Versor de polarização circular

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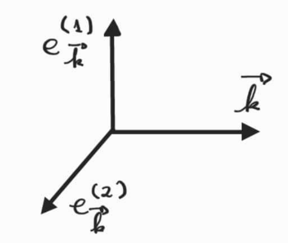
Na aula 23, na quantização do campo eletromagnético, escrevemos os campos elétrico e magnético em termos dos versores de polarização circular $\hat{\mathbf{e}}_{\mathbf{k}\lambda}$, tal que:

$$\hat{\mathbf{e}}_{\mathbf{k}\pm} = \mp\frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}^{(1)}_{\mathbf{k}} \pm \ i\hat{\mathbf{e}}^{(2)}_{\mathbf{k}}\right) \tag{1}$$

onde $\hat{\mathbf{k}}$ é a direção de propagação da onda e $\hat{\mathbf{e}}^{(1)}_{\mathbf{k}}$ e $\hat{\mathbf{e}}^{(2)}_{\mathbf{k}}$ são as direções de polarização lineares.



$$\hat{\mathbf{e}}^{(1)}_{\mathbf{k}} \times \hat{\mathbf{e}}^{(2)}_{\mathbf{k}} = \hat{\mathbf{k}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{e}}^{(1)}_{\mathbf{k}} = \hat{\mathbf{e}}^{(2)}_{\mathbf{k}}$$

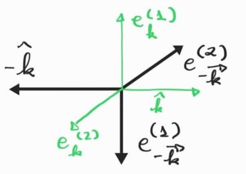
$$\hat{\mathbf{e}}^{(2)}_{\mathbf{k}} \times \hat{\mathbf{k}} = \hat{\mathbf{e}}^{(1)}_{\mathbf{k}}$$

Queremos mostrar que:

$$\hat{\mathbf{e}}^{*}_{\mathbf{k}\lambda} \cdot \hat{\mathbf{e}}_{\pm\mathbf{k}\lambda} = \pm\delta_{\lambda\lambda'}$$

$$\hat{\mathbf{e}}^{*}_{\mathbf{k}\lambda} \times \hat{\mathbf{e}}_{\pm\mathbf{k}\lambda} = \pm i\lambda\delta_{\lambda\lambda'}\hat{\mathbf{k}}$$

Definindo:



$$\hat{\mathbf{e}}^{(2)}_{-\mathbf{k}} \times \hat{\mathbf{e}}^{(1)}_{-\mathbf{k}} = -\hat{\mathbf{k}}$$

$$\hat{\mathbf{e}}^{(1)}_{-\mathbf{k}} \times (-\hat{\mathbf{k}}) = \hat{\mathbf{e}}^{(2)}_{-\mathbf{k}}$$

$$-\hat{\mathbf{k}} \times \hat{\mathbf{e}}^{(2)}_{-\mathbf{k}} = \hat{\mathbf{e}}^{(1)}_{-\mathbf{k}}$$

Isso nos dá:

$$\hat{\mathbf{e}}_{-\mathbf{k}\pm} = \pm\frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}^{(1)}_{\mathbf{k}} \pm \; i\hat{\mathbf{e}}^{(2)}_{\mathbf{k}}\right) = -\hat{\mathbf{e}}_{\mathbf{k}\pm}$$

Por fim, vamos escrever:

$$\hat{\mathbf{e}}_{\mathbf{k}\pm} = \mp\frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}^{(1)}_{\mathbf{k}} \pm \; i\hat{\mathbf{e}}^{(2)}_{\mathbf{k}}\right)$$

$$\hat{\mathbf{e}}_{\mathbf{k}\lambda} = (-\lambda)\frac{1}{\sqrt{2}}\left(\hat{\mathbf{e}}^{(1)}_{\mathbf{k}} + \; i\lambda\hat{\mathbf{e}}^{(2)}_{\mathbf{k}}\right)$$

Com $\lambda = \pm 1$

Com essas definições, o cálculo dos produtos escalar e vetorial é direto:

$$
\begin{aligned}
\hat{\mathbf{e}}^*_{\mathbf{k}\lambda} \cdot \hat{\mathbf{e}}_{\pm\mathbf{k}\lambda'} &= \frac{1}{2}(-\lambda)\left(\hat{\mathbf{e}}^{(1)}_{\mathbf{k}} - \ i\lambda\hat{\mathbf{e}}^{(2)}_{\mathbf{k}}\right) \cdot (-\lambda')\left(\hat{\mathbf{e}}^{(1)}_{\pm\mathbf{k}} + \ i\lambda'\hat{\mathbf{e}}^{(2)}_{\pm\mathbf{k}}\right) \\
&= \frac{\lambda\lambda'}{2}\left(\hat{\mathbf{e}}^{(1)}_{\mathbf{k}} \cdot \hat{\mathbf{e}}^{(1)}_{\pm\mathbf{k}} + \lambda\lambda'\hat{\mathbf{e}}^{(2)}_{\mathbf{k}} \cdot \hat{\mathbf{e}}^{(2)}_{\pm\mathbf{k}}\right) \\
&= \frac{\lambda\lambda'}{2}(\pm 1 \pm \lambda\lambda') = \pm\frac{\lambda\lambda'}{2}(1 + \lambda\lambda') \\
&= \pm\delta_{\lambda\lambda'}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathbf{e}}^*_{\mathbf{k}\lambda} \times \hat{\mathbf{e}}_{\pm\mathbf{k}\lambda'} &= \frac{1}{2}(-\lambda)\left(\hat{\mathbf{e}}^{(1)}_{\mathbf{k}} - \ i\lambda\hat{\mathbf{e}}^{(2)}_{\mathbf{k}}\right) \times (-\lambda')\left(\hat{\mathbf{e}}^{(1)}_{\pm\mathbf{k}} + \ i\lambda'\hat{\mathbf{e}}^{(2)}_{\pm\mathbf{k}}\right) \\
&= \frac{\lambda\lambda'}{2}\left(i\lambda'\hat{\mathbf{e}}^{(1)}_{\mathbf{k}} \times \hat{\mathbf{e}}^{(2)}_{\pm\mathbf{k}} - i\lambda\hat{\mathbf{e}}^{(2)}_{\mathbf{k}} \times \hat{\mathbf{e}}^{(1)}_{\pm\mathbf{k}}\right) \\
&= i\frac{\lambda\lambda'}{2}(\pm\lambda' - (\mp)\lambda)\hat{\mathbf{k}} = \pm i\frac{\lambda\lambda'}{2}(\lambda' + \lambda)\hat{\mathbf{k}} \\
&= \pm i\frac{\lambda\lambda'}{2}2\lambda\delta_{\lambda\lambda'}\hat{\mathbf{k}} = \pm i\lambda\delta_{\lambda\lambda'}\hat{\mathbf{k}}
\end{aligned}
$$