letters

Approaches for improving students' understanding of quantum mechanics

In the first sentence of their article "Improving Students' Understanding of Quantum Mechanics" (Physics Today, August 2006, page 43), Chandralekha Singh, Mario Belloni, and Wolfgang Christian refer to Richard Feynman's well-known assertion that nobody understands quantum mechanics. But thereafter they ignore it, and apparently assume that student misconceptions when learning quantum mechanics are not connected with foundational issues. I argue the contrary, that Feynman's statement should be a central concern in all efforts to improve quantum pedagogy. If we teachers do not understand a topic, we pass our own misconceptions on to our students and make the subject much more difficult for them.

Many conceptual difficulties, including those Feynman was referring to, arise from the problem of introducing probabilities into quantum theory in a useful and consistent way. Textbooks avoid the problem by assigning probabilities to macroscopic measurement outcomes rather than to microscopic quantum systems. Although that approach avoids inconsistencies, it gives rise to some serious misconceptions: Measurements are somehow special and unrelated to other quantum phenomena; they require a "classical" apparatus that functions outside the scope of quantum mechanics (where is one to find such a thing nowadays?); they produce physical effects at long distances; one can say nothing sensible about what a quantum system is doing in the absence of measurements; measurements can be used to predict the future of the measured system but tell us nothing about its past; and so forth. These misconceptions are not unrelated to those that Singh and coauthors have reported.

Letters and opinions are encouraged and should be sent to Letters, Physics Today, American Center for Physics, One Physics Ellipse, College Park, MD 20740-3842 or by e-mail to ptletter@aip.org (using your surname as "Subject"). Please include your affiliation, mailing address, and daytime phone number. We reserve the right to edit submissions.

Students learning quantum mechanics could benefit greatly if their instructors used advances in our understanding that have occurred during the 40 years since Feynman unashamedly confessed his perplexity. He seems to have reacted favorably to a preliminary version of the new ideas (see the letter from Murray Gell-Mann and James Hartle in Physics Today, February 1999, page 11), so he might have appreciated the more mature form now available. In brief, we now know how to consistently assign probabilities directly to microscopic systems without referring to measurements, and we can show that under appropriate conditions a properly constructed measurement apparatus, described in fully quantum terms, will reveal properties the measured system possessed *before* the measurement took place. In such circumstances the probabilities of measurement outcomes are the same as those of the measured properties, and measurements are no longer an essential conceptual tool: One can think directly in physical terms about what the quantum system is doing at different times. This gets rid of a major source of student difficulties and misconceptions.

Consider the example reported by Singh and coauthors in which students used a calculation employing $\langle A\rangle = \langle\psi|A|\psi\rangle$ to find the expectation of an observable A, rather than simply using a probability distribution they had worked out previously. (In that article, A was the energy, but the same principle applies to any observable.) This failure is not surprising given that textbooks lack a good discussion of how to assign probability distributions to observables. So the student memorizes an independent formula $\langle A\rangle = \langle\psi|A|\psi\rangle$, which is a good way of calculating something that comes up in homework and exams, but whose physical significance is not particularly clear (to student or instructor). What the student should be taught is that A is the quantum counterpart of a random variable in ordinary probability theory, and its average can be obtained from its probability distribution in exactly the same way. Defining $\langle A\rangle$ in this manner before introducing $\langle\psi|A|\psi\rangle$ as a convenient formula for calculating it would make things clearer. But quantum textbooks do not contain the necessary tools, and for good reason. With two noncommuting observables A and B, it is easy to poke either of them into the $\langle\psi|A|\psi\rangle$ formula, whereas assigning probabilities leads into a vast swamp, which work on quantum foundations has shown to be filled with nasty paradoxes ready to bite the unwary. Retreating to macroscopic measurements allows textbook writers to avoid the swamp, but with a serious loss in clarity of thought and physical intuition. It is better to drain the swamp of its root cause: a failed attempt to meld classical and quantum modes of reasoning, instead of consistently applying quantum concepts at all levels, microscopic and macroscopic, which is something we now know how to do.

