On the Motions of Electricity in Bodies of Molecular Constitution

Wilhelm Weber

Editor's Note: An English translation of Wilhelm Weber's 1875 paper "Ueber die Bewegung der Elektricität in Körpern von molekularer Konstitution".[1]

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[1] [Web75].

Contents

Chapter 1

On the Motions of Electricity in Bodies of Molecular Constitution

Wilhelm Weber[2,3,4,5]

In my First Treatise on Electrodynamic Measurements in the year 1846,[6,7] I presented a general law of *electric force* that encompassed both electrostatics and electrodynamics, and I later showed how it implied some special consequences, namely: *first of all*, I published the general *potential law* of electric forces in these *Annalen*, Vol. 73, p. 229 (1848),[8,9] *secondly*, the *energy principle* and its connection with those general laws of electric force, in the last Treatise in Electrodynamic Measurements in the year 1871,[10,11] and *thirdly* and finally, on the *capacity for doing work*[12] for two electric particles, as an equivalent vis viva,[13] in the Jubilee volume of these *Annalen* in the year 1874, p. 199.[14,15] Some further discussions of those topics shall be appended here that could not find their place in the latter Treatises. In particular,

[2][Web75].

[3]Translated by D. H. Delphenich, feedback@neo-classical-physics.info and http://www.neo-classical-physics.info/index.html. Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[4]The Notes by Wilhelm Weber are represented by [Note by WW:]; the Notes by H. Weber, the editor of the fourth volume of Weber's *Werke*, are represented by [Note by HW:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[5][Note by HW:] Annalen der Physik und Chemie, edited by J. C. Poggendorff, Vol. 156, Leipzig, p. 1-61.

[6][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 25.

[7][Note by AKTA:] [Web46] with a partial French translation in [Web87] and a complete English translation in [Web21a].

[8][Note by HW:] Ibid., Vol. III, p. 245.

[9][Note by AKTA:] [Web48] with English translations in [Web52], [Web66], [Web19] and [Web21c].

[10][Note by HW:] Ibid., Vol. IV, p. 247.

[11][Note by AKTA:] [Web71] with English translations in [Web72] and [Web21b].

[12][Note by AKTA:] In German: *Arbeitsfähigkeit*. This expression can also be translated as working capacity.

[13][Note by AKTA:] We translate the German *lebendige Kraft*, literally "living force," by the Latin term *vis viva* (plural *vires vivae*) also meaning "living force." Originated by Gottfried Leibniz (1646-1716) in the 17th century, the *vis viva* of a body of mass m moving with velocity v relative to an inertial frame of reference was defined as mv^2, that is, twice the modern kinetic energy. During the XIXth century many authors, including Hermann von Helmholtz (1821-1894) and Wilhelm Weber, defined the *vis viva* as $mv^2/2$, that is, like the modern kinetic energy.

[14][Note by HW:] Ibid., Vol. IV, p. 300.

[15][Note by AKTA:] [Web74].

after some prefatory remarks and addenda to the last two Treatises that are concerned with the objections that have been raised against the general law of electric force that was presented in the First Treatise, *the motions of electricity in bodies of molecular constitution* shall be treated since the results of so many other researches lead to their consideration and necessitate that they themselves should be the subject of a thorough special investigation that can hardly be avoided, despite the narrow limitations that mathematics seems to presently impose on that investigation.

1.1 Remarks on the Basic Laws of Electricity that were Exhibited in the Treatise on Electrodynamic Measurements in the Year 1871, Section 4

In the second and third Section of the cited Treatise in the year 1871,[16] the *law of electric force*, which was presented in the Treatise from the year 1846, was considered, and in the context of its connection with the much-simpler *law of the electric potential* in these *Annalen*, Vol. 73, p. 229. However, since even the latter law is still lacking in the simplicity that one would desire in a *fundamental law*, that potential law was analyzed more precisely in the fourth Section of that same Treatise, in which an attempt was made to resolve that law into components that would possess the simplicity of fundamental laws, namely, the law of the dependency of the potential of two particles upon the distance between them when they are *in the same state of relative motion*, and the law of the dependency of the potential on the relative motion *at a certain distance*, but in that way, the determination of that distance would essentially necessitate a third law, namely, the *law of electrostatics*, which already possesses the desired simplicity of a fundamental law.

The fundamental laws of electricity, including electrostatics, that are presented in the fourth Section are the following *three*:

First law: When two electric mass-particles ε and ε' at a distance of r from each other are in a state of relative rest, they will exert a force on each other that is directly proportional to the product of their masses $\varepsilon\varepsilon'$ and inversely proportional to the square of that distance r^2 in the direction of r, so it is equal to $\mu^2 \cdot [\varepsilon\varepsilon'/r^2]$. — If one sets $\mu\varepsilon = \pm e$ and $\mu\varepsilon' = \pm e'$, where the upper or lower sign will be valid according to whether the mass-particle carries positive or negative electricity, respectively, so the expression for the force ee'/r^2 will be positive or negative according to whether force is one of repulsion or attraction.

Second law: When two electric particles e and e' are found in a state of relative rest or relative motion of equal magnitude (so they will possess equal relative vires vivae) at the distances r' and r'' at different points in time, they will do amounts of work V' and V'' by their reciprocal action when both particles can be brought from the given distances r' and r'' to infinity, that are in inverse proportion to the given distances, i.e.:

$$V' : V'' = r'' : r' \ .$$

Third law: The work U that would be done under the action of the forces that the particles e and e' exert upon each other when the particles are at a *certain distance apart proportional*

[16][Note by AKTA:] See footnote 11.

to ee', i.e., $\rho = ee'/a$,[17] in which they possess a certain vis viva x, that will define a constant sum when combined with U, namely, a, when they are shifted to an infinite distance, i.e.:

$$U + x = a \ .$$

In regard to those laws, it should be noted that, *first of all*, the quantity U that was introduced into the third of them, which defines the constant sum a with the vis viva x, just like the other two quantities x and a, is *always positive*, regardless of whether the two electric particles e and e' have *the same* or *unequal* types.

That *positive* value results from the equation that is implied by the second law $U = [r/\rho]V$, (in which[18]

$$V = \left(\frac{ee'}{r}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right]\right)$$

expresses the *potential* of the two particles e and e', and $\rho = ee'/a$ is a *distance that can be determined from the nature of the electricity and the particles* e *and* e'), since the quantities V and ρ will be either both positive or both negative according to whether e and e' have the same or different types, respectively, so the quotient V/ρ will then remain *always positive.* It should be pointed out that when one has any concerns about whether the calculation admits a *negative value of* ρ for the distance between two points, as was remarked in a Note on the third law, instead of $\rho = ee'/a$, which is negative when the electric particles have unequal types, one can set $\rho = \mu\varepsilon \cdot \mu\varepsilon'/a$, which is *always positive*, but then at the same time, U must not be set equal to the *work* done

$$\left(\frac{ee'}{\rho}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right]\right) \ ,$$

which is *negative* for particles of unequal type, but must be equated to the *absolute value* of this work:

$$\pm\left(\frac{ee'}{\rho}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right]\right) = \mu^2\left(\frac{\varepsilon\varepsilon'}{\rho}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right]\right) \ .$$

Secondly, it should be noted that when the fundamental law of electrostatics is added to the electrodynamic one, as was done here, one of the electrodynamic laws, namely, the law of the dependency of the potential of both particles on the distance between them *for the same relative motion*, can be dropped completely since it is, in fact, essentially included in the other two laws already and can be derived from them.

That is because the *first* law, viz., the fundamental law of electrostatics, will imply the *potential* V for $x = 0$ as a function of the three variable quantities e, e', r, and indeed they are proportional to three factors E, E', and R, each of which includes only one of those quantities, so the value of V at $x = 0$ will be determined from the following equation:

[17] [Note by WW:] In place of ee', one can set $\mu\varepsilon \cdot \mu\varepsilon'$ (namely, the *absolute value* of ee'), and in that way, one can ensure that r always has a *positive* value. However, one would then have to take the *absolute value* of the work done U in order for the stated law to be valid.

[18] [Note by AKTA:] The next equation should be understood as:

$$V = \left(\frac{ee'}{r}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr}{dt}\right]^2\right) \ .$$

The same thing should be understood in some of the following equations.

$$V = A \cdot EE'R\ .$$

From the *third* law, however, it follows that the *potential* V for $r = \rho$ depends on the variable quantities e, e', x, and [they] are proportional to three factors E, E', X, each of which contains only *one* of these quantities, according to which V, for $r = \rho$, is determined by the following equation:

$$V = B \cdot EE'X\ .$$

Now, one has $E = e$, $E' = e'$, $R = 1/r$, $X = (1 - x/a)$ in that, and in addition, that gives the value of the constant A as equal to the value of X when $x = 0$, and the value of the constant B as equal to the value of R when $r = \rho$.

One can conclude from this that E, E', R, X, or e, e', $1/r$, $(1-x/a)$, are always factors of V, and that gives only the possibility of yet another factor, namely, the factor $(1 + f(r,\ x))$, in which $f(r,\ x)$ must be a function of r and x that vanishes for $r = \rho$, as well as for $x = 0$.

Thus, $V = EE'RX = (ee'/r)(1 - x/a)$ is, in any event, the *simplest* way of determining V that the first and third laws admit and is implied by those two laws independent of the second law, which can itself be derived from the determination of $V = (ee'/r)(1 - x/a)$ that was just achieved. That is because that determination will imply that for two values of V, namely, V' and V'', which are true for equal values of x, but different values of r, namely, r' and r'', one has the following proportion:

$$V' : V'' = \frac{ee'}{r}\left(1 - \frac{x}{a}\right) : \frac{ee'}{r''}\left(1 - \frac{x}{a}\right) = r'' : r'\ ,$$

which is *entirely identical* to the second law.

However, one complication of the law that would arise by adding a factor of $(1 + f(r,\ x))$ is deemed to be allowable with no proven necessity of any sort.

That implies the *result* that instead of the three fundamental laws that were cited above, only two of them will already suffice, namely:

1. The fundamental law of electrostatics, and

2. The principle of energy;[19]

since one easily sees that the fundamental law that was placed last above as the *third one* is the *principle of energy* itself, whose essence consists of the fact that the *relative vis viva* x of two particles e and e' will indeed be bigger or smaller, but that in addition to that vis viva of the two particles, an *equivalent vis viva* will also be present that will experience a reduction with each increase in the vis viva, and conversely, such that the *sum of that vis viva and the simultaneously-present equivalent one will have a constant value* that will be denoted by a. At the same time, one sees that the quantity that was denoted by U in the statement of the fundamental law above is the *equivalent vis viva that is present in addition to the vis viva* x *of the particle-pair.*

C. Neumann arrived at a similar result to the one that was found here in his "Principles of Electrodynamics", Tübingen, 1868,[20] in such a way that quite the same thing that was

[19][Note by AKTA:] What Weber called here the principle of energy is nowadays called the principle of the conservation of energy.

