On Galvanometry

Wilhelm Weber

Editor's Note: An English translation of Wilhelm Weber's 1862 paper "Zur Galvanometrie".[1]

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[1] [Web62].

Contents

Chapter 1

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In the ever-expanding technical applications of galvanism, various proposals have already been made for the introduction of *galvanic resistance units*[5] (etalons or standards) in order to meet the manifold needs arising therefrom, and it is likely that the serious efforts made by experts will succeed in not only identifying and establishing, to the widest extent and in the most perfect way, the measurement rules corresponding to this purpose, but also to bring them to a practical and successful implementation as soon as possible.

All *galvanic cells* used for chemical analysis, galvanoplastic, telegraphic and other technical purposes, even if they are called constant,[6] are constantly subject to smaller and often larger changes, which one must get to know more closely in order to master them. But even if these cells were completely unchangeable, their *effects* would now be larger and sometimes smaller, depending on the variety of applications that are made of them. To control these *effects* therefore requires not only knowledge of the cell itself, but also of all bodies through which the current of the cell is to pass, namely, knowledge of their *resistance*. That is why *resistance measurements* have become indispensable for all practical applications, especially for the construction and testing of electrical telegraphs, especially given their increasing extent and complexity of conditions.

However, a *resistance standard* is required for the resistance measurements. Without measuring with such standards, the bodies through which the current is to be conducted can be described in various ways; but after a measurement made with such standards, *a single number* is enough to express everything essential more completely and more precisely than is possible through all descriptions. Because the resistance measurements often reveal differences and changes in the bodies that cannot be recognized even from the most precise description.

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[2] [Web62].

[3] Translated and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[4] The Notes by Wilhelm Weber are represented by [Note by WW:]; the Notes by H. Weber, the editor of the fourth volume of Weber's *Werke*, are represented by [Note by HW:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[5] [Note by AKTA: In German: *galvanischer Widerstandsmaasse*. The word "Maass", nowadays written as "Maß", can be translated as: measurement, measure, dimension, unit, standard etc.

[6] [Note by AKTA:] That is, supposed to produce a constant electromotive force between its terminals, or to produce a constant electric current when connected to a closed circuit with a fixed resistance.

Basically, such a *standard* was put into use at an early stage by comparing the various bodies through which currents were to be conducted with copper wires, the length and cross-section of which were measured. It is obvious that this was based, even if only tacitly, on the resistance of a copper wire with a length equal to the linear unit and a cross-section equal to the area unit as *resistance standard*. However, the explicit determination of a certain standard of resistance was first raised by Jacobi in Petersburg in 1846.

Jacobi said:[7]

> It is no less important than the absoluteness of the current measurements that the physicists express the size of the conducting resistance in terms of a common unit. But there can be no absolute determination here, because it seems that there are differences in the resistances of even the chemically purest metals that cannot be explained by a difference in dimensions alone. All of these difficulties are eliminated if you let a copper or other wire of your choice wander around the physicists and ask them to refer their resistance measuring instruments to it and in the future to only give their measurements according to this measurement.

Of such a resistance standard chosen by Jacobi (a copper wire 25 feet long and weighing $22\,337\frac{1}{2}$ milligrams) a number of copies were actually made and used for resistance measurements. However, be it that the necessary care was not taken in the production, or be it that such resistance standards suffer changes over time, very significant differences have later emerged between these copies.

Therefore, in 1860, Siemens in Berlin,[8] with special consideration of the increasingly urgent needs of technical physics, tried to set up a *new resistance standard* that would meet all requirements and be easily represented by anyone and with the necessary precision, based on the use of the resistance of *mercury*, as that metal which can be obtained or produced anywhere with great ease in sufficient, almost perfect purity and, as long as it is liquid, does not assume any other molecular properties that modify its conductivity, and is also less dependent on temperature changes than other metals in its resistance, and finally offers particular convenience for use due to the size of its specific resistance.

With the creation of this new *resistance standard*, Siemens also combined the presentation of *resistance scales*, as necessary and indispensable mediators between the standard and the objects to be measured, and has constructed them to such an extent and perfection that all resistances can be formed with the greatest ease and accuracy, which according to his measure can be expressed by integer numbers from 1 to 10 000.

Finally, in England too, there are currently plans to establish a certain standard of resistance and it is hoped to ensure its general distribution and application, as well as all the scientific and technical purposes that can be achieved thereby, by establishing an institution under the combined protection of the British Association and the Royal Society, from which every experimenter in the whole world should, at his request, be provided with a *resistance standard*, which is not only valid for a precisely determined temperature, but also with an indication of its variation for a specific change in temperature, and whose *galvanic significance is finally given by a precise indication of the force required to excite a specific current in it*.

[7][Note by AKTA:] Moritz Hermann von Jacobi (1801-1874). A French version of this letter can be found in [Jac51].

[8][Note by AKTA:] E. W. v. Siemens (1816-1892), [Sie60] with English translation in [Sie61]. See also [GT19].

A long time ago I dealt with more precise measurements for this latter purpose, namely, to investigate the *galvanic significance of a conductor*, by determining the force required to produce a certain current, under the title of *absolute resistance measurements.* For example, the galvanic significance of the Jacobian resistance standard was established by stating that in order to excite in it a current intensity equal to 1 according to the the Gaussian units,[9] an electromotive force according to Gaussian measure of 5980 million units is required.[10,11,12] I have a similar determination from another copper circuit which I submitted to the Royal Society [of Göttingen] in 1853.[13,14,15] These previous provisions, however, had been more concerned with the method and the significance of the results to be obtained with them than with the extreme finesse of the quantitative execution, which had been achieved only on a trial basis with the aids and instruments available for other purposes.

But if these absolute resistance measurements are to find further application, if they are to be used to give a *lasting* expression to all quantitative results of important galvanic observations and research, then a similar situation arises as with other fundamental determinations, namely, the need to carry out at least one absolute resistance measurement according to the strictest methods, with the most perfect instruments and with all the art of the finest observation. This is a task which can only be completely solved by very skilful hands, with the most undisturbed leisure and with more solid facilities than are now available for physical research. The fact that only one such measurement is required, but one which must be carried out with the greatest precision, is readily apparent from the fact that the resistances of all bodies can be accurately compared with the resistances of *a single standard*, and that therefore only a precise knowledge of the *absolute value of this single standard* is required in order to transfer the advantages of all relations given by absolute values to all bodies in general.

Apart from these advantages which the knowledge of the absolute value of such a standard resistance can confer, the task of this measurement also offers interest in itself because of the influence which it has on the development of science. The development of almost all of *galvanometry* can be linked to this task, and all advances in galvanometry can be tested in solving this task. Once the goal to be achieved has been defined after gaining insight into the possibility of the solution, every more perfect solution is almost more important as proof of the progress of galvanometry than for its own immediate benefit.

A finer design of absolute resistance measurement not only fills essential gaps in galvanometry, but also brings many scattered investigations into a closer context. Conversely, if a higher level of galvanometry were to be achieved by other means, the result would be a finer execution of absolute resistance measurement. We will now take a closer look at some of these *galvanometric investigations* that serve to carry out the absolute resistance measurement in more detail.

A distinction is made between *galvanometers* and *galvanoscopes.* Those to which the tangent galvanometer[16] belong are only used for stronger currents, the intensity of which

[9][Note by AKTA:] That is, a current intensity equal to 1 according to the absolute system of units initiated by Carl Friedrich Gauss (1777-1855).

[10][Note by WW:] Abhandhingen der Königl. Sächs. Gesellschaft der Wissenschaften, I, p. 252.

[11][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 351.

[12][Note by AKTA:] [Web52, p. 352 of Weber's *Werke*] with English translation in [Web21a].

[13][Note by WW:] Abhandlungen der Königl. Gesellschaft der Wissenschaften zu Göttingen, Vol. 5.

[14][Note by HW:] Wilhelm Weber's *Werke*, Vol. II, p. 319.

[15][Note by AKTA:] [Web53c, p. 319 of Weber's *Werke*]. See also [Web53a] and [Web53b].

[16][Note by AKTA:] In German: *Tangenten-Boussole*. The tangent galvanometer was invented by Johan

is thus obtained in terms of a precisely known absolute measure. These,[17] on the other hand, serve to observe the slightest traces of currents, of which nothing else can be perceived. The great sensitivity of the latter is only achieved by very tightly surrounding the [magnetized] needle with its multiplier, which means that the more precise knowledge of the scale is lost, which arises automatically from the construction of the tangent galvanometer. In order to use such a galvanoscope for measurements, some observation is required as a measure of the sensitivity of the instrument in addition to observing the deflection caused by the current. As a rule, one tries to establish this standard once and for all by making corresponding observations on galvanometers and galvanoscopes. Apart from the fact that such corresponding observations do not produce an exact result because of the very different sensitivity of the two instruments, the scale of sensitivity for very sensitive galvanoscopes is generally not constant at all and therefore cannot be determined in advance. On the other hand, the observation of the *deflection* can be combined with another observation, namely, that of *vibration damping*, which directly provides this yardstick.

This connection makes it possible to use the most sensitive galvanoscopes for the most precise measurements, which is the necessary condition for carrying out absolute resistance measurements. However, galvanoscopes for this use require a construction that differs from ordinary galvanoscopes and the theory of which must be specially developed. This development is also of interest because it paves the way for the use of the most sensitive galvanoscopes in many other fine researches.

The present purpose therefore requires such a construction which allows the *deflection* and *damping* to be observed simultaneously with the greatest accuracy, whereas with ordinary galvanoscopes only the finest observation of the deflection was decisive for the construction. But what increases the deflection does not always increase the attenuation and vice versa. In addition, the deflection and attenuation must not exceed certain limits if they are to be capable of the finest determination. It is now the consideration of damping that particularly requires the use of strong magnets as galvanoscope needles, to which is added the need for a longer period of oscillation and a less variable resting point for the galvanoscope needle. This establishes the use of an astatic system formed by two strong magnets,[18] the period of oscillation of which is regulated by the length and strength of the metal wire used for suspension.

The absolute measurement of a *standard resistance* does not only depend on the accuracy of the *galvanometric measurements*, but also on the accuracy of our knowledge of the *Earth's magnetism in absolute value at the place and at the time of those galvanometric measurements*. The ultimate goal of galvanometric measurements is therefore that the inevitable uncertainty in the *absolute value of the standard resistance*, which arises from the determination of the Earth's magnetism, is not noticeably increased by the galvanometric

Jakob Nervander (1805-1848) and the sine galvanometer by Claude Servais Mathias Pouillet (1790-1868), [Ner33], [Pou37] and [Sih21]. Friedrich Kohlrausch discussed measurement of currents with the tangent and sine galvanometers, [Koh83, Chapters 64 and 65, pp. 188-192].

[17] That is, galvanoscopes.

[18] [Note by AKTA:] The adjective "astatic" is used in physics with the meaning of something having no tendency to take a definite position or direction. An astatic needle can be a combination of two parallel magnetized needles having equal magnetic moments, but with their poles turned opposite ways, that is, in antiparallel position. The arrangement protects the system from the influence of terrestrial magnetism. It was invented by André-Marie Ampère (1775-1826), [Amp21] and [LA98]. An earlier system composed of a single magnetized needle had also been created by Ampère, [Amp20c, p. 198] with Portuguese translation in [CA09, p. 133], [Amp20a, p. 239] and [Amp20b, p. 2], see also [AC15, p. 57].

measurement. Explaining and examining how this goal is to be achieved is the main purpose of this paper, which will be followed by some discussion of the *copying* of resistance standards and other questions relating to the determination and meaning of the resistance standard.

I - The Method of Absolute Resistance Measurement

1.1 Ratio of an Electromotive Force to a Current Intensity

A galvanic current i, which is moved with its ponderable carrier against a conductor with the velocity v, exerts an *electromotive force* e on the conductor according to the induction law discovered by Faraday,[19] which is proportional both to the intensity of the inducing current i and to the velocity of the inducing movement v. The ratio of this electromotive force to the product of the intensity of the inducing current in the velocity of the inducing movement, e/iv, therefore has a value that is independent of both the intensity i and the velocity v, namely, this value is determined from geometrically given relationships between the current carrier and the conductor as a *pure numerical value*, that is, independent of the spatial dimensions used for the geometric dimensions, as well as of the dimensions of the electromotive forces, current intensities and velocities. If one considers a single length element α of the inducing current i, which is moved with the velocity v against the length element of the conductor α', at the moment when the distance of both elements from each other $= r$, and denote the four angles which are determined by the directions of the two elements, $[\alpha]$, $[\alpha']$, on the direction of their connecting line $[r]$ and on the direction of movement of the current element $[v]$, with $\vartheta = [r,\ \alpha]$, $\vartheta' = [r,\ v]$, $\varepsilon = [\alpha,\ v]$, $\varphi = [r,\ \alpha']$, then according to the well-known law applicable to voltaic induction,[20] the electromotive force e, which is exerted by the element α of the inducing current i on the induced element α',

$$e = iv \cdot \frac{\alpha\alpha'}{r^2} (3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon)\cos\varphi \ ,$$

or it is the ratio

$$\frac{e}{iv} = \frac{\alpha\alpha'}{r^2} (3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon)\cos\varphi \ ,$$

whose value is hereafter obtained expressed in a *pure number*, since the ratios of two lines α/r and α'/r as well as the cosines of the angles are pure numbers.

[19][Note by AKTA:] Michael Faraday (1791-1867). See [Far32a] with German translation in [Far32b] and [Far89], and Portuguese translation in [Far11].

[20][Note by AKTA:] The expression utilized by Weber, *Volta-Induktion*, had been first suggested by Faraday himself in paragraph 26 of his first paper on electromagnetic induction of 1831, [Far32a, § 26, page 267 of the *Great Books of the Western World*] with German translation in [Far32b] and Portuguese translation in [Far11, p. 159]:

> For the purpose of avoiding periphrasis, I propose to call this action of the current from the voltaic battery, *volta-electric induction*.

In this English translation of Weber's 1862 paper we utilized the expressions *Volta-induction* and *voltaic induction* for this class of phenomena which is nowadays called Faraday's law of induction.

If we now call those ratios of the current carrier and the conductor to each other, under which this number is $= 1$, the *normal ratios*, it follows that under these normal ratios the ratio of the electromotive force to the current intensity, e/i, is equal to the velocity v with which the current carrier is moved, or that

$$\frac{e}{i} = v \ .$$

In general, one can see from this that the quotient of any electromotive force divided by any current intensity is equal to any velocity, which is expressed by the following theorem: *an electromotive force is related to a current intensity as a path length is related to a time.*

The same proposition also results directly from the terms associated with *electromotive forces* e and *current intensities* i in the theory of galvanism.

If ε denotes the quantity of positive or negative electricity in the unit of length of the conductor by electrostatic measure (in parts of that quantity which exerts on an equal quantity in the unit of distance a force which would give the unit of velocity to the ponderable unit of mass in the unit of time), and u the velocity at which the electricity moves in the conductor, so i is proportional to εu and is obtained by multiplying it by the factor $[1/c]\cdot\sqrt{8}$, where c denotes a *constant velocity* known from the fundamental law of electric action, as shown in Volume 5 of the Treatises of the Mathematical-Physical Class of Königl. Saxons. Societies of Sciences,[21,22] page 264, which has been found $= 439\,450\cdot 10^6$ millimeter/(second).

Furthermore, if f denotes the difference between the force acting on the positive electricity contained in the induced conductor in the direction of the conductor and the force acting on the negative electricity contained therein, expressed in parts of that force which would give the unit of velocity to the ponderable unit of mass in the unit of time, then the electromotive force acting on the induced conductor is proportional to f and is calculated by multiplying it by the obtained factor $[c/\varepsilon]\cdot\sqrt{1/8}$.

These meanings of i and e are the same, according to which a current of the intensity $= 1$, when passing around the unit of area, exerts the same effects with the unit of the magnetic moment, and according to which further the unit of the magnetic force on a closed conductor, while it is rotated in such a way that the projection of the area enclosed by it on the plane perpendicular to the direction of the magnetic force grows uniformly in the unit of time around the unit of area, exerts the unit of electromotive force. These meanings of i and e are to be used as the basis for all *absolute measurements* because of their relationship to magnetism.

According to these values of e and i, which must also be taken as a basis for the absolute resistance measurements, the ratio is

$$\frac{e}{i} = \frac{\frac{fc}{\varepsilon}\sqrt{\frac{1}{8}}}{\frac{\varepsilon u}{c}\sqrt{8}} = \frac{c^2}{8u}\cdot\frac{f}{\varepsilon^2} \ .$$

If the electrostatic force which the amount of positive or negative electricity contained in a piece x of the conductor, $= \varepsilon x$, exerts on an equal amount at the distance x, is called f', then it is known that

$$f' = \frac{\varepsilon x\cdot\varepsilon x}{x^2} = \varepsilon^2 \ ,$$

[21] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 652.

[22] [Note by AKTA:] [KW57, p. 652 of Weber's *Werke*] with English translation in [KW21].

consequently,

$$\frac{e}{i} = \frac{c^2}{8u} \cdot \frac{f}{f'} .$$

Now the ratio of two forces f/f', as well as the ratio of two velocities c/u, are expressed by pure numbers, from which it follows that

$$\frac{1}{8} \cdot \frac{c}{u} \cdot \frac{f}{f'} = n$$

is a pure numerical factor, and from this it follows that e/i is a velocity n times larger than the velocity c.

1.2 Representation of a Velocity Equal to the Resistance of a Conductor

According to Ohm's law of the galvanic circuit,[23] the current intensity i is directly proportional to the electromotive force e acting on the circuit, and inversely proportional to the resistance w of the circuit, and, if the resistance standard is chosen accordingly, it can be set

$$i = \frac{e}{w} ,$$

from which it follows that the quotient

$$\frac{e}{i} = w$$

for any given circuit has a *constant* value, which is called its *absolute resistance*.

This *resistance*, because it is the quotient of an electromotive force divided by a current intensity, must, according to the previous Section, be equal to a certain *velocity*, and it is of interest not only to determine this velocity by its size, but also to actually represent it that way, as it corresponds to the ratio e/i in all physical relationships.

Give the conducting wire the shape of a circle, which is placed parallel to the magnetic meridian plane and rotated around its horizontal diameter, while a small compass is placed in the center of the circle. This compass is then deflected from the magnetic meridian according to known laws, the more the faster the circle is rotated; for the currents induced in this rotation by the vertical component of the Earth's magnetism in the circle act on the compass and exert on it a directing force perpendicular to the plane of the meridian, the *mean value of which for the duration of half a revolution* increases proportionally with the angular velocity.[24] — During the duration of half a revolution, this directing force is of course variable, from which it follows that the needle cannot remain at rest, but must fluctuate within certain limits; however, the shorter the duration of half a revolution becomes during accelerated rotation compared to the oscillation period given to the needle by the Earth's magnetic directing force, the more those limits come closer to one another, and the above needle fluctuation can be reduced to such an extent that it becomes completely

[23] [Note by AKTA:] Georg Simon Ohm (1789-1854). Ohm's law is from 1826: [Ohm26a], [Ohm26c], [Ohm26d], [Ohm26b] and [Ohm27] with French translation in [Ohm60] and English translation in [Ohm66].

[24] [Note by AKTA:] In German: *Drehungsgeschwindigkeit*.

imperceptible and the needle appears completely still. — The *velocity* at which the conductor particles must be moved by rotation at a distance equal to that of the radius of the circle, so that this *mean value* is π^2 times greater than the vertical directional force exerted directly on the compass by the vertical component of the Earth's magnetism, is the *velocity equal to the resistance of the conductor wire.*

However, this *mean value* and this vertical directing force exerted directly by the Earth on the compass behave like the tangents of the deflections v and I they produce, where v is the horizontal deflection of the compass and I is the geomagnetic inclination observed during the rotation. So if one observes that at n revolutions in the unit of time, $\tan v / \tan I = \pi^2$, then the resistance of the circular conductor will be

$$w = 2n\pi r \ ,$$

if r denotes the radius of the circular conductor.

This velocity represented in this way, which is equal to the resistance, actually has the same physical relationships as the ratio of the electromotive force to the current intensity or the resistance of the conductor; because it can be proven that this velocity, as well as this resistance, is completely independent of both the strength and direction of the Earth's magnetic force, which has an inducing effect on the conductor, and also of the strength of the compass, on which the Earth's magnetism and the currents induced in the conductor act.

The following explanations serve to prove this relationship between the velocity and resistance just described.

If φ is the angle which the circular plane forms with the meridian plane, $d\varphi/dt$ is the angular velocity and r is the radius of the circle, then one obtains the electromotive force exerted on the circle by the vertical component of the Earth's magnetism T', according to the meaning given in Section 1.1,

$$e = \pi r^2 \cdot T' \cdot \cos\varphi \frac{d\varphi}{dt} \ .$$

If the angular velocity $d\varphi/dt = \rho$ is constant, that is, $e = \pi r^2 T' \rho \cos\varphi$, proportional with $\cos\varphi$, then, according to Ohm's laws, the intensity of the current i induced in the conductor is also proportional to $\cos\varphi$, and can be set

$$i = i_0 \cos\varphi \ ,$$

where i_0 has a constant value.

According to electromagnetic laws, this induced current exerts a rotational moment on the needle m in the center of the circular conductor,[25] which, when the circular plane is vertical, or $\varphi = 0$, and also when the deflection of the needle from the magnetic meridian $v = 0$, from the known theory of the tangent galvanometer would be represented by the quotient of the product of the length of the conductor $2\pi r$ into the current intensity i and into the needle magnetism m, divided by the square of the circle radius r^2, so $= 2\pi im/r$; but if φ and v are different from zero, then, as can be easily shown, this quotient must still be multiplied by $\cos\varphi \cos v$, whereby the rotational moment[26] exerted on the needle by the

[25] [Note by AKTA:] That is, on the needle with a magnetic moment m.

