

A Brief History of Quantum Mechanics

13.1 Social Context in Central Europe During the 1920s

To continue the building analogy of Chap. 1, the theoretical foundations of physics were shaken at the beginning of the twentieth century. These tremors preceded those of society as a whole. The historian Eric Hobsbawm has written [88]:

The decades from the outbreak of the First World War to the aftermath of the second, were an Age of Catastrophe for this society [...] shaken by two world wars, followed by two waves of global rebellion and revolution [...]. The huge colonial empires, built up before and during the Age of the Empire, were shaken, and crumbled to dust. A world economic crisis of unprecedented depth brought even the strongest capitalistic economies to their knees and seemed to reverse the creation of a single universal world economy, which had been so remarkable an achievement of nineteenth-century liberal capitalism. Even the USA, safe from war and revolution, seemed close to collapse. While the economy tottered, the institutions of liberal democracy virtually disappeared between 1917 and 1942 from all but a fringe of Europe and parts of North America and Australasia, as Fascism and its satellite authoritarian movements and regimes advanced.

Since quantum mechanics was developed for the most part in Northern and Central Europe (see Table 13.1), we will devote most of our attention here to the conditions prevailing at that time in Germany and Denmark.

Hobsbawm's description applies particularly well to the case of Germany. While the Anglo-Saxon world and the wartime neutrals more or less succeeded in stabilizing their economies between 1922 and 1926, Germany was overwhelmed in 1923 with economic, political, and spiritual crises. Hunger riots erupted everywhere, as the value of the mark plunged to 10^{-12} of its pre-1913 value. Additional difficulties arose from a repressed military putsch in North

Table 13.1. Publications in quantum mechanics. July 1925–March 1927 [90]

Country	Papers written	Country	Papers written
Germany	54	France	12
USA	26	USSR	11
Switzerland	21	Netherlands	5
Britain	18	Sweden	5
Denmark	17	others	7

Germany, a separatist movement in the Rhineland, problems with France on the Rhur, and radical leftist tendencies in Saxony and Thuringia. In the East, Soviet Russia did not fare better.

A cultural movement against dogmatic rationalism gained ground in German society after the war. The most widely read book opposed causality to life, and assimilated physics into causality [89]. Moreover, a profound division along political, scientific, and geographic lines started to grow in the German physics community. Right wing physicists were in general chauvinistic, ultra-conservative, provincial, anti-Weimar, and anti-Semitic. They were interested in the results of experiments and dissociated themselves from quantum and relativity theory. On the opposite side, the Berlin physicists were labeled as liberal and theoretical. Note, however, that the German physicists of that time, with the possible exception of Einstein and Born, could only be labeled as liberal or progressive in comparison with Johannes Stark and Philipp Lenard. The adjective “theoretical” (appearing also in the name of Bohr’s Institute in Copenhagen) would be translated today as “fundamental.” Although the main theoretical center was in Berlin, strong theoretical schools also flourished in Göttingen and Munich. The start of Nazi persecutions in the thirties and the exclusion of Jews from the first group had consequences in the world distribution of physicists devoted to the most fundamental aspects of physics.

After the First World War (1918) German physicists had been excluded from international collaborations, and the lack of foreign currency made it almost impossible to purchase foreign journals and equipment. However, a new national organization, the *Notgemeinschaft der Deutschen Wissenschaft*, created in 1920 under the direction of Max von Laue and Max Planck, was instrumental in the provision of funds for scientific research. Atomic theorists in Berlin, Göttingen, and Munich received sufficient funds to support the work of physicists like Heisenberg and Born. The foreign boycott was not observed by Scandinavia and the Netherlands. Bohr kept friendly relations with his German colleagues (see p. 210).

Denmark had been on the decline at least since 1864, when it was defeated by Prussia and Austria with the resultant loss of about one-third of its

territory. The years after the war represented a period of unprecedented turmoil in Denmark as well. For the first time in 400 years, this country teetered on the brink of revolution, although of a kind that was different from those experienced in neighboring countries. Disputes over the shift of the border with Germany, social struggles between town and country, fights for extensive reforms in employment conditions. All these difficulties added to the loss of wartime markets, and to trade deficits and inflation. In spite of such hardships, Bohr's new institute was inaugurated in 1921.