[20][Note by AKTA:] [Neu68a] with English translation in [Neu21b]. See also [Neu68b] and [Neu69] with English translation in [Neu21a].

achieved here by means of the *principle of energy*, in conjunction with the fundamental law of electrostatics, was achieved by the *law of propagation of the potential* that he posed, in conjunction with the fundamental law of electrostatics. In so doing, a connection between that principle of energy and the law of propagation of the potential was derived that would seem to lead to an explanation for the one in terms of the other. For that law of propagation of the potential, one can see also the *Mathematischen Annalen*, Vol. I, p. 317 and the *Abhandlungen der König. Sächs. Gesellschaft der Wissenschaften*, XVIII, p. 103, et seq.[21]

1.2 Remarks on the Essay in the Jubilee Volume of These Annalen, p. 199

After distinguishing the properties of individual particles from the properties of particle-pairs, the theorem was expressed in the cited essay[22,23] that a system of *three* or more particles would possess no properties that were not included already in the properties of individual particles and pairs, and it was accordingly proposed that it would be a peculiarity of a true *fundamental law* that nothing should come under consideration in it beyond the *nature* and *mutual relationships* of the particles that form a pair, and the *work that is done by their interaction under any change of distance under those relationships.* For a *fundamental law* that is represented in that way, only the *time* t, the *relative distance* r between the two particles, their *relative velocity* dr/dt, and functions of those quantities should then come under consideration as variable quantities.

Having assumed that, one must infer the demand that must be imposed upon the *principle of energy* as a fundamental law, that it must be valid for a particle-pair under all relationships that they might be found in, regardless of whether they alone are present or arbitrarily-many particles besides them, and that in the latter case, neither the nature nor the relationships between the other particles can come under consideration in the expression of the principle.

Therefore, the *principle of energy*, as a fundamental law, can deal with only the particle-pairs themselves and the *energies* that are associated with them exclusively. One such energy is the relative vis viva of the particle-pair which is called its *energy of motion*. However, since that energy of motion of a particle-pair changes, the principle of energy will necessarily assume the existence of *another energy* for the particle-pair in order for an *unvarying total energy* to be possible. That *other energy* must likewise change, and in such a way that a reduction in it will always be linked with an increase in the energy of motion, and conversely. The essence of that *second energy* will then consist of the idea that *new vis viva will be created, or existing vis viva will be annihilated* as a result of that increase or decrease, respectively.

However, *new vis viva will be created or existing vis viva will be annihilated by work*, for example, by the work that is done by the interaction of the two particles themselves under any change in the distance between them. Nonetheless, the actual existence of such work assumes the *capacity to do work*[24] that is based in the interaction of the particles. That would imply the capacity to create or annihilate vis viva, and that *capacity to do work* that is to be measured according to the amount of vis viva that can be created or annihilated is

[21] [Note by AKTA:] [Neu69] with English translation in [Neu21a].

[22] [Note by HW:] Wilhelm Weber's *Werke*, Vol. IV, p. 300.

[23] [Note by AKTA:] [Web74].

[24] [Note by AKTA:] In German: *Arbeitsvermögen*. This expression can also be translated as ability do to work, working capacity, work capacity, energy capability or energy capacity.

the *second energy of the particle-pair*, which will be larger or smaller according to whether the *first energy*, namely, the vis viva of the particle-pair, is smaller or larger, respectively, such that the *sum of both energies*, namely, the *vis viva* and the *capacity to do work* of the particle-pair will remain unchanged.

We then infer the following detailed conditions for the definition of the *capacity to do work as a form of energy* from that: *First of all*, the capacity to do work of two particles e and e' is a *property that they always possess* for a *given energy of motion* (that is, for a given relative vis viva of the particles).

Secondly, the magnitude of that property will be determined by the work that would be done as a consequence of the interaction of both particles *under a certain change in distance between them that must be determined more precisely.* However, that *change in distance between them that must be determined more precisely* is not one that *actually* takes place or even can take place (namely, it must begin with the distance r that is present), but only a *virtual change in distance* that begins with a distance ρ at which the particles must be thought of being shifted to, and which is determined completely independently of the existing distance r. That is because the work that would be done by a change in distance that begins with the existing distance r could not serve to determine the magnitude of the capacity to do work since it would depend upon r, and therefore would also assume different values with r *for an unchanged relative vis viva of the particles*, whereas for a fictitious shift of the particles to an always-equal distance ρ that can be determined independently of the existing distance r, a change in distance can be thought of as taking place between fixed limits ρ and ρ' at which, as a consequence of the interaction of both particles, *an always-equal amount of work would be done whose absolute value can serve as a measure of a property that belongs to the particle-pair (since positive or negative means the same thing for the capacity to do work). It is then self-explanatory that the relative vis viva of the particles must be thought of as remaining unchanged under that fictitious shift of the particles from the distance r to the distance ρ.*

If R denotes the force of repulsion that results from the interaction of the two particles e and e', and $\rho - \rho'$ denotes the imagined change in distance between the particles that results from the fictitious shift, then the *capacity to do work* U of those particles will be represented by the formula:

$$U = \pm \int_{\sigma=\rho}^{\sigma=\rho'} R d\sigma \ ,$$

in which the upper or low sign applies according to whether the two particles have the same or different types, respectively.

In that formula, R is a function of σ, but not always the same one, which would be the case only if $d\sigma/dt = 0$, so from the law of electrostatics $R = ee'/\sigma^2$, it would always be the same function of σ. When $d\sigma/dt$ is non-zero, R will be a function of σ and $d\sigma/dt$, and $d\sigma/dt$ will not always be the same function of σ, which is explained by the facts that, *first of all*, the *initial values* of σ and $d\sigma/dt$ can be given arbitrarily, so very different values of $d\sigma/dt$ can be given for the same value of σ; and *secondly*, for equal changes in distance, the relative velocity $d\sigma/dt$ can experience very different changes according to the variety of *external forces* that act upon the particles.

Now, if R is a function of σ and $d\sigma/dt$, then the *indefinite integral* $\int R d\sigma$ will also be such a function. However, the *definite integral* $\int_{\sigma=\rho}^{\sigma=\rho'} R d\sigma$ will depend upon merely the initial and final values of σ, namely, ρ and ρ', and the differential quotients that belong to those values.

Now, should the *definite integral* $\int_{\sigma=\rho}^{\sigma=\rho'} Rd\sigma$ express the *capacity for two particles to do work when they possess the relative velocity* r', then it was remarked already that the relative velocity r' existing at the distance r must be thought of as remaining unchanged under the fictitious shift of the particles to the distance ρ, such that $d\sigma/dt = 0$ is given for $\sigma = \rho$, which will determine the dependency of the capacity to do work U on r'.

However, *an equal dependency of the capacity to do work* U on the value of $d\sigma/dt$ that belongs to the final value $\sigma = \rho'$ would also exist, except for the case in which $\rho' = \infty$, in which case such a dependency would not need to exist. It would follow from this that one must set $\rho' = \infty$ since the formula for U, as the definition of the *capacity to work by two particles that possess a relative velocity* r', can depend upon no other relative velocity besides r', namely, the one that the particles actually possess at the moment at which their capacity to do work is considered.

When one sets $\rho' = \infty$, the *capacity to do work* U of the particles e and e' will then be expressed by the formula:

$$U = \pm \int_{\sigma=\rho}^{\sigma=\infty} Rd\sigma \ ,$$

in which R is a function of σ and $d\sigma/dt$, and $d\sigma/dt$ is a function of σ that possesses the value r' for $\sigma = \rho$, i.e., the relative velocity of the particles whose capacity to do work is to be determined.

Finally, as far as the quantity ρ is concerned, it will be determined in such a way that only *a finite distance between two electrical particles* can be given that can be determined by merely the nature of the electricity in general and of the two particles in particular, but quite independently of the existing distance r, namely, from the unvarying total energy a that belongs to the particle pair, and from the forces of repulsion $\mu^2\varepsilon^2$ and $\mu^2\varepsilon'^2$ that are exerted by each of the two mass-particles ε and ε' on an equal particle at a separation distance of one unit according to the fundamental law of electrostatics, so according to the formula $\rho = \mu^2 \cdot [\varepsilon\varepsilon'/a]$. If one sets $\mu\varepsilon = \pm e$ and $\mu\varepsilon' = \pm e'$, where the upper or lower sign is true according to whether the mass-particle belongs to positive or negative electricity, respectively, then ρ, which is always positive, can be set equal to $\pm ee'/a$, in which the upper or lower sign will be true according to whether both particles have the same or different types, respectively.

Remark. Should U itself be expressed in the formula in such a way that R is a function of σ and $d\sigma/dt$, and $d\sigma/dt$ is a function of σ that possesses the value r' for $\sigma = \rho$, i.e., the *given* relative velocity of the particle whose *capacity to do work* is to be determined, then, *first of all*, R could be set equal to $R(\sigma,\ d\sigma/dt)$, *secondly*, in order for one to refer to R as a function of σ, $d\sigma/dt$ would be set equal to $f(\sigma)$, and *thirdly* and finally, in order for one to distinguish the function $f(\sigma)$ that should, in fact, assume *the given value* r' for $\sigma = \rho$ here, from any other function $f(\sigma)$ that can assume *the given value* r' for other values of σ, the value of σ for which the function f *assumes the given value* can be added to the function symbol f, in particular, so $f_\rho(\sigma)$ will be written in place of $f(\sigma)$ here. One can then express the *capacity to do work* as:

$$U = \pm \int_{\sigma=\rho}^{\sigma=\infty} R[\sigma,\ f_\rho(\sigma)] d\sigma \ ,$$

and the *given relative velocity*:

$$r' = f_\rho(\rho) \ .$$

However, when the two particles are thought of as being shifted to a separation distance $\sigma = \rho$ for the purpose of determining their capacity to do work U from that distance $\sigma = r$ at which they are actually found, while preserving the relative velocity r' that they actually possess, as a consequence of their interaction, *an amount of work would also be done under the actual shift from the existing distance* $\sigma = r$ to $\sigma = \infty$, *that will be called the potential of the particles* and which one cares to denote by V. With the notation that was given for U, one will get *the expression for that potential*:

$$V = \int_{\sigma=r}^{\sigma=\infty} R[\sigma,\ f_r(\sigma)] d\sigma \ ,$$

and the *given relative velocity*:

$$r' = f_r(r) \ .$$

From the given definition of the *capacity to do work* by two particles e and e', as the *energy of work*, which will determine the energies of the two particles e and e' completely, in conjunction with the *relative vis viva as an energy of motion*, and from the *principle of energy* that was expressed as the *law of the unvarying sum of both energies*, one can now pose the problem:

> Determine the force R with which two arbitrarily-moving electric particles e and e' act upon each other reciprocally from the *principle of energy* in conjunction with the *fundamental law of electrostatics*.

Because the force with which two electric particles e and e' act upon each other, *when their relative vis viva is zero*, is to be determined from the *fundamental law of electrostatics*, as well as the *principle of energy*, *which will vary under the interaction of two electric particles e and* e' when their relative vis viva is not equal to zero, but equal to x (namely, the *increase in energy of motion* of the two particles by an amount x that is coupled by a *decrease in the energy of work* by the same amount x), it would seem to emerge from this that one must be able to derive the general law of the force with which two electric particles e and e' act upon each other reciprocally that will encompass electrostatics, as well as electrodynamics, from the *principle of energy in conjunction with the fundamental law of electrostatics*.