[26] [Note by AKTA:] In German: *Drehungsmoment*. This expression can also be translated as rotatory action, moment of force, or torque.

induced current is obtained[27]

$$= \frac{2\pi i m}{r} \cdot \cos\varphi \cos v = \frac{2\pi m}{r} i_0 \cos v \cdot \cos\varphi^2 \ .$$

The *mean value* of this rotational moment for the duration of half a revolution $= \pi/\rho$ results from this

$$\frac{2\pi m}{r} i_0 \cos v \cdot \frac{\rho}{\pi} \int_0^{\pi/\rho} \cos\varphi^2 dt = \frac{2\pi m}{r} i_0 \cos v \cdot \frac{1}{\pi} \int_0^{\pi} \cos\varphi^2 d\varphi = \frac{\pi m}{r} i_0 \cos v \ .$$

Now, further, the rotational moment exerted by the Earth on the needle, when T denotes the horizontal component of the Earth's magnetism, is,

$$= Tm \sin v \ ,$$

which, if v suffers no perceptible change, can be taken as constant. This rotational moment exerted by the Earth on the needle must then be equal to the *average value* of that exerted by the induced current, that is,

$$Tm \sin v = \frac{\pi m}{r} i_0 \cos v \ ,$$

consequently

$$i_0 = \frac{rT}{\pi} \tan v \ ,$$

$$i = \frac{rT}{\pi} \tan v \cdot \cos\varphi \ .$$

But now

$$e = \pi r^2 T' \rho \cdot \cos\varphi \ ,$$

consequently

$$\frac{e}{i} = \frac{\pi^2}{\tan v} \cdot \frac{T'}{T} \cdot r\rho \ ,$$

or, because $T'/T = \tan I$, if I denotes the geomagnetic inclination,

$$\frac{e}{i} = \frac{\tan I}{\tan v} \cdot \pi^2 r \rho \ .$$

Finally, if $2n\pi$ denotes the value of ρ for which

$$\frac{\tan v}{\tan I} = \pi^2 \ ,$$

is observed, then

[27][Note by AKTA:] The next equation should be understood as:

$$= \frac{2\pi i m}{r} \cdot \cos\varphi \cos v = \frac{2\pi m}{r} i_0 \cos v \cdot \cos^2\varphi \ .$$

The same meaning should be understood in similar equations.

$$\frac{e}{i} = 2n\pi r \ ,$$

that is, the *velocity* with which the conductor particle located at the distance r from the axis of rotation moves in its circular path represents the *resistance of the conductor* $w = e/i$.

If we denote the resistance of the *unit length* of the conductor by the name of its *specific resistance*, then the specific resistance of a circular conductor is equal to a certain *angular velocity* of this conductor, namely, since $2\pi r$ is the length of the conductor, the angular velocity n, for which

$$\frac{\tan v}{\tan I} = \frac{\pi}{2}$$

is observed.

1.3 Determination of the Resistance from the Ratio $\int edt/\int idt$ for an Induction Shock

From the possibility of actually representing those velocities that are equal to the resistances of conducting wires, the possibility of *measuring* these velocities, and thus also the resistances equal to them, is recognized. These measurements are called *the absolute resistance measurements*.

Although the possibility of absolute resistance measurements is apparent from this, it is by no means the most precise and refined method of actual execution, on which the practical significance of these measurements depends; rather, their determination requires further discussion, which must be preceded by the execution.

The conducting wire mentioned in the previous Section, which is brought into the shape of a circle and can be rotated around its horizontal diameter, together with the compass located in the center of the circle, essentially forms the same instrument that was already discussed under the name of the *Induction-Inclinatorium*[28] in the "Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1837" (Results from the observations of the magnetic association in 1837), pages 81-96.[29,30] Through this *Induction-Inclinatorium*, the resistance measurements can be reduced to velocity measurements. But an exact execution of these velocity measurements, as is self-evident, requires a completely uniform angular velocity, the representation of which, although not impossible, is associated with great practical difficulties. It is therefore of the greatest importance for the execution of an accurate absolute resistance measurement that it be made independent of the representation and measurement of such a completely uniform angular velocity.

The method of achieving this purpose is generally based on, instead of obtaining a certain average value of the electromotive force e and the current intensity i for a long time by

[28] [Note by AKTA:] In German: *Induktions-Inklinatorium*. The dip circle, dip needle, inclinometer or inclinatorium is an instrument used to measure the angle between the horizon and terrestrial magnetism (the dip angle). It consists essentially of a magnetic needle pivoted at the center of a vertical graduated circle. Weber's *Induktions-Inklinatorium* is a new instrument which he presented in 1837, [Web38]. It offered a novel way to circumvent the two main problems with dip circles: the effect of gravity, and the need to reverse the polatiry of the needle, [WSH03].

[29] [Note by HW:] Wilhelm Weber's *Werke*, Vol. II, pp. 75-88.

[30] [Note by AKTA:] [Web38].

continued uniform rotation, and measuring the same during this time, trying to represent precisely determined and measurable integral values $\int edt$ and $\int idt$, but limited to a very short time, under circumstances in which the quotient e/i remains constant for all time elements dt, although e and i vary. Exact measurement of the integral values $\int edt$ and $\int idt$ then results in the quotient $\int edt / \int idt = e/i$, equal to the desired resistance of the conductor wire w, whereby it does not matter, whether the short period of time over which those integrals extend, which do not need to be measured at all, is slightly larger or smaller, since the result is completely independent of this.

1.4 Execution with the Induction-Inclinatorium

The method given in the previous Section could now be easily carried out using the *Induction-Inclinatorium* in the following manner. Instead of setting the circle formed from the conductor wire into a continuous, uniform rotation, you just turn it a little, for example halfway around, most expediently starting from the horizontal position of the circle up to the horizontal position again, and in a very short time, which is referred to by the name of an induction shock.[31] The integral value $\int edt$ for such an induction shock is easy to determine; because according to Section 1.2, $e = \pi r^2 T' \cos\varphi \cdot d\varphi/dt$, therefore the integral value of edt taken from $\varphi = -\pi/2$ to $\varphi = +\pi/2$ is

$$\int edt = 2\pi r^2 T' \ ,$$

if r denotes the radius of the circle and T' the vertical component of the Earth's magnetism.

The integral value $\int idt$ can also be determined very simply by determining the angular velocity at which the compass is set by such an induction shock; because if this angular velocity is denoted by γ, the magnetism and the moment of inertia of the compass are denoted by m and k,[32] then we have

$$\int idt = \frac{2rk}{\pi^2 m} \cdot \gamma \ .$$[33]

Now, with a needle set in vibration, the greatest angular velocity γ (at the moment when it passes through the equilibrium position) is related to the greatest deflection from the equilibrium position, that is, to the elongation width α, as π to the period of oscillation of

[31][Note by AKTA:] In German: *eines Induktionsstosses*. This expression can also be translated as inductive shock, kick, blow or hit.

[32][Note by AKTA:] That is, if the magnetic moment and the moment of inertia of the needle are represented by m and k, respectively.

[33][Note by WW:] According to Section 1.2, the horizontal torque exerted on the needle by the induced current $i = i_0 \cos\varphi$ was $= [2\pi m/r] \cdot i_0 \cos v \cos\varphi^2$, hence, if at the moment of the induction shock the needle is at rest and $v = 0$, [this expression becomes] $= [2\pi m/r] \cdot i_0 \cos\varphi^2$. This rotational moment divided by the moment of inertia k gives the angular acceleration of the needle $d\gamma/dt = [2\pi m/rk] \cdot i_0 \cos\varphi^2$. From this we get, if the angular velocity of the circle $d\varphi/dt$ is denoted by ρ, $d\gamma = [2\pi m/rk] \cdot [i_0/\rho] \cdot \cos\varphi^2 d\varphi$, and the integral value thereof, between $\varphi = -\pi/2$ and $\varphi = +\pi/2$, $\gamma = [\pi^2 m/rk] \cdot [i_0/\rho]$, therefore $i = i_0 \cos\varphi = [rk/\pi^2 m] \cdot \rho\gamma \cos\varphi$, from which $idt = [rk/\pi^2 m] \cdot \gamma \cos\varphi d\varphi$, and the integral value thereof, between the limits $\varphi = -\pi/2$ and $\varphi = +\varphi/2$, $\int idt = [2rk/\pi^2 m] \cdot \gamma$ is obtained. The angular velocity ρ of the circle has been assumed to be constant; but one can easily see that the result would remain unchanged even if ρ were changing; because i_0 would then also be variable, but the ratio i_0/ρ would remain constant.

the needle t,[34] or it is $\gamma = [\pi/t] \cdot \alpha$, so

$$\int idt = \frac{2rk}{\pi mt} \cdot \alpha \ .$$

From this we get, since $\int edt = 2\pi r^2 T'$, the desired resistance of the conductor wire

$$w = \frac{\int edt}{\int idt} = \frac{\pi^2 mrtT'}{k\alpha} \ .$$

If T denotes the horizontal component of the Earth's magnetism and I the inclination, then it is known that $T'/T = \tan I$ and $mT/k = \pi^2/t^2$; consequently

$$w = \frac{\pi^4 r}{\alpha \cdot t} \tan I \ .$$

If the wire formed, instead of a simple circle, a ring composed of n windings of equal size, isolated from each other, one would find:

$$w = \frac{n^2 \pi^4 r}{\alpha \cdot t} \cdot \tan I \ .$$

1.5 Separation of the Inductor from the Galvanometer

As simple as the method of absolute resistance measurement with the *Induction-Inclinatorium* described in the previous Section appears, it does not prove itself in practice. For, *firstly*, the angular velocity imparted by a single half revolution of the circle (induction shock) of the needle and the elongation width thereby produced are far too small to be observed and measured with an ordinary compass; even the finest magnetometric observations would not be sufficient for this purpose if the compass could be replaced by a magnetometer equipped with a mirror and scale, the placement of which would, incidentally, be associated with great practical difficulties in the center of the rotating circle. *Secondly*, however, there is also the fact that with this method the horizontality of the needle axis would have to be completely guaranteed; otherwise, as one can easily see, when the circle rotates about its horizontal diameter, the induction of the vertical component of the Earth's magnetism would be mixed with the induction of the vertical component of the needle magnetism.

These reasons therefore make it seem far more expedient to form two circles from the conductor wire instead of one circle, one of which is used as an inductor and is rotated, the other serves as a multiplier and is fixed. This separation gives you a free hand for the most expedient arrangement of the inductor as well as the multiplier required for the galvanometer, where each can then be constructed much more perfectly on its own, without having to take the other into account. On this separation of the inductor from the multiplier is based the method developed in the First Volume of the Treatises of the Königl. Saxon. Society of Sciences,[35,36] about which the following comment will suffice here.

[34] [Note by AKTA:] In German: *Schwingungsdauer*. Gauss and Weber utilized the old French definition of the period of oscillation t which is half of the English definition of the period of oscillation T, that is, $t = T/2$, [Gil71, pp. 154 and 180]. For instance, the period of oscillation for small oscillations of a simple pendulum of length ℓ is $T = 2\pi\sqrt{\ell/g}$, where g is the local free fall acceleration due to the gravity of the Earth, while $t = T/2 = \pi\sqrt{\ell/g}$.

[35] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 301.

[36] [Note by AKTA:] [Web52] with English translation in [Web21a].

The calculation of the resistance of the conductor wire from the observations is changed only slightly by the separation of the inductor from the multiplier, namely, only as a result of the fixed position in which the separated multiplier, which no longer takes part in the rotation of the inductor, remains, according to which, *firstly*, the horizontal rotational moment $= [2\pi m/r] \cdot i_0 \cos\varphi$ exerted on the needle by the induced current $i = i_0 \cos\varphi$ is found (instead of that value $= [2\pi m/r] \cdot i_0 \cos\varphi^2$ stated in the Note to Section 1.4), from which follows $\int idt = [rk/2\pi m] \cdot \gamma$; and *secondly*, the elongation width α can no longer be determined from the angular velocity γ according to the law $\gamma = [\pi/t] \cdot \alpha$ listed in Section 1.4, because this law is only valid for one freely oscillating needle that does not suffer any attenuation, which was the case in Section 1.4, because the multiplier connected to the inductor was always in a horizontal position before and after the induction shock. If, on the other hand, the multiplier, which is separated from the inductor, remains in its vertical position parallel to the meridian plane during the entire needle oscillation, the oscillating needle suffers damping and the elongation width α is then to be determined from the angular velocity γ according to the laws developed by Gauss in the "Resultaten aus den Beobachtungen des magnetischen Vereins in Jahre 1837" (Results from the observations of the magnetic association in 1837).[37,38] If γ is determined according to these laws from the observed elongation width and from the simultaneously observed decrease in the oscillation arcs of the needle, the following equation results for the calculation of w, namely, either for simple circles of the same radius r, both as inductor as well as multiplier:

$$w = \frac{4\pi^4 r}{\gamma \cdot t^2} \cdot \tan I \ ,$$

or for a ring composed of n turns of radius r as an inductor and for a ring composed of n' turns of radius r' as a multiplier:

$$w = \frac{4nn'\pi^4}{\gamma \cdot t^2} \cdot \frac{r^2}{r'} \cdot \tan I \ .$$

1.6 Damping as a Measure of the Sensitivity of the Galvanometer

The freedom to make the radius of the multiplier windings r' smaller than that of the inductor windings r, and to increase the number of multiplier windings n', which is obtained by the separation of the multiplier from the inductor discussed in the previous Section, then acquires greater importance in that, *firstly*, in the fixed position of the multiplier, the substitution of the compass with a magnetometer is no longer obstructed, *secondly*, that, in addition, the angular velocity γ imparted to the needle by an induction shock can be given a size appropriate for finer observation. For from the equation at the end of the previous Section it can be seen that if r' gets half the value and n' gets twice the value, otherwise under exactly the same conditions, with unchanged wires, the angular velocity γ obtained by an induction shock assumes a value four times greater. Only in this way is it possible to give the elongation range α of the needle, which depends on γ, the size necessary for accurate measurement, when induction is as weak as the Earth magnetism offers.

[37][Note by HW:] Gauss' *Werke*, Vol. V, p. 389.

[38][Note by AKTA:] [Gau38a, p. 389 of Gauss' *Werke*].

But it is clear that if the multiplier encloses the needle tightly, instead of forming a wide circle around it as in a tangent galvanometer, the law valid for the tangent galvanometer, according to which the angular velocity γ given by an induction shock to the needle was determined, namely, the equation $\int idt = [rk/2\pi m] \cdot \gamma$ listed in Section 1.5, according to which $\gamma = [2\pi m/rk] \cdot \int idt$ (or, for a plurality of turns n' of radius r', $\gamma = [2n'\pi m/r'k] \cdot \int idt$) no longer applies because then the difference in the position of the various turns from which the multiplier is composed and the way in which the magnetism is distributed in the needle gain influence and must be taken into account more precisely; but even then γ remains proportional to $\int idt$ and the constant ratio $\gamma / \int idt$, which can be called the sensitivity coefficient of the galvanometer and can be denoted by f, can easily be determined for any given galvanometer by observation, by simultaneously measuring γ and $\int idt$. However, it should be noted that the constancy of the coefficient $\gamma / \int idt = f$ is necessarily linked to the immutability of the instrument, an immutability that cannot be attributed to such sensitive galvanoscopes with tightly surrounding multipliers in the long term, which is why, as already noted in the Introduction, the sensitivity of such instruments cannot be determined in advance. So the coefficient f, or the sensitivity of the instrument, must be determined for the moment of observation itself.

Such a determination is made by the observations combined according to the *throwback method*[39] (which was discussed in more detail in the First Volume of the Treatises of the Royal Saxon Society of Sciences, page 349),[40,41] which determine the *angular velocity* imparted by an induction shock to the needle and at the same time affect their *damping*; because this damping is proportional to the square of the coefficient f. If from such observations the damping resulting from the closure of the circuit is determined by the value of the logarithmic decrement (according to the natural system), then if w is the resistance of the circuit, k is the moment of inertia of the needle and τ denotes the period of oscillation under the influence of damping,

$$f^2 = \frac{2w}{k\tau} \cdot \lambda \ .$$ [42]

[39][Note by AKTA:] In German: *Zurückwerfungsmethode*. See [Gau38a], [Web39] and [WK68, p. 108, Note 13].

[40][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 441.

[41][Note by AKTA:] [Web52, p. 441 of Weber's *Werke*] with English translation in [Web21a].

[42][Note by WW:] According to the *electromagnetic* law, as stated above, the angular velocity of the needle was given by the currents induced by an induction shock

$$\gamma = f \cdot \int idt \ ,$$

so, during the induction shock, $d\gamma = fidt$. It is therefore the angular acceleration imparted by the current i in the multiplier of the needle $d\gamma/dt = fi$, and consequently, if k denotes the moment of inertia of the needle, that angular velocity exerted by the current i in the multiplier on the needle is $= kfi$.

However, if $i = 1$ is set in it, the expression of this torque gives, according to *magnetoelectric* law, the factor which, multiplied by the angular velocity of the needle γ, is equal to the *electromotive force* exerted by the moving needle on the multiplier, $= kf\gamma$, from which, according to Ohm's law, the current generated by the moving needle in the multiplier follows $i = kf\gamma/w$. If one now puts this value of i into the equation $d\gamma = fidt$, one finds that the damping caused by closing the circuit retards the angular velocity of the needle and that this retardation is $d\gamma/dt = [kf^2/w] \cdot \gamma$. Now the differential equation of the oscillating needle (see results from the observations of the magnetic association, 1837, page 74) [[Note by HW:] Gauss' *Werke*, vol. V, page 389][[Note by AKTA:] [Gau38a, p. 389 of Gauss' *Werke*].] is $d^2x/dt^2 + 2\varepsilon dx/dt + n^2x = 0$, where the angular velocity $\gamma = dx/dt$ and the rotation retardation resulting from the damping is set $[kf^2/w] \cdot \gamma = 2\varepsilon dx/dt$, hence $kf^2/w = 2\varepsilon$. — From this differential equation it follows $x = p + Ae^{-\varepsilon t} \sin\left(t\sqrt{n^2 - \varepsilon^2} - B\right)$, according

But now $f = \gamma / \int idt$, and according to Sections 1.3 and 1.4, $w = \int edt / \int idt$ and $\int edt = 2n\pi r^2 T'$; consequently, if f, $\int edt$ and $\int idt$ are eliminated from these four equations, we get,

$$w = \frac{8(n\pi r^2 T')^2}{k\gamma^2 \tau} \cdot \lambda \ .$$

With regard to the execution of the observations, it should be noted that, *firstly*, with strong attenuation, it may occur that the period of oscillation τ with a closed circuit cannot be directly determined precisely, and that it is therefore necessary to observe the period of oscillation t with an open circuit for this purpose; *secondly*, even with an open circuit a still perceptible attenuation occurs very often, which is determined by observing the logarithmic decrement λ_0. If the circuit is closed, λ is added to λ_0, and the observed logarithmic decrement is $\lambda_0 + \lambda = \lambda_1$. Under such conditions, $\lambda = \lambda_1 - \lambda_0$ and $\tau = t_0\sqrt{(\pi^2 + \lambda_1^2)/(\pi^2 + \lambda_0^2)}$ must be substituted in the above equation to represent the resistance w in its dependence on the observed values t_0, λ_0 and λ_1, namely:

$$w = \frac{8(n\pi r^2 T')^2}{k\gamma^2 t_0} \cdot (\lambda_1 - \lambda_0)\sqrt{\frac{\pi^2 + \lambda_0^2}{\pi^2 + \lambda_1^2}} \ .$$

1.7 Induction by the Horizontal Component of Terrestrial Magnetism

In a similar manner as the separation of the multiplier from the inductor specified in Section 1.5 can be used to obtain a galvanometer with a closely surrounding *multiplier* of the highest sensitivity for the measurement, in the same way this separation can also serve to give the *inductor* a more suitable and advantageous arrangement.

The radius of the inductor windings no longer needs to be limited by the galvanometer, but can be increased as much as is compatible with a rapid and slight rotation of the inductor, whereby the induction shocks are significantly increased. Because the strength of the induction shock has been found $\int edt = 2n\pi r^2 T'$ and one can easily see that this value is increased m times, even if the wire length remains unchanged, if the radius r of the inductor windings is taken to be m times larger and consequently the number n of inductor windings m times smaller.

In addition, as a result of the inductor being separated from the multiplier, the reason why the inductor rotation had to occur by the *horizontal* diameter of the inductor when the inductor and multiplier were combined no longer applies, namely, the reason that when the inductor rotates there is only an induction due to the Earth's magnetism, and not at the same time through needle magnetism, because the latter would be difficult to determine or eliminate. As a result of the separation of the inductor from the multiplier, the rotation can also occur around the *vertical* diameter of the inductor, whereby the induction is made dependent on the *horizontal* component of the Earth's magnetism T, instead of on the *vertical* component T'. By substituting T with T', the equation at the end of the previous Section turns into the following:

to which the oscillation lasts $\tau = \pi/\sqrt{n^2 - \varepsilon^2}$ and the decrementum logarithimicum naturale is $\lambda = \varepsilon\tau$. According to this, $kf^2/w = 2\varepsilon = 2\lambda/\tau$, or $f^2 = [2w/k\tau] \cdot \lambda$, which had to be proven.

$$w = \frac{8(n\pi r^2 T)^2}{k\gamma^2 t_0} \cdot (\lambda_1 - \lambda_0) \cdot \sqrt{\frac{\pi^2 + \lambda_0^2}{\pi^2 + \lambda_1^2}} \, .$$

This exchange offers the advantage that the measurement of the horizontal component of the Earth's magnetism T is sufficient, while in the other case the measurement of the *inclination* I was also necessary in order to be able to determine the *vertical* component $T' = T \tan I$.

II - Construction of the Galvanometer

1.8

From the overview of the method of absolute resistance measurement given in the previous Section, it is clear that the construction of the galvanometer is important for carrying out such a measurement. What is important is not just a high degree of sensitivity, but also that this degree of sensitivity can be precisely determined from the damping observations of the needle vibrations.

The theory of the galvanometer has often been discussed from different sides, according to the variety of purposes for which it was intended to serve. Closely related to the purpose of absolute resistance measurement, with which we are concerned here, is the use of the galvanometer, discussed in the Fifth Volume of these Treatises[43,44] for the measurement of *magnetic inclination* carried out with the help of *induction*, which also in the place mentioned has one application connected to the resistance measurement itself. The rules given there for the construction of the galvanometer also apply here, for example that the resistance of the multiplier should be almost the same as that of the rest of the circuit, to which the inductor belongs. However, in the case of the galvanometer there, it was mainly only a question of the sensitivity or size of the deflection of the galvanometer needle set in vibration by an induction shock, whereas here it is also about the size of the attenuation, which is to be used to precisely determine that sensitivity. This use of damping has already been discussed there, on the occasion of the application to resistance measurement; however, it still requires further discussion in order to determine what is the highest degree of accuracy in this *determination of sensitivity* and how it can be achieved.