The scientific and the social crisis during the first part of the twentieth century were both very profound. However, the first one was over by the end of the 1920s. The second one continued *in crescendo* until the aftermath of the Second World War (1945).

13.2 Prehistory of Quantum Physics ($1860 \leq t \leq 1900$)

Gustav Kirchhoff is at the origin of both radiation and matter branches of quantum physics.¹ In 1860 he showed that the emissive power of a black-body $E(\nu)$ depends only on the frequency ν and on the temperature T and challenged both experimentalists and theoreticians to find such dependence [93]. This search proved to be full of difficulties. Only in 1893 Wilhelm Wien demonstrated his displacement law and in 1896 he proposed the exponential dependence for the function $f(\nu/T)$ in (13.1) [94]. In 1900 Planck modified this dependence with an extremely successful guess (13.2), that still holds today

$$E(\nu) = \nu^3 f(\nu/T) \quad f(\nu/T) = \alpha \exp[-\beta\nu/T] \quad (13.1)$$

$$\rightarrow \frac{h\nu^3}{c^2} \frac{1}{\exp[h\nu/kT] - 1}, \quad (13.2)$$

where the Planck constant h was introduced [2].

Analytical spectroscopy was also started by Kirchhoff in 1860, in collaboration with Robert Bunsen [95]. The Balmer formula fitting the frequencies of the discrete hydrogen spectrum dates from 1885:

$$\nu_n = cR_H \left(\frac{1}{4} - \frac{1}{n^2} \right), \quad n = 3, 4, \dots, \quad (13.3)$$

where R_H is the Rydberg constant [7]. No significant progress was made in understanding Balmer's formula for 28 years.

¹ The sources [90–92] have been used extensively for this chapter.

13.3 Old Quantum Theory ($1900 \leq t \leq 1925$)

13.3.1 Radiation

Planck justified his law by means of an unorthodox way of counting partitions plus the quantum hypothesis: the (fictitious) oscillators of a black-body have energy [2]

$$\epsilon = h\nu . \quad (13.4)$$

In 1905, Einstein showed that the expression for the increase of entropy with volume of a gas composed of noninteracting molecules becomes identical to the same quantity for monochromatic radiation obeying Wien exponential law (13.1), if in such expression the number of molecules n is replaced by $E/h\nu$, where E is the total energy. Thus, from purely thermodynamic arguments, Einstein concluded that "... it seems reasonable to investigate whether the laws governing the emission and transformation of light are also constructed as if light consisted of ... energy quanta" [3]. This proposal constituted a revolution, in view of the so far wholly accepted Maxwell's wave theory of light. Moreover, Einstein endowed Planck's oscillators with physical reality.

On the basis of (13.4), Einstein interpreted the photoelectric effect as the total transfer of the photon² energy to the electron, whose energy E is given by

$$E = h\nu - W , \quad (13.5)$$

where W is the work function of the metal. This relation was only confirmed experimentally in 1914 by Robert Millikan [96], although even then Millikan did not conclude in favor of Einstein's "bold, not to say reckless hypothesis." In fact, from 1905 to 1923 Einstein was the only physicist seriously considering the existence of light quanta.

From 1906 to 1911, quantum theory was Einstein's main concern (even more than the theory of relativity). He contributed to the specific heat of solids and to energy fluctuations of black-body radiation. In 1909, he foretold: "It is my opinion that the next phase of theoretical physics will bring us a theory of light which can be interpreted as a kind of fusion of the wave and the emission theory."

Between 1916 and 1917 Einstein made fundamental contributions to the theory of radiation [58]. Combining classical thermodynamics and electromagnetism with Bohr's first two quantum postulates (Sect. 13.3.2), and assuming thermal equilibrium between atoms and radiation field, he derived:

- The concepts of spontaneous and induced emission and absorption, from which he could obtain Planck radiation law (13.2) (see Sect. 9.5.4[†]).
- The momentum of the light particle $h\nu/c$ which, together with the energy (1905), completes the properties of a quantum particle.