The *principle of energy* will then give the following formulas, namely, *first of all*, *the formula for the energy of motion* (or relative vis viva) of two particles that possess the masses ε and ε':[25]

[25] [Note by AKTA:] The following formula should be understood as:

$$\xi = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'} \cdot \frac{d\sigma^2}{dt^2} \ ,$$

from which it will follow that when the relative velocity at the existing separation distance $\sigma = r$ is denoted by r' and the relative vis viva by x, one will have:

$$x = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'} \cdot r'^2 \ ; \qquad (1)$$

secondly, the formula for the energy of work, namely:

$$U = \pm \int_{\sigma=\rho}^{\sigma=\infty} R[\sigma,\ f_\rho(\sigma)] d\sigma \ , \qquad (2)$$

in which $f_\rho(\sigma) = d\sigma/dt$ denotes a function of σ that possesses the given value r' for $\sigma = \rho$, and *thirdly*, the *law of constant total energy*, which is expressed by the following formula:

$$x + U = a \ . \qquad (3)$$

One must add a *fourth* formula to those *three* formulas that are given by the *principle of energy*, namely, the formula for the *fundamental law of electrostatics*, which is the law for the force of repulsion R by which two particles e and e' in a state of relative rest act upon each other at a separation distance of σ:

$$R = \frac{ee'}{\sigma^2} \ . \qquad (4)$$

For $\xi = 0$, when one also has $x = 0$, the expression for the electrodynamic force $R[\sigma,\ f_\rho(\sigma)]$ will go to the expression for the electrostatic force $R = ee'/\sigma^2$, and one will find from equation (2) and equation (3), for $x = 0$,

$$U = \pm \int_{\sigma=\rho}^{\sigma=\infty} \frac{ee'}{\sigma^2} d\sigma = \pm\frac{ee'}{\rho} = a \ .$$

Moreover, if one denotes the value of r' in equation (1) for $x = a = \pm ee'/\rho$ by c, which will give

$$\pm\frac{ee'}{\rho} = \frac{1}{2}\left[\frac{\varepsilon\varepsilon'}{(\varepsilon+\varepsilon')}\right] \cdot c^2 \ ,$$

and substitutes the value of

$$\frac{1}{2}\left[\frac{\varepsilon\varepsilon'}{(\varepsilon+\varepsilon')}\right] = \pm\frac{ee'}{\rho c^2}$$

that it gives in equation (1), then one will find that:

$$x = \pm\frac{ee'}{\rho} \cdot \frac{r'^2}{c^2} \ .$$

$$\xi = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'} \cdot \left(\frac{d\sigma}{dt}\right)^2 \ .$$

If one now substitutes that value of x and the previously-found values of $a = \pm ee'/\rho$ into equation (3) and appeals to equation (2), then one will get:

$$U = \pm\frac{ee'}{\rho}\left(1 - \frac{r'^2}{c^2}\right) = \pm\int_{\sigma=\rho}^{\sigma=\infty} R\,[\sigma,\ f_\rho(\sigma)]\,d\sigma\ .$$

Now, one has identically:

$$-d\cdot\frac{ee'}{\sigma}\left(1 - \frac{1}{c^2}\frac{d\sigma^2}{dt^2}\right) = \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\frac{d\sigma^2}{dt^2} + \frac{2\sigma}{c^2}\frac{d^2\sigma}{dt^2}\right)d\sigma\ ,$$

from which the indefinite integral will follow:

$$-\frac{ee'}{\sigma}\left(1 - \frac{1}{c^2}\frac{d\sigma^2}{dt^2}\right) = \int\frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\frac{d\sigma^2}{dt^2} + \frac{2\sigma}{c^2}\frac{d^2\sigma}{dt^2}\right)d\sigma\ .$$

Now, when one substitutes $d\sigma/dt = f_\rho(\sigma)$ in that, which denotes a function of σ that possesses the given value r' when $\sigma = \rho$, then that will give:

$$-\frac{ee'}{\sigma}\left(1 - \frac{1}{c^2}\,[f_\rho(\sigma)]^2\right) = \int\frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\,[f_\rho(\sigma)]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right)d\sigma\ ,$$

and if one considers that integral between the limits $\sigma = \rho$ and $\sigma = \infty$, one will get:

$$\frac{ee'}{\rho}\left(1 - \frac{1}{c^2}\,[f_\rho(\rho)]^2\right) = \int_{\sigma=\rho}^{\sigma=\infty}\frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\,[f_\rho(\sigma)]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right)d\sigma\ ,$$

and as a result, since $f_\rho(\rho) = r'$, and one has

$$\left[\frac{ee'}{\rho}\right]\left(1 - \frac{r'^2}{c^2}\right) = \int_{\sigma=\rho}^{\sigma=\infty} R\,[\sigma,\ f_\rho(\sigma)]\,d\sigma\ ,$$

then

$$\int_{\sigma=\rho}^{\sigma=\infty} R\,[\sigma,\ f_\rho(\sigma)]\,d\sigma = \int_{\sigma=\rho}^{\sigma=\infty}\frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\,[f_\rho(\sigma)]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right)d\sigma\ .$$

The simplest assumption for satisfying that formula consists of setting:

$$R\,[\sigma,\ f_\rho(\sigma)] = \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\,[f_\rho(\sigma)]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right)\ .$$

In the expression for the *capacity to do work* U, [the expression] $f_\rho(\sigma)$ was set equal to $d\sigma/dt$ in order to define a function of σ that possessed the given value r' for $\sigma = \rho$.

Now, in the expression for the *potential* V, [the expression] $f_r(\sigma)$ was likewise set equal to $d\sigma/dt$ in order to define a function of σ that possessed the given value r' for $\sigma = r$, one would then get in a similar way:

$$R\,[\sigma,\ f_r(\sigma)] = \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\,[f_r(\sigma)]^2 + \frac{2\sigma}{c^2}\cdot\frac{d\cdot f_r(\sigma)}{dt}\right)\ .$$

However, in the latter case, where $\sigma = r$ and $d\sigma/dt = r'$ are the separation distance and velocity that is actually present, one does not care to use the function symbol $f_r(\sigma)$ at all but leaves $d\sigma/dt$ unchanged in the formula. One can also drop the special notation for the force of repulsion between both particles as a function of σ and $d\sigma/dt$ that involves adding those variables under the function symbol R, namely, $R(\sigma,\ d\sigma/dt)$, and merely set it equal to R. Now, having done that, one can represent the common law for the force R with which two arbitrarily-moving electric particles e and e' act upon each other by the following formula:

$$R = \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\frac{d\sigma^2}{dt^2} + \frac{2\sigma}{c^2}\frac{d^2\sigma}{dt^2}\right) .$$

Finally, the *principle of energy* that was exhibited here shall be applied to the law that C. Neumann exhibited in his "Principles of Electrodynamics," Tübingen, 1868, p. 37,[26] and in the *Berichten der Königl. Sächs. Gesellschaften der Wissenschaften*, 1871, Art. 20, p. 399,[27] since proving its agreement with the principle above is not without its own special interest.

Neumann expressed that law in the latter reference in the form:

> "If a system of arbitrarily-many particles $M + \mu$ moves under the action of given external forces, then the following formula will be true for each time element dt:
>
> $$d(T + U^0 + U - V) = dS ,$$
>
> i.e., for every time interval, the increase in the *energy* of the system will be equal in magnitude to the work that is consumed by the system during that time interval. In that way, one understands the *energy* of the system to mean that expression $T+U^0+U-V$, which depends upon only its instantaneous state (i.e., the coordinates and velocities), and in which T denotes the vis viva, U^0 denotes the ordinary potential of the system, U denotes the electrostatic potential, and V denotes the electrodynamic one."

The *capacity to do work*, or the *energy of work* U, of two electric particles e and e' (which possess any well-defined relative vis viva x at any well-defined distance r from each other) was found to be:

$$U = \int_{\sigma=\rho}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right) d\sigma ,$$

in which $f_\rho(\sigma) = d\sigma/dt$ denotes a function of σ whose value for $\sigma = \rho$ is given by the existing vis viva x, namely, by the equation:

$$\pm\frac{ee'}{\rho c^2}\left[f_\rho(\rho)\right]^2 = x \qquad \text{or} \qquad f_\rho(\rho) = c\sqrt{\pm\frac{\rho x}{ee'}} .$$

The value of U above can now be represented as a sum of two terms, namely:

$$U = \int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right) d\sigma$$

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[26] [Note by AKTA:] See footnote 20.

[27] [Note by AKTA:] [Neu71, p. 399].

$$+ \int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2} \left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{d \cdot f_\rho(\sigma)}{dt}\right) d\sigma \ .$$

Now, since nothing about the function $f_\rho(\sigma)$ is generally definite beyond merely its value for $\sigma = \rho$, which is implied by the equation $[\pm ee'/\rho c^2][f_\rho(\rho)]^2 = x$, namely, $f_\rho(\rho) = c\sqrt{\pm\rho x/ee'}$, then $f_\rho(\sigma)$ can be defined to be very different functions of σ, in general.

The function $f_\rho(\sigma)$ will *actually be defined precisely* only when one is dealing with an *actual displacement* of the particles e and e' for which all relationships upon which the function $f_\rho(\sigma)$ depends are actually given. However, one cannot speak of an *actual displacement* of the particles from ρ to ∞ when they are not even found at a distance ρ apart, but at a distance r. Nonetheless, for the purpose of defining U, it suffices to only imagine displacing the particles from ρ to ∞, once one has imagined *previously* displacing them from r to ρ, and indeed in such a way that the relative vis viva of the particles at the distance ρ would again be the same as it was when the distance was r, namely, $x = [\pm ee'/\rho c^2][f_\rho(\rho)]^2$, and in that way, the value of the function $f_\rho(\sigma)$ will be determined for $\sigma = \rho$.

If one would now like to imagine, moreover, that the further displacement of the particles, namely, initially from ρ back to r, takes places only under the reciprocal action of the particles, but *without the action of external forces*; then since $f_\rho(\sigma)$ is given for $\sigma = \rho$, namely, $f_\rho(\rho) = c\sqrt{\pm\rho x/ee'}$, the value of $f_\rho(\sigma)$ for a value of σ that is different from ρ (e.g., for $\sigma = r$) will be found to be equal to $c\sqrt{\pm\rho y/ee'}$, in which y is determined by the following equation:

$$y - x = \int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2} \left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{d \cdot f_\rho(\sigma)}{dt}\right) d\sigma \ ,$$

that is, the change in the relative vis viva during the change in distance from ρ to r is equal to the work done by the forces of interaction along the path that was taken.

However, when not only the forces that result from the interaction act upon the particles during their change in distance, but also other *external forces* P, and they likewise seek to get further apart from each other (or closer together), y would be determined by the following equation:

$$y - x = \int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2} \left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{d \cdot f_\rho(\sigma)}{dt}\right) d\sigma + S \ ,$$

when $S = \int_{\sigma=\rho}^{\sigma=r} P d\sigma$ denotes the work done by the *external forces*.