1.9 Limits on the Size of the Deflection and Attenuation

Even if the *deflection* and *attenuation* could be increased at will, partly by tightly surrounding the galvanometer needle with the multiplier, partly by increasing the needle magnetism, certain limits should not be exceeded if the accuracy of the resistance measurement is not to be reduced instead of increased.

For as far as the *deflection* is concerned, which must not go beyond the scale with which it is to be measured, its enlargement in all *magnetometric* observations is governed by the rule that it should always remain limited to small deflection angles, according to which

[43] [Note by HW:] Wilhelm Weber's *Werke*, Vol. II, p. 277.
[44] [Note by AKTA:] [Web53c].

the length of the scale length is determined. Otherwise the most important advantage of these observations would be lost, which is that small deflections are always needed and sufficient even for the finest measurements, which avoids many disturbing influences and greatly simplifies the calculation of the observations.

Likewise, with regard to the *attenuation*, which can be determined from the *difference* in the size ratio of two successive oscillation arcs from unity, it is clear that this *difference* must have a size sufficient for precise determination, but must not be so large that, on the other hand, the unity itself disappears because otherwise either the first oscillation arc would be too large to be measured with the scale, or the second would be too small for an accurate measurement. Between these two limits there must be a case where the accuracy that can be achieved in determining the damping is a maximum.

If the larger oscillation arc, which should be almost equal to the limit value set by the length of the scale, is denoted by a and the smaller one by x, then the size of the damping is found proportional to $\log[a/x] = \lambda$, and the accuracy that can be achieved in determining the damping is represented by the quotient of the smallest measurable change in x, divided by the corresponding change in λ expressed in parts of λ. The value of x for which the absolute value of this quotient is a maximum is given by the equation

$$\left(\frac{\lambda dx}{d\lambda}\right)^2 = x^2 \left(\log \frac{a}{x}\right)^2 = \text{maximum},$$

from which follows $a : x = e : 1$ if $e = 2.71828$ denotes the base of natural logarithms. This results in the rule that for the determination of the damping it is most advantageous to construct the galvanometer in such a way that the ratio of two consecutive arcs of the vibrating needle is equal to or at least comes close to the ratio $e : 1$.

1.10 Unifilar and Bifilar Suspension of the Galvanometer Needle

The suspension of the galvanometer needle can be either *unifilar* or *bifilar* and only a closer consideration of the observations to be made can give preference to the choice of one or the other suspension.

If the galvanometer needle is set into vibration by an induction shock, that is, if it is given a certain angular velocity γ at the moment when it is in the rest position, it is well known that it is not enough to observe the deflection or the *first* elongation of the needle a, but rather it is necessary, especially to determine the attenuation, to also observe the *second* elongation of the needle b, to the opposite side from the rest position. However, for the purpose of accurate measurement, these two observations must be repeated more frequently. It is now clear that, instead of waiting until the needle has reached complete rest between every two repetitions, there is great advantage in carrying out a system of such repetitions without interruption in a continuous succession, which is useful if one considers that although the needle at the moment of each induction shock should be in the position where it could remain in equilibrium if it had no movement, it is not necessary for the purpose of these observations that it really be in equilibrium. Rather, the needle can have an angular velocity at this moment, if the latter is always the same in all repetitions at the moment of each induction shock. The method of arranging such observation system was given by Gauss and can be found in the First Volume of the Treatises of the mathematical-physical class of the Königl. Saxon. Society of Sciences, p. 349, discussed in more detail under the name of the *throwback method*.[45,46] It follows from this that the precise implementation of such an observation system requires, *firstly*, that the duration of an induction shock forms a very small fraction of the oscillation period of the needle, *secondly*, that the moment of each induction shock coincides as precisely as possible with the moment when the needle is in the position in which it could remain in equilibrium if its angular velocity were zero. However, it is clear that the fulfillment of these two requirements can only be achieved with a longer oscillation period of the needle, for example 20 to 30 seconds, which means that the construction of the galvanometer must be adjusted accordingly.

If such a longer period of oscillation is to be produced by *unifilar* suspension of the needle, and if the needle is to have the strongest possible magnetism in relation to its size for the purpose of damping, then the necessity of a larger needle is obvious, for example from 600 to 900 millimeters in length, which also requires a corresponding expansion of the multiplier in the direction of the needle. With such an extension of the multiplier, a sufficiently strong *damping* can be achieved through the associated increase in the magnetism of the needle; the size of the *deflection* caused by an induction shock, on the other hand, decreases so quickly as the needle and multiplier are lengthened that it can happen that they are no longer sufficient for precise measurements. With larger needles, under normal circumstances one can calculate that the size of the deflection is inversely proportional to the length of the needle, for example with a 600 to 900 millimeter long needle, which would require an oscillation period of 20 to 30 seconds, the deflection would be 4 to 6 times smaller than with a 150 millimeter long needle.[47] If it were found that the deflection, under otherwise favorable conditions, remained of a size sufficient for finer measurements, there would be no essential reason for its use to discard the *unifilar* suspension. However, if it should turn out that the reduced deflection was no longer sufficient, one would be forced to use *bifilar* suspension.

This *bifilar* suspension can then be set up in such a way that the resulting static directive

[45] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 441.

[46] In German: *der Zurückwerfungsmethode*. See the footnote 39 and [Web52, p. 441 of Weber's *Werke*] with English translation in [Web21a]. See also [Gau38a], [Web39] and [WK68, p. 108, Note 13].

[47] [Note by WW:] For the present consideration it is sufficient to consider only a single turn of the multiplier in the vertical plane of the needle, and only two points of the needle, which can be called the North pole and the South pole, the distance from each other being $= l$. Let the multiplier turn form a semicircle of radius r around each of these points, both connected by two parallel pieces of length l. The angular velocity which is imparted to the needle by an impulse is then composed of several parts, namely, that which comes from the two semicircles acting on the needle poles located at their centers, which is obtained $= [\pi m/kr] \cdot \int idt$ when m denotes the magnetic moment and k denotes the moment of inertia of the needle; further from that which comes from the two parallel connecting pieces, $= [2lm/rk\sqrt{l^2+r^2}] \cdot \int idt$, and finally from that which comes from each semicircle by acting on the needle pole located at the center of the other semicircle, which, however, can be considered vanishing if r is very small compared to l. According to this, if r is very small compared to l, $\gamma = (\pi + 2)(m/kr) \int idt$ can be set. Now, for two homogeneous needles of similar shape, the largest magnetic moments they can assume, $m : m'$, behave like the cube [of their pole distances $l : l'$], their moments of inertia $k : k'$ behave like the fifth power of their pole distances $l : l'$, or it is $m : m' = l^3 : l'^3$ and $k : k' = l^5 : l'^5$, from which follows the ratio of their angular velocities

$$\gamma : \gamma' = l'^2 : l^2 \ .$$

And since their deflections $\alpha : \alpha'$ are in the composite ratio of this angular velocity and the period of oscillation proportional to the pole distance, we get from this

$$\alpha : \alpha' = l' : l \ .$$

force S is greater than the magnetic directive force D,[48] and that (if the needle poles are inverted) the oscillation period of the needle depends only on the difference $S - D$, which makes it possible to regulate and extend it as desired. A longer period of oscillation produced in this way, combined with a relatively strong magnetism of the needle, not only makes it possible to produce a galvanometer with very high sensitivity, but also to increase the damping so as to be able to determine it as accurately as the deflection of the needle. Finally, even in cases where the main purpose just described could be achieved by *unifilar suspension of larger magnets*, this *bifilar* suspension still offers the special advantage of making it possible to construct the galvanometer *on a smaller scale without compromising the accuracy of the measurements*, which is often of great importance for practical application.

1.11 Astatic Needle System with Unifilar Suspension

However, the same purpose for which, according to the previous Section, the *bifilar* suspension was particularly suitable and, especially when smaller needles were involved, seemed to deserve preference over the *unifilar* suspension, can also be achieved by the *unifilar* suspension if the simple needle is substituted with an *astatic needle system*, that is, with a system of two identical needles that are firmly connected to each other, one of which is enclosed by the multiplier, the other with the opposite position of the poles outside, either above or below the multiplier.[49] The magnetic directing forces of the two connected needles then cancel each other out, and the static directing force can be regulated by choosing a suitable wire for suspension so that the most appropriate period of oscillation is obtained. In addition to all the advantages that could be achieved by *bifilar* suspension of a simple needle, this device offers a special advantage in that the influence of many otherwise unavoidable external disturbances is completely avoided. Such external disturbances are particularly due to the *declination variations of the Earth's magnetism*. Although these variations are generally small, even during the short duration of the observations, it should not be overlooked that they occur with a needle which is directed by the mere *difference in the static and magnetic directing force*, which was the case with a sensitive bifilar suspension, they are magnified as many times as that difference is contained in the whole magnetic directing force. As a result, the equilibrium position of the needle can often be subject to a rapid and considerably large change, which disrupts the precise execution of the induction shocks and greatly reduces the certainty and agreement that otherwise characterize these observations. These disturbances are completely avoided in the described astatic needle system, if one pays attention to the exact equality of the needles and the parallelism of their axes; because it is obvious that the variations in the Earth's magnetism have no influence at all on the behavior of such a needle system. Thus, if only the *static* directing force is constant, the equilibrium position of such a system is completely unchangeable and allows the most precise execution of the observations.

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[48][Note by AKTA:] In German: *statische Direktionskraft* and *magnetische Direktionskraft*. The concept of "directive force", also called directing force or directional force, was introduced by Gauss in 1838, [Gau38b, p. 4] with English translation in [Gau41b, p. 254]. Consider, for instance, a compass needle of magnetic moment m. Utilizing Gauss and Weber's terminology, let T be the horizontal component of the Earth's magnetic force. The torque τ exerted by the Earth on the needle when it is deflected by an angle θ relative to the local magnetic meridian is given by $\tau = mT \sin\theta$. The so-called magnetic directive force is here given by mT.

[49][Note by AKTA:] See footnote 18.

1.12 Theory of the Multiplier

After the preceding discussions about the construction of the galvanometer in general and in particular about the construction of the galvanometer needle and its suspension, we move on to the main task, namely, the theory of the multiplier, which for the present purpose requires a more complete development than has hitherto been given of it.

The main findings of previous discussions can be summarized briefly as follows. If, *firstly*, it is the multiplier of a tangent galvanometer, which is supposed to form a circular ring of a very large radius, against which the dimensions of the needle as well as those of its own cross-section are very small, then it is clear that the shape of this cross-section is of very little influence. If, however, this shape is not to be left to arbitrary determination, firstly, for practical reasons, the expediency of giving the cross-section the shape of a rectangle is self-evident, and secondly, for the ratio of the two sides of this rectangle the simple rule follows: if the area of the rectangle is given and is expressed in parts of the square of the ring radius $= 2\varepsilon^3$, the height of the rectangle to the base behaves like $\varepsilon : 2$.

If, *secondly*, it is a question of the multiplier of a galvanoscope, which is intended to enclose the needle as tightly as the free movement of the needle permits, then only the shape of the winding which initially encloses the needle is to be regarded as given. As a rule, this winding will form a figure enclosed by two parallel lines and two semicircles. For practical reasons, it may then be assumed that a number of coils of the same shape and size lie side by side on the surface of a column having that figure as its base and form the lowest layer of the multiplier coils over which all the other coils are wound in the form of parallel layers. The whole multiplier thus takes the shape of a ring, for which the shape of its cross-section has to be determined even more closely.

This determination is obtained by considering the moment of rotation exerted on the needle by any winding. When considering the moment of rotation in this way, it is sufficient (as in the Note to Section 1.10) to consider only two points on the needle, which can be described as the North pole and the South pole, and whose distance, $= l^0$, can be set equal to the length of the two parallel sides of the winding. The moment of rotation which the current i passing through the winding exerts on the needle is then composed of several parts, namely, in the first place, that which is exerted by the two semicircles on the needle poles lying in their axes, if the axis of a semicircle is understood to be the perpendicular erected in its center point on its plane; in the second place, from that which comes from the two parallel sides of the winding; in the third place, from that which is exerted by each semicircle on the needle pole located in the axis of the other semicircle. If r denotes the radius of the semicircles and x the length of the perpendicular falling from a needle pole onto the plane of the semicircle, so the *first* part is found $= \pi r^2 mi/(r^2+x^2)^{3/2}$, the *second* $= 2rl^0 mi/[(r^2+x^2)\sqrt{l^{02}+r^2+x^2}]$, the *third* $= 2rmi\int(l^0\cos\varphi + r)d\varphi/(l^{02}+r^2+x^2+2rl^0\cos\varphi)^{3/2}$, where the integral value is to be taken between the limits $\varphi = 0$ and $\varphi = \pi/2$. If we then prefer to consider the two main cases, namely, in the first place, where $l^0 = 0$ or the multiplier turns form circles, and in the second place, where l^0 is so large that x and r can be regarded as vanishing in comparison, then in the first case the second part is $= 0$ and the third is equal to the first, $= \pi r^2 mi/(r^2+x^2)^{3/2}$, in the second case the third part is $= 2rmi/l^{02}$. If we now denote the quotient of the total moment of rotation of a turn divided by its length by the name of the specific moment of rotation, then all turns for which the specific moment of rotation is equal, in the case where $l^0 = 0$, that is, when the multiplier turns are circular, are given by the following equation:

$$\frac{1}{2\pi r} \cdot \frac{2\pi r^2 mi}{(r^2+x^2)^{3/2}} = \text{constant;}$$

in the case when l^0 is very large compared to r and x, that is, when the multiplier turns are very elongated, by the following equation:

$$\frac{1}{2(l^0+\pi r)}\left(\frac{\pi r^2 mi}{(r^2+x^2)^{3/2}} + \frac{2rl^0 mi}{(r^2+x^2)\sqrt{l^{02}+r^2+x^2}} + \frac{2rmi}{l^{02}}\right) = \text{constant,}$$

what can also be set

$$\frac{mi}{2l^0}\left(\frac{\pi r^2}{(r^2+x^2)^{3/2}} + \frac{2r}{r^2+x^2}\right) = \text{constant.}$$

In the former case, if one sets the constant $= mi/d^2$, or, by taking d as the unit of length, $= mi$, one obtains

$$\frac{r}{(r^2+x^2)^{3/2}} = 1\ ;$$

in the latter case, if one sets the constant $= mi/2l^0 d$ or, by taking d as the unit of length, $= mi/2l^0$, one obtains

$$\frac{\pi r^2}{(r^2+x^2)^{3/2}} + \frac{2r}{r^2+x^2} = 1\ .$$

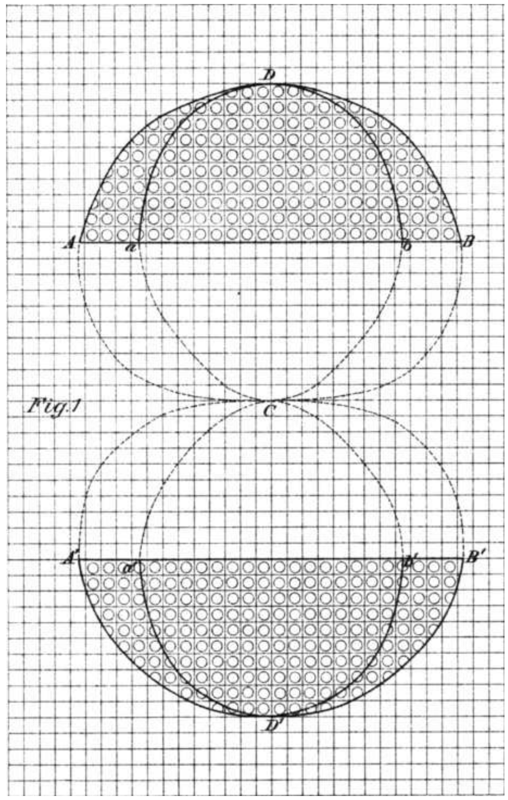
If you finally set $r^2 + x^2 = \rho^2$ in both equations, this results

for the first case, $r = \rho^3$,

for the second case, $\pi r = \rho(\sqrt{1+\pi\rho} - 1)$.

But the sensitivity of the galvanometer increases in proportion to the torque exerted by the multiplier on the needle, and from the latter it is clear that if its value is to be a maximum, the specific torque of all the turns on the outer surface must be the same. It follows from this that the outer limit of the cross section of the multiplier must be determined according to the above equations if the galvanometer is to have the greatest sensitivity. The inner limit of the cross section of the multiplier is given by the space that must be left free for the needle.

Figure 1 of Plate I represents the cross-section of the multiplier in both cases. The given inner boundary is indicated by the lines AB and $A'B'$; ADB and $A'D'B'$ are the outer boundaries in the former case, aDb and $a'D'b'$ in the latter case.



If, however, as with the multiplier of a tangent galvanometer, a *rectangular cross-section* is preferred, for which only the ratio of the two sides of the rectangle is to be determined, the following equation results for the former case, where the multiplier turns are circular, if the radius of the cylindrical space to be left free for the needle is set equal to 1 and the side of the rectangle parallel to the radius is denoted by a and the side perpendicular to it by $2b$:

$$\frac{2}{(1+a)^2-1}\int_1^{1+a}\frac{r^2dr}{(r^2+b^2)^{3/2}}=\frac{1}{b}\int_0^b\frac{(1+a)dx}{([1+a]^2+x^2)^{3/2}}\ ,$$

or when the integration is executed,

$$\log\frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}}=\frac{3(1+a)^2-1}{2(1+a)\sqrt{(1+a)^2+b^2}}-\frac{1}{\sqrt{1+b^2}}\ .$$

According to this,

for small values of a, $b=\sqrt{a}$,

for $a=1$, $b=1.1444$,

for $a=\infty$, $b=0.4413\cdot a$.

For the latter case, the following equation results in the same way:

$$\frac{1}{a}\int_1^{1+a}\frac{\pi r^2dr}{(r^2+b^2)^{3/2}}+\frac{1}{a}\int_1^{1+a}\frac{2rdr}{r^2+b^2}=\frac{1}{b}\int_0^b\frac{\pi(1+a)^2dx}{([1+a]^2+x^2)^{3/2}}+\frac{1}{b}\int_0^b\frac{2(1+a)dx}{(1+a)^2+x^2}\ ,$$

from which is obtained

$$\log\frac{(1+a)^2+b^2}{1+b^2}+\pi\log\frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}}$$
$$=\frac{\pi(1+2a)}{\sqrt{(1+a)^2+b^2}}-\frac{\pi}{\sqrt{1+b^2}}+\frac{2a}{b}\arctan\frac{b}{1+a}\;.$$

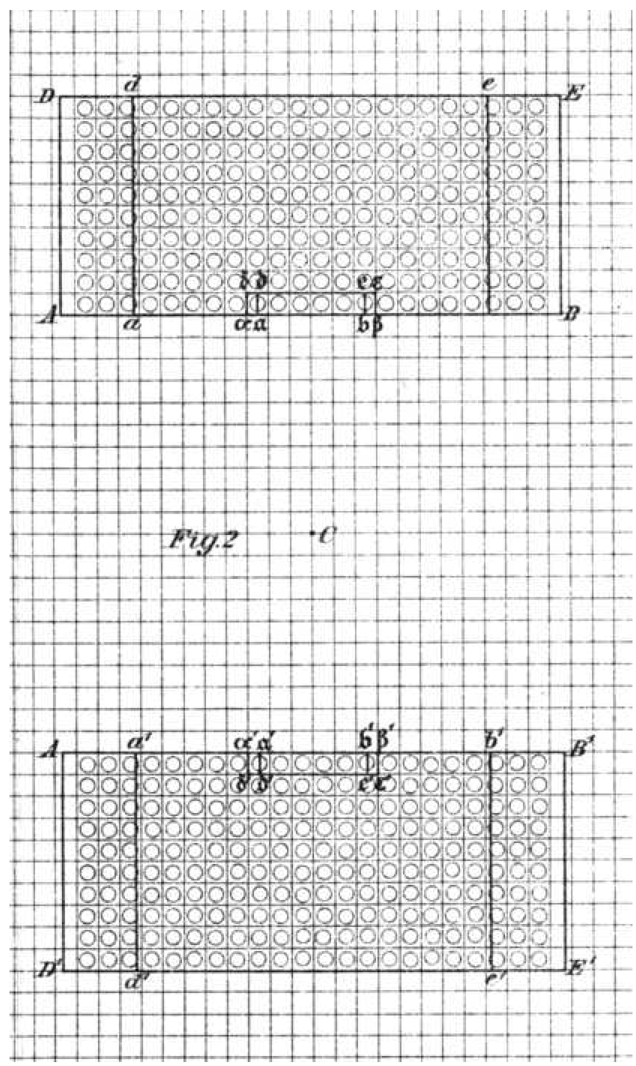
According to this,

for small values of a, $b^2=\frac{3}{2}\cdot\frac{2+\pi}{4+3\pi}\cdot a=0.5745\cdot a$,

for $a=1$, $b=0.8322$,

for $a=\infty$, $b=0.3435\cdot a$.

Figure 2 of Table I represents these cross sections when $a=1$.



The given inner boundary against the space left free for the needle is indicated by the lines AB and $A'B'$; $ADEB$ and $A'D'E'B'$ represent the two rectangular cross sections of a circular [multiplier], $adeb$ and $a'd'e'b'$ those of an elongated multiplier. The former cross section $ADEB$ is more than twice as large as the square of the distance c of the needle axis from the multiplier, the latter $adeb$ is close to $5/3$ of the same square. If a smaller multiplier is to be formed with the same size of the space left for the needle, then $\alpha\delta\varepsilon\zeta$ and $\alpha'\delta'\varepsilon'\zeta'$

represent the cross sections of the circular [multiplier], $\mathfrak{abcd}$ and $\mathfrak{a}'\mathfrak{b}'\mathfrak{c}'\mathfrak{d}'$ those of an elongated multiplier, of which one of the former is close to 1/16, one of the latter is close to 1/21 of the square of c.