² The term "photon" was only coined during the 1920s.

- The need of a probabilistic description, inherent to the concept of spontaneous emission.

Einstein's 1917 paper carried the seeds of many developments in physics. However, he did not work by himself two rather immediate consequences:

1. The scattering of an atom and a light particle. Such experiments made by Arthur Compton verified both the energy and momentum conservation in these processes and thus confirmed the validity of the light quantum hypothesis³ (1923, [4]).
2. Satyendra Bose's derivation of Planck's law using symmetric states was translated and submitted for publication by Einstein in 1924 [48]. The same year Einstein applied Bose's ideas to an ideal gas of particles [39] (see Sect. 7.5[†] on Bose–Einstein condensation).

However, a last storm over the light quanta was on the way (see Sect. 13.5.2).

13.3.2 Matter

In 1911, the work of Rutherford's young colleagues Hans Geiger and Ernest Marsden showed conclusively that a hydrogen atom consists of one electron outside the positively charged nucleus, where almost all the mass is concentrated [6]. At that time, electrons were supposed to be just particles. (Electron wave behavior was experimentally verified in [5].)

Like Einstein in 1905, Bohr was aware that the postulates of his 1913 model [8] were in conflict with classical physics:

- An atom displays stationary states of energy E_n that do not radiate.
- Transitions between stationary states are accompanied by monochromatic radiation of frequency ν satisfying the Balmer series

$$h\nu = E_n - E_m. \quad (13.6)$$

This assumption implied a renunciation of causality because of the absence of any directive for the transition.

- For large values of n , the quantum frequency ν should agree with the classical frequency of light irradiated by the rotating electron. This correspondence principle constituted the main link between classical and quantum theory. (See Fig. 4.2 as an illustration of the survival of the correspondence principle in quantum mechanics.)

³ However, explanations based on classical electromagnetic fields and quantized processes of emission and absorption could only be completely ruled out after experimental evidence that there is no lower limit on the accumulation time of light energy in the photoelectric effect [97] or on the nonexistence of correlations of a photon with itself [98].

The derivation of the Rydberg constant as a function of the mass and the charge of an electron and of Planck's constant, and the correct helium ion/hydrogen ratio to five significant figures, commanded the attention of the physics community.

By means of his precise determination of X-ray energies, Henry Moseley gave further support to the Bohr model both through the assignment of a Z value to all known elements and the prediction of the still missing elements [100]. James Frank and Heinrich Hertz further confirmed the model by using the impact of electrons on atoms to excite their atomic spectrum [101]. The Bohr model appeared to work, in spite of its assumptions. In order to joke about the situation with the old quantum theory, Bohr used to tell the story of a visitor to his country home who noticed a horseshoe hanging over the entrance door. Puzzled, he asked Bohr if he really believed that this brought luck. The answer was: "Of course not, but I am told it works even if you don't believe in it." [92].

Bohr developed his model during a postdoctoral stay at Rutherford's laboratory (Manchester, UK). He was appointed professor of physics at the University of Copenhagen in 1916 and the Universitetets Institut for Teoretisk Fysik (today, Niels Bohr Institutet) was inaugurated in 1921. Unlike Einstein and Dirac, Bohr seldom worked alone. His first collaborator was Hendrik Kramers (Netherlands), followed by Oscar Klein (Sweden). During the 1920s, there were 63 visitors to the Institute who stayed more than one month, including ten Nobel Laureates. The flow of foreign visitors lasted throughout Bohr's life. He became both an inspiration and a father figure.

Arnold Sommerfeld established an important school in Munich. In 1914, it was found that every line predicted by the Balmer formula is a narrowly split set of lines. By taking into account the influence of relativity theory, Sommerfeld showed that the orbits of the electrons are approximate ellipses displaying a perihelion precession [102]. Sommerfeld's work was also one of the first attempts to unite the quantum and relativity theories, a synthesis still not completely achieved.

In Göttingen, Born did not turn his attention to atomic theory until around 1921. Heisenberg and Jordan were his assistants.