Among all of the conceivable cases up to now, one also finds the case in which one has:

$$\int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2} \left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma + S = 0 \ ,$$

for the value at $\sigma = r$, which is equal to the actual distance between the particles that possess the relative vis viva for which one desires to know U, so one has $y = x$. That means that the vis viva of the two particles at the end of the change in distance can be equal to the one at the beginning only when the work done by the *forces of interaction* during the change in distance is canceled by the work done by the *external forces*.

However, if the relative vis viva of the two particles at the distance $\sigma = r$, which is denoted by y, will be the same *at the end* of the change of distance that is given in the integral:

$$\int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma$$

as it was *at the beginning* (namely, it is equal to x), then that will explain the fact that the same value of the vis viva x will also be valid for the distance $\sigma = r$ *at the beginning* of the further change in distance from r to ∞ that is given in the integral:

$$\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma \ .$$

However, that explains the fact that the difference between the functions $f_\rho(\sigma)$ and $f_r(\sigma)$ vanishes, and:

$$\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma$$

denotes the same work as:

$$\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_r(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_r(\sigma)}{dt}\right) d\sigma \ ,$$

namely, the work that would be done as a consequence of the interaction of the particles that possess the given vis viva x under a change in distance from r to ∞. One will then have:

$$\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma$$
$$= \int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_r(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_r(\sigma)}{dt}\right) d\sigma = V \ .$$

Thus, one can now set the first part of U, namely:

$$\int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma = -S \ ,$$

and the second part of U, namely:

$$\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma = V \ ,$$

which will give:

$$U = V - S \ ,$$

and if one substitutes that value in the equation:

$$U + x = a \ ,$$

then one will get the following equation:

$$V + x - S = a \ .$$

When those particles are found at a distance of r_1 and possess the relative vis viva x_1, that will imply the following equation in the same way:

$$V_1 + x_1 - S_1 = a \ ,$$

from which one will get the following differential equation for small values of $r - r_1$ and $x - x_1$:

$$dV + dx - dS = 0 \ ,$$

which is the same equation that Neumann exhibited in the place cited, except that Neumann had denoted the vis viva by T and the potential by $U - V$, since it is composed of an electrostatic and an electrodynamic part, and finally, for the case in which the electric particles are endowed with ponderable masses, he added the potential U^0 that results from the interaction of those ponderable masses, so he expressed the same law in the following equation:

$$d(T + U^0 + U - V) = dS \ .$$

1.3 On the Objections that were Raised Against the Fundamental Law of Electric Action

When the fundamental law of electric action, which says that the interaction between two electric particles e and e' (expressed in electrostatic units) will result in the force of repulsion:

$$R = \left[\frac{ee'}{r^2}\right]\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right] + \left[\frac{2r}{c^2}\right]\left[\frac{d^2r}{dt^2}\right]\right) \ ,$$

is considered in regard to its connection with the principle of energy that was developed here, that will explain the fact that in all applications of that law that should be made in order to determine the later relationships between the particles from their *initial relationships*, those *initial relationships* cannot be assumed to be completely arbitrary. They cannot be assumed to be such that they would already contain contradictions to the principle at their basis, which would be the case, for example, when two electric particles are ascribed an *initial relative vis viva* that would already be greater in its own right than the total energy of the particles according to the principle of the unchanging total energy.

By the assumption of such *initial relationships* that contradict the principle that was posed, one can generally arrive at consequences whose admissibility one can rightfully object to, and which might seem to contradict the law, but that is not truly the case. One might return to some of the objections that Helmholtz raised against the law above.[28] For example, Helmholtz arrived at the consequence of the law above that two particles with a relative velocity that is initially finite, but greater than c (which would imply that the relative vis viva of the particles would be greater than the entire sum of energy, which is unchangeable in principle), would attain an infinite vis viva during a finite change in distance, and would therefore do an infinitely-large amount of work. One could also infer the possibility of a *perpetuum mobile* from that.[29]

[28][Note by AKTA:] [Hel73].

[29][Note by AKTA:] That is, the possibility of a perpetual motion.

Now, those consequences will generally go away by themselves, when *each energy* in the principle that was posed is coupled with an *essentially-positive quantity*, and when they are all taken together, they will define a *finite and unchanging sum.* However, even when one accepts a value for energy that is negative and increases to infinity, those consequences would not necessarily lead one to reject the law above, since the basis for declaring those consequences to be inadmissible would no longer exist, in fact. That would then explain the fact that when *one energy* is negative and becomes *negatively infinite*, *another energy* must likewise be present that would be positive and become positively infinite. Now, if that energy that increases to infinity were the *energy of motion*, then there would be an inexhaustible source of *vis viva*, so one would be able to product all of the effects that Helmholtz declared to be inadmissible.

The objections against the fundamental law of electric action that were just considered, which are based upon the fact that the principle of energy that was posed is not recognized and that the initial relationships between the electric particles that would contradict it must be assumed, can be combined with yet another objection that is based upon the fact that Helmholtz believed that he had proved that the distance ρ that he referred to as *critical* was not always a *molecular* distance. However, he had proved that only by assigning a meaning to the distance ρ that was completely different from the one that it was given for the purpose of defining the *energy of work* U for two particles e and e'. Namely, $\rho = ee'/a$ depended upon merely on the nature of electricity and on the two particles e and e', namely, on the three quantities a, e^2 and e'^2, which denoted the constant total energy of the particle-pair and the electrostatic forces of repulsion that the two particles will each exert upon a particle that is equal to it, at a unit distance.

Helmholtz said (in the place cited, p. 43):[30] "The value of the distance ρ is $\rho = 2ee'/c^2\mu$." Therefore, Helmholtz was setting μ equal to $2a/c^2$. Helmholtz then continued with: "If the electric particle is endowed with only its proper mass, then e/μ will have any well-defined value β. If μ also included ponderable mass then one will have $e/\mu < \beta$." That explains why, according to Helmholtz, ρ is also a quantity that depends upon the ponderable mass that the electric particle e is endowed with, so it has an entirely different meaning from the one that it had in the law that I proposed. Helmholtz continued further: "However, when $b = 2e/c^2\mu$ is also an exceptionally small quantity, ρ cannot depend upon b alone, but one will have $\rho = be'$, and e' can still have any arbitrary magnitude, and as a result, ρ as well. It should probably be noted in that regard that if we would like to envision e' in the form of a spherical mass of a certain density that is an insulator that is carrying an electric fluid either through it or on it, when e' increases, the diameter of that ball will increase by either $\sqrt[3]{e'}$ or $\sqrt[2]{e'}$ according to whether e' is distributed throughout the interior or across the surface, respectively, but ρ will increase like e' itself, and that by appropriately increasing e' we can give the magnitude ρ any finite size and its end point any distance from the surface of the electrical mass e'."

The description given here by Helmholtz of the electric particle e' clearly shows how different it is, according to Helmholtz's conception, from every *atom* that actually exists in nature, given its size and mass. One easily sees that when one would like to imagine *atoms with planetary masses*, instead of the atomic bodies with immeasurably-small masses that actually exist in nature, as one is free to do, obviously, the molecular and atomic distances in that imaginary world would not be as immeasurably-small as they are in the real world. The fact that such giant atoms would conform to the fiction of fixed couplings of ponderable

[30][Note by AKTA:] [Hel73, p. 43].

atoms with each other and could be produced by electric ones is self-explanatory. However, that could probably not be said about the fundamental law of electric action, which has no connection whatsoever to such fictions.

If the objections that Helmholtz cherished in relation to the possibility of a perpetuum mobile, as well as in regard to measurable magnitudes for the critical distance ρ, seem to originate mainly in differences in his fundamental concepts and pictures, then things will behave differently when one makes the following objection. One objection that Helmholtz raised consisted essentially of the fact that as Helmholtz believed he had proved, it would follow from the fundamental law that I had proposed that "in certain cases, a force that acts forwards on the (driven) point μ will accelerate it backwards, and conversely."

However, the proof rests essentially on the fact that in Borchardt's Journal, Vol. 75, p. 47,[31] as well as in *Monatsberichte der Akademie der Wissenschaften zu Berlin*, 1872, April 18, p. 253,[32] Helmholtz spoke of a *vis viva* equal to[33]

$$\frac{1}{2}\left(\mu - \left[\frac{1}{c^2}\right]\left[\frac{ee'}{r}\right]\cos\vartheta^2\right)q^2 \ ,$$

in which q is the velocity with which the mass μ moves, but the quantity $-[1/c^2][ee'/r]\cos\vartheta^2$ is not at all an *actually-existing mass*, much less a mass that moves with the velocity q. I have not been able to guess what Mr. Helmholtz has meant by saying of the magnitude

$$\left(\mu - \left[\frac{1}{c^2}\right]\left[\frac{ee'}{r}\right]\cos\vartheta^2\right) \ ,$$

not that it *is* the mass moving with the velocity q, but that it *represents* the mass (Borchardt's Journal, Vol. 75, p. 48), or that it *stood in* for that mass (*Monatsbericht*, 1872, April 18, p. 253), and therefore I do not understand how Helmholtz, with the aid of this comparison, has managed to find that "a consequence of Weber's law is that in certain cases, a force that points forward will accelerate the point μ backwards, and conversely."

It is just as hard for me to grasp how that quantity, which only *replaces* or *stands in* for a mass, can *collide* with another mass that actually exists and how its motion *after the collision* can be determined from the laws that would be valid if one were dealing with *actually-existing masses that move with velocity* q.

In Borchardt's Journal, as well as in the *Monatsberichte der Berliner Akademie*, Helmholtz had referred to the equation for vis viva that he developed from my fundamental law, and in the case of just *one* mass-point μ with the electric quantum e moving in a space that is bounded by a spherical surface with radius R that is uniformly endowed with electricity, it will reduce to the following equation, in which ε denotes the quantum of electricity per unit area of the spherical surface, namely:

$$\frac{1}{2}\left(\mu - \frac{4\pi}{3c^2}\cdot R\varepsilon e\right)q^2 - V + C = 0 \ .$$

[31][Note by AKTA:] [Hel73, p. 47].

[32][Note by AKTA:] [Hel72a, p. 253] with English translation in [Hel72b], see also [Hel82].

[33][Note by AKTA:] The next equation should be understood as:

$$\frac{1}{2}\left(\mu - \left[\frac{1}{c^2}\right]\left[\frac{ee'}{r}\right]\cos^2\vartheta\right)q^2 \ .$$

V denotes the potential of the *non-electric* forces, and dV/ds denotes the *driving force* that Helmholtz referred to. Differentiating that equation will give:

$$\left(\mu - \frac{4\pi}{3c^2} \cdot R\varepsilon e\right) q\frac{dq}{ds} - \frac{dV}{ds} = 0 \ ;$$

so when dV/ds is positive, i.e., with Helmholtz's assumption, *with a forward driving force*, and at the same time $(\mu - [4\pi/c] \cdot R\varepsilon e)$ is *negative*, q will decrease, that is, μ will be *accelerated backwards*.