Finally, when designing the cross section of the multiplier, particular attention must be paid to ensuring that the *proportionality of the observed deflections with the current intensities* is maintained within the widest possible limits, which is very important for measurement purposes. In this respect it is sufficient to note that this proportionality is the more perfect the larger, with the same cross-section, the side $2b$ is in relation to a; but, given the small extent of deflections in magnetometric observation, such an increase in the side $2b$ compared to a need not occur at the expense of sensitivity, which would be the case if b exceeded the value determined above. On the other hand, it must be completely avoided, which often happens, that the multiplier breaks into two parts separated by a space in order make room for the needle to be suspended. Between the needle enclosed by the multiplier and the suspension thread or the needle to be connected astatically, it is always easy to establish a hook-shaped, sufficiently strong and free connection that is guided around the cross section of the multiplier, which allows sufficient scope for the needle movement without bumping into it, so that it no need to break through the multiplier to suspend the needle.

1.13

In many cases, for the galvanometer to be constructed, it is not only the scope that must be left free for the needle, which is, in the case of a circular shape, the radius c of the cylinder space enclosed by the multiplier, but also the wire itself to be used for the multiplier with its given volume v. In all such cases, the provisions developed in the previous Section shall suffice; because from the given radius c and volume v both a and b can then be calculated, namely, for circular multipliers according to the two equations

$$\log \frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}} = \frac{3(1+a)^2-1}{2(1+a)\sqrt{(1+a)^2+b^2}} - \frac{1}{\sqrt{1+b^2}} \ ,$$

$$v = 2\pi(2+a)abc^3 \ .$$

However, the situation is different if, as in the present case, the choice of wire is completely free in order to achieve the highest sensitivity and greatest attenuation.

This choice, apart from the specific nature (usually copper), refers only to the cross-section and volume of the wire. But since Section 1.8 already states that it is advantageous to make the resistance of the wire to be used for the multiplier the same as that of the rest of the circuit to which the inductor belongs, this freedom is reduced either to the choice of the cross-section from which the volume is determined, or the choice of volume from which the cross section is determined. It therefore remains to determine how this choice can be made most expediently to increase *sensitivity* and *attenuation*.

For the selection of the volume, from which the wire thickness is to be determined in each case, it is first of all important to consider that the *sensitivity* also increases rapidly with increasing volume at the beginning, but that this growth is not uniform, but decreases until it disappears completely, whereupon even the case occurs where the sensitivity decreases with increasing volume. There is therefore a certain value of the volume for which the sensitivity is greatest.

In order to determine this value, the expression of sensitivity must be developed more precisely. The angular velocity f which is given to the needle by the unit of current in the time unit serves as a measure of sensitivity; therefore f must first be developed.

According to the electromagnetic law, the rotational moment exerted on the galvanometer needle m by the element α of a multiplier turn through which the current unit passes is denoted by, when r is the radius of the turn and x is the distance of the axis of rotation of the needle from the plane of the turn, both expressed in parts of the radius c of the space left free for the needle, in the case of the circular shape,

$$= \frac{\alpha m}{c^2} \cdot \frac{r}{(r^2 + x^2)^{3/2}} \, .$$

If you multiply this expression by dx, the integral value of it from $x = -b$ to $x = +b$, divided by $2b$, gives the *mean* rotational moment of all current elements corresponding to the same value of r

$$= \frac{\alpha m}{c^2} \cdot \frac{1}{r\sqrt{r^2 + b^2}} \, ,$$

from which the *average* rotational moment of all multiplier turns corresponding to the same value of r is obtained by multiplying by $2\pi rc/\alpha$, namely,

$$= \frac{2\pi m}{c} \cdot \frac{1}{\sqrt{r^2 + b^2}} \, .$$

If one now multiplies this expression by dr, the integral value from $r = 1$ to $r = 1 + a$, divided by a, gives the *mean* rotational moment of all turns of the entire multiplier

$$= \frac{2\pi m}{ac} \log \frac{1 + a + \sqrt{(1+a)^2 + b^2}}{1 + \sqrt{1 + b^2}} \, ,$$

from which the rotational moment of the entire multiplier is obtained by multiplying by the length l of the entire multiplier wire and dividing by the average length of all its turns $2\pi c(1 + a/2)$.

The quotient of this rotation moment, divided by the moment of inertia k of the needle, is the desired expression of f, or it is

$$f = \frac{2l}{(2+a)ac^2} \cdot \frac{m}{k} \log \frac{1 + a + \sqrt{(1+a)^2 + b^2}}{1 + \sqrt{1 + b^2}} \, .$$

If w now denotes the given absolute resistance of the multiplier wire, and $\varkappa$ the given specific resistance of the metal (copper) of which it is made, then the cross-section of the wire, according to Ohm's law, is $= [\varkappa/w] \cdot l$, that is, the volume of the entire multiplier

$$\frac{\varkappa}{w} l^2 = 2\pi(2+a)abc^3 \, .$$

If you put the resulting value of l into the above expression of f, you get

$$f = 2\frac{m}{k}\sqrt{\frac{2\pi w}{c\varkappa}} \cdot \sqrt{\frac{b}{(2+a)a}} \cdot \log \frac{1 + a + \sqrt{(1+a)^2 + b^2}}{1 + \sqrt{1 + b^2}} \, .$$

Since m, k, w, c, $\varkappa$ are given quantities, the sensitivity f only changes with the value of a and becomes a maximum if

$$\sqrt{\frac{b}{(2+a)a}} \cdot \log \frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}} = \text{maximum},$$

where b is given as a function of a by the first equation given at the beginning of the Section. If one then adds the second equation given there for v, and the equation resulting from Ohm's law, $l^2 = [w/\varkappa] \cdot v$, for the wire length l (from which the cross section $= v/l$ results), the four elements a, b, v, l can be determined, in which all regulations for the construction of the multiplier are completely contained.

If one initially only takes into account the equation

$$\sqrt{\frac{b}{(2+a)a}} \cdot \log \frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}} = \text{maximum},$$

but assumes $b/(1+a) = \beta$ as known or given, then, setting $r = 1+a$, one can write

$$\sqrt{\frac{r}{r^2-1}} \cdot \left(\log r + \log \frac{1+\sqrt{1+\beta^2}}{1+\sqrt{1+(\beta r)^2}} \right) = \text{maximum},$$

from which follows through differentiation[50]

$$\frac{1+r^2}{2r\frac{1}{2}\left(r^2-1\right)^{3/2}} \left(\log r + \log \frac{1+\sqrt{1+\beta^2}}{1+\sqrt{1+(\beta r)^2}} \right)$$
$$- \sqrt{\frac{r}{r^2-1}} \cdot \left(\frac{1}{r} - \frac{\beta^2 r^2}{1+(\beta r)^2+\sqrt{1+(\beta r)^2}} \right) = 0 \ ,$$

which can be traced back to

$$\log \frac{r+\sqrt{r^2+b^2}}{1+\sqrt{1+b^2}} = \frac{3r^2-1}{1+r^2} \cdot \frac{1}{\sqrt{1+b^2}} - \frac{1}{\sqrt{1+b^2}} \ .$$

If one now also adds the equation given at the beginning of this Section, setting $r = 1+a$, namely,

$$\log \frac{r+\sqrt{r^2+b^2}}{1+\sqrt{1+b^2}} = \frac{3r^2-1}{2r\sqrt{r^2+b^2}} - \frac{1}{\sqrt{1+b^2}} \ ,$$

you get a third equation from these two equations

[50][Note by AKTA:] In the original:

$$\frac{1+r^2}{2r\frac{1}{2}\left(r^2-1\right)\frac{3}{2}} \left(\log r + \log \frac{1+\sqrt{1+\beta^2}}{1+\sqrt{1+(\beta r)^2}} \right)$$
$$- \sqrt{\frac{r}{r^2-1}} \cdot \left(\frac{1}{r} - \frac{\beta^2 r^2}{1+(\beta r)^2+\sqrt{1+(\beta r)^2}} \right) = 0 \ .$$

$$(1 + r^2)\sqrt{1 + b^2} = 2r\sqrt{r^2 + b^2} \ ,$$

out of which follows

$$b^2 = \frac{3r^2 + 1}{r^2 - 1} \ ;$$

and if one substitutes this value of b^2 into the first equation, one finds the determination of r

$$\log \frac{r\sqrt{r^2 - 1} + r^2 + 1}{\sqrt{r^2 - 1} + 2r} = \frac{(r^2 - 1)^{3/2}}{r(1 + r^2)} \ .$$

From this equation one finds $r = 3.0951$, from which $a = r - 1$, $b = (3r^2 + 1)/(r^2 - 1)$, $v = 2\pi(2 + a)abc^3$, $l = \sqrt{wv/\varkappa}$ can be calculated, namely,

$$\begin{aligned} a &= 2.095\,1 \ , \\ b &= 1.861\,78 \ , \\ v &= 100.364 \cdot c^3 \ , \\ l &= 10.018\,2 \cdot \sqrt{\frac{w \cdot c^3}{\varkappa}} \ . \end{aligned}$$

All of these regulations for the construction of the multiplier result solely from the requirement for the *greatest sensitivity*, without any consideration of the *attenuation*, and it therefore remains to discuss in particular how the *attenuation* behaves in such a multiplier. If we now note in relation to the *damping* that it generally depends not only on the multiplier, but also on the needle magnetism, but that in our case, where we are only dealing with the *theory of the multiplier*, the needle magnetism should be considered as given, it can easily be proven that under these conditions, with a given needle magnetism, it increases with the sensitivity and at the same time reaches the highest value with it, so that through the same construction of the multiplier, through which the highest sensitivity is produced, the strongest damping will also be obtained. If the measure of sensitivity f given above is also referred to again, then, as already stated in Section 1.6, $f^2 = [2w/k\tau] \cdot \lambda$, where τ is the period of oscillation with a closed circuit (which is related to the oscillation period t with an open circuit as $\sqrt{1 + \lambda^2/\pi^2} : 1$) and w the resistance of the entire circuit, to which belong the multiplier and inductor, further $e^\lambda : 1$ was the ratio of two consecutive oscillation arcs, the exponent of which $\lambda = [k\tau/2w] \cdot f^2$, for a given oscillation period, can be taken as a measure of the damping. This measure of attenuation is therefore, with a constant value of the factor $k\tau/2w$, proportional to the square of the sensitivity, from which it follows that the highest sensitivity also corresponds to the strongest attenuation.

For the case of the highest sensitivity, see page 29[51]

$$f = 2\frac{m}{k}\sqrt{\frac{2\pi w}{c\varkappa}} \cdot \sqrt{\frac{b}{(2 + a)a}} \cdot \log \frac{1 + a + \sqrt{(1 + a)^2 + b^2}}{1 + \sqrt{1 + b^2}} = 1.792\,27 \cdot \frac{m}{k}\sqrt{\frac{w}{c\varkappa}} \ ,$$

[51][Note by AKTA:] Page 29 corresponds to page 49 of Vol. 4 of Weber's *Werke*.

where w denotes the *multiplier resistance*; the resistance of the *entire circuit* was twice as great, so that according to this designation

$$\lambda = \frac{k\tau}{4w} f^2 = 0.803\,056 \cdot \frac{\tau}{c} \cdot \frac{m^2}{k\varkappa} \ .$$

Now, according to Section 1.9, the ratio should be $e^\lambda : 1 = e : 1$, consequently

$$\lambda = 0.803\,056 \cdot \frac{\tau}{c} \cdot \frac{m^2}{k\varkappa} = 1 \ .$$

Therefore, if the attenuation corresponding to the highest sensitivity were too strong, only the galvanometer needle would need to be substituted with a weaker one, whereby it can easily be set up so that m/k, and consequently the sensitivity f, remains unchanged with this substitution.

1.14

For the sake of simplicity, only the rules for constructing a multiplier with *circular turns* have been developed in the previous Section, although *elongated* multipliers are usually used for finer observations; however, it is clear that similar provisions for the latter can be derived from Section 1.12, which were also developed for *elongated* multipliers. For practical use, however, such a more precise development will rarely be necessary, but it will suffice to set up the cross-section of an *elongated* multiplier in accordance with the regulations given in Section 1.12, but otherwise to follow the regulations given in the previous Section for circular multipliers, only with the modification that the expression of the sensitivity f is reduced in the ratio of

$$1 : \frac{2+\pi}{2\pi}\sqrt{\frac{\pi c(1+a/2)}{\pi c(1+a/2)+l^0}} \ ,$$

after which the ratio of two consecutive oscillating arcs is also to be altered, namely, its exponent is to be set:

$$\lambda = 0.803\,056 \cdot \left(\frac{2+\pi}{2\pi}\right)^2 \cdot \frac{\pi c(1+a/2)}{\pi c(1+a/2)+l^0} \cdot \frac{\tau}{c} \cdot \frac{m^2}{k\varkappa} \ ,$$

consequently, because according Section 1.9, $e^\lambda : 1 = e : 1$,

$$0.803\,056 \cdot \left(\frac{2+\pi}{2\pi}\right)^2 \cdot \frac{\pi c(1+a/2)}{\pi c(1+a/2)+l^0} \cdot \frac{\tau}{c} \cdot \frac{m^2}{k\varkappa} = 1 \ .$$

It is sufficient here, as in the Note to Section 1.10, to consider only a single turn of the multiplier, whose moment of rotation is reduced for large values of l^0 in the ratio of $2\pi : 2{+}\pi$. In addition, by increasing the mass of the wire through the extension of the multiplier, with unchanged cross-section and resistance, in the ratio of $\pi c(1+a/2) : \pi c(1+a/2)+l^0$, the number of turns and, proportionally, both the moment of rotation and the sensitivity of the whole multiplier, decrease in the ratio of $\sqrt{\pi c(1+a/2)+l^0} : \sqrt{\pi c(1+a/2)}$.

Finally, it should be noted that in the theory of the multiplier developed here, also for the sake of simplicity, only a simple needle *enclosed by the multiplier* has been considered.

For an *astatic needle system*, according to Section 1.11, the results found can therefore only be directly applied if the needle not enclosed by the multiplier, which can easily be done, is kept so far away from the multiplier, that its effect on the same needle disappears against the one on the enclosed needle. As a rule, however, this is not the case, but the two needles are usually suspended symmetrically at an equal distance from the upper side of the multiplier, which results in an increase in both sensitivity and attenuation. With an *elongated* multiplier, the sensitivity is increased by a ratio of almost 3 : 4 and the attenuation by a ratio of 9 : 16.

III - Galvanometric Observations

1.15

After discussing the method of absolute resistance measurement and the construction of the galvanometer, we move on to the consideration of the *observations*, which can be broken down into *galvanometric* and *magnetic*, but of which the latter only concern the intensity of the horizontal component of the Earth's magnetism, its determination in terms of absolute value has been dealt with so completely by Gauss that the necessary observations do not require any further consideration.

If the *galvanometric observations* were to be made with a galvanometer constructed according to the regulations of the previous Section, the *standard* whose resistance is primarily concerned, namely, the etalon intended for general use and serving to compare the resistances of all bodies, must actually be available and given; for it is intended to provide the standard according to which the galvanometer is to be constructed, so that the resistance of the circuit formed by the multiplier of the galvanometer and the inductor becomes equal to its resistance, which is necessary so that both can be directly compared with each other.

If this *standard resistance* was really present and given and its absolute value was at least approximately known, namely, $W = 2w$, then, according to the rules developed in the previous Section, a more specific approach to the construction of the whole galvanometric apparatus, the galvanometer as well as the inductor, could easily be made in the following way.

For example, assume the radius of the space to be left free for the galvanometer needle $c = 20$ millimeters. In the case of a *circular multiplier*, the volume of the multiplier would then be $= 802\,912$ cubic millimeters according to the equation $v = 100.364 \cdot c^3$. This wire would then have to be wound according to the rule that $b = 1.861\,78$ in the form of a ring with a clear diameter of $2c = 40$ millimeters and a height of $2bc = 74.471\,2$ millimeters, after which the outer ring diameter $2(1+a)c = 123.804$ millimeters. — In the case of an *elongated* multiplier, which is necessary in order to be able to use stronger needles and thereby produce stronger damping, the specified cross section would have to be changed slightly according to the rules developed in the previous Section, namely, to increase the outer diameter slightly, but to reduce the ring height slightly, whereby the size of the cross section would also suffer a small change. But since the size of the cross-section that is then precisely determined would correspond to a maximum value of the sensitivity, where small changes in the cross-section have only an insignificant influence on the sensitivity, it is sufficient for the practical purpose of the survey to adhere to the unchanged size of the cross-section calculated for a circular multiplier, for which it is necessary, however, to increase the volume of the multiplier in the ratio of $\pi c(1 + a/2) : \pi c(1 + a/2) + l^0$, if l^0 denotes the length of the parallel sides connected between the two half rings. Now $\pi c(1 + a/2) = 128.65$ millimeters, therefore, for $l^0 = 128.65$ [mm], the volume would be $= 1\,605\,824$ [mm^3], for $l^0 = 385.95$, the volume $= 3\,211\,648$ cubic millimeters.

If one assumes the average density $= 6$ for copper, but taking into account the wrapping used for insulation and the gaps remaining in the cylindrical wire shape, the wire mass would be estimated at $9\,634\,944$ [mg] for $l^0 = 128.65$ [mm], while for $l^0 = 385.95$ it would be estimated at $19\,269\,888$ milligrams. This wire would therefore have to be wound on a multiplier frame, which would be formed by two semicircles, each of 20 millimeters in radius, and by two parallel sides, each of the length $l^0 = 128.65$ or $l^0 = 385.95$ millimeters, and that would leave a ring-shaped space for the wire, which should be slightly smaller in height than the value $2bc$ given for a circular multiplier, that is, a space of almost 70 millimeters in height. The length l of the wire to be formed from the given mass would be obtained from the equation $l^2 = [w/\varkappa] \cdot v$, where $\varkappa$ denotes the [specific] resistance of the wire with 1 millimeter length and 1 square millimeter cross section, for which the value $\varkappa = \frac{1}{6} \cdot 2\,000\,000$ can be calculated in round numbers, assuming a density $= 6$. This results in the value of $l = 2.1949\sqrt{w}$ for $l^0 = 128.65$, but $l = 3.104\sqrt{w}$ for $l^0 = 385.95$. For example, if the given standard resistance was $W = 2w = 2 \cdot 10^{10}$, then the specified mass for the case $l^0 = 128.65$ would result in a wire length of $219\,490$ millimeters, which would form 426 turns, for the case $l^0 = 385.95$ a wire length of $310\,400$ millimeters would be produced, which would form 302 turns.

As far as the manufacture of the *astatic needle system* is concerned, the moment of inertia k of the same can be divided into that of the two needles and that of their fixed connecting piece together with the mirror and other accessories. The latter can be considered as given since it is independent of the choice of needles and is assumed, for example, for millimeters and milligrams as space and mass measures, $= 20 \cdot 10^6$. The length of the needles, which must depend on the length of the multiplier, can also be estimated at a maximum of 150 millimeters for $l^0 = 128.65$ and a maximum of 400 millimeters for $l^0 = 385.95$, after which can be calculated the moment of inertia of the two needles in the former case $= \frac{1}{12} \cdot 150^2 \cdot p$, in the latter case $= \frac{1}{12} \cdot 400^2 \cdot \left(\frac{8}{3}\right)^3 p$, where thin and homogeneous needles of similar shape are assumed and the mass of the smaller pair of needles is denoted by p. According to this, the moment of inertia of the entire astatic needle system, for $l^0 = 128.65$, is $k = 20 \cdot 10^6 + \frac{1}{12} \cdot 150^2 \cdot p$, for $l^0 = 385.95$, $k = 10 \cdot 10^6 + \frac{1}{12} \cdot 400^2 \cdot \left(\frac{8}{3}\right)^3 \cdot p$. The oscillation period of the system can be adapted to the needs of the observation by appropriately choosing the suspension wire, after which the oscillation period $\tau = 30$ seconds is assumed. Since the [specific] resistance of the wire with a length of 1 millimeter and a cross-section of 1 square millimeter has now been assumed to be $\varkappa = \frac{1}{6} 2\,000\,000$, the equation in Section 1.14 results in

$$0.803\,056 \cdot \left(\frac{2+\pi}{2\pi}\right)^2 \cdot \frac{\pi c(1 + a/2)}{\pi c(1 + a/2) + l^0} \cdot \frac{\tau}{c} \cdot \frac{m^2}{\varkappa k} = 1\ ,$$

for $l^0 = 128.65$ the value of the magnetism of a needle m is equal to the geometric mean of the two numbers $824\,880$ and $20 \cdot 10^6 + 1875p$; for $l^0 = 385.95$ is equal to that of the two numbers $1\,649\,760$ and $20 \cdot 10^6 + 252\,840p$. So if the mass of a smaller needle $\frac{1}{2}p = 50\,000$ milligrams, its magnetic moment should be $= 13\,083\,000$, hence for each milligram on average 261.66 units; the mass of the larger needle would then be $948\,160$ milligrams with a similar shape and its magnetic moment should be $= 204\,310\,000$, hence 215.48 units for each milligram. However, magnetic needles of this size and strength are easy to prepare.

According to the comment made at the end of Section 1.14, these provisions only apply to an *astatic needle system* if the outer needle is kept far enough away from the multiplier. However, if the arrangement is such that the upper side of the multiplier lies symmetrically between both needles, the change in the above equation is that the unit in the second term is substituted with the fraction 9/16, according to which the magnetic moments of the needles are obtained in the ratio 4 : 3 smaller, namely, for the smaller needle = 9 812 250 [units], for the larger needle 153 232 500 units.