Another misconception reported in the Physics Today article, that measurement of a physical observable causes the system to be stuck forever in the measured eigenstate, is hardly surprising when students are taught that measurements and wavefunction collapse are part of the axiomatic, and thus unanalyzable, structure of quantum theory. Instead, they need to think about measurements as quantum physical processes, governed by the same laws as the rest of the quantum world, and learn how to use conditional probabilities to relate measurement outcomes to the past as well as the future behavior of a measured system. Once again, outdated ideas make the subject harder to learn.

For 10 years I have been teaching advanced undergraduate and beginning graduate quantum mechanics courses and courses in quantum information, using the new perspective in which quantum mechanics is based on probabilistic laws of universal validity, with measurements being only one application. Reactions have generally been positive, though the students show signs of shock when I tell them that by the end of the course, provided they do their homework, they will understand some aspects of quantum mechanics better than Feynman did. Presenting the new ideas takes somewhat longer than the material they replace, but not enormously so. Some time will be regained in courses that include an introduction to quantum entanglement and Einstein-Podolsky-Rosen, since circuitous arguments invoking Bell's inequality and the like, which can leave students quite confused, are replaced by a short, clear treatment of the essentials.

Although I can see the value of computer simulations of Schrödinger's equation, I think it is more effective to first introduce students to basic quantum dynamics, both unitary and stochastic, through the use of "toy models." I included various examples in *Consistent Quantum Theory*.[1] The properties of such models are easily worked out with a pencil on a small sheet of paper, like the back of an envelope. Working through them helps students master new concepts and get rid of certain misconceptions about quantum measurements.

The fact that students in my courses have been able to learn how to apply probabilities consistently to microscopic systems, in a way that disposes of numerous difficulties and conceptual paradoxes, suggests it might be worthwhile for other teachers to invest some time in learning post-Feynman ideas. The main difficulty is the absence of a textbook. I have used reference 1 as a supplement, though it is not ideal. It has no exercises, although a few are available on the corresponding website. I would be happy to hear from anyone skilled in textbook writing who wants to revise an older one or start something new.

In conclusion, I strongly favor every effort to improve students' understanding of quantum mechanics, and I consider the research reported by Singh and coauthors a valuable contribution to that end. However, if we want our students to genuinely understand quantum mechanics and not simply calculate things, I believe a much bigger step forward is possible by combining the efforts reported in the article with advances in quantum foundations.

Reference

1. R. B. Griffiths, *Consistent Quantum Theory*, Cambridge U. Press, New York (2002). Some chapters and a few exercises are available at http://quantum.phys.cmu.edu.

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In their article, Chandralekha Singh, Mario Belloni, and Wolfgang Christian focus exclusively on "functional understanding of quantum mechanics," which they claim "is quite distinct from the foundational issues alluded to by Feynman."

But are the foundational and the functional really so distinct? The work of other physics education researchers suggests not. For example, in a classic article, Alan Van Heuvelen discusses students' prevalent and frustrating use of "primitive formula-centered problem-solving strategies"[1] and suggests that physical, intuitive understanding developed through qualitative diagrams and models "must come before students start using math in problem solving. The equations become crutches that short-circuit attempts at understanding." Van Heuvelen also urges that "instead of thinking of [problems] as an effort to determine some unknown quantity, [teachers] might . . . encourage students to think of the problem statement as describing a physical process—a movie of a region of space during a short time interval or of an event at one instant of time." I suspect Singh, Belloni, and Christian would agree with this advice. They comment that such "qualitative understanding of quantum mechanics is much more challenging than facility with the technical aspects."