However, in that way, Helmholtz was considering only *one part of the driving force*, namely, the one that was implied by the potential of the *non-electric* forces. Nonetheless, Helmholtz had not considered the *other part of the driving force at all*, which is implied by the *electric potential* $([4\pi/6c^2]R\varepsilon e \cdot q^2)$ and which Helmholtz combined with the vis viva $\frac{1}{2}\mu q^2$, merely because it had the common factor of q^2, since he said: "*q will decrease under a forward driving force, or μ will be accelerated backwards, when* $(\mu - [4\pi/3c^2]R\varepsilon e)$ *is negative.*" That should really read: μ will be accelerated backwards by a forward-pointing *non-electric* force when $(\mu - [4\pi/3c^2]R\varepsilon e)$ is negative. However, if the *total driving force* is included in the calculation, instead of merely one *part* of the driving force, then one will get:

$$\mu q\frac{dq}{ds} - \left(\frac{4\pi}{3c^2}R\varepsilon e \cdot q\frac{dq}{ds} + \frac{dV}{ds}\right) = 0 \ ,$$

upon differentiating the equation above, in which $([4\pi/3c^2]R\varepsilon e \cdot q[dq/ds] + dV/ds)$ is the *total driving force*, and it will follow from this that:

$$dq = \frac{ds}{\mu q}\left(\frac{4\pi}{3c^2} \cdot R\varepsilon e \cdot q\frac{dq}{ds} + \frac{dV}{ds}\right) \ ,$$

that is, when one recalls that $ds/\mu q$ is always positive, *for a forward-pointing total force* (electric and non-electric combined), *μ will always accelerate forwards*, and conversely, in which it is totally irrelevant whether $(\mu - [4\pi/3c^2]R\varepsilon e)$ has positive or negative value.

Once one has gotten around the apparent inconsistencies in the consequences of my fundamental law that Helmholtz inferred in that way, all that still remains will be a surprising result, namely that according to that law, a *non-electric* force that *retards* the motion of a particle μ and can be represented by a negative value of dV/ds, indirectly results in an electric force equal to $[4\pi/3c^2]R\varepsilon e \cdot q[dq/ds]$, which accelerates the particle μ in its motion, and indeed it will *accelerate* that particle more than it is retarded by the former force.

However, the *immediate basis* for that electric force $([4\pi/3c^2]R\varepsilon e \cdot q[dq/ds])$ does not lie in the force dV/ds, but, according to the fundamental law, it will lie in the existing *relative acceleration*, which is represented by $q[dq/ds]$ here and from which that force will be obtained in the specified manner by multiplying by $[4\pi/3c^2]R\varepsilon e$. However, according to the general laws of motion, that acceleration $q[dq/ds]$ will itself result, not from *one* force, but from *all existing forces*, so not merely the *non-electric* force dV/ds, but also the *electric* force $([4\pi/3c^2]R\varepsilon e \cdot q[dq/ds])$ itself, namely, upon dividing the sum of both forces by μ, that will give:

$$q\frac{dq}{ds} = \frac{1}{\mu}\left(\frac{dV}{ds} + \frac{4\pi}{3c^2}R\varepsilon e \cdot q\frac{dq}{ds}\right) \ .$$

Now, in general, the values of the *acceleration* $q[dq/ds]$ and the *electric force* $([4\pi/3c^2]R\varepsilon e\cdot q[dq/ds])$ will then be represented as also depending indirectly upon just the given *non-electric force* dV/ds, namely:

$$q\frac{dq}{ds} = \frac{1}{\mu - \frac{4\pi}{3c^2}R\varepsilon e}\cdot\frac{dV}{ds}\ ,$$

so:

$$\frac{4\pi}{3c^2}R\varepsilon e\cdot q\frac{dq}{ds} = \frac{\frac{4\pi}{3c^2}R\varepsilon e}{\mu - \frac{4\pi}{3c^2}R\varepsilon e}\cdot\frac{dV}{ds}\ .$$

Therefore, when the given value of dV/ds is *negative*, for a very small negative value of $(\mu - [4\pi/3c^2]R\varepsilon e)$, the *given backwards-driving, non-electric* force that acts upon a mass μ that moves with a velocity of q will imply a much larger *forward-driving electric* force that acts upon that same mass.

That explains why the denominator $(\mu - [4\pi/3c^2]R\varepsilon e)$ can be zero or negative for only a positive value of εe, i.e., only when the electricity that the spherical surface is endowed with has the same type as the electricity that the moving ponderable mass is endowed with. That will then imply that when $\mu > [4\pi/3c^2]R\varepsilon e$, the electric force $([4\pi/3c^2]R\varepsilon e\cdot q[dq/ds])$ will have the *same direction* as the non-electric force dV/ds, and its *magnitude*, which is equal to the other force when $[4\pi/3c^2]R\varepsilon e = \frac{1}{2}\mu$, will increase with increasing values of $[4\pi/3c^2]R\varepsilon e$, until it becomes *infinite* when $[4\pi/3c^2]R\varepsilon e = \mu$ and then *changes sign*.

Such a jump in the *magnitude and direction* of the electric force, namely, from $+\infty$ to $-\infty$, can be truly undesirable when it actually follows from the law as a loss of continuity. However, such a jump will not actually occur at all, according to the law, since in fact the mass μ with its charge e cannot remain inside of the spherical space for so long as a result of the ever-increasing acceleration given to it until $[4\pi/3c^2]R\varepsilon e = \mu$, but before that, it must be driven up to the spherical surface that is composed of a *fixed insulator*, whose resistance will once more bring it to rest.

As one sees from this, those consequences include no inconsistencies whatsoever, and can even prove to be unexpected only under circumstances that are so highly exceptional that one cannot even think of them presenting themselves in reality. That is because if one imagines that electric charges that can actually exist in reality might amount to, say, 10 electrostatic units of charge per milligram of ponderable carrier, so $e/\mu = 10$, and one further imagines that the same charge is distributed over each square millimeter of the surface, so $\varepsilon = 10$, then the fact that $[4\pi/3c^2]R\varepsilon e > \mu$ will impose the demand that one must have a spherical insulator whose radius would be $R > 3c^2/400\pi > 46\cdot 10^{19}$ millimeters, i.e., it would have to be 3 million times greater than the distance from the Earth to the Sun.

Furthermore, other even-more remarkable, but still not inconsistent, consequences of the fundamental law of electric action were already pointed out in the First Treatise on Electrodynamic Measurements in the year 1846,[34,35] in particular, the fact that the *interaction of two bodies* will depend indirectly on the *presence of a third body*, which will result in forces that Berzelius had referred to by the name of catalytic.[36]

[34][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 212.

[35][Note by AKTA:] [Web46, p. 212 of Weber's *Werke*] with a partial French translation in [Web87] and a complete English translation in [Web21a].

[36][Note by AKTA:] Jöns Jacob Berzelius (1779-1848). See [Ber36c], [Ber36a] and [Ber36b].

However, if those consequences also find no analogues in the consequences that are inferred from other laws, then one might very well pose the question of whether that lack of an analogue is a drawback or an advantage, since in order to explain many realms of phenomena, and in particular ones that have a closer relationship to the molecular constitution of bodies; obviously, not all consequences can be inferred from the law of gravitation and from all of the other laws that are proposed by analogy with it. Therefore, laws of some other type would then seem necessary.

1.4 Identity of the Moving Parts that are Contained in All Bodies, Whose Motion is Heat, Magnetism or Galvanism

One subdivides all ponderable bodies into solid, liquid, and gaseous ones and distinguishes between the statics and dynamics of those bodies according to whether one considers them to be in a state of rest or motion, respectively. However, when one speaks of the rest state in the statics of a body, one does not at all mean a state of rest for *all* of the parts that are enclosed within the boundary of that body, but only the *ponderable* bodies that are enclosed within that boundary. Without that restriction, one could never speak of the rest state of a body since any body will include not only its ponderable parts, but also other parts that will never be at rest.

That is because, *first of all*, the more precise study of all *electrical* phenomena that are observed in ponderable bodies has led to the fact that moving parts are present in the interior of all of those bodies (even so-called solid ones and ones that are found to be at rest), namely, electric ones, and that the motions of these parts in the interior of that body is the basis for all of the galvanic and electrodynamic phenomena and effects that exist in that body.

Secondly, the closer study of all *magnetic* phenomena that are observed in ponderable bodies such as paramagnetism, as well as diamagnetism, likewise led to the fact that moving parts were present in the interior of all of those bodies that one had sought for a long time to distinguish from the electric fluids by calling them *magnetic fluids*. It was asserted that those magnetic fluids could be *distributed* in the interiors of those bodies in various ways according to the variety of the situations, but that they could come to rest and equilibrium under steady-state conditions. The basis for magnetic phenomena lies in the distribution of that magnetic fluid without requiring the continuing motion of it. However, further investigation implied that such magnetic fluids at rest could not be the basis for *all* magnetic phenomena (e.g., paramagnetism and diamagnetism), no matter how they might be distributed. Nonetheless, *all* of those phenomena can be explained by the presence of *continuously moving parts in the interiors of ponderable bodies*, and indeed the same parts whose motion were the basis for all galvanic and electrodynamic phenomena and effects, namely, the *electrical ones*.

Thirdly and finally, it must be added that the study of the *temperature* that is associated with any ponderable body has also led to the facts that moving parts are present in the interior of all of those bodies and that the basis for all of the temperature-related phenomena (i.e., *heat*) that are observed in those bodies consists of the motion of those parts.

Now, if the moving parts that are included in all ponderable bodies whose motions are the basis for all galvanic effects include no other parts beyond ones whose motions are the basis for all magnetic effects (e.g., paramagnetic and diamagnetic), then that would strongly suggest that the parts that are included in all ponderable bodies whose motion represents *heat* would also be identical to the parts that are included in all ponderable bodies whose motion represents *magnetism*, and as a result, they are also identical to the parts that are included in all ponderable bodies whose motion represents *galvanism*. Namely, if one must also generally recognize the presence of parts in the interior of the body that move, while the ponderable parts remain at rest, then one would have many more reservations about the presence of *several types of such parts*, and indeed in the smallest parts of the body, assuming that one could separate the ones that belong to each of them and that there would fewer prospects for a closer study of each of them individually. — That suggested identity was also confirmed by facts that should be considered in more detail in what follows.

1.5 Identity of the Vis Viva that is Created in a Current by the Electromotive Force and the Heat that is Created by the Current in a Conductor

The creation of heat by the galvanic current in a current conductor has been the subject of many investigations that established the law that the *mechanical equivalent of the heat created* during the time element dt is equal to the product of dt with the square of the current intensity i and the resistance w of the conductor through which the current flows, both of which are measured in absolute magnetic units.[37] However, it should be noted that most of the measurements that have been performed in that regard were not actually reduced to absolute units at all and that this reduction, when it was even attempted, still did not attain the desired precision and certainty, since resistance scales with a precisely-guaranteed reduction to absolute units have been lacking up to now. That is because the only scales of resistance that are useful for such purposes up to now are the ones that were first implemented recently by Siemens,[38] and the only precisely-guaranteed reduction of those scales to absolute resistance units was first given by Kohlrausch (Supplementary Volume VI, 1873, p. 1).[39]

Strictly speaking, only the law of the *proportionality* of the heat created and the product i^2wdt was considered after that, and in order to be able to set them *equal to each other*, one would require even finer absolute measurements than the ones that were performed up to now. Meanwhile, we would like to assume that *equality* for the time being, as other physicists have done, even though it has still not been proven with sufficient precision, but only approximately.