In 1924 Pauli had published 15 papers on topics ranging from relativity to the old quantum theory, the first one before entering the University of Munich. In 1922, he went for a year to Copenhagen. Later he held a position at Hamburg. He made the assumption of two-valuedness for electrons and stated the exclusion principle [35] (Sect. 7.1) so important for understanding the properties of atoms, metals, nuclei, baryons, etc.

The crucial experimental results of Stern and Gerlach were known in 1921 [17]. A proposal concerning spin was made⁴ by two Dutch students from the University of Leiden, Uhlenbeck, and Goudsmit, who also suggested the existence of $m_s = \pm 1/2$ as a fourth quantum number [31] (Sect. 5.2.2). They explained the anomalous Zeeman effect by including the factor of two appearing in (5.25), which was accounted for in [103]. After receiving objections from Henrik Lorentz, Uhlenbeck, and Goudsmit considered withdrawing their paper, but it was too late. (Their advisor, Paul Ehrenfest, also argued that the authors were young enough to be able to afford some stupidity.) The two-component spin formalism (5.24) was introduced by Pauli in 1927 [32].

Dirac, and independently Enrico Fermi, developed quantum statistics for antisymmetric wave functions [49, 50].

However, until 1925, there were almost as many setbacks as successes in the application of the model. For instance, the spectrum of He proved to be intractable, in spite of heroic efforts by Kramers, Heisenberg and others. The final blow was the negative result of the BKS proposal (Sect. 13.5.2).

13.4 Quantum Mechanics (1925 $\leq t \leq$ 1928)

Periodically, Bohr used to gather his former assistants together at the Institute in Copenhagen. In 1925, the ongoing crisis in quantum mechanics was examined by Bohr, Kramers, Heisenberg, and Pauli. A few months later, back at Göttingen, Heisenberg found a way out of the impasse [9]. He succeeded in formulating the theory in terms of observable quantities, doing away with the concepts of orbits and trajectories (see Sect. 2.1). Heisenberg found a correspondence between the coordinate $x(t)$ and the double array x_{nm} (n, m labeling quantum states). $x_{nm}(t)$ was interpreted as a sort of transition coordinate, and hence an allowed observable. In order to represent $x^2(t)$, he made the crucial assumption that $(x^2)_{nm} = \sum_p x_{np}x_{pm}$. Heisenberg solved the simple but nontrivial problem of the harmonic oscillator by showing that the Hamiltonian is given by $H_{mn} = E_n\delta_{nm}$, where the E_n reproduce the correct eigenvalues (Sect. 3.2).

Born and Jordan realized that the arrays (x_{nm}) were matrices and arrived at the fundamental commutation relation (2.16) in its matrix form (3.45) [104] (see Fig. 13.1). Born, Heisenberg, and Jordan wrote a comprehensive text on quantum mechanics, which included unitary transformations, perturbation

⁴ The combination of the Pauli principle and of spinning electrons prompted Ralph Kronig to propose the idea of half-integer spin. However, he was dissuaded from publishing it by Pauli and others, on the grounds that models for electrons carrying an intrinsic angular momentum $\hbar/2$ either required the periphery of the electron to rotate with a velocity much larger than the velocity of light c , or the electron radius to be much larger than the classical value.



Fig. 13.1. The 1930 Copenhagen Conference. In the front row: Klein, Bohr, Heisenberg, Pauli, Gamow, Landau, and Kramers. (Reproduced with permission from the Niels Bohr Archive, Copenhagen)

theory, the treatment of degenerate systems, and commutation relations for the angular momentum operators [10].

Many of these results were also obtained by Dirac [11], who introduced the idea that physical quantities are represented by operators (of which Heisenberg's matrices are just one representation), the description of physical states by vectors in abstract Hilbert spaces, and the connection between the commutator of two operators with the classical Poisson bracket.

In 1926, Pauli and Dirac independently reproduced the results for the hydrogen atom of old quantum theory using the new matrix mechanics [105, 106].