But isn't the main barrier to such intuitive, qualitative understanding the nature of quantum mechanics itself—at least, the version of the theory advocated by Niels Bohr, Werner Heisenberg, and virtually every textbook writer since? Why should we expect students to invest the time and energy necessary to, say, visualize the time-dependence of $|\psi|^2$ when we also preach the ambiguous and contradictory Copenhagen dogma that ψ does not represent anything physically real, yet still provides a complete description of physical reality? Why are we surprised that students are confused about, and don't take seriously, something that we assure them is, at best, some kind of algorithmic fantasy? Is there really any difference between "shut up and calculate" and "plug and chug"?

Why not teach them Bohmian mechanics—an alternative (deterministic) version of quantum theory in which particles are particles (and really exist, all the time) and the same dynamical laws apply whether anyone is looking or not?[2] About this alternative theory John S. Bell asked, "Why is [it] ignored in text books? Should it not be taught . . . as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism are not forced upon us by experimental facts, but by deliberate theoretical choice?"[3]

If we really want to help students understand quantum mechanics, the first step is to reject the confusion-spawning foundational vagueness, ambiguity, and philosophical absurdity of Copenhagen quantum theory, and adopt a clearer, more scientific, less fuzzy version. (See Sheldon Goldstein's two-part article "Quantum Theory Without Observers," PHYSICS TODAY, March 1998, page 42, and April 1998, page 38.) The first step, in short, is to present them with a theory that *can* be understood.

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2. S. Goldstein, in *Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/qm-bohm.
3. J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics: Collected Papers on Quantum Philosophy*, Cambridge U. Press, New York (2004), p. 173.

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A quite different approach from the one presented by the authors of "Improving Students' Understanding of Quantum Mechanics" may be appropriate at least for some classes of students. It might be called the pragmatic approach, teaching students to deal with a wide variety of problems while minimizing philosophical discussion. I took this approach for several years while teaching a course for graduate engineers at Stanford.[1] The resulting course was surprisingly orthogonal to the traditional quantum course. Solving the Schrödinger equation became a minimal part of the subject; rather, tight-binding expansions allowed the student to use simple algebra to obtain a meaningful understanding of atoms, molecules, and solids. Transition rates and shake-off excitations provided understanding of a wide variety of phenomena.

I took the defensible stance that all of quantum mechanics is the direct consequence of a single assertion, wave–particle duality. The uncertainty principle and the Pauli principle are consequences, not independent conjectures. Quantum theory does not tell us that there will be a particle of spin ½ with the mass and charge of an electron, but it indicates how such a particle will behave if there is one. When the consequences seem puzzling, it is fair to say that one is simply having difficulty with the starting assertion.

Reference

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Authors Singh, Belloni, and Christian demonstrate how visualizations can help students learn some of the most difficult and counterintuitive principles in the physics curriculum. But as two surveys have shown, there are broader roles for computation in that curriculum that ought to be, but currently are generally not being, used to help prepare physics students for their likely work environments.

An August 2002 survey by the American Institute of Physics (available at http://aip.org/statistics/trends/reports/bachplus5.pdf) looked at physics bachelor graduates in the nonacademic workplace at least five years beyond their graduation. The results revealed a significant gap between their computational preparation as undergraduates and the computational demands of their work. The AIP survey does not detail these demands, but from my own experiences in engineering research and development environments, I've found that they include constructing and validating numerical models as well as interpreting results from running those models. In short, holders of physics bachelor's degrees must be able to think about their physics in computational terms.

The other survey, completed by Robert Fuller from the University of Nebraska–Lincoln, provides some answers to how much computation is included in today's physics curricula of colleges and universities nationwide. The answers indicate wide variability in the degree of computation amid a widespread agreement by faculty on the importance of integrating computation into their courses. Fuller concludes that physics departments in the US generally acknowledge the need for more computation in their curricula, but most are not meeting the need in a systematic way. This gap—between acknowledged need and community response—is consistent with AIP's survey findings. The September/October 2006 issue of *Computing in Science and Engineering* gives Fuller's report and provides some examples of possible ways to close the gap. They include the "lone wolf" who is the sole interested person in the department; the "persuasive pioneer," implementer of a full computational physics undergraduate major; and a range of cases in between.