However, according to the *magnetic units* used in the formulation of this law, the resistance of the conductor is now known as $w = e/i$, where e denotes the electromotive force and i denotes the intensity of the current produced by this [electromotive] force in the conductor. The mechanical equivalent of the heat generated by the current in the time element dt can therefore also be represented by $eidt$ instead of i^2wdt.

[37][Note by AKTA:] This result is due to James Prescott Joule (1818-1889), [Jou41b], and [Jou41a] with French translation in [Jou42]. A detailed analysis of Joule's paper can be found in [MS20] and [Mar22]. The German expression *magnetische Maass* is being translated here as magnetic unit.

[38][Note by AKTA:] E. W. v. Siemens (1816-1892), [Sie60] with English translation in [Sie61]. See also [GT19].

[39][Note by AKTA:] [Koh73].

Furthermore, that implies that *according to magnetic units*, *first of all*, the current intensity is $i = 2Eu \cdot \sqrt{2}/c$,[40,41] where $2Eu$ denotes the sum of the product of the positive electricity $+E$ that is contained in a unit length of the conductor in electrostatic units with its velocity $+u$, and the product of the negative electricity $-E$ that is contained in a unit length of the conductor with its velocity $-u$.

Secondly, that implies that *according to magnetic units*, the electromotive force is $e = [f/E] \cdot [c/\sqrt{2}]$,[42] in which f denotes one-half the difference between forces (expressed in mechanical units) that act upon the positive and negative electricity in the conductor in the direction of the conductor, and E denotes the number of electrostatic units that are contained in a unit length of the conductor as positive or negative electricity.

The mechanical equivalent of the heat created by the current in the conductor will follow from that:

$$i^2 wdt = eidt = 2fudt = (+f) \cdot (+udt) + (-f) \cdot (-udt) ,$$

which is equal to the sum of the products of the forces that act upon each streaming particle with the path length from that particle in the direction of the force that acts upon it, i.e., it is equal to the *work done by the current*.

Now, if no other force acts upon the electricity that flows in the conductor than the electromotive force, then that would explain why the vis viva of the electricity that flows must increase and why the magnitude of that increase is given by the magnitude of the *work done by the current*. That increase in the vis viva of the current that flows will then further imply an increase in the velocity with which the flowing current moves. Therefore, if the current that flows in the conductor is not combined with any other motion than the motion of the current, then that would imply a continual decline in the current intensity, but that would contradict the *steady current* that was assumed here, which would require a steady electromotive force in order to produce it, according to Ohm's law.[43]

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[40][Note by WW:] See the Fourth Treatise on Electrodynamic Measurements, 1857, p. 264 [Note by HW: Wilhelm Weber's *Werke*, Vol. III, p. 652], in which the ratio of the magnetic unit of the current intensity to the mechanical one is given as $= c\sqrt{2} : 4$. — When only positive electricity is considered, as is also customary in the determination of the direction of the current, the current intensity in mechanical units will be expressed by Eu, where E denotes the positive electricity per unit length of the conductor in electrostatic units, and u denotes the velocity with which it moves. See the First Treatise on Electrodynamic Measurements, 1846, Section 21, p. 114 and the following [Note by HW: Wilhelm Weber's *Werke*, Vol. III, p. 152]. — That will imply the current intensity in magnetic units as $i = Eu \cdot 2\sqrt{2}/c$.

[41][Note by AKTA:] [KW57, p. 652 of Weber's *Werke*] with English translation in [KW21]. See also [Web46, p. 152 of Weber's *Werke*] with a partial French translation in [Web87] and a complete English translation in [Web21a].

[42][Note by WW:] One understands the electromotive force that is exerted upon a conductor to mean the difference between forces that would act upon the positive and negative electricity in the conductor if each unit length of the conductor contained positive and negative electricity, which are expressed in mechanical units. Indeed, when each unit length contains an *electrostatic unit* of positive and negative electricity, the electromotive force that acts upon the conductor would be expressed in *mechanical units*. By contrast, if it included a *magnetic unit* of positive and negative electricity, which would then be $c/[2\sqrt{2}]$ times bigger than the electrostatic unit, then the electromotive force that acts upon the conductor would be expressed in *magnetic units*. — Now, if one lets $2f$ denote the difference between forces that actually act upon the positive and negative electricity in the conductor, as expressed in mechanical units, and lets E denote the number of electrostatic units that are included in each unit length as positive or negative electricity; then that will imply that the electromotive force that is exerted upon the conductor is equal to $2f/E$ in *mechanical units*, and it will be equal to $e = [f/E] \cdot [c/\sqrt{2}]$ in *magnetic units*.

[43][Note by AKTA:] Georg Simon Ohm (1789-1854). Ohm's law is from 1826: [Ohm26a], [Ohm26c],

All that will then remain in the case that was assumed here is that the electricity in conductor is *not always found in mere current motion*, but that this current motion will go to a *different* motion in the course of time, and conversely.

Now, if that *other* motion is the motion of the electricity around the ponderable molecules that are the basis for all magnetic (paramagnetic and diamagnetic) phenomena, which is always present in the conductor and in which a large enough amount of electricity takes part to make the amount of streaming electricity vanish, then that alone would imply that the streaming electricity must always start out from the foregoing molecular current with a smaller velocity than it got from the following molecular current as a result of the acceleration that it had experienced along its path under the electromotive force. However, it would also imply that the increase in vis viva at the next station that is achieved by the streaming electricity in that way will likewise be given to the molecular current, such that with a steady current, only the vis viva of the *molecular currents* will increase. That increase in vis viva is nothing but the heat that is created by the current in the conductor, which can be inferred from the fact that it is equal to the mechanical equivalent of the heat created, which has been proved at least approximately, as was remarked before. — That confirms the suggestion that was expressed at the conclusion of the previous Section that the moving parts that are included in all ponderable bodies whose motion is one of *heat*, are *identical* to the parts that are included in all ponderable bodies whose motion is one of *magnetism*. There are no other moving parts in the interior of the body that are independent of the ponderable ones than those, namely, the *electric* parts.

1.6 Motion of Electricity in Conductors

If the electricity in all bodies is found to be in continual motion, and especially around the ponderable molecules, and those motions are the basis for all *galvanic*, *magnetic*, and *thermal phenomena*, then that will also be true of electricity in *conductors*, particularly in *metallic conductors*, which are distinguished from all other bodies by their *galvanic* behavior, as well as in regard to *thermal conduction*,[44] and finally, some of them like iron and bismuth are also distinguished by their *magnetism* or *diamagnetism*, the basis for which is obviously to be sought in the special circumstances under which the electricity in those bodies is found.

Electrical current motions take place mainly in *metallic conductors*, and indeed *purely electric ones* (namely, ones for which only the electricity flows without the participation of the ponderable parts) take place *only* in metallic conductors. That is because in *non-metallic*, so-called moist or *decomposable conductors*,[45] no current will flow without *electrolytic* action, i.e., not without the participation of ponderable parts in the current, and indeed some ponderable parts will participate in the flow of positive electricity, while other ones will participate in the flow of the negative electricity.

Now, the *steadiness* of the electrical currents in metallic conductors requires a more detailed explanation. Namely, it is known from Ohm's law that a *steady current* can exist in a closed conductor only under the steady and ongoing action of a certain *electromotive force*, and from the previous Section, such an electromotive force must *accelerate* the electricity that flows in its direction, so the current intensity would also change.

[Ohm26d], [Ohm26b] and [Ohm27] with French translation in [Ohm60] and English translation in [Ohm66].

[44] [Note by AKTA:] In German: *Wärmeleitung*.

[45] [Note by AKTA:] In German: *in nicht metallischen sogenannten feuchten oder zersetzbaren Leitern.*

However, if, as was stated in the previous Section, the current in the conductor consists of nothing but *current elements* in which the current motion is continuous only from one conducting molecule to another and when an electric particle arrives at another conduction molecule by way of that current, it will mix with the electricity that exists there and move around that molecule, so it *goes over* from a flowing motion to a rotational motion, while any other particle of the electricity that is present there will conversely *go over* from a rotational motion to a flowing motion, which will define a second current element, etc., then that will explain why acceleration of the electricity in each current element must indeed occur because of electromotive force, but that no increase in intensity of the total current needs to occur in that way when, in fact, the electric particles in all current elements will begin their flowing motion with a velocity that is always equal, *but smaller*, and conclude that motion in it with a velocity that is always equal, *but larger*.

It emerges from this that in metallic conductors, the *transition* from rotational motion to flowing motion and conversely must play a special role for electric particles. That is because that transition should mediate the conduction of electricity in its own right.

In addition, *electrical conduction* and *thermal conduction* are closely related in metallic conductors, and it is clear that if heat is really identical with the vis viva of the electricity constantly moving inside the ponderable bodies, then *thermal conduction in metallic conductors*, as well as electric conduction, must be mediated by the *transition* from rotational movement to current movement, and vice versa.

Now, if the basis for the capability of a metallic conductor to conduct electricity and heat lies in the fact that the electric parts that are found to be in rotational motion can be *converted* into current motion and conversely, then one must ask what that transition depends upon and why it takes place in *conductors*, but not in *insulators*. To that end, we shall go on to the molecular motions of two electric particles of *different types* that were considered in the last Treatise on Electrodynamic Measurements (in Vol. 10 of the *Abhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften*, 1871, Section 16)[46,47] and to the variety of molecular *constitutions of bodies* that are based upon them.

Namely, if we restrict ourselves to systems that consists of pairs of particles, one of which $-e$ is negatively electrical and bound to a ponderable mass, while the other one $+e$ is positively electrical and moving around the former, then such systems can differ from each other to varying degrees by the following properties:

First property: Every such system will assign a well-defined, and in fact negative, value to ρ (namely, when one sets $ee'/a = \rho$ and makes the signs of e and e' depend upon whether the particles that they denote belong to positive or negative electricity) that can be very different for different systems. It is then a *property* of such systems that each of them assigns a well-defined value to ρ, or to ρc^2, by which they can be distinguished from other systems.

Second property: From Section 11, in the place cited, one has that $r^2\alpha^2 = r_0^2\alpha_0^2$ (when r_0 and α_0 denote the initial values of the distance between the two particles and their relative velocity in the direction perpendicular to their connecting line, respectively, while r and α denote the current values) is a *constant* of the system, at least as long as no other forces act upon the particles than the ones that result from their interaction. That *constant* is a *second property* that can likewise serve to distinguish between different systems. However, this does not mean that *permanent* distinctions are made; instead, transitions from one system to another can take place as a result of *external* influences.

[46] [Note by HW:] Wilhelm Weber's *Werke*, Vol. IV, p. 279.

[47] [Note by AKTA:] [Web71, p. 279 of Weber's *Werke*] with English translation in [Web72] and [Web21b].

Third property: For a *steady-state* system, the distance between two particles can indeed vary, but there must be a finite least distance r_0, as well as a greatest one r^0, that depends upon the former. Now, the value of the least distance r_0 can be different for different systems and can therefore be considered to be a *third property* that will serve to distinguish between different systems but is likewise subject to variation as a consequence of external influences.