Finally, as far as the *inductor* is concerned, it should be constructed in such a way that the length of wire $2\pi\Sigma r$ used for it has a resistance which is equal to the resistance w of the multiplier after subtracting the resistance of the connecting wires; so, if all wires are of the same type, it is $2\pi\Sigma r = l - l'$, if l' denotes the length of the two connecting wires. The integral value of the current produced by an induction shock is then, with inducing Earth magnetism T, $\int idt = [2\pi T/2w]\cdot\Sigma r^2$ and consequently the angular velocity imparted to the astatic needle system by such an induction shock, according to Section 1.13, is $\gamma = f\cdot\int idt = 2\pi T\cdot\Sigma r^2\cdot\sqrt{1/wk\tau}$, if τ is the period of oscillation with a closed circuit and if in the ratio $e^\lambda : 1$ of two consecutive oscillation arcs, λ is $= 1$. This angular velocity γ, when given to the needle in the rest position, results in the size of the first subsequent deflection $\alpha = \gamma\tau\cdot\frac{1+e^2}{\sqrt{1+\pi^2}}\cdot e^{-\left(\frac{3}{2}+\frac{1}{\pi}\arctan\frac{1}{\pi}\right)}$, or, if we replace the value for γ,

$$\alpha = 2\pi T\cdot\Sigma r^2\cdot(1+e^2)\sqrt{\frac{\tau}{(1+\pi^2)wk}}\cdot e^{-\left(\frac{3}{2}+\frac{1}{\pi}\arctan\frac{1}{\pi}\right)}\ .$$

If the radius of all inductor windings were the same and their number n, then the total wire length would be $l - l' = 2n\pi r$ and $\Sigma r^2 = nr^2 = [(l-l')/2\pi]\cdot r$. After substituting this value, one obtains from the previous equation

$$r = \frac{\alpha}{(1+e^2)(l-l')T}\cdot\sqrt{\frac{(1+\pi^2)wk}{\tau}}\cdot e^{\frac{3}{2}+\frac{1}{\pi}\arctan\frac{1}{\pi}}\ ,$$

where α can be taken as large as the scale with which the deflections of the needle are to be observed allows, since the deflections to be observed using the throwback method never quite reach the value α. So if, as is usually the case with magnetometers, a 1 meter long scale is needed at a distance of 5 meters from the mirror, $\alpha = 1/20$ can be set, and $T = 1.81$ (as currently in Göttingen), so one obtains, since $\tau = 30$ has been assumed,

$$r = 0.009\,799\cdot\frac{\sqrt{wk}}{l-l'}\ .$$

In the example already given, where $W = 2w = 2\cdot 10^{10}$, this results in the *first* case, namely, with a multiplier for which $l^0 = 128.65$, $l = 129\,490$ and $k = 20\cdot 10^6 + 1875p$, for $p = 100\,000$ and $l' = 10\,000$, $r = 67.38$ millimeters; in the *second* case, namely, with a multiplier for which $l^0 = 385.95$, $l = 310\,400$ and $k = 20\cdot 10^6 + 252\,840p$, for the same values of p and l', $r = 518.9$ millimeters. However, it should be noted that in the case of an astatic needle system it was assumed that the outer needle was so far away from the multiplier that its effect on it disappeared compared to that on the enclosed needle. But if both needles lay symmetrically against the upper side of the multiplier, r would have to be reduced in almost the ratio 4 : 3 and in the first case it should be $r = 50.53$, in the second case it should be $r = 389.18$ millimeters.

However, if these found radii do not fit correctly for the exact measurement or convenient rotation of the inductor, you only need to use a different type of wire to construct the inductor than the one used for the multiplier. For example, if you want the radius to have the value μr instead of r, you take a wire whose length and cross-section are related to those of the multiplier wire as $1 : \mu$; the resistance and the sum of the circular areas enclosed by all the turns, and consequently also the effects produced by an induction shock, remain completely unchanged.

With an apparatus manufactured in this way, in accordance with all the prescribed rules, the *galvanometric observations* would be made from which (in connection with the measurement of the horizontal geomagnetic force carried out with the instruments of a magnetic observatory and traced back to the place and time of the galvanometric observations) the given *standard-resistance* should be determined according to its absolute value. — At present, however, it is not possible to manufacture such an apparatus where there is still a lack of the standard, which is in general use and which serves to compare the resistances of all bodies.

If, as already noted, it is not a question of the definitive determination of such an *standard-resistance*, but only of testing the *galvanometric observations* required for this, which require an accuracy that is at least equal to that of the other observations, then observations also suffice made with an apparatus which does not conform to all prescribed rules, if only the deviations either have little influence on the accuracy or if their influence can be easily and precisely determined. The following observations are therefore sufficient for this purpose.

1.16

The following observations were made with an apparatus that deviated from the given regulations only in that an already existing and precisely known inductor was used, the resistance of which was significantly greater than that of the multiplier. The resulting influence on the observations is easy to determine and will be discussed in more detail below. This deviation is therefore not of any significant disadvantage if it is simply a matter of testing the security and accuracy that can be achieved in *galvanometric observations*, and the following observations made with it can be used quite well for this purpose.

The following Table contains a set of such observations made by the *throwback method*,[52] arranged in five columns in such a way that, in order of time, the *first* column contains observations 1, 5, 9 etc., the *second* contains observations 2, 6, 10 etc., the *third* contains observations 3, 7, 11 etc., the *fourth* contains observations 4, 8, 12 etc. The observations in the first two columns are the elongations of the galvanometer needle that initially precede and follow the *negative* induction shocks. The observations in the last two columns are the elongations that initially precede and follow the *positive* induction shocks. The numbers are the scale divisions observed at the moment of greatest elongation.

[52]See footnote 39.

1.	2.	3.	4.
661.5	600.9	844.8	904.9
659.6	598.7	842.7	903.0
657.2	596.2	840.0	900.4
654.5	593.5	837.5	897.7
751.7	590.4	834.8	894.8
649.5	588.3	832.6	892.7
647.3	586.2	830.5	890.8
645.4	584.7	828.2	889.0
643.3	582.3	826.7	886.7
641.4	580.4	824.3	884.5
639.2	578.0	822.3	882.6
637.5	576.6	820.3	881.3
635.2	574.8	819.0	879.7
634.2	573.9	817.9	879.3
632.8	573.1	816.1	877.0

The resting position of the needle can be determined from four consecutive observations for the moment falling symmetrically in the middle between these observations. The average between two such successive periods of rest then gives the period of rest which corresponds to the moment of elongation in between. In this way, the resting positions corresponding to all individual elongation observations in the previous Table were calculated and the elongation widths resulting after their deduction were compiled in the following Table: the first two and the last two data in this Table are determined by adding two preceding and two following observations made solely for this purpose.

1.	2.	3.	4.
−92.49	−152.46	+92.01	+152.63
−92.14	−152.54	+92.00	+152.91
−92.24	−152.57	+91.89	+152.96
−92.29	−152.64	+92.05	+152.99
−92.29	−152.89	+92.15	+152.69
−92.07	−152.74	+92.10	+152.74
−92.14	−152.74	+92.04	+152.76
−92.16	−152.35	+91.64	+153.00
−92.21	−152.74	+92.19	+152.66
−92.10	−152.52	+91.93	+152.70
−92.05	−152.76	+91.99	+152.68
−92.00	−152.49	+91.66	+153.18
−92.54	−152.57	+91.95	+152.89
−92.36	−152.47	+91.75	+153.43
−92.75	−151.94	+91.49	+152.91
−92.255	−152.561	+91.923	+152.875

At first sight there is a very satisfactory agreement among these observations, which becomes even more clear when one considers that every small disturbance which slightly reduces or enlarges the elongation before an induction shock causes an enlargement or reduction of the next following elongation. The averages from the first two and the last two columns, in which these opposite influences almost balance each other, therefore provide an even more definite test of the accuracy of the results which can be obtained by this method of observation. These averages are as follows.

1. 2.	Difference from the mean.		3. 4.	Difference from the mean.
−122.475	+0.067		+122.320	+0.079
−122.340	−0.068		+122.455	−0.056
−122.405	−0.003		+122.425	−0.026
−122.465	+0.057		+122.520	−0.121
−122.590	+0.182		+122.420	−0.021
−122.405	−0.003		+122.420	−0.021
−122.440	+0.032		+122.400	−0.001
−122.255	−0.153		+122.320	+0.079
−122.475	+0.067		+122.425	−0.026
−122.310	−0.098		+122.315	+0.084
−122.405	−0.003		+122.335	+0.064
−122.245	−0.163		+122.420	−0.021
−122.555	+0.147		+122.420	−0.021
−122.415	+0.007		+122.590	−0.191
−122.345	−0.063		+122.200	+0.199
−122.408	+0.095		+122.399	+0.089

From this you can see that the difference between the individual values and their mean is on average less than 1/10 part of the scale.

1.17

According to the fineness of the observation method examined in the previous Section, it is sufficient for further consideration to stick solely to the four averages of the observed elongations, namely, corresponding to the four columns:

−92.255	−152.561	+91.923	+152.875

The first of the third and the second of the fourth elongation should now be oppositely equal. The reason why this is not exactly true is that as a result of the gradual decrease in the rest position of the galvanometer needle, which is immediately apparent from the observations cited in the previous Section, the individual induction shocks do not occur exactly at the moment when the needle passed the rest position, but in a moment when the needle passed a slightly higher scale point (namely, the one which had been resting shortly before), from which it easily follows that with a uniformly decreasing resting point, the first and third elongations are almost as small as the second and fourth had to be found too big. This gradual decline in the rest position was a result of the elastic after-effect[53] of the recently suspended iron wire supporting the galvanometer needle, and would disappear in time. It did not seem necessary to wait for this time because, given the uniformity of the sinking, there would be no detrimental influence on the measurement. It is also easy to take

[53] [Note by AKTA:] In German: *der elastischen Nachwirkung*. This effect was discovered by Weber in 1835, [Web35] and [Dör91].

this influence into account by adding a small correction to the observed elongations. So if, instead of the observed elongations, you put

$-92.255 + x$	$-152.561 - x$	$+91.923 + x$	$+152.875 - x$

and $x = 0.1615$, you get the corrected values:

-92.0935	-152.7225	$+92.0845$	$+152.7135$,

which do not deviate from the required symmetry by 1/100 part of the scale.

For further consideration, where only the difference of the fourth and second elongation $= 2a'$ and that of the third and first $= 2b'$ are taken into account, these corrections are not necessary at all because these differences are independent of this, namely:

$$2a' = 305.436 \ ,$$
$$2b' = 184.178 \ .$$

Let r now denote the distance of the mirror from the scale and set

$$a' = r \tan 2\varphi \ ,$$
$$b' = r \tan 2\varphi' \ ,$$

so φ and φ' are the deflection angles of the galvanometer needle and, if they are small, can be equated to the elongations labeled a and b in Section 1.10, from which the angular velocity γ and the logarithmic decrement λ should be calculated. For larger values of φ and φ', however, a and b, which should remain proportional to the angular velocities, grow like the sines of the half deflection angles, after which

$$a = 2 \sin \frac{1}{2}\varphi \ ,$$
$$b = 2 \sin \frac{1}{2}\varphi' \ ,$$

is to be set. Strictly speaking, these exact expressions[54] for needles oscillating without damping would have to be accompanied by a small correction resulting from the hitherto undeveloped theory of needle oscillations for larger oscillation arcs under the action of damping,

[54][Note by WW:] For needles that oscillate without damping, according to Section 1.4, a is the product of the oscillation period t divided by π into the highest oscillation velocity. However, the greatest oscillation velocity is $= \gamma/2$, when γ denotes the entire change brought about by the induction shock, because without damping the needle velocity before and after the induction shock (in the throw-back method) should be oppositely equal, therefore $a = [t/2\pi] \cdot \gamma$. However, a needle whose period of oscillation is $= t$ oscillates just like a simple pendulum of length $l = gt^2/\pi^2$, which, when at the lowest point of its path, has the angular velocity $\gamma/2$, rises to the height $\frac{1}{4}\gamma^2 l^2/2g = l \sin\text{vers}\, \varphi$, from which $\gamma/2 = [2\pi/t] \cdot \sin \frac{1}{2}\varphi$ follows, so $a = 2 \sin \frac{1}{2}\varphi$.

which, however, may be neglected in the case of oscillation arcs as small as those observed with magnetometer needles.

If you now expand a and b into powers of a'/r and b'/r, you get

$$a = \frac{1}{2} \cdot \frac{a'}{r} - \frac{11}{64} \cdot \frac{a'^3}{r^3} + \dots ,$$

$$b = \frac{1}{2} \cdot \frac{b'}{r} - \frac{11}{64} \cdot \frac{b'^3}{r^3} + \dots .$$

In the observations described in the previous Section, from which $a' = 152.718$, $b' = 92.089$ were obtained in parts of the scale, the distance of the mirror from the scale was $r = 3245$ scale divisions; consequently,

$$a = 0.023\,503 ,$$
$$b = 0.014\,186 .$$

1.18

The galvanometer observations described in Section 1.16 must also be accompanied by the auxiliary observations with the *open* circuit, regarding the *period of oscillation* and the *decrease in the oscillation arcs*. These observations gave

Number of the Oscillation	Time	Oscillation arc
0.	$11^h\ 34'\ 59.15''$	464.0
17.	$45'\ 23.20''$	376.3
34.	$55'\ 47.39''$	307.6

These observations yield the oscillation period $= 36.7109''$ reduced to infinitely small arcs according to clock time adjustment, or, after reducing the clock time adjustment to mean time, the oscillation period with the *open circuit* was

$$t_0 = 36.7061'' .$$

This also results in the logarithmic decrement for the decrease in the oscillation arcs when the circuit is open

$$\lambda_0 = 0.012\,09 .$$

In addition to these observation results are the constants of the instruments, namely, the moment of inertia K of the galvanometer needle, and, for the inductor, the value of $S = \Sigma \pi r^2$, if r denotes the radius of the inductor windings, and finally the horizontal intensity of the Earth's magnetism found by *magnetic measurements* at the time and place of the galvanometer observations, namely,

$$
\begin{aligned}
K &= 1\,132\,000\,000\ , \\
S &= 39\,216\,930\ , \\
T &= 1.816\,4\ .
\end{aligned}
$$

The inductor was the same as that described in the Fifth Volume of these Treatises[55],[56] and was used for inclination measurements, where the value of S was also given. The value of the moment of inertia K and the horizontal intensity of the Earth's magnetism T were determined according to the instructions given by Gauss in the *Intensitas*.[57]

These results of the auxiliary observations, added to those of the galvanometric ones, finally give in *absolute value* all the elements for determining the resistance of the circuit formed by the multiplier and inductor at the temperature of 17.5 degrees of the 100-divisions scale at which the galvanometer observations were made. Because first of all, according to the theory of the throwback method, it follows from the values listed

$$
a = 0.023\,503, b = 0.014\,186, t_0 = 36.706\,1, \lambda_0 = 0.012\,09\ ,
$$

$$
\gamma = \frac{\sqrt{\pi^2 + \lambda_0^2}}{t_0} \left(a\sqrt{\frac{a}{b}} + b\sqrt{\frac{b}{a}} \right) \cdot \left(\frac{b}{a} \right)^{\frac{1}{\pi} \arctan\left(\frac{1}{\pi} \log \mathrm{nat}\ \frac{a}{b}\right)} = 0.003\,330\,4\ ,
$$

$$
\lambda_1 = \log \mathrm{nat}\ \frac{a}{b} = 0.504\,87\ ;
$$

then according to Section 1.7, where $S = n\pi r^2$ was set, with the given values of K, S, T you get the resistance of the entire circuit

$$
w = \frac{8S^2T^2}{K\gamma^2 t_0} \cdot (\lambda_1 - \lambda_0) \cdot \sqrt{\frac{\pi^2 + \lambda_0^2}{\pi^2 + \lambda_1^2}} = 42\,855 \cdot 10^6 \frac{\text{millimeter}}{\text{second}}\ .
$$

1.19 Thomson's Standard and Other Etalons

Even if there is not yet a generally accepted and used resistance standard to which the result derived from the previous observations could be related, there are several standards which have gained particular interest by the studies made with them or by the measurements

[55][Note by HW:] Wilhelm Weber's *Werke*, Vol. II, p. 323.

[56][Note by AKTA:] [Web53c, p. 323 of Weber's *Werke*].

[57][Note by AKTA:] Gauss' work on the intensity of the Earth's magnetic force reduced to absolute measure was announced at the Königlichen Societät der Wissenschaften zu Göttingen in December 1832, [Gau32] with English translation in [Gau33a], [Gau37a] and [Gau21a], see also [Rei02, pp. 138-150].

The original paper in Latin was published only in 1841, although a preprint appeared already in 1833 in small edition, [Gau41a] and [Rei19]. Several translations have been published. There are two German versions, one by J. C. Poggendorff in 1833 and another one in 1894 translated by A. Kiel with notes by E. Dorn; a French version by Arago in 1834; two Russian versions, one by A. N. Drašusov of 1836 and another one by A. N. Krylov in 1952; an Italian version by P. Frisiani in 1837; an English extract was published in 1935, while a complete English translation by S. P. Johnson was published in 2003 and 2021; and a Portuguese version by A. K. T. Assis in 2003: [Gau33b], [Gau34], [Gau36], [Gau37b], [Gau94], [Gau35], [Gau52], [Gau75], [Gau03] and [Gau21b], and [Ass03].

reduced to them. The specific reason for the observations described above was to determine the absolute resistance value of such an etalon, which is referred to as Thomson's *standard*. It was available for this purpose in two copies, kindly communicated by Professor William Thomson at Glasgow,[58] one in copper wire, the resistance of which, according to Professor Thomson, increases by 36/10000 with each grade of the 100-divisions scale, and the other in nickel-silver wire, the resistance of which increases by 36/100000 with each grade. Both were guaranteed to be exactly the same for the temperature of 16.3 degrees.

With this special purpose in mind, it was particularly important to ensure that, even if not all of the requirements made in the previous Section could be met in these observations, at least the resistance equality of the circuit formed from the multiplier and inductor with the standard was achieved as closely as possible, in order to put the result found for that circuit in the most direct and precise relationship with this standard resistance. This equality had been established up to 1/1850, by which the standard resistance was smaller, as was shown by a very precise comparison with the copper copy made at 16.6 degrees.

This is followed by the resistance of the copper copy of Thomson's *standard* for 17.5 degrees of the 100-divisions scale from the results found in Section 1.18

$$= 42\,832\,000 \frac{\text{meter}}{\text{second}}$$

or for 0 degree temperature

$$= 40\,293\,000 \frac{\text{meter}}{\text{second}} \,.$$

Another standard was the Siemens standard already discussed in the Introduction,[59] which formed the basis of a Siemens resistance scale made of nickel-silver wires as a unit. This Siemens *unit of resistance*, given in a nickel silver wire at 15 degrees of the 100-divisions scale, was, according to Siemens, exactly equal to the resistance of a mercury column with a length of 1 meter and a cross section of 1 square millimeter at 0 degree temperature. If this new silver wire was connected to the circuit formed by the multiplier and inductor, the resistance of the circuit was increased in the ratio of 1 : 1.2395, from which the resistance of this new silver wire is obtained at 17.5 degrees on the 100-divisions scale = 10 266 000 meter/second, hence at 15 degrees, that is, the Siemens standard resistance itself, = 10 257 000 meter/second. However, it is easy to see that the determination of the Siemens standard resistance derived from this comparison is not quite as accurate as that of the preceding Thomson standard resistance, because the influence of unavoidable observation errors on the ratio of two different resistances is much greater than when comparing two very close resistances.

Finally, Siemens also compared its standard resistance with Jacobi's *standard resistance*, whose absolute value, as already mentioned in the Introduction,[60] was found to be = 5 980 000 meters/second after an earlier, less precise measurement. Siemens found that its standard resistance to the Jacobian would be 1000 : 661.8, from which the Jacobian standard resistance would follow = 6 788 000 meters/second. However, Siemens himself noted that he did not have the Jacobi normal standard at his disposal, but that he had obtained several copies of it, which, however, differed greatly from one another. This is probably the main reason for the deviation of over 12 percent, since the error in the earlier measurement can be estimated

[58][Note by AKTA:] William Thomson (1824-1907), also known as Lord Kelvin, was a British mathematician, physicist and engineer.

[59][Note by AKTA:] See footnote 8.

[60][Note by AKTA:] See footnote 7.

at a maximum of 2 to 3 percent. Anyway, this case can serve as evidence of how important an appropriately established Institution would be for the general dissemination of consistent, precisely tested resistance standards and scales.

1.20

If only the *galvanometric observations* referred to in Section 1.16 are considered, the mean error of a distance measured on the scale is 0.092 part of the scale, according to which the distances considered in Section 1.17 are obtained as:

$$
\begin{aligned}
2a' &= 305.436 \pm 0.092 \;, \\
2b' &= 184.178 \pm 0.092 \;.
\end{aligned}
$$

From this it follows that:

$$
\begin{aligned}
a &= 0.023\,503 \left(1 \pm \frac{1}{3320}\right) \;, \\
b &= 0.014\,186 \left(1 \pm \frac{1}{2000}\right) \;,
\end{aligned}
$$

and finally the angular velocity γ and the logarithmic decrement λ_1 of the decrease in vibration obtained by an induction shock to the needle, as considered in Section 1.18

$$
\begin{aligned}
\gamma &= 0.003\,330\,4 \left(1 \pm \frac{1}{2554}\right) \;, \\
\lambda_1 &= 0.504\,87 \left(1 \pm \frac{1}{865}\right) \;.
\end{aligned}
$$

Could the sensitivity of the galvanometer be increased according to the regulations developed in the previous Section, so that the distance

$$2a' = 1000 \pm 0.92$$

would be obtained, and at the same time the damping could also be increased so that $\lambda_1 = 1$, that is,[61]

$$2b' = 367.9 \pm 0.92 \;;$$

in the same way, the mean error of γ would be

[61] [Note by AKTA:] In the original we have:

$$2b' = 367.9 \pm 092 \;;$$

$$\pm \frac{1}{7127}\gamma \ ,$$

the mean error of λ_1 can be obtained

$$= \pm \frac{1}{3753}\lambda \ ,$$

which may be regarded as the highest degree of accuracy with which these results of galvanometric observations can be determined, but only when the sensitivity and attenuation are regulated in the manner indicated. Errors in the scale division and in the measurement of the distance of the mirror from the scale are not taken into account at all because they are not to be counted as purely galvanometric observations, but as auxiliary observations.

Finally, the description of the apparatus with which the observations just considered were made shows how easy it would actually be to produce the required sensitivity and attenuation. For it has already been mentioned in Section 1.16 that this apparatus differed essentially from the rules prescribed in the previous Section only in that an already existing, exactly known inductor was used, the resistance of which was significant, namely, almost four times greater than that of the multiplier. Therefore, if only the cross-section of the inductor wire had to be increased in the ratio of $3 : 8$, the resistance of the entire circuit would have been reduced in the ratio of $8 + 2 : 3 + 2$, whereby the sensitivity as well as the attenuation would have been increased in the opposite ratio, namely, in that of $1 : 2$.

It even turns out that the sensitivity as well as the attenuation could easily be increased far beyond the prescribed limits, whereby the regulations given in Section 1.9 regarding these limits would actually come into effect.