I believe the physics community needs to reconceive the canon of the undergraduate physics curriculum to include a significant role for computation. Whether or not they learn their physics principles with computation embedded, students will need to put their knowledge to productive use in their work. Today that usually means through computation.

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In their article "Improving Students' Understanding of Quantum Mechanics," the authors present the following survey question: "By definition, the Hamiltonian acting on any allowed state of the system Ψ will give the same state back, i.e., $H\Psi = E\Psi$. . . . Explain why you agree or disagree." This wording appears to be ambiguous, since an "allowed state of the system" seems to connote an eigenstate. Perhaps better wording would be "the Hamiltonian acting on any wavefunction Ψ," or even better, "acting on a wavefunction Ψ in Hilbert space," rather than referring to the state as "allowed."

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I have found the recent articles on improving physics education very helpful—please keep them coming! Although I am a physics undergraduate looking toward a future in research, such articles have influenced me at least as much as your articles on physics innovations.

I was very lucky to have an outstanding advanced placement physics teacher. His explanations and guidance were simple yet effective, and he led the class through his entire thought process when working out examples. Although most of the students were not going into physics or engineering, almost all were able to understand the material. His brilliant instruction was one of the factors that made me choose to be a physics major.

On the other hand, I am privy to the horror stories of my friends taking introductory physics for science majors under other instructors. The range of experiences, from stunning to devastating, have encouraged me to focus on teaching as well as research. Please, keep the physics education articles coming. At a time when our country is facing a lack of science education, how physics is taught may be one of the most important areas to study.

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Singh, Belloni, and Christian reply: We appreciate the number and quality of the responses to our article. They indicate a strong interest, which we share, in the teaching of upper-level courses such as quantum mechanics. Our article focused on the concept of time evolution to illustrate a variety of difficulties students face; we barely scratched the surface of the breadth and depth of teaching and learning issues in a standard quantum mechanics course.

We value highly the perspectives on fundamental issues from Robert Griffiths and Travis Norsen, who raised similar concerns from different viewpoints. Foundational issues in quantum mechanics are not emphasized in most undergraduate or graduate quantum mechanics curricula. Griffiths has argued that the lack of proper grounding in foundational issues is the source of many student misconceptions in quantum mechanics. The consistent histories approach[1] or Bohm's interpretation[2] may be conceptually "cleaner," but our research has shown that many of the difficulties—for example, the confusion between the time-independent and time-dependent Schrödinger equation—are not foundational but conceptual.

As a practical matter, non-Copenhagen interpretations are not widely incorporated in quantum mechanics textbooks. We have argued that there are ways to improve student understanding within the current framework—surely, these general methods will work if and when the physics community has collectively adopted new ways of thinking about quantum mechanics.

Physics education research is well-established now, and a controlled study involving two quantum mechanics classes taught by the same instructor might be worthwhile. One class could use the standard Copenhagen interpretation while the other uses the consistent histories approach. An important question, then, is this: If both classes cover approximately the same amount of material and students in both classes are given the surveys we have developed, do students in one class significantly outperform those in the other? In addition to the written surveys, a subset of students from both classes could be interviewed to further ascertain their level of understanding. If students using the consistent histories approach significantly outperform those learning the standard Copenhagen interpretation, it may be worthwhile to develop interactive tutorials similar to those discussed in the article but using the consistent histories approach.

In response to Travis Norsen, we note that we agree with Alan Van Heuvelen, whom Norsen cites, and our approach is consistent with his advice.[3] However, intuition and foundational issues are not exactly the same things. Although a deep understanding of foundational issues may improve intuition, we can help our students develop qualitative, conceptual understanding of many aspects of quantum theory without first having to clarify every foundational issue. Our research suggests that the nature of physical intuition is not well understood, though intuition is important.[4]

As Philip Shemella has suggested, we have used other wordings for the question of interest, including the wording he recommends. Our findings are unchanged. During interviews, the interviewer has often rephrased the question when a student was unable to answer correctly. The responses were qualitatively unchanged.