Zurich-based Schrödinger did not belong to the Copenhagen–Göttingen–Munich tradition. In 1925 he came across de Broglie's suggestion [28] that the wave–corpuscle duality should also be extended to material particles, satisfying the momentum–frequency relation (4.32). This relation is reproduced if the momentum and the energy are replaced by the differential operators (4.4) and (9.5) and if such operators act on the plane wave solutions (4.30) and (9.9). Upon making the same substitution in a general Hamiltonian, Schrödinger derived the time-independent and the time-dependent equations that bear his name [12]. Quantization was obtained through the requirement that the wave function should be single-valued [as in (5.49)]. Schrödinger presented his derivation as a step towards a continuous theory, the integers (quantum numbers) originating in the same way as the number of nodes in a

classical vibrating string. Schrödinger's formulation gained rapid acceptance, both because of the answers that it produced and because it was built from mathematical tools that were familiar to the theoretical physicists of the time. Schrödinger hoped that quantum mechanics would become another branch of classical physics: waves would be the only reality, particles being produced by means of wave packets. This expectation turned out to be wrong.

In 1926, Schrödinger also proved that the matrix and the differential formulation are equivalent. Since physicists understood how to transcribe the language of wave mechanics into matrix mechanics, both of them were referred to as quantum mechanics.

The probability interpretation of $|\Psi(x, t)|^2$ soon became apparent (Sect. 4.1.1), and it is usually considered part of the Copenhagen interpretation. However, Born was the first to write it explicitly [27]. In his paper on collision theory, he also stated that $|c_i|^2$ (2.6) was the probability of finding the system in the state i . He emphasized that quantum mechanics does not answer the question: what is the state after a collision? Rather it tells us how probable a given effect of the collision is. Determinism in the atomic world was thereby explicitly abandoned.

In 1926 Heisenberg was able to account for the He problem using the Schrödinger equation plus the Pauli principle plus spin (Sect. 8.3) [107].

The relativistic generalization of the Schrödinger time-dependent, two-component spin formalism (5.24) encountered some difficulties. In 1928, Dirac produced an equation, linear in both the coordinates and time derivatives, with the properties that:

- It is Lorentz invariant
- It satisfies a continuity equation (4.16) with positive density ρ (which previous attempts at relativistic generalization had failed to do)
- It encompasses spin from the start
- It reproduces the accuracy of the Sommerfeld model for the H atom, which was more accurate than the (new) quantum mechanics [108].

The price that Dirac had to pay for this most beautiful product of twentieth century mathematical physics was that it turned out to be a four-component rather than a two-component theory (5.24). Its interpretation including the additional two components is beyond the scope of this exposition.

A few comments are in order:

- Quantum mechanics and its traditional interpretation were developed over only a few years (1925–1928). The rate of publication in this period was such that many physicists complained about the impossibility of keeping up to date. Moreover, communication delays certainly hampered the ability of non-European physicists to contribute.
- Quantum mechanics was developed under a very unfavorable social context (see Sect. 13.1).

- Unlike previous scientific cornerstones, quantum mechanics was the result of the coherent effort of a group of people, mostly in Northern and Central Europe. Table 13.1 shows the number of papers written in each country during the period of major activity in the creation of quantum mechanics. It reflects both the predominance of Germany and the number of visitors, especially in the case of Denmark. It also reminds us that the scientific center of gravity was only transferred to the other side of the Atlantic after the Second World War (1939–1945).
- The Bohr Festspiele took place at Göttingen in 1922. After Bohr’s speech, the 20-year old Heisenberg stood up and raised objections to Bohr’s calculations. During a walk in the mountains that same afternoon, Bohr invited Heisenberg to become his assistant in Copenhagen. This anecdote points out the extreme youth (and self-confidence) of most of the contributors to quantum mechanics. In 1925 Dirac was 23 years old, Heisenberg was 24, Jordan 22, and Pauli 25. The “elders” were Bohr (40), Born (43), and Schrödinger (38).

13.5 Philosophical Aspects

13.5.1 Complementarity Principle

Neither the Heisenberg nor the Schrödinger formulations improved the contemporary understanding of wave–particle duality. In 1927, Heisenberg answered the question: can quantum mechanics represent the fact that an electron finds itself approximately in a given place and that it moves approximately with a given velocity, and can we make these approximations so close that they do not cause mathematical difficulties? [24]. The answer was given in terms of the uncertainty relations (2.37) and (9.34) (see the last paragraph of Sect. 2.6). Heisenberg’s paper was the beginning of the discussion of the measurement problem in quantum mechanics (Chap. 12), on which so many volumes have been written.