Finally, it follows that it is not due to the *galvanometric observations* that the absolute value of a given standard resistance would not be obtained with great certainty and precision if the prescribed rules were followed; because the error arising from these observations alone would, according to the above information, only be about 1/2530 of the entire resistance; on the contrary, it will be difficult to carry out the other observations, especially the magnetic ones, to determine the intensity of the horizontal geomagnetic force at the place and time of the galvanometric observations, with corresponding accuracy, from which it follows that the unavoidable uncertainty in the *absolute value of the given standard resistance* resulting from the determination of Earth magnetism would not be significantly increased by the uncertainty of the *galvanometric measurement* carried out in accordance with the prescribed rules, so that the main purpose of the present Treatise, namely, to explain how to achieve that objective, may be regarded as fulfilled.

IV - Copying Methods

1.21

It follows from the two preceding paragraphs that a galvanometric apparatus for absolute resistance measurement can be manufactured with the greatest practicality only for the measurement of *a certain standard resistance*, which is sufficient because the comparability of the resistances of all bodies to one another only requires the exact knowledge of *one* such standard resistance in absolute terms in order to indirectly gain knowledge of the absolute resistance values of all bodies and to make all possible applications of it.

However, the same rules apply to the selection and determination of such an *standard resistance* as to the selection and determination of *fundamental standards*[62] *of other types of quantities*: only those types of quantities are suitable for establishing basic standards, from which existing dimensions can be preserved unchanged, moved from one place to another at will, and reproduced using a method of the finest copying. Wherever these conditions can be met, the establishment of such a *fundamental standard* would appear to be desirable because of the practical importance of the simplification of measurements that can thereby be achieved, which at the same time increase in finesse and accuracy. But where these conditions cannot be satisfied, the determination of a *measure* is necessary, but this does not have to be a *fundamental standard*, but can also be a *derived measure*, namely, *an absolute measure reduced to the basic measures of other types of quantities*. For example, velocities are one of those types of quantities that are not suitable for establishing a fundamental standard because a velocity cannot be maintained unchanged, cannot be moved arbitrarily and cannot be copied exactly. On the other hand, straight lines and body masses are, as is well known, particularly suitable for establishing fundamental standards.

Whether *galvanic resistances* are suitable for establishing a *basic measurement* in a certain etalon or standard also depends only on whether an existing resistor can be preserved unchanged, moved from one place to another at will and reproduced using a method of the finest copying. If there is an unchanging resistance attached to a certain metal wire (like an unchanging length on a rod or an unchanging mass on a weight), then it is self-evident that the existing resistance of the wire can be maintained unchanged with the wire and moved from one place to another; it remains only to be proved that it can also be reproduced by the finest copying methods.

If one considers that the most important and essential thing about a *basic measure*, in addition to its immutability for all times and places, is the *general use* of it, and if one considers the great difficulties which such a general introduction encounters, then for this reason it might seem expedient to limit the number of such *basic measures* as much as possible and to expand the use of the *derived measures*, namely, the *absolute measures* reduced to the basic measures of other types of quantities; but on closer examination one will easily see that, instead of substituting *absolute measures* for *basic measures*, it is more expedient to bring the *basic measures into quite exact agreement with the absolute measures* by which they could possibly be replaced because the *absolute measure* can then represent the *basic measure* where it is not widely used.

The *absolute measure*, as can be seen from the example of the absolute resistance measure, *does not allow any direct application*, but every absolute measurement determination is always *mediated* by certain laws of the dependence of different types of quantities observed on an object at the same time on one another, and therefore requires a planned *combination of different observations*, the execution of which requires greater effort and work, and also sets stricter limits on accuracy, than if the results are based directly on simple observations. On the other hand, a *basic measure* of quantities which are suitable for this purpose *is directly applicable*, combined with greater simplicity of measurement and greater fineness of results, whereby it should be particularly emphasized that the *freedom in the choice of such a basic measure* also allows a really existing *absolute measure to be used, or a higher or lower decimal unit of the same, to take the same quantity, or at least one which comes very close to it, as the basic measure after the finest examination, and thus to preserve all the advantages*

[62][Note by AKTA:] In German: *Grundmaassen*. This expression can also be translated as fundamental measures or basic dimensions. See also footnote 5.

connected with the use of absolute measures which the laws of the dependence of different kinds of quantities considered simultaneously on one object confer on each other.

For example, the *derived measure* we use for resistance is the *absolute measure* traced back to the *basic measure of space, time and mass*, namely, *millimeters*, *seconds* and *milligrams* have been assumed as basic measures for the latter types of quantities. If one persists with these latter basic measures, so the freedom available when choosing a *basic measure for resistance* can very well be used, a really existing resistance which, after the most precise examination, is equal to *a higher decimal unit of that absolute measure* or at least comes very close to it (which is the case with Siemens' measure, which is almost 10^{10} times larger than that absolute measure, which actually takes place approximately), to be taken as the *basic measure*, whereby all the advantages associated with the use of that *absolute measure* would also be preserved for the use of this basic measure.

But it must be possible to really apply such a basic standard to *general or at least very extensive application.* However, the only way to do this is to *reproduce the standard by copying*, if a method is available which allows all copies made afterward to be considered *completely identical* for all practical purposes. Without a doubt, the establishment of a basic measurement for galvanic resistors is also very desirable, but above all it is necessary to check whether the *coping methods of the resistors* meet the stated purpose. This test becomes even more necessary in view of our investigation, because the wire circuit of our galvanometric apparatus itself is in no way suitable to serve as a basic measure, since it cannot be moved from one place to another at will. This wire circuit should therefore also be a *copy of the basic measure*, which for all practical purposes may be regarded as completely identical to the basic measure, so that all determinations obtained for this circuit also apply to the *basic measure.*

1.22 Copying Methods Without Current Splitting

Copying is based on the judgment about the equality or inequality of two quantities. Directly from the definition of resistance arises a first method of judging the equality or inequality of two resistances, namely, according to the following principle: *the resistance of two conductors is the same if the same currents are excited in them by the same electromotive forces.* The accuracy of the method based on this principle is, however, limited: (1) by the accuracy that can be achieved when assessing the equality of two electromotive forces acting on two different conductors; (2) by the accuracy with which one can observe and compare the current intensities in two different conductors. Closer examination easily shows that this sets much narrower limits to the comparison of resistances than would appear permissible when copying measure-standards.

A *second* method is that of *connection* according to the following principle: *if two conductors are connected successively in the same circuit, in which the same electromotive force always acts, the resistance of the two conductors is the same if the current intensities are the same.* — In addition to the definition of resistance, Ohm's law of summation of the resistances of conductors through which the same current passes is used. — Even according to this method, the accuracy of the comparison of two resistances is limited by the accuracy that can be achieved by observing the current intensities, which, even when using the finest galvanometers, generally does not meet the requirements to be made when copying measure-standards, even if it is sufficient for many practical purposes.

Finally, the *third* method to be discussed in more detail in the following Sections is that of *current division*, whereby two cases can be distinguished, namely, that of *single* and *double* division. The method implemented in Wheatstone *bridge* or *balance* is based on *double current division*, but a closer look at this method should be preceded by a brief discussion of the method based on *single current division*.[63]

1.23 Copying Methods With Simple Current Division

In order to determine the accuracy achievable by the *simple current splitting* method in comparing two resistors with each other, it is necessary to go back to the principle of this method. This principle is as follows: *if a current splits into two branch currents, and both branch currents, each through a multiplier through which it passes, act on the same magnetic needle but in opposite directions, then the resistance of two conductors passed through by these branch currents is equal if the total effect observed at the magnetic needle is not altered by the exchange of the two conductors.* — The total effect may be greater or smaller, or even zero, from which it is obvious that the accuracy achievable by this method in the comparison of two resistors is completely independent of the magnitude of the observed total effect. — With this method, in addition to the laws listed in the previous Section, Ohm's laws of *current branching* are also used.

1.24

The principle stated in the previous Section can be easily proven in the following way. A current i produced by the electromotive force e passes through the conductor c, Figure 3 of Table I,[64] which is divided into two branch currents i_1 and i', of which the former passes through the conductors which have the resistors a and α, the latter going through the conductors which have the resistors b and β.

[63][Note by AKTA:] In German: *Wheatstone'schen Brücke oder Waage*. The so-called Wheatstone bridge was invented by S. H. Christie (1784-1865) in 1833 and popularized by C. Wheatstone (1802-1875) in 1843, [Chr33], [Whe43, p. 325] with French translation in [Whe44b] and German translation in [Whe44a]. See also [Eke01].

[64][Note by AKTA:] In Figure 3 we should have $a(i_1)$ instead of $a(i)$.

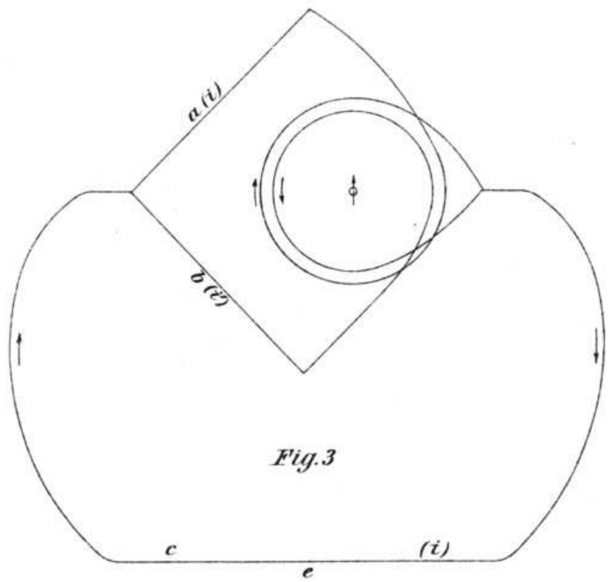


Fig. 3

a and b are the resistances that are to be compared with each other, which is why the arrangement is made so that the two conductors that have these resistances can be exchanged with each other. The conductors with the resistances α, β form multipliers for one and the same magnetic needle, which, however, is deflected by the branch current passing through the conductor α in the opposite direction than that passing through the conductor β. — A current in the conductor α of intensity $= 1$ keeps the needle in a deflection $= m$; a current in the conductor β of intensity $= 1$ keeps the needle in a deflection $= -n$. m and n can therefore be called the sensitivity coefficients of the two multipliers. — Finally, the two conductors α, β reunite with the conductor c, closing the circuit.

The following three equations result from Ohm's laws of current division

$$\frac{e}{i} = c + \frac{(a+\alpha)(b+\beta)}{a+\alpha+b+\beta} \ ,$$

$$i_1 + i' = i \ ,$$

$$i_1 : i' = (b+\beta) : (a+\alpha) \ ,$$

in addition, there is the determination of the total deflection of the needle by the two branch currents, which may be referred to as:

$$A = mi_1 - ni' \ .$$

From these four equations, if i, i_1 and i' are eliminated,

$$A = \frac{m(b+\beta) - n(a+\alpha)}{c(a+\alpha+b+\beta) + (a+\alpha)(b+\beta)} \cdot e \ .$$

If one further denotes the total deflection of the needle after exchanging a and b with A', so is

$$A' = \frac{m(a+\beta) - n(b+\alpha)}{c(a+\alpha+b+\beta) + (b+\alpha)(a+\beta)} \cdot e \ .$$

From this it follows that if the deflection $A = A'$ is found,

$$(b-a)\cdot[(m+n)(a+b+\alpha+\beta)c + m(a+\beta)(b+\beta) + n(a+\alpha)(b+\alpha)] = 0 \ .$$

But since the second factor enclosed in brackets consists of a sum of positive quantities and therefore cannot disappear, the first factor must

$$b - a = 0 \ ,$$

from which it follows that if the deflection $A = A'$ is found, the resistances a and b are equal, which was to be proven.

1.25

According to the method just considered, the accuracy of comparing the two resistances a and b with each other is completely independent of the size of the observed total effect A, and A can therefore generally have a larger or smaller value or to be zero; however, carrying out such a comparison is much easier if A is quite small or zero, from which it follows for $a = b$ that the ratio of the sensitivity coefficients $m : n$ should be almost equal to the ratio of the resistances $a + \alpha : \alpha + \beta$ in the branch currents, which can best be achieved if both multipliers are made from exactly the same wires, which are wound together in such a way that they form exactly the same turns. The differences $m - n$ and $\beta - \alpha$ will then, if they do not disappear completely, at least be very small. If one now denotes the smallest value of the difference $A - A'$, which can still be observed with certainty, by $\triangle$ and the corresponding value of the difference $b - a$ by x, then the value of x/a can be developed, which gives the smallest fraction up to which the equality of the resistances a and b can be guaranteed using this method.

From the values of A and A' found in the previous Section, the following equation easily results:

$$\frac{\triangle}{ex} = \frac{A - A'}{e(b-a)}$$

$$= \frac{(m+n)[c(a+b+\alpha+\beta) + ab + \alpha\beta + \frac{1}{2}(a+b)(\alpha+\beta)] + \frac{1}{2}(a+b+2\beta)(m-n)(\beta-\alpha) + n(\beta-\alpha)^2}{[c(a+b+\alpha+\beta) + ab + \alpha\beta + \frac{1}{2}(a+b)(\alpha+\beta)]^2 - \frac{1}{4}(b-a)^2(\beta-\alpha)^2} \ ,$$

for which, considering that the differences $b - a$, $m - n$, $\beta - \alpha$ are always very small,

$$\frac{\triangle}{ex} = \frac{m+n}{c(a+b+\alpha+\beta) + ab + \alpha\beta + \frac{1}{2}(a+b)(\alpha+\beta)} \ ,$$

or even more simply

$$\frac{\triangle}{ex} = \frac{2m}{(a+\alpha)(a+\alpha+2c)}$$

can be set, from which is obtained

$$\frac{x}{a} = \frac{(a+\alpha)(a+\alpha+2c)}{2mea}\triangle \ .$$

1.26

After the determination found for the accuracy that is required for resistance comparisons using the method of simple current division, rules for the appropriate construction of the devices and the limits of the accuracy that can be achieved can easily be specified in more detail. In general, it is clear that the rules developed in the second Part apply to the construction of the galvanometer and especially the double multiplier required for it, according to which the multiplier space can be viewed as given, that is, the *product* of the length in the cross section of the multiplier wires. Since according to Ohm's law the *ratio* of length to cross section is proportional to the resistance α, this results in an n^2 value of the resistance α with n times the length. With n times the length, the number of multiplier turns, and thus also the sensitivity m, is increased n times. According to this, m and α can be determined in their dependence on n by the equations

$$m = nm_0 \ , \qquad\qquad \alpha = n^2\alpha_0 \ .$$

If you put these values of m and α into the equation of the previous Section, you get

$$\frac{x}{a} = \frac{\triangle}{2m_0 ea} \cdot \frac{a(2c+a) + 2(c+a)\alpha_0 n^2 + \alpha_0^2 n^4}{n} \ .$$

It can be seen from this that the accuracy of the resistance comparison depends primarily on the choice of multiplier wires, by which the value of n is determined, and that there is a value of n and consequently of α for which that accuracy is greatest or the fraction x/a is smallest, namely,

$$n = \sqrt{\frac{a+c}{3\alpha_0}\left(2\sqrt{1 - \frac{3}{4}\frac{c^2}{(a+c)^2}} - 1\right)} \ ,$$

$$\alpha = \frac{1}{3}(a+c)\left(2\sqrt{1 - \frac{3}{4}\frac{c^2}{(a+c)^2}} - 1\right) \ .$$

In addition, the accuracy increases the smaller c/a becomes, which means that at the same time α and x/a approach certain limit values, namely,

$$\begin{aligned} \alpha &= \frac{1}{3}a \ , \\ \frac{x}{a} &= \frac{8}{9} \cdot \frac{a}{me} \triangle \ . \end{aligned}$$

Now $[3/4] \cdot me/a$ is the value to which, for $\alpha = [1/3] \cdot a$, the value of $me/[a + \alpha + c]$ approaches the more, the smaller c/a; but $me/[a+\alpha+c]$ is the deflection of the needle when the branch current passing through the conductors b and β is released, and would be easily measured if at the high sensitivity m the length of the scale was sufficient. However, the high sensitivity m can be compensated for by a small electromotive force e. If, for example, one finds for an electromotive force $\varepsilon = e/100$ (if, for example, a thermomagnetic element is set for a Grove's element)[65] the deflection $m\varepsilon/[a+\alpha+c] = 1000\triangle$, then in the limit case $[3/4]\cdot me/a = 100000\triangle$, hence $x/a = [8/9]\cdot[a/me]\cdot\triangle = 1/150000$ is the smallest fraction up to which the equality of the resistances a and b can be guaranteed.

It follows from this that the copying method with simple current division allows a duplication of resistance etalons or standards, which may be considered completely identical for all practical applications.

1.27 Copying Method with Double Current Division

The same thing that can be achieved by simple current division according to the previous discussion can also be achieved by double current division, namely, with the Wheatstone *bridge* or *balance*.[66]

The Wheatstone balance consists of a closed conductor, of which four points, Figure 4 of Table I, A, B, C, D, are also connected crosswise.

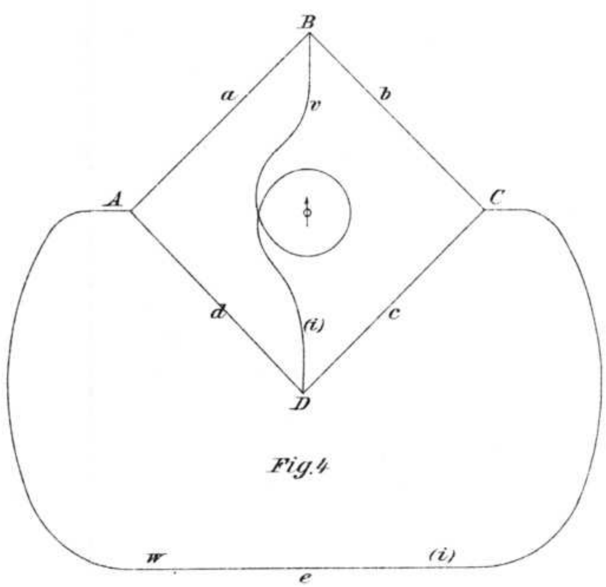


Fig.4

Let the resistors AB, BC, CD, DA be denoted in sequence by a, b, c, d; furthermore with w the resistance of the conductor connecting the first point A with the third C, in which the electromotive force e (a pile) acts and which may therefore be called the undivided conductor; with v the resistance of the conductor connecting the second point B with the fourth D, which is called the bridge and forms the multiplier of a galvanometer; let i denote the current intensity in the undivided conductor, i' denote the current intensity in the bridge. — If the bridge were missing, a current in the undivided conductor from A to C would form the two branch currents ABC and ADC and, in the whole, the resistance $w' = w + (a+b)(c+d)/[a+b+c+d]$; if the undivided conductor were missing, a current in the bridge from B to D would form the two branch currents BAD and BCD and, in the whole, the resistance $v' = v + (a+d)(b+c)/[a+b+c+d]$. — Finally, the real resistance which the current produced by e finds in its entire circuit is denoted by W.

[65][Note by AKTA:] The Grove voltaic cell, element, battery or pile was named after its inventor, William Robert Grove (1811-1896), [Gro39].

[66][Note by AKTA:] See footnote 63.

It is known how, in the theory of the Wheatstone balance, the ratio of the current intensity in the bridge i' to the intensity of the undivided current i is determined from the ratios of the resistances a, b, c, d to the resistance of the bridge v, namely, by the equation

$$\frac{i'}{i} = \frac{ac - bd}{(a+d)(b+c) + (a+b+c+d)v} = \frac{ac - bd}{(a+b+c+d)v'} \ .$$

It follows from this that if the current in the bridge i' disappears, $ac - bd = 0$ or $a : b = d : c$. If i' does not disappear, its value, and consequently that of $(ac - bd)$, should at least be very small. Assuming this, we add to that special equation for the Wheatstone balance the general one given by Ohm's law, namely,

$$i = \frac{e}{W} \ ,$$

and develop the total resistance W in a series progressing after powers of $(ac - bd)$, where, however, with the assumed small value of $(ac - bd)$ all members containing a power greater than the square of this magnitude may be considered as vanishing. You then get

$$W = w + \frac{(a+b)(c+d)}{a+b+c+d} - \frac{b(c+d) + c(a+b)}{d(b+c) + v(c+d)} \cdot \frac{1}{b} \left(\frac{ac - bd}{a+b+c+d} \right)^2 \ ,$$

where $w + (a+b)(c+d)/[a+b+c+d] = w'$. From this it finally follows

$$i' = \frac{ac - bd}{(a+b+c+d)v'} \cdot \frac{e}{W} = \frac{(ac - bd)e}{(a+b+c+d)v'w'} \ .$$

If, as in the previous Section, the deflection of the needle produced by the unit of current intensity in the bridge is called m, then the deflection produced by the current i' is

$$A = mi' = \frac{(ac - bd)me}{(a+b+c+d)v'w'} \ .$$

If the two resistances a and b are now to be compared with each other, then one sets their very small difference $a - b = x$ and also $c - d = \delta$, which also has only a small value for small values of A. Then you get

$$A = \frac{a\delta + cx}{2(a+c)} \cdot \frac{me}{v'w'} \ ,$$

and if a and b are substituted, we get

$$A' = \frac{a\delta - cx}{2(a+c)} \cdot \frac{me}{v'w'} \ ;$$

because the factor $me/[a+b+c+d)w']$ remains completely unchanged in this substitution, as you can see, if you set $w + (a+b)(c+d)/[a+b+c+d]$ for w'; v' remains unchanged at least for small values of x and δ, because $v' = v + (a+d)(b+c)/[a+b+c+d] = v + (a+c)/2 - (\delta + x)/2$ then changes into $v + (b+d)(a+c)/[a+b+c+d] = v + (a+c)/2 - (\delta - x)/2$; finally $(ac - bd) = a\delta + cx$ becomes $(bc - ad) = a\delta - cx$. So it is

$$A - A' = \frac{mec}{(a+c)v'w'} \cdot x \ ,$$

consequently, if $A - A' = \triangle$ denotes the smallest value of the deflection difference that can still be observed with certainty, then one obtains the smallest fraction up to which the equality of the resistances a and b according to these observations can be guaranteed, namely,

$$\frac{x}{a} = \frac{(a+c)v'w'}{meac} \cdot \triangle = \frac{(a+c+2v)(2ac+(a+c)w)}{2meac} \cdot \triangle \,.$$

The smaller the resistances of the bridge v and of the undivided conductor w, the smaller this fraction is, and the smaller v and w become, the closer it approaches the value of

$$\frac{x}{a} = \frac{a+c}{me} \cdot \triangle \,.$$

Now $me/[a+c]$ is the value that $me/[a+c+v+w]$ approaches the smaller $v+w$ becomes; but $me/[a+c+v+w]$ is the deflection of the needle when the branch currents passing through b and d are released, and can be easily observed and measured, even at high sensitivity m of the galvanometer, if the larger electromotive force e used in the observations A and A', as already stated in the previous Section, is substituted with a smaller electromotive force, for example, $\varepsilon = e/100$. If the deflection $m\varepsilon/[a+c+v+w]$ then has a measurable size, for example, $1000\triangle$, then in the limit case $me/[a+c] = 100000\triangle$, consequently $x/a = 1/100000$ the smallest fraction up to which the equality of the resistances a and b can be guaranteed.

