16\pi}}(3V_z^2 - 1)$$

Exercicio 2

$$
\begin{aligned}
Y_2^{\pm 1} &= \mp\sqrt{\frac{15}{8\pi}}(sen(\theta)cos(\theta))e^{\pm i\phi} \\
&= \mp\sqrt{\frac{15}{8\pi}}[sen(\theta)cos(\theta)](cos(\phi) \pm isen(\phi)) \\
&= \mp\sqrt{\frac{15}{8\pi}}cos(\theta)[sen(\theta)cos(\phi) \pm isen(\theta)sen(\phi)] \\
T_{\pm 2}^{(1)} &= \mp\sqrt{\frac{15}{8\pi}}V_z(V_x \pm iV_y)
\end{aligned}
$$

Exercicio 3

$$Y_2^{\pm 2} = \sqrt{\frac{15}{32\pi}} sen^2(\theta) e^{\pm 2i\phi}$$

$$= \sqrt{\frac{15}{32\pi}} sen^2(\theta)(cos(2\phi) \pm isen(2\phi))$$

$$= \sqrt{\frac{15}{32\pi}} sen^2(\theta)[cos^2(\phi) - sen^2(\phi) \pm 2isen(\phi)cos(\phi)]$$

$$= \sqrt{\frac{15}{32\pi}} sen^2(\theta)[cos^2(\phi) \pm 2isen(\phi)cos(\phi) + (isen(\phi))^2]$$

$$= \sqrt{\frac{15}{32\pi}} [(sen(\theta)cos(\phi))^2 \pm 2i(sen(\theta)sen(\phi))(sen(\theta)cos(\phi)) + (isen(\theta)sen(\phi))^2]$$

$$T_{\pm 2}^{(2)} = \sqrt{\frac{15}{32\pi}} (V_x^2 \pm 2iV_xV_y + (iV_y)^2)$$

$$T_{\pm 2}^{(2)} = \sqrt{\frac{15}{32\pi}} (V_x \pm iV_y)^2$$