As most theoretical physicists would have done, Heisenberg derived his uncertainty principle from the (matrix) formalism (Sect. 2.6). Bohr had the opposite attitude. While being duly impressed by the existence of at least two formulations predicting correct quantum results, he insisted on first understanding the philosophical implications, rather than the formalisms. His main tools consisted of words, which he struggled continually to define precisely. Bohr pointed out that theories – even quantum theories – were checked by readings from classical instruments. Therefore, all the evidence has to be expressed within classical language, in which the mutually exclusive terms “particle” and “wave” are well defined. Either picture may be applied in experimental situations, but the other is then inapplicable. The idea of complementarity is that a full understanding of this microscopic world comes only from the possibility of applying both pictures; neither is complete in itself.

Both must be present, but when one is applied, the other is excluded. These ideas were stated for the first time at the Como Conference, September 1927. Bohr continued to reformulate the presentation of the concept of complementarity throughout the rest of his life [22]. He defined the term “phenomenon” to include both the object of study and the mode of observation.

13.5.2 Discussions between Bohr and Einstein

Many histories of science display a sequence of continuous successes, thus ignoring the many frustrations accompanying creative processes. The discussions between Bohr and Einstein about problems of principle illustrate the difficulties inherent to changes in the description of the physical world, even in the case of our greatest forefathers.

The first meeting between Bohr and Einstein took place in 1920, on the occasion of Bohr’s visit to Berlin. Verification of general relativity through the bending of light had taken place shortly before. Thus Einstein was on the zenith, while Bohr was only a rising star. Although they interchanged affectionate compliments, the subject of their Berlin conversations remains unclear. As many other physicists, at that time Bohr did not believe in light quanta, and this disbelief continued even after Compton’s experiment (1923). In 1924 there appeared a paper signed by Bohr, Kramers and the American physicist John Slater with the following contents [109]:

- Since the simultaneous validity of the (continuous) wave theory of light, and the description of matter processes involving (discrete) energy transitions is incompatible with conservation of energy in individual events, this principle is given up, as well as conservation of momentum. They hold only statistically.
- Statistical independence of the processes of emission and absorption in distant atoms is also assumed.
- The mediation of virtual fields produced by virtual oscillators is proposed. However, the paper describes neither formal mechanisms governing the behavior of these entities, nor their interaction with real fields. In fact, (13.6) is the only mathematical expression included in the paper.

Born, Klein and Schrödinger reacted positively. Einstein and Pauli were against. However, two experiments on Compton scattering ended the BKS speculation during the following year. They concerned the time-interval between the electron recoil and the scattered photon, and momentum conservation in individual processes. The BKS proposal marked the end of old quantum mechanics.

Einstein’s initial appraisal of Heisenberg’s and Schrödinger’s quantum mechanics appears to have been positive. However, the approval was withdrawn after Born’s probabilistic interpretation. Einstein never accepted limitations to our knowledge arising from first principles of the theory.

Quantum mechanics was discussed at the V Solvay Conference (1927) in Brussels, attended by all founders. Einstein expressed his concern over the extent to which the causal account in space and time had been abandoned in quantum mechanics. The discussions centered on whether a fuller description of phenomena could be obtained through the detailed balance of energy and momentum in individual processes. For instance, in the case of a beam of particles passing through a slit in a diaphragm, Einstein would suggest that the indetermination principle could be invalidated by measuring the momentum of the recoiling slit. During the evenings Bohr would explain how the inherent uncertainty in the location of the slit due to its recoil restored Heisenberg's principle. Bohr systematically emphasized the need to fully specify the measuring apparatus in any experiment. It was on this occasion that Einstein asked whether God had recourse to playing dice, to which Bohr replied by calling for great caution in ascribing attributes to Providence in everyday language.