It follows from this that the copying method with double current division allows an almost as accurate test of the equality of two resistances a and b as that with single current division, and therefore also enables a duplication of resistance etalons or standards, which for all practical applications can be considered completely identical; but in this respect the method of double division cannot be given any preference over the method of single division. — The method of double division only has a peculiar value when it is not a question of testing *equality*, but rather of determining the unknown ratio of two very different resistances $a : b$, which then, with disappearing deflection A, a known resistance ratio $d : c$ is recognized as equal; however, the accuracy of the result is made dependent on the exact knowledge of the resistance ratio $d : c$, which must be given.

V - On the General Principles of Resistance Measurement

1.28

The principles of galvanic resistance measurement were derived from the nature of galvanic resistance, which is a *property of ponderable bodies*, for example, a copper wire, and therefore had to be derived from the definition given by this property. Such a definition was first established on the basis of Ohm's law, which determines the dependence of the *current intensity* in a ponderable body on the *electrical forces* acting on the electricity contained therein. According to the principles derived from this definition, the method by which the resistance of a given body (a copper wire) can be determined most accurately has been developed in the first Parts of this Treatise. In the last Part it was finally discussed how the resistances of other bodies could be most accurately compared with the resistance thus researched.

All these investigations were based on the first definition of conduction resistance, which is based on the well-known Ohm's law of experience derived from related measurements of *electromotive forces* and *current intensities*, namely, that no matter how different the electromotive forces e and current intensities i, as long as the ponderable body remains the same to which these forces and these currents belong, the quotient e/i always has the same value, while it assumes different values for different bodies, according to which the *constant* value of the quotient e/i for each body is a *property of the body* which can serve to distinguish it from other bodies and is called its *conduction resistance*.

The *property of a ponderable body*, which is hereafter referred to as resistance, must have its *causes* in the peculiar nature of the ponderable body itself, and must in itself be independent of the forces that act on the electrical fluids contained in it, as well as of the movements, into which these fluids are thereby put; however, these *causes*, which lie in the nature of the ponderable body itself, have not yet been investigated. We therefore only know the *effect* of its resistance *from experience*, and from this we only know that, *for a given electromotive force*, it consists of *a certain current intensity*.

But if resistance itself is a property founded in the nature of the ponderable body itself, then *other effects* can also exist that can be proven through experience; for example, the case could take place that such an effect, which can be proven by experience, would be present in any *given* current that passes through the body, regardless of where it comes from or by what forces it is produced. Such a really existing effect, which takes place with every *given* current passing through a body, is called *electrical work*,[67] and the only question is how this effect can be observed and how its dependence on the conduction resistance of the body can be proven.

As experience shows, a current produces *heat* in the wire through which it passes, and heat is, according to the mechanical theory of heat, vis viva equivalent to *work*.[68] If one can now consider the heat generated by a current as *electrical work*, then this electrical work is measurable, as is the current that produces it. Joule and Lenz finally based an *empirical law* on these related measurements of the *intensity of the currents* and the *heat* they generate in the same way as Ohm's law on the related measurements of *electromotive forces* and *current intensities*, namely, the law that no matter how different current intensities i, and no matter how different heat productions A, as long as the ponderable body remains the same, to which those currents and these heat productions belong, the quotient A/i^2 always has the same value, which therefore also, as a *property of the ponderable body*, can serve to distinguish it from other bodies for which this quotient has different values.[69]

If this *second* property could now be considered identical to the *first one*, which was called *resistance* (experience really shows the proportionality of both quotients), then a *second definition of resistance* would be obtained, from which a new one would arise, completely independent of the previously considered principles for resistance measurement. The devel-

[67][Note by AKTA:] In German: *Stromarbeit*. This expression can also be translated as current work.

[68][Note by AKTA:] I am translating the German expression *lebendige Kraft*, literally "living force," by the Latin expression *vis viva* (plural *vires vivae*) also meaning "living force." Originated by Gottfried Leibniz (1646-1716) in the 17th century, the *vis viva* of a body of mass m moving with velocity v relative to an inertial frame of reference was defined as mv^2, that is, twice the modern kinetic energy. During the XIXth century many authors, including Hermann von Helmholtz (1821-1894) and Wilhelm Weber, defined the *vis viva* as $mv^2/2$, that is, like the modern kinetic energy.

[69][Note by AKTA:] James Prescott Joule (1818 - 1889) and Heinrich Friedrich Emil Lenz (1804-1865). See [Jou41b]; [Jou41a] with French translation in [Jou42]; [Len43c], [Len43a], [Len43b], [Len44b] and [Len44a]. See also [MS20] and [Mar22].

opment of a method of resistance measurement based on these new principles would initially have to involve research into, *firstly*, the accuracy of the heat measurement methods to be used, *secondly*, the determination of the equivalence of heat with work, and *thirdly*, the examination of the prerequisite that all electrical work is converted into heat. However, before we go into this new, broad area of research, we still need to discuss in more detail what can be achieved independently of the consideration of heat, based solely on the known general electrical laws.

1.29 Electrical Work According to Electrical Laws

Work is only spoken of when the points of action of forces are moving. The work A of such a point is the product of the component of the force acting upon it, according to the direction of its motion, in the path it has travelled. However, work can be taken in two senses: it means either the work itself or the thing that is worked. According to the given definition, A is work in the latter sense, while work in the former sense is represented by the differential quotient of A with respect to time, that is, is expressed by dA/dt.

With a galvanic current i in a conductor element α, all particles of the electrical fluids contained in α are points of action of the electromotive forces, and these points of action move partly forward and partly backwards in the direction of element α.[70] The work A or dA/dt of all these points of application is the work of the galvanic current i in the conductor element α. The fact that the moving points of application of the forces in this case do not have a ponderable mass is of no importance for the work itself, according to the given definition.

The amount of *positive* electricity contained in the element α is denoted by $+\alpha\varepsilon$, and the force acting on it, which is proportional to it according to electrical law and directed forward in the direction α, is denoted by $+f$, where f is the numerical value which indicates how often the force which gives the ponderable mass unit the unit of velocity in the unit of time is contained therein. — The amount of *negative* electricity contained in the element α is denoted by $-\alpha\varepsilon$, and the force acting on it, directed backwards in the direction α, is denoted by $-f$. — The velocity at which these electrical masses move forward and backward in the direction α shall be denoted by $\pm u$. According to the given definition, the work of *positive* electricity in the element α, during time t, is

$$A' = (+f) \cdot (+ut) = +fut \ ;$$

the work of the *negative* electricity in the element α during the same time,

$$A'' = (-f) \cdot (-ut) = +fut \ ;$$

consequently the entire work of the galvanic current in the element α, during the time t,

$$A = 2fut \ .$$

But for work, taken in the sense of working, you get

[70][Note by AKTA:] That is, according to the usual assumption at that time, positive particles would move in one direction relative to the body of the conductor, while negative particles would move in the opposite direction.

$$\frac{dA}{dt} = 2fu \ .$$

$2f$ is called the *absolute separating force*[71] acting on the electricity in the element α, u is the *absolute current velocity*,[72] both of which can neither be observed nor measured directly.

On the other hand, the so-called *electromotive force* e and the *current intensity* i acting on α are observed and measured according to the absolute units determined earlier.

If the electrical work in α is to be determined, the relationship between the separating force $2f$ and electromotive force e, and also the relationship between the current velocity u and the current intensity i, must be given, which has already been discussed in Section 1.1. It is, as stated there (where only f meant the absolute force of separation, denoted here by $2f$),

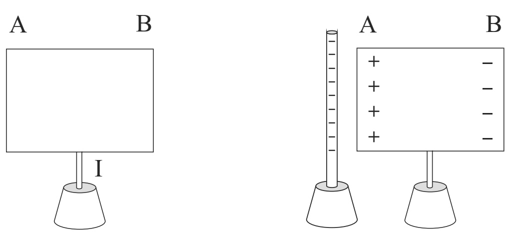
$$\frac{i}{u} = \frac{\varepsilon}{c}\sqrt{8} \ ,$$

$$\frac{e}{2f} = \frac{c}{\varepsilon}\sqrt{\frac{1}{8}} \ ,$$

where c is a *constant velocity* known from the fundamental laws of electrical action, namely, $c = 439\,450 \cdot 10^6$ millimeters/second.

[71][Note by AKTA:] In German: Scheidungskraft. This expression can also be translated as "force of separation" or "segregating force".

I present here a simple example of a separating force. Consider a metal plate AB insulated from the ground by a dielectric support I as in Figure (a) of this footnote:



(a) (b)

If a negatively charged straw is placed close to side A of the plate, the charges on the plate become separated as illustrated in Figure (b). Side A of the plate becomes positively electrified, while side B becomes negatively electrified. This polarization of the plate is caused by the electric force of the negatively electrified straw acting on the free electrons of the plate. I presented several interesting experiments on this topic made with simple material, together with many quotes from original sources, in the 2 volumes of the book *The Experimental and Historical Foundations of Electricity* which is available in English, Portuguese, Italian and Russian: [Ass10a], [Ass10b], [Ass15], [Ass17], [Ass18a], [Ass18b] and [Ass19].

Another effect of a separating force takes place in electrolysis. The electric forces in general are proportional to the charge q of the test particle on which they are acting. A positively electrified particle with $q > 0$ experiences a force in one direction, while a negatively electrified particle with $q < 0$ will be forced in the opposite direction. If these particles are free to move as in electrolysis, a double current will be produced due to this separating electric force. That is, the positive particles will move in one direction and the negative particles will move in the opposite direction.

[72][Note by AKTA:] In German: *die absolute Stromgeschwindigkeit*. From the context of Weber's discussion, this so-called absolute current velocity is the velocity of the electrified particles relative to the body of the conductor, that is, their drift velocity.

This results in $2fu = ei$; consequently, the *resistance according to the second definition*, namely, the quotient of the electrical work dA/dt divided by the square of the current intensity,

$$\frac{1}{i^2} \cdot \frac{dA}{dt} = \frac{2fu}{i^2} = \frac{e}{i} ,$$

is identical to the *resistance according to the first definition*, namely, the quotient of the electromotive force e divided by the current intensity i,

$$\frac{e}{i} = w .$$

So the *electrical work* in a current conductor is $dA/dt = wi^2$, where i denotes the current intensity and w denotes the resistance of the conductor according to the absolute units determined earlier. Conversely, the resistance of a current conductor can be defined in absolute terms as the *work of the unit of current* in the conductor. If, in some way, electrical work wi^2 and current intensity i can be observed independently of one another and measured according to the established absolute units, then from these two measurements one can find the *resistance* according to absolute measure $w = wi^2/i^2$, without any knowledge of the *electromotive force* e by which the current was produced. These principles result in an essentially new method of absolute resistance measurement.

It has already been noticed how the observation and measurement of the heat generated by a current in a conductor can be used to determine the work of the current independently of the intensity of the current; but there is another way where it is not necessary to use the assumptions of the mechanical theory of heat, but where the basic electrical law is sufficient, according to which measurable *work of ponderable bodies* can be converted into electrical work, so that electrical work can be determined by measuring the work of moving ponderable body. However, the closer discussion of this method of measuring the work of electricity should be preceded by a brief consideration of the *maximum of the electrical work*, which results directly from the determination of the work of the electricity given according to electrical laws.

1.30 Maximum of Electrical Work

Let there be a voltaic cell or some other electric motor, which, depending on the difference, does a larger, sometimes a smaller electrical work in the conductor through which it is connected; we search the conductor for which this electrical work is a maximum.

If we denote the resistance of the conductor by w and the current intensity by i, then the electrical work in this conductor according to electrical laws, as shown in the previous Section, is $= wi^2$. According to Ohm's laws, if e denotes the electromotive force and w' denotes the resistance of the given electric motor, the current intensity $i = e/(w' + w)$, therefore, $wi^2 = e^2w/(w' + w)^2$. The conductor is then found for which the electrical work is a maximum if

$$\frac{e^2 w}{(w' + w)^2} = \text{maximum}$$

is set for a variable value of w, from which follows

$$\frac{[(w'+w)^2e^2 - 2e^2w(w'+w)]}{[w'+w]^4} = 0 \ ,$$

that is, $w = w'$. This means that the electrical work in the conductor is greatest when the resistance of the conductor is equal to the given resistance of the electric motor; but this greatest value itself is $= e^2/4w'$, while the entire electrical work, taken together in the conductor and in the electric motor, is $= e^2/2w'$, that is, twice as large. If $w > w'$, the work transferred to the conductor would be more than half of the total electrical work, but would still be smaller, with reduced total electrical work, than if $w = w'$.

However, the *maximum of the entire electrical work* takes place when no conductor is needed at the end of the circuit, and therefore no transfer of electrical work to such a conductor is possible, but the electric motor is closed in itself. This greatest value of the entire electrical work is $= e^2/w'$, which is four times greater than the electrical work that can be transferred to other conductors. This is related to the strong heating of self-contained cells, especially when these cells have a very low resistance in relation to their electromotive force, as is the case, for example, with Grove cells.

Incidentally, it is easy to see that the law previously established for galvanometers, namely, that their sensitivity, regardless of the size and shape of their multiplier, is always greatest when the resistance of the multiplier wire is equal to the resistance of the rest of the circuit, can be considered as individual case or special application of the more general law found for the maximum of the electrical work transferred to conductors.

1.31 Conversion of the Work of Moving Ponderable Bodies into Electrical Work through Electrical Interaction

If a closed conductor is moved against a solenoid, that is, against another closed conductor on which a given electromotive force e acts, then the fundamental law of electrical action results in partly electromotive forces which move the electrical fluids in their ponderable conductors (induction forces according to Faraday),[73] partly in forces that move the electrical fluids with their ponderable conductors (electrodynamic forces according to Ampère).[74]

The *former* or the inductive forces according to Faraday are

1. the electromotive force ε' acting on the *closed conductor* according to the law of *voltaic induction as a result of the movement* of the closed conductor against the solenoid;[75]

2. the electromotive force η' acting on the *closed conductor* according to the law of *voltaic induction as a result of the change in current* in the solenoid;

[73] [Note by AKTA:] See footnote 19.

[74] [Note by AKTA:] Ampère's masterpiece was published in 1826, [Amp26] and [Amp23]. There is a complete Portuguese translation of this work, [Cha09] and [AC11]. Partial English translations can be found at [Amp65] and [Amp69]. Complete and commented English translations can be found in [Amp12] and [AC15]. A huge material on Ampère and his force law between current elements can be found in the homepage *Ampère et l'Histoire de l'Électricité*, [Blo05].

[75] [Note by AKTA:] See footnote 20.

3. the electromotive force ε acting on the *solenoid* according to the law of *voltaic induction as a result of the movement* of the closed conductor against the solenoid;

4. the electromotive force η acting on the *solenoid* according to the law of *voltaic induction as a result of the current change* in the closed conductor.

The *latter*, or the electrodynamic forces according to Ampère, are the attractive or repulsive forces exerted by all current elements of the solenoid on all current elements of the closed conductor.

According to this overview, one has to distinguish, *firstly*, the electrical work dA'/dt of the current i' excited by the electromotive forces $(\varepsilon' + \eta')$ in the *closed conductor*, *secondly*, the electrical work dA''/dt of the current i'' excited by the electromotive forces $(\varepsilon + \eta)$ in the *solenoid*, *thirdly*, finally, the work dA'''/dt carried out by the moving *ponderable* particles of the closed conductor, on which the attractive and repulsive forces exerted by the current elements of the solenoid act.

If the resistance of the closed conductor is denoted by w', then

$$\frac{dA'}{dt} = w'i'^2 = \frac{(\varepsilon' + \eta')^2}{w'} \ ;$$

if the resistance of the *solenoid* is denoted by w and if e is the *given constant electromotive force in the solenoid*, and $i = e/w$ is the intensity of the current excited by this force, then we have

$$\frac{dA''}{dt} = w(i + i'')^2 - wi^2 = \frac{(e + \varepsilon + \eta)^2 - e^2}{w} \ ;$$

finally, if we denote by f the sum of the components of all the attractive and repulsive forces exerted on a moving *ponderable* particle of the closed conductor by all the current elements of the solenoid, according to the direction of the movement, and [if we denote] by v the velocity of this movement, then we have

$$\frac{dA'''}{dt} = \sum fv \ .$$

If one now substitutes here the values of the electromotive forces ε, η, ε', η' known from the general fundamental electrical laws, as well as the electrodynamic forces f, then it must be proved that

$$\int \left(\frac{dA'}{dt} + \frac{dA''}{dt} + \frac{dA'''}{dt} \right) dt = 0 \ ,$$

if the integration is extended over the entire period after which all ponderable particles of the closed conductor return to their previous position with an unchanged velocity.

We restrict ourselves here to considering the simple case where the solenoid as well as the closed conductor are *circles* whose radii may be denoted by r and r'. The distance between the two circle centers is R and is so large that r and r' can be considered vanishing. The connecting line R is perpendicular to the solenoid plane, and the closed conductor rotates around its diameter perpendicular to R, with uniform velocity $d\alpha/dt = \gamma$, where α denotes the angle that the perpendicular made on the plane of the closed conductor makes with R. If you then set $\pi^2 r^2 r'^2 / R^3 = a$, the following expressions for the electromotive forces can easily be derived from the fundamental laws of electrical action:[76]

[76][Note by AKTA:] The expression $\cos\alpha^2$ should be understood as $\cos^2\alpha$.

$$\begin{aligned}
\varepsilon' &= -2a\gamma\frac{e+\varepsilon+\eta}{w}\cdot\sin\alpha\ ,\\
\eta' &= -2a(1-\cos\alpha)\frac{d\varepsilon+d\eta}{wdt}\ ,\\
\varepsilon &= -2a\gamma\frac{\varepsilon'+\eta'}{w'}\cdot\frac{\sin\alpha}{1+3\cos\alpha^2}\ ,\\
\eta &= -2a\sqrt{\frac{1}{3}}\left(\frac{\pi}{3}-\arctan\left[\cos\alpha\cdot\sqrt{3}\right]\right)\frac{d\varepsilon'+d\eta'}{w'dt}\ .
\end{aligned}$$

If one now expands $(\varepsilon'+\eta')$ and $(\varepsilon+\eta)$ in series according to increasing powers of a, one obtains the first members of these series, against which all subsequent members disappear,[77]

$$\begin{aligned}
\varepsilon'+\eta' &= -2a\gamma\frac{e}{w}\cdot\sin\alpha\ ,\\
\varepsilon+\eta &= 4a^2\gamma^2\frac{e}{ww'}\left(\frac{\sin\alpha^2}{1+3\cos\alpha^2}+\sqrt{\frac{1}{3}}\left(\frac{\pi}{3}-\arctan\left[\cos\alpha\cdot\sqrt{3}\right]\right)\cos\alpha\right)\ ,
\end{aligned}$$

and from this, likewise developed,

$$\begin{aligned}
\frac{dA'}{dt} &= 4a^2\gamma^2\cdot\frac{e^2}{w^2w'}\cdot\sin\alpha^2\ ,\\
\frac{dA''}{dt} &= 8a^2\gamma^2\cdot\frac{e^2}{w^2w'}\cdot\left(\frac{\sin\alpha^2}{1+3\cos\alpha^2}+\sqrt{\frac{1}{3}}\left(\frac{\pi}{3}-\arctan\left[\cos\alpha\cdot\sqrt{3}\right]\right)\cos\alpha\right)\ ,
\end{aligned}$$

or, since the differential coefficient

$$\frac{d\left[\sin\alpha\cdot\arctan\left(\cos\alpha\cdot\sqrt{3}\ \right)\right]}{d\alpha}=\cos\alpha\cdot\arctan\left(\cos\alpha\cdot\sqrt{3}\right)-\frac{\sin\alpha^2\cdot\sqrt{3}}{1+3\cos\alpha^2}\ ,$$

we have

$$\frac{dA''}{dt}=8a^2\gamma^2\cdot\frac{e^2}{w^2w'}\cdot\sqrt{\frac{1}{3}}\left(\frac{\pi}{3}\cos\alpha-\frac{d\left[\sin\alpha\arctan\left(\cos\alpha\cdot\sqrt{3}\ \right)\right]}{d\alpha}\right)\ .$$

Finally, if one denotes the distance of any ponderable particle of the closed conductor from its axis of rotation by ρ, then the moment of rotation exerted by the solenoid on the closed conductor is $D=\sum f\rho$, and the velocity at which the ponderable particle moves in its circular orbit (the tangent of which coincides with the direction of the force f), $v=\rho\gamma$; consequently, at *constant angular velocity* γ,

$$\frac{dA'''}{dt}=\sum fv=\sum f\rho\gamma=\gamma\sum f\rho=\gamma D\ .$$

[77][Note by AKTA:] The expression $\sin\alpha^2$ should be understood as $\sin^2\alpha$.