As Griffiths, Norsen, and Walter Harrison imply, the use of simulations and results from physics education research to address functional issues is just a single prong in what should be a multi-pronged approach to the teaching of quantum mechanics. We agree that addressing foundational issues is just as important.

In addition to the approach taken in textbooks by Griffiths and Harrison, Richard Robinett's quantum text[5] relates pedagogical quantum models to modern experimental realizations of these systems and emphasizes connections to classical mechanics.

We agree with Norman Chonacky that a discussion of the broader role of computation in the physics curriculum is needed. We encourage interested readers to attend the American Association of Physics Teachers topical conference Computational Physics for Upper Level Courses, to be held in July 2007 (see http://www.opensourcephysics.org/CPC/index.html). Its purpose is to identify problems in which computation helps students understand key physics concepts.

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Uncertainty over weakening circulation

Barbara Goss Levi's Search and Discovery story (Physics Today, April 2006, page 26) discusses evidence of weakening ocean circulation and its possible connection to global warming. The Atlantic Ocean circulation across 25° N latitude has been used as a benchmark for characterizing the mass and heat transport from the tropics to the northern latitudes. The upper portion of this transport includes the Gulf Stream, which is at least partially responsible for a moderate climate in Europe. A weakening of the Atlantic meridional overturning circulation and of the Gulf Stream might have the unpleasant consequence of cooling Europe's climate.

The Physics Today piece is based on analysis of work by Harry Bryden, Hannah Longworth, and Stuart Cunningham,[1] which concluded that the Atlantic meridional overturning circulation slowed by about 30% between 1957 and 2004. Their work inspired speculations that the anthropogenic increase in carbon dioxide may be responsible for the weakening of heat transport from the tropics, and that such an effect has now been detected.

The conclusion that the Atlantic meridional overturning circulation has decreased by 30% does not follow from the data presented by Bryden and coauthors, but is based on an incorrect treatment of measurement errors.

According to Bryden and coauthors, the 1957 transport in a layer shallower than 1000 m was 22.9 ± 6 Sverdrups (1 Sv = 10^6 m^3/s) compared with the transport of 14.8 ± 6 Sv in 2004. The ± 6 Sv represents an uncorrelated error of each measurement. Bryden subtracts the two quantities and presents the results as 8.1 ± 6 Sv (instead of 8.1 ± 12 Sv or ± 8.5 Sv, depending on the character of errors), which is an incorrect result. It is a mystery how such an error was missed by Levi and by the editors and reviewers of the original paper. The observed change of 8.1 Sv is well within the uncertainty of the measurement. The correct conclusion from the data presented in Bryden's paper should have been that no statistically significant change in Atlantic meridional overturning circulation at 25° N between 1957 and 2004 has been detected. Such a conclusion is in agreement with the earlier analysis of essentially the same data (between 1957 and 1999) by Alexandre Ganachaud and Carl Wunsch.[2]

Research also failed to detect any slowing,[3,4] and one of the relevant papers[4] concludes that "there is no sign of any Meridional Overturning Circulation slowdown trend over the past decade, contrary to some recent suggestions."[1]

In defense of Bryden and his coauthors, I must share a comment from a personal communication I had with Bryden shortly after his *Nature* paper was published. Bryden's paper as submitted for publication to *Nature* included a question mark at the end of the title, suggesting only a possibility that the circulation might be slowing down. On the editor's insistence, the question mark was removed, and the title was changed into a positive statement that caused a considerable stir.

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Postscript on Chandra and Eddington

The letters from Arthur Miller and Kameshwar Wali (Physics Today,