If one now denotes the quotient for such a system (in which $\rho = ee'/a$ has a negative value, as was remarked before) that is defined from the three constants ρc^2, $r_0^2 \alpha_0^2$ and r_0 by dividing the second one by the product of the first and last ones by $-n$, so one sets:

$$n = -\frac{r_0 \alpha_0^2}{\rho c^2} \ ,$$

then according to Section 16, in the place cited, that will give the following equation of motion, in which u denotes the relative velocity of both particles, namely:

$$\frac{\rho - r}{\rho} \cdot \frac{u^2}{c^2} = \left(\frac{r}{r_0} - 1 \right) \cdot \left(n \left[\frac{r_0}{r} + 1 \right] - 1 \right) \ .$$

It follows from this that for $u = 0$, one has either $r = r_0$ or $r = [n/(1-n)] r_0 = r^0$.

Moreover, that will imply the difference between *steady-state* and *non-steady-state* systems by way of the values of n. Namely, a *steady-state system* for which r_0 is the least value of r and r^0 is the greatest one will exist for only $1 > n > 1/2$, i.e., when the value of n lies between $1/2$ and 1. That is because for $n > 1$ and $n < 0$, no value at all will exist for r^0, which is essentially positive, and for $1/2 > n > 0$, one will get $r = r^0 < r_0$, i.e., the equation would no longer allow one to find the larger of the two values of r for which $u = 0$ from the smaller one but conversely, the smaller from the larger.

All *steady-state* systems will then split into two classes, namely, the ones for which $1/2 < n < 1 - \varepsilon$, which are *insulators*, and the ones for which $1 - \varepsilon < n < 1$, which are *conductors*. The ε in that is determined in such a way that when $n = 1 - \varepsilon$, the larger value of r for which $u = 0$, and which was denoted by r^0, is large enough that the moving particle will enter into the sphere of action of the neighboring system, and therefore go from one system to another. If one sets that value of r^0 equal to $(1 + \mu) r_0$ and observes that in general, one will have $r^0 = [n/(1-n)] r_0$, then one will get the equation $1 + \mu = n/(1-n)$ for $n = 1 - \varepsilon$, and as a result $\varepsilon = 1/(2 + \mu)$.

For the value $n = 1 - \varepsilon$ at which the *transition from an insulator to a conductor* will take place, the conductance[48] will be equal to 0, and it will increase with n when the latter is greater than $1 - \varepsilon$ and increasing.

1.7 Two Types of Heat Transfer in Ponderable Bodies

The considerations of the previous Section were essentially built upon the laws of motion of two electric particles that are left to only their own interaction. If other particles were present, then they would be assumed to be far enough away that their influence would almost vanish in comparison to that of the two particles under consideration. Only in the case where the two particles of a pair grow ever further apart must there be a limit beyond which the influence of other particles will become greater than the interaction of the particles in question. However, the laws of motion that apply to that transition are still not known

[48] [Note by AKTA:] In German: *Leitungsvermögen*.

completely and have not been developed in general. That is why the only result that was cited in the foregoing Section was that the two particles that had defined a pair up to a point in time will separate from each other and combine with other particles into new pairs.

Now, if the *heat* is the vis viva of moving particles in the interior of ponderable bodies, and if those moving particles are positively electrical ones that move around negatively electrical ones in the ponderable parts, then that will explain why in *metallic conductors* (as they were defined in the previous Section), one will find *heat propagation by conduction*[49] at the boundary surface between two conducting elements, and indeed in opposite directions simultaneously, namely, isolated positive electric particles will cross the boundary with the tangential velocity of their rotational motion about a molecule on one side of it and mix with the rotating electricity of a molecule on the other side of the boundary surface, and conversely. That *propagation of heat in metallic conductors*, which results from the transfer of vis viva through *all of its carrier*, is called the *propagation of heat by emission*, or more briefly, *thermal conduction*.

However, *heat transfer* likewise takes place in *insulators*, i.e., the transfer of vis viva from a molecule on one side of the boundary surface between two elements of the insulator to a molecule on the other side, and conversely, but without the electrical particles that are the carriers of that vis viva crossing that boundary surface in their own right. That *second type of heat transfer*, as it is found in insulators, namely, by the *transfer of vis viva without the transfer of its carriers*, is called the *transfer of heat by radiation*, or more briefly, *thermal radiation*.[50] It takes place from one ponderable body to another through empty space, e.g., in outer space.

It is known that the same thing is true for that transfer of heat by radiation in empty space or in insulators that is true for the radiation of light, namely, that it is mediated by the propagation of waves, which then assumes the existence of a medium that is capable of propagating waves. Up to now, one has sought to learn about the nature of that medium from the laws of wave propagation, as they were found from observations of light phenomena. However, if that medium consists of electricity, and one possesses more detailed knowledge about its constitution, then it would be possible to develop the laws of that wave propagation from the fundamental law of electric action and to explain the light phenomena in terms of it, which has actually being attempted in various ways, but we will not go further in that direction here.

1.8 On the Concept of Thermoelectricity Developed by Kohlrausch

We have distinguished between two types of heat transfer, namely, conduction and radiation, which coincide with two types of transfer for electrical motion, namely, the transfer of that motion either with or without its carrier. The *former* type of transfer takes place in *metallic conductors*, in which the electricity can also be set into current motion by *electromotive forces*.

The electric currents, however, which take place, even if the particles are not driven by electromotive forces, but merely by following the laws according to which they move around

[49][Note by AKTA:] In German: *Wärmeverbreitung durch Leitung*. This expression can be translated as heat transfer by conduction or heat propagation by conduction.

[50][Note by AKTA:] In German: *Wärmestrahlung*.

each other by virtue of their interaction, whereby they move away from each other until they exceed the molecular boundaries, differ substantially from the electric currents produced by electromotive forces in that, in the former, the same amount of electricity passes forward through the interface of *two identical and equally hot* molecules, while in the case of currents produced by *electromotive forces*, a greater amount of electricity passes through the interface in the direction the force than in the opposite direction. Those oppositely equal currents cancel each other out, so that no current in the narrower sense remains, because current in the narrower sense only means the difference between the two opposite currents.

For *equal, but unequally-warm* molecules in a metallic conductor on which no electromotive forces otherwise act and through which a steady-state current in a closed loop is produced, a larger amount of electricity can indeed go forwards through the boundary surface from the warmer molecule to the colder one during a moment than the amount that goes through it backwards, but that moment lasts only long enough for the excess of electricity that arrives at the colder molecules to generate a charge that exerts an electromotive force at the location of the boundary surface that will drive just as much electricity from the colder molecule backwards through the boundary surface as would go forwards without it, such that equality will once more be established in that way.

Once equality has been established, the *electric current* that goes through the boundary surface in a moment will vanish. By contrast, the *heat current* can still persist, namely, when the particles that come from the warmer molecules move *with greater velocity* than the ones that come from the colder molecules. One sees from this that the close connection between heat current and electric current, which is based upon the fact that both of them originate in the electricity that goes through the boundary surface, *does not at all imply* that no heat current can exist without electrical current, or conversely.

However, from the concept of *thermoelectricity* that Kohlrausch developed in the *Nachrichten der Königl. Gesellschaft der Wissenschaft zu Göttingen*, 1874, pp. 65,[51] such a connection between heat currents and electrical currents should exist *in reality*, in such a way that it would be the case that when electricity and heat are two bodies that are connected to each other by cohesive forces, one would very well speak of the transfer of heat by electricity, as well as the transfer of electricity by heat. However, heat is not a body, but the vis viva of a body, and as a result, *heat current* is the transfer of vis viva from one location to another, *either* by its carrier, as in metallic conductors, *or* without a carrier, as in insulators. It is only in the *former* case, namely, metallic conductors, that one can explain why it is possible for the connection that Kohlrausch proposed to exist. In the *latter* case, such a connection would not be possible since no electrical current would exist at all then.

In a unit time, an electric mass ε (in milligrams) with a velocity of α will go through an element f of the boundary surface between two metallic conducting elements from the warmer molecules on one side of it to the colder molecules on the other side. A mass ε' with velocity α' will go backwards through the same element of the boundary surface from the colder molecules to the warmer ones. In that way, an *electric current* that goes through f will be given, and likewise a *heat current* through f, each of which will have an intensity of $i = (\varepsilon - \varepsilon')$ in mechanical units (with the milligram as the unit of mass) and $W = (\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ as its mechanical equivalent.

The following cases are then possible, in general:

(1) $\varepsilon = \varepsilon'$,[52] in which a heat current of intensity $\varepsilon(\alpha^2 - \alpha'^2)$ would exist with no electrical

[51] [Note by AKTA:] [Koh74].

[52] [Note by AKTA:] In the original: $\varepsilon = \varepsilon$.

current;

(2) $\varepsilon\alpha^2 = \varepsilon'\alpha'^2$, in which an electric current of intensity $(\varepsilon - \varepsilon')$ would exist with no heat current.

(3) When one has neither $\varepsilon = \varepsilon'$ nor $\varepsilon\alpha^2 = \varepsilon'\alpha'^2$, but a certain relationship exists between $(\varepsilon - \varepsilon')$ and $(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ that remains constant under changes of temperature in the conductor, but which will vary with the different types of conductors.

The third case essentially agrees with the theory of thermoelectricity that Kohlrausch developed.

Namely, Kohlrausch made the assumption that the ratio of the intensities of the electric and heat currents $(\varepsilon - \varepsilon')/(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ was constant for each conductor, but depended upon the nature of the conductor, and he denoted it by α, which made the current intensity $i = \alpha W$, if W denotes the intensity of the heat current. Kohlrausch deduced the law of thermo-electromotive forces from that assumption, namely, that the thermo-electromotive forces depend upon only the temperature at the location where contact exists and that they are proportional to the temperature difference, as well as Peltier's law of heat production,[53] which says that heat will be produced (absorbed, respectively) at the location of contact between two conductors according to whether the current goes to a conductor of smaller (larger, respectively) thermoelectric constant.

The third case, namely, that a certain relationship exists between $(\varepsilon - \varepsilon')$ and $(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$, will obviously be satisfied when one sets $\alpha^2 = \alpha'^2$. However, under that restriction, the derivation of the law of thermo-electromotive forces that Kohlrausch gave found no application to thermopiles in which each of the conductors that define a closed loop possess different temperatures at its two ends since each temperature difference in a (homogeneous) conductor can have its basis in only the differences between the values of α^2 and α'^2. Therefore, the *first* law, namely, the law of thermo-electromotive forces, cannot be derived from the invariance of $(\varepsilon - \varepsilon')/(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ that results from $\alpha^2 = \alpha'^2$, although the *second* law, namely, Peltier's law of heat production (absorption, respectively), probably can.