However, the rotational moment D exerted by the solenoid on the closed conductor is according to Ampère's law

$$D = 2a\gamma \sin\alpha \cdot \frac{e+\varepsilon+\eta}{w} \cdot \frac{\varepsilon'+\eta'}{w'} \ ,$$

and if you insert the found values of $(\varepsilon+\eta)$ and $(\varepsilon'+\eta')$ here, and expand into powers of a, you get the first term against which the others disappear,

$$D = -4a^2\gamma \cdot \frac{e^2}{w^2 w'} \cdot \sin\alpha^2 \ ,$$

consequently

$$\frac{dA'''}{dt} = -4a^2\gamma^2 \cdot \frac{e^2}{w^2 w'} \cdot \sin\alpha^2 = -\frac{dA'}{dt} \ .$$

For dA''/dt this results in the integral value $\int [dA''/dt] \cdot dt$ for the time of a whole revolution of the closed conductor, that is, for the time after which all ponderable particles with unchanged velocity return to their previous positions, with *constant* angular velocity $d\alpha/dt = \gamma$,

$$\int \frac{dA''}{dt} dt = \int 8a^2\gamma \cdot \frac{e^2}{w^2 w'} \cdot \sqrt{\frac{1}{3}} \left(\frac{\pi}{3} \cos\alpha - \frac{d\left[\sin\alpha \cdot \arctan\left(\cos\alpha \cdot \sqrt{3}\ \right)\right]}{d\alpha} \right) d\alpha \ ,$$

which is equal to zero when taken between the boundaries α and $\alpha + 2\pi$. Since $dA'/dt + dA'''/dt = 0$, therefore also $\int (dA'/dt + dA'''/dt)\, dt = 0$, it follows from this, between the specified limits,

$$\int \left(\frac{dA'}{dt} + \frac{dA''}{dt} + \frac{dA'''}{dt} \right) dt = 0 \ ,$$

which was to be proved.

It can be seen from this, in relation to the *work of the ponderable particles* of the closed conductor, that at every time interval dt there is a loss of work due to the *damping* caused by the induction,

$$\frac{dA'''}{dt} dt = -4a^2\gamma^2 \cdot \frac{e^2}{w^2 w'} \cdot \sin\alpha^2 dt \ ,$$

which must be replaced by a *driving force* acting on the closed conductor if the angular velocity γ is supposed to remain unchanged. On the other hand, at the same time interval dt there is a gain in *electrical work in the closed conductor*, namely

$$\frac{dA'}{dt} dt = +4a^2\gamma^2 \cdot \frac{e^2}{w^2 w'} \cdot \sin\alpha^2 dt$$

of the same amount, from which it follows that through the mediation of electrical interactions, a pure conversion of *work from ponderable bodies* into *electrical work* has taken place.

If it were to result from observation that the angular velocity γ really remained completely unchanged, and if the *driving forces* were measured, which would have to act on the rotating closed conductor in order to keep this angular velocity unchangeable, both with the solenoid *open* (whereby the driving force required to overcome the resistance of the air and the friction is determined), as well as with the solenoid *closed* (whereby the driving force required to overcome the electrical damping is determined together with that required to overcome the resistance of the air and the friction), then there would be the difference of both measured driving forces, multiplied by the angular velocity γ, which is also easy to measure, the value of

$$-\frac{dA'''}{dt} = \frac{dA'}{dt} ,$$

that is, the value of the *electrical work in the closed conductor*, which the current i' induced therein performed in the unit of time.

If this measurement of the *electrical work* dA'/dt were finally combined with the measurement of the *current intensity* i', the *resistance of the closed conductor*, in absolute value, would result:

$$w' = \frac{1}{i'^2} \cdot \frac{dA'}{dt} .$$

1.32 Determination of the Electrical Work by Means of Heat Measurement, According to Experiments by Becquerel and Lenz

If the resistance of a conductor is to be determined in absolute terms, but not according to the previously used method, by measuring the *electromotive force* and the *current intensity*, but according to the last given method, by measuring the *electrical work* and the *current intensity*, then in general it is as shown, two ways open, depending on the difference in the method by which the *electrical work* is measured. *Electrical work* can be measured, *firstly*, by measuring the *work of moving ponderable bodies*, which is converted into electrical work, which was discussed in the previous Section, and *secondly*, by measuring the *heat* into which the electrical work is converted.

The *first method* was of particular interest because it was based solely on the known laws belonging to the pure theory of electricity. The way in which it is carried out was explained using a simple example in the previous Section, but in reality this would not lead to any practical results. At least the most favorable conditions for the observations required using this method should first be discussed in more detail, but this will not be discussed here because it is easy to overlook in advance that even then, the resistance of the air and the friction of solid bodies against each other are always dependent conditions under which all *ponderable bodies* that we observe move, the measurement of the work done by them, or the driving force necessary to maintain their movement, could not be carried out precisely enough even under the otherwise most favorable conditions.

The *latter method*, in which the laws of the mechanical theory of heat have to be taken as aid, therefore seems practically the only one from which one may expect such precise determinations of *electrical work* as would be necessary to determine a conduction resistance from *electrical work* and *current intensity* as precisely as from *electromotive force* and *current intensity*. It is therefore of interest to take a closer look at what has been achieved in this way in recent times through the numerous experiments carried out on it, particularly by Becquerel and Lenz.

Edmond Becquerel states in his Treatise: *Des lois du dégagement de la chaleur pendant le passage des courants électriques à travers les corps solids et liquides* (The laws of heat production during the passage of electric currents through solid and liquid bodies) (Annales de Chimie et de Physique, 1843, Volume IX)[78] that, according to his experiments, a current which, if it were passed through water, would produce 3.383 cubic centimeters of oxyhydrogen gas[79] every minute, at a temperature of 0° and a barometer reading of 0.76 meter, would produce in a platinum wire 44 centimeters long and weighing 0.422 grams, as much heat per minute as 2.18523 grams of water need to raise its temperature by 1 degree.

If we add to this information the determination found by Joule,[80] based on the mechanical theory of heat, according to which the amount of heat which can warm 1 kilogram of water from 0° to 1° when converted into mechanical work has a working quantity of 423.55 kilogram-meters, it is found that the heat generated in *every minute* by the specified current in the platinum wire, when converted into mechanical work, has a working quantity of $2.18523 \cdot 0.42355$ kilogram-meters, so the heat generated *every second* gives the 60th part of this. This results in the *electrical work* according to the *absolute work measure*, which we reduce to millimeters, milligrams and seconds as the basic measures of length, mass and time (according to which the gravity $g = 9811$ millimeters/second^2) as given by

$$wi^2 = \frac{1}{60} \cdot 9811 \cdot 2.18523 \cdot 0.42355 \cdot 10^9 = 151340 \cdot 10^6 \ .$$

As far as the *intensity of the current* is concerned, we use the statement that the intensity of a current that decomposes 1 milligram of water in one second, is $106\frac{2}{3}$ times larger than the absolute intensity unit (see Treatises of the mathematical-physical class of the Royal Saxon Societies of the Sciences, Vol. 3, p. 224).[81,82] If you now calculate that 1 milligram of water decomposes and produces 1.8568 cubic centimeters of oxyhydrogen at a temperature of 0° and a barometer reading of 0.76 meters, then the intensity of the current described, which produces 3.383 cubic centimeters of oxyhydrogen *every minute*, in absolute units, is

$$i = \frac{1}{60} \cdot \frac{3.383}{1.8568} \cdot 106\frac{2}{3} = 3.2391 \ .$$

From these determinations the *absolute resistance of the described platinum wire* finally emerges

$$w = \frac{wi^2}{i^2} = \frac{151340 \cdot 10^6}{3.2391^2} = 14425 \cdot 10^6 \ .$$

This resistance, multiplied by the mass of a millimeter-long piece of wire $= \frac{422}{440}$ and divided by the length of the wire $= 440$ expressed in millimeters, gives the resistance of a platinum wire of 1 millimeter length and 1 milligram mass according to Ohm's laws, that is, the *specific resistance of platinum*

$$p = 31\,443\,000 \ .$$

Lenz, in his Treatise: *Ueber die Gesetze der Wärmeentwickelung durch den galvanischen Strom* (On the law of the heat development through the galvanic current) (Poggendorff's

[78][Note by AKTA:] Edmond Becquerel (1820-1891) was a French physicist, see [Bec43].

[79][Note by AKTA:] In German: *Knallgas*. This expression can also be translated as explosive gas.

[80][Note by AKTA:] See footnote 69.

[81][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 614.

[82][Note by AKTA:] [KW57, p. 614 of Weber's *Werke*] with English translation in [KW21]. See also [Web41] and [Web42] with English translation in [Web21b, p. 199, footnote 357].

Annalen, 1843-44, Vols. 59, 61)[83] gives the time for heating 1 gram of water to 1° R[84] through a wire of resistance = 1, through which a current = 1 passes, as $57\frac{1}{2}$ minutes (due to a misprint, as it appears, $5\frac{3}{4}$ seconds is given), whereby *the unit of resistance* has been attributed to a copper wire of 6.358 feet length and 0.0336 inches in diameter, at a temperature of 15°, and *the unit of intensity* has been attributed to a current whose electrolytic action = 41.16 cubic centimeters of oxyhydrogen per hour, at a temperature of 0° and a barometric pressure of 760 millimeters.

According to the mechanical theory of heat, in accordance with Joule's determination already mentioned, the heat generated by the assumed unit of current every second in the described copper wire, when converted into mechanical work, gives a *work quantity* = $[5/4] \cdot [1/(60 \cdot 57.5)] \cdot 0.42355$ kilogram-meter, that is, according to *absolute work dimensions* (reduced to millimeters, milligrams and seconds as the basic measures of length, mass and time), the *electrical work*

$$wi^2 = 9811 \cdot \frac{5}{4} \cdot \frac{1}{60 \cdot 57.5} \cdot 0.42355 \cdot 10^9 = 1506 \cdot 10^6 \ .$$

Furthermore, for the *assumed unit of current*, the electrolytic action of which corresponded to 41.16 cubic centimeters of oxyhydrogen gas per hour, the value is found after reduction to *absolute measure*

$$i = \frac{1}{3600} \cdot \frac{41.16}{1.8568} \cdot 106\frac{2}{3} = 0.65683 \ .$$

From these determinations the *absolute resistance of the described copper wire* finally results

$$w = \frac{wi^2}{i^2} = \frac{1506 \cdot 10^6}{0.65683^2} = 3490 \cdot 10^6 \ .$$

If you calculate the mass of the described 6.358 English feet = 1938 millimeter long copper wire to 9889 milligrams by assuming the density of the copper = 8.921, then according to Ohm's laws you get w by multiplying the resistance found by the mass of a 1 millimeter long piece, = $\frac{9889}{1938}$, and dividing by the length of the wire expressed in millimeters, = 1938, the resistance of a copper wire 1 millimeter long and 1 milligram mass, that is, the *specific resistance of the copper*

$$\varkappa = 9\,190\,000 \ .$$

This result, compared with that derived from Becquerel's experiments, would show that the resistivity of copper would be about $3\frac{1}{2}$ times smaller than that of platinum, while it is known from numerous direct comparisons that it is still much smaller, namely, according to Arndtsen's experiments, if one reduces the information given for equal wire lengths of the same cross-section to equal wire lengths of the same mass, and thereby assumes the density ratio of copper to platinum as 1 : 2.244, 15.22 times smaller, and after Matthiessen's experiments are 15.93 times smaller, on average 15.575 times smaller.[85] According to this,

[83] [Note by AKTA:] See footnote 69.

[84] [Note by AKTA:] That is, 1° Réaumur.

[85] [Note by AKTA:] Adam Frederik Oluf Arndtsen (1829-1919) was a Norwegian professor and physicist who was trained by Weber at Göttingen in 1857. Augustus Matthiessen (1831-1870) was a British chemist and physicist. See also [RA11].

the specific resistance of copper

$$\varkappa = \frac{p}{15.575} = 2\,018\,800$$

would be calculated from Becquerel's experiments, which is quite close to the average of the values found so far in other ways for various types of copper, but is exceeded by $4\frac{1}{2}$ times in magnitude by the value derived from Lenz's experiments.

However, Lenz himself remarks at the place cited in relation to the absolute magnitude of the result derived from his experiments:

> This result is merely an approximation, and can only serve as a rough estimate, for neither the absolute quantity of the spirit nor its heat capacity have been determined with certainty. My present experiments had no other purpose than to determine the law of the heating of metal wires; for the exact determination of the absolute value of this heating, I intend to undertake special experiments.

It is therefore very likely that with the care Lenz otherwise took in all respects in these experiments, simply because the attention was less focused on *absolute* value determinations, some accidental confusion took place in the value of the reduction coefficients, which had no influence on the sole purpose of establishing the laws, which is to blame for the above great deviation in *absolute* value; for a closer examination of the experiments clearly shows that the determination of the resistance of a body using this method can be carried out, which also seems to be confirmed by the good agreement of the result derived from Becquerel's experiments with those found by other means; however, in order to obtain completely reliable and precise results in this way, the *heat measurement methods* would have to be very perfected and sharper determinations about the *equivalence of heat and work* than are currently available would have to be obtained, and even then, the absolute resistance measurement of a *conductor wire* using this method would not achieve the accuracy of the result that can be obtained by measuring electromotive force and current intensity.

But if one divides the galvanic conductors into *metallic ones, which cannot be decomposed by the current*, and *moist ones, which can be decomposed*, it follows that with moist decomposable conductors, for example with *water*, an inverse relationship to that with *conductor wires* takes place, namely, that a *direct* determination of the resistance of wet conductors by measured electromotive force and current intensity is almost impossible, and what's more, even an *indirect* determination by comparing the unknown resistance of the wet conductor with that known resistances of a conductor wire, because of the so-called *polarization* of the metal surface touching the wet conductor, finds great difficulties. It is known that, despite all the effort and care expended, the resistance conditions of wet conductors are still very poorly researched. For these reasons, the *other method of resistance measurement* gains the greatest importance for this research, namely, through measured electrical work (heat) and measured current intensity, because when applied to *wet conductors* it has just as great advantages over the former, as the *former*, when applied to *conductor wires*, owned before the second. These advantages are based not only on the more perfect heat measurement methods applicable to wet conductors (water), but primarily on the independence of the entire measurement from the consideration of the *electromotive force*, which must always be considered as variable in all circuits where wet conductors are connected, because the influences of polarization can be reduced, but not completely eliminated. However, the *electromotive force* cannot be precisely determined with such irregular changes.

This equally important and interesting application that this second method finds in the absolute resistance measurement of *moist, decomposable conductors*, should be reserved for a special discussion, since it has no closer connection with the subject of this Treatise.

1.33 On the Conversion of Electrical Work into Heat

The electrical work is linked to the movement of the electrical fluids; according to the mechanical theory of heat, heat is also linked to the movement of a body, which, however, is usually distinguished from electrical fluids. A closer insight into the way in which electrical work is converted into heat therefore first requires that the movements of the electrical fluids be followed closely to the end, in order to get to know the conditions under which the transition of the movement of the electrical fluids into the movement of another medium takes place. The ideal assumption of the superposition of several substances continuously and uniformly distributed in the space of the conductor, namely, the ponderable substance of the conductor, the two electric fluids and also that of a so-called heat medium, however appropriate it may be for many other purposes in the case of action at a distance effects, would not seem to be permissible; rather, it is easy to see the need to assume that the ponderable substance of the conductor is concentrated in individual molecules, which are surrounded by electrical particles which, in the case of a current, move from one molecule to the other. The separation of an electrical particle from a molecule must then take place either more slowly or more quickly, depending on the different magnitude of the electromotive force by which the current is produced, on which the number of electrical particles separating from the molecule in a certain time depends. The work of each electrical particle in the separation movement, as a result of the forces exerted on it by the molecule, may or may not depend on the velocity of the separation; an oppositely equal work will always be done by the same particle in its union movement with the following molecule, so that these two work quantities compensate each other. But as soon as the electric particle is separated from the first molecule, it will, driven by the electromotive force f, pass through the space α to the second molecule and thereby perform the work $f\alpha$. The sum of all of these work quantities, $\sum f\alpha$, forms the total electrical work in the conductor. Each electric particle therefore enters the area of the following molecule with a vis viva that is greater than the value equivalent to $f\alpha$ when it left the area of the previous molecule, which means that the value of the vires vivae in the area of all molecules taken together must be increased by an amount equivalent to the entire electrical work. However, according to the mechanical theory of heat, such an increase in the vires vivae in all molecules taken together, which is equivalent to the electrical work, is also the heat generated by the current in the conductor, and the only question is whether it is completely identical to that, that is, whether it consists in the continuous movement of those electrical particles themselves, or whether the movement supplied to each molecule is transmitted from the electrical particles which they brought with them to other body particles, for example to the particles of a special medium located in the area of the same molecule and only after this transfer emerges as heat, where the laws of transfer would then have to be researched and a closer account given as to why the same vis viva only emerges as heat when it is linked to the particles of the heat medium instead of to electrical particles.

One can easily see that the assertion of such a transfer of the vis viva brought by electrical particles to the particles of another medium located in the region of the molecule does not encounter insignificant difficulties, especially because the continuance of the motion of the electric particles in the region of such a ponderable molecule would then have to be consistently denied. If the electrical particles that carry the electrical work with them, upon entering the area of a ponderable molecule, must immediately give up the electrical work they have brought with them, and not just partially, but entirely, to other material particles (to the particles of the heat medium), for the same reason, any movement given to the electrical particles in the area of ponderable molecules, regardless of where it may come from, must be immediately withdrawn from them again, so that no *persistent movement of electrical particles* in the area of ponderable molecules would be possible. This would even make the possibility of electric current in the ponderable body doubtful; for an electric particle, even if it were driven by electromotive forces no matter how large it was, could not get into any major movement if every movement that was created was immediately transmitted from it to the particles of the heat medium.

It is clear from this, that the assertion that all electrical work is transferred to the heat medium of ponderable molecules is, above all, in total contradiction with the assertion of the existence of *persistent electrical molecular currents*, as first put forward by Ampère.[86] So whoever denies with Ampère the real existence of two magnetic fluids and is thereby forced to assert *persistent electrical molecular currents*, must not admit this transfer, and he needs to admit it all the less because nothing can be cited that would be gained through such a transfer. At least according to the mechanical theory of heat, it is clear that in relation to heat, in principle, nothing else is *directly* relevant except the vis viva present in the molecules, for which the nature of their material support is indifferent. Only *indirectly*, according to the mechanical theory of heat, could the nature of the material carrier of the vis viva that forms the essence of heat come into consideration, namely, insofar as the forces of interaction of the particles of this carrier, partly with one another and partly with other particles, and consequently the transfer- or propagation laws (the laws of heat radiation, temperature communication and temperature equalization among different ponderable molecules) would depend on it.

If also the *heat ether in the empty space* is at least indirectly defined by the laws of wave propagation attributed to it, like the light ether, and its existence and distribution, also in the interior of the ponderable bodies, in the empty spaces between the molecules, cannot be abstracted without rejecting the whole wave theory of radiant heat, then there is no further relationship between the ponderable body molecules (with everything that lies in their area and belongs to it) and that ether than, *on the one hand*, the wave excitation in the ether (the heat radiation), *on the other hand*, the wave attenuation (heat absorption) must come from the ponderable molecules, for which a special heat medium is no more necessary in the molecules than air needs to be contained in the metal of the bell, which emits sound waves through the air medium.

All of these considerations can be briefly summarized as follows. Since, according to the mechanical theory of heat, an increase in the temperature of the ponderable molecules requires an increase in the vis viva in the molecules, since this increase in the vis viva is given by the electric particles which enter with greater velocity into the area of the molecules and exit with lower velocity, which form the current, since furthermore this increase in vis viva *remains* unweakened *according to the theory of persistent electrical molecular currents*, while the particles are in the area of the molecules, then it seems that there can be no question of a *conversion* of electrical work into heat, but that *electrical work* accumulated in the molecules then seems to itself have to be viewed as the *heat* contained in the molecules.

It is, of course, obvious that the laws of the relationships summarized under the name

[86][Note by AKTA:] See footnote 74.

of *heat radiation* and *heat absorption* between the electricity in a persistent molecular flow around the individual molecules and the heat ether in the surrounding space still require a more detailed justification based on the nature of both media; but those laws would also require just the same justification if the so-called heat medium were substituted for electricity. While in the latter case such justification has not even been attempted, as far as the former case is concerned one can use the astute investigation carried out by C. Neumann: *Explicare tentatur quomodo fiat ut lucis planum polarizationis per vires electricas vel magneticas declinetur*, Halis Saxonum, 1858,[87] as such a first attempt; for it suggests that what Neumann says about the relationships between persistent electrical molecular currents and light ether will also apply in a similar way to the relationships between persistent electrical molecular currents and heat ether.

According to his premises, Neumann found that there could be no effect of electrical molecular currents on *resting ether particles*; however, it should be noted that these premises, in accordance with the purpose of Neumann's investigation, which was limited to the influence of the molecular currents on the already existing wave trains propagated through the ether between the molecules, relate to the effects of the molecular currents in very small distances, but still allowed the admission of an *ideal* conception of molecular currents, according to which they are viewed as a *superposition of oppositely equal currents of positive and negative electricity*, which is apparently not permitted when it comes to the excitation of new wave trains by the electrical molecular currents, which can only take place in the *ether layer immediately adjacent* to the molecular currents. For this ether layer, the positive and negative electrical particles moving in opposite directions can no longer be considered as coinciding. If one then imagines, for example, that the negative fluid is firmly connected to the molecule, and the positive fluid is only understood as a molecular flow, or vice versa (a way of thinking that is recommended because it can exist with the persistence of the molecular flows without electromotive forces), it is clear that the difference in the position and behavior of the two electrical fluids in the area of the molecule no longer needs to be taken into account even at very small distances (as Neumann considers), on which the admissibility of that *ideal* conception of molecular currents is based, however, it can be of importance for the *immediately adjacent ether layer*, especially if the electrical fluid in the molecular flow was *not continuously and uniformly* distributed around the molecule.

But if there really is a disturbance of the equilibrium in the *immediately adjacent ether layer*, and consequently an excitation of ether waves, then it is obvious that this will be repeated with every revolution of the electricity around the molecule, that is, the *wave period* with the *revolution period of the electric particles* in the molecular current must match. In the case of *luminous molecules*, however, the duration of the wave trains emitted by them is known precisely from optical experiments; if the assumed relationship between electrical molecular currents and the light ether, according to Neumann's idea, were confirmed, it would then be possible to obtain more detailed information from optical experiments about the behavior of the electricity forming the molecular currents. — In any case, Neumann's investigation was so successful in its initial development for optics, *to explain the rotation of the plane of polarization through galvanic and magnetic forces*, that one can hope that the further pursuit and development of the theory of persistent electrical molecular currents in their relationship to the light or heat ether and its wave movement will lead to many other insights concerning the important and still so little researched connection between *electricity*, *heat* and *light*.

[87][Note by AKTA:] [Neu58], see also [Neu63].

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