At the next Solvay meeting in 1930 (Fig. 13.2), Einstein claimed that a control of energy and time could be achieved using relativity theory. He proposed the device represented in Fig. 13.3, consisting of a box with a hole on a wall and a clock inside, such that a single photon might be released at a known moment. Moreover, it would be possible to measure the energy of the photon with any prescribed accuracy by weighing the box before and after the event, and making use of the relativity equation $E = mc^2$. Bewilderment among quantum physicists lasted until the next day, when Bohr came up with an answer based on general relativity: since the rate of the clock depends on its position in a gravitational field, the lack of precision in the box displacement generates an uncertainty Δt in the determination of time, while the indeterminacy in the energy ΔE is obtained through the position–momentum relation (2.37). The product $\Delta t \Delta E$ satisfies the Heisenberg time–energy uncertainty relation.

Starting then, Einstein did not attack the consistency of quantum mechanics. However, in 1935 Einstein, Podolsky and Rosen presented a profound argument pointing to the incompleteness of quantum mechanics [16]. They considered a system consisting of two entangled but spatially separated particles. An adaptation of their argument to the case of spin entanglement is presented in Sect. 11.2. Until his end Einstein believed that, although quantum results were correct, indeterminacies appeared because some parameters characterizing the system were unmeasurable. These hidden variables should be incorporated into the description of physical systems. A more fundamental theory should bear to quantum mechanics a relation similar to the one existing between classical and statistical mechanics [110].

Most of the physics community rejected the EPR conclusion because of Bohr's reply, which was based on his concept of "phenomenon." According to Bohr, the two (mutually exclusive) experimental setups were not specified in the EPR definition of reality [81].



Fig. 13.2. Einstein and Bohr leaving the Solvay meeting of 1930. (Reproduced with permission from the Niels Bohr Archive, Copenhagen)

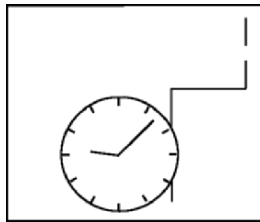


Fig. 13.3. Sketch of the thought experiment proposed by Einstein in order to reject the time–energy uncertainty relation. (Reproduced with permission from the Niels Bohr Archive, Copenhagen)

Probably the best description of Einstein's and Bohr's respective positions is stated in Bohr's presentation on the occasion of Einstein's seventieth birthday [111] and in Einstein's answer in the same volume [110].

13.6 Recent History

Rather than dwell on philosophical interpretations of equations, most physicists proceeded to carry out many exciting applications of quantum mechanics [86]:

This approach proved stunningly successful. Quantum mechanics was instrumental in predicting antimatter, understanding radioactivity (leading to nuclear power), accounting for the behavior of materials such as semiconductors, explaining superconductivity, and describing interactions such as those between light and matter (leading to the invention of the laser) and of radio waves and nuclei (leading to magnetic resonance imaging). Many successes of quantum mechanics involve its extension, quantum field theory, which forms the foundations of elementary particle physics

On the other hand, the controversy over the EPR experiment did not die down, but the important issue turned out to be the locality of quantum mechanics, not its completeness. In 1952 Bohm was able to reproduce the quantum mechanical predictions for systems of the EPR-type using a deterministic hidden-variable formulation, including nonlocal interactions between spatially separated particles [112].

In 1964 Bell proved that nonlocality is an inherent characteristic of hidden variable theories reproducing the results of quantum mechanics [74]. Moreover, he showed that the correlations between measured properties of any classical, deterministic, local two-particle theory would obey a mathematical inequality. On the contrary, the same measurements would violate such an inequality if the two particles were in an entangled quantum state. Three-particle entangled states are predicted to display even more spectacular contradictions.

Following instrumental improvements in the production of entangled photons, Bell's contribution was followed by many proposals of possible experiments. They culminated in the results of [76] and [77], which are in conclusive disagreement with the results of local hidden-variable theories. The quantum mechanical correlations cannot be reproduced within the realm of local realism (Sect. 11.2).

However, one may not conclude that these tests have *proven* the validity of quantum mechanics. It is worthwhile to remember that experiments can only prove that a theory is incorrect, if their results contradict the predictions of the theory. The validity of quantum mechanics should be further tested by checking consequences of modifications of quantum principles [such as the linearity of the Schrödinger equation (9.5)] against experiment.