Namely, if one has two different metallic conductors, and one lets m denote the quotient $(\varepsilon - \varepsilon')/(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ for one conductor that is constant for $\alpha^2 = \alpha'^2$, while the other one is denoted by n, then one will get the total heat that goes through the boundary surface between the last two elements of the first conductor as being equal to $m(\varepsilon - \varepsilon')$, while the total heat that goes through the boundary surface between the first two elements of the second conductor will be equal to $n(\varepsilon - \varepsilon')$. If a current of magnitude $(\varepsilon - \varepsilon')$ passes through the closed circuit formed by both conductors, then at the point where the first conductor touches the second, a total heat $(m - n)(\varepsilon - \varepsilon')$ will be *produced*. By contrast, at the other contact location, namely, the one where the second conductor contacts the first one, the total heat $(n - m)(\varepsilon - \varepsilon')$ will be produced, or what amounts to the same thing, the total heat $(m - n)(\varepsilon - \varepsilon')$ will be *absorbed* there.

However, in addition to the three cases that were cited above, there remains a fourth case to consider, namely, in addition to the cases in which one has either $\varepsilon = \varepsilon'$, $\varepsilon\alpha^2 = \varepsilon'\alpha'^2$, or $\alpha^2 = \alpha'^2$ in the quotient, there remains the case:

(4) One again has neither $\varepsilon = \varepsilon'$ nor $\alpha^2 = \alpha'^2$, but there is a dependency of the ratio α^2/α'^2 on the ratio ε/ε'; where, for example, α^2/α'^2 is equal to some power of ε/ε'.

Namely, in a metallic conductor, an increase in temperature will imply an increase in heat, i.e., from our assumption, that would be an increase in the vis viva of the moving electric parts in the conductor, which would imply an increase in the velocity of those parts, since

[53][Note by AKTA:] Jean Charles Athanase Peltier (1785-1845). See [Pel34].

their total amount or mass can suffer no change. Now, that increase in the velocity will also be true for the moment when they cross the boundary between two neighboring molecules whose velocity will be denoted by α. Therefore, α will increase with the temperature of the conductor. However, that increase in the velocity of all moving parts that is coupled with an increase in temperature assumes that the amount or mass of the particles ε that pass through the boundary surface per unit time increases in such a way that a simultaneous increase in α and ε will take place, which was assumed in the fourth case.

The equation:

$$\frac{\alpha^2}{\alpha'^2} = \left(\frac{\varepsilon}{\varepsilon'}\right)^n ,$$

will then give the following equation for the intensity of the heat current:

$$\varepsilon\alpha^2 - \varepsilon'\alpha'^2 = \varepsilon\alpha^2\left(1 - \frac{\varepsilon'}{\varepsilon}\cdot\frac{\alpha'^2}{\alpha^2}\right) = \varepsilon\alpha^2\left(1 - \left[\frac{\varepsilon'}{\varepsilon}\right]^{n+1}\right) .$$

If one divides that intensity of the heat current by the intensity of the electric current $(\varepsilon - \varepsilon') = \varepsilon(1 - \varepsilon'/\varepsilon)$, then one will get that the ratio of the two intensities is:

$$= \alpha^2\left(1 + \frac{\varepsilon'}{\varepsilon} + \ldots + \left[\frac{\varepsilon'}{\varepsilon}\right]^n\right) .$$

One sees from this that when $n = 0$, the fourth case will coincide completely with the third case that was considered already since the two cases will give:

$$\frac{\varepsilon - \varepsilon'}{\varepsilon\alpha^2 - \varepsilon'\alpha'^2} = \frac{1}{\alpha^2} .$$

The next case to consider is $n = 1$, for which one will get:

$$\frac{\varepsilon - \varepsilon'}{\varepsilon\alpha^2 - \varepsilon'\alpha'^2} = \frac{1}{\alpha^2\left(1 + \frac{\varepsilon'}{\varepsilon}\right)} .$$

However, the fact that the differences between two neighboring molecules in a conductor are always very small will further imply that the value of ε'/ε is only slightly less than 1, at least for weak currents in good conductors such that one will get approximately

$$\frac{\varepsilon - \varepsilon'}{\varepsilon\alpha^2 - \varepsilon'\alpha'^2} = \frac{1}{2\alpha^2}$$

in the fourth case that was just considered. The assumption that Kohlrausch made in this case that the intensity ratio of the electric and heat currents $(\varepsilon - \varepsilon')/(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ was a constant whose value depends upon only the nature of the conductor is also true in this case, at least *approximately*. It will then follow in this fourth case that one can deduce the same consequences that Kohlrausch had deduced from his assumption *approximately*, and in particular, the law of thermo-electromotive forces that those forces depend upon only the temperature at the location where contact exists, and they are proportional to the temperature differences at those locations.

If the derivation of the law of thermo-electromotive forces that Kohlrausch gave also made no appeal to any *contact action*, then that would explain the fact that such a contact action is not completely excluded in that way either, but such a thing might possibly be added.

1.9 Resistance to Conduction and Maximum Current Intensity

If the electricity in metallic conductors with a molecular constitution are actually to be in the state of motion that was given in Section 1.6, namely, the positive electric parts rotate around the negative parts that are endowed with ponderable masses, although they do not always remain in the same circular orbit in that, but begin from a smallest circular orbit whose radius increases so they approach another molecule and finally go over to that molecule, then that will yield a dependency of the current intensity on the electromotive forces in such conductors that would not agree completely with the one that is given by Ohm's law, but deviate from it by the fact that the current intensity does not always increase uniformly with the electromotive force, but will finally approach a certain limiting value that it cannot exceed. However, that limiting value will be attained only when the directions of all of the parts that go over to a current motion, no matter how different they might be to begin with, are all brought into the direction of the electromotive force in the shortest time through ever-increasing values of that force. The intensity of the current would not be able to increase further then, so it would have attained its maximum. Attempts to decide whether the intensities of the currents that are excited by very large and small electromotive forces in the same conductor are always proportional to those electromotive forces would then be of greatest importance.

In Figure 1, let A be a molecule from which positive electric particles are ejected in all directions with the same velocity α. Let one such direction be AB, and let ξ be the length of the path that the particle would traverse in a time interval t due to its velocity α. However, a constant (electromotive) force acts upon that particle in a direction that is parallel to AC and subtends an angle of ψ with AB, so the particle would then traverse a path of length η in the time interval t that increases in proportion to t^2 or ξ^2 by that alone.

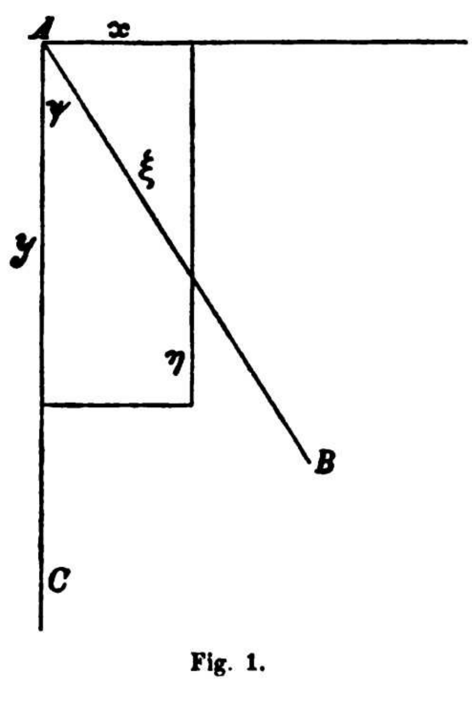

Fig. 1.

One then sets:

$$\eta = a\xi^2 \ ,$$

and furthermore:[54]

$$x = \xi \sin\psi \ ,$$

$$y = \xi \cos\psi + \eta = x \cot\psi + \frac{a}{\sin\psi^2} \cdot x^2 \ ,$$

$$r^2 = x^2 + y^2 \ ,$$

from which one will get:

$$y = \cot\psi \cdot \sqrt{r^2 - y^2} + \frac{a}{\sin\psi^2} \cdot (r^2 - y^2) \ .$$

That ballistic motion[55] comes to an end when the distance of the particle from A has become equal to r, as the particle then reaches the neighboring molecule. That distance r is independent of the direction of the ballistic motion and can be taken to be equal for all particles that are thrown from A, so it will be referred to as the mean molecular distance.

First of all, because the greater the electromotive force proportional to a is, the more all the other members of the equation above disappear against the member which has a as the factor, that for increasing electromotive force y^2 approaches a limit value, namely:

$$y^2 = r^2 \ ,$$

which will be the same for all particles that are ejected from A. That would then imply that the lengths of the paths that are traversed by all particles in the direction of the force will then be equal, namely, $= r$.

If ε denotes the mass of the particles that are emitted from A per unit time, and n denotes the number of molecules that are contained in an element of the conductor of length r; then $n\varepsilon$ will be the mass of positive electricity that would go in the direction of the electromotive force through the boundary surface between two successive molecular layers per unit time when the electromotive force is increased to infinity, i.e., the limiting value of the current intensity in mechanical units when based upon the mechanical unit of mass (viz., the milligram), in which one must note only that since the electricity cannot be determined in such mass units, in the determinations of the intensity in terms of so-called mechanical units, one does not prefer to express the total electricity in the mass units of mechanics (i.e., milligrams), but in *electrostatic* units.

Now, if σ denotes the number of *electrostatic* units that go to the unit mass of mechanics (milligram), then one will get the limiting value of the current intensity in so-called mechanical units equal to $n\varepsilon\sigma$, or when one adds the notation for the reduction of the mass unit to the three basic units of mechanics (namely, mass M, distance R, and time T), it is:

[54][Note by AKTA:] The equation for y should be understood as:

$$y = \xi \cos\psi + \eta = x \cot\psi + \frac{a}{\sin^2\psi} \cdot x^2 \ .$$

[55][Note by AKTA:] In German: *Wurfbewegung*. This expression can also be translated as throwing motion.

$$= n\varepsilon\sigma \cdot \left[\sqrt{\frac{MR^3}{T^4}}\right] .$$

In *electrostatic* units, an amount of electricity will be determined by a force (that this amount of electricity exerts on an equal amount of electricity) that is equal to $f \cdot [MR/T^2]$ and a distance (at which that force is exerted) equal to $r \cdot [R]$ and is expressed by:

$$r\sqrt{f} \cdot \left[\sqrt{\frac{MR^3}{T^2}}\right] .$$

However, the current intensity in mechanical units is the quotient of one such amount of electricity that goes through the cross-section of the conductor divided by the duration of that transfer, which equals $t \cdot [T]$, so that intensity will be equal to

$$r\frac{\sqrt{f}}{t} \cdot \left[\sqrt{\frac{MR^3}{T^4}}\right] .$$

In the present case, one has $r\sqrt{f}/t = n\varepsilon\sigma$.

Moreover, in that determination of the limiting value of the current intensity, it was assumed that the electromotive force itself had no effect on the number of electric particles that were emitted from the molecules.

Now, if by contrast the electromotive force or the quantity a that is proportional to it is very small, then one can set the y^2 in the last term of the equation that was found:

$$y = \cot\psi \cdot \sqrt{r^2 - y^2} + \frac{a}{\sin\psi^2} \cdot (r^2 - y^2) ,$$

which has a as a factor, equal to the approximate value that the equation will give when $a = 0$, namely, $y^2 = r^2\cos\psi^2$. One will then get:

$$y = \cot\psi \cdot \sqrt{r^2 - y^2} + ar^2 ,$$

and that will likewise give approximately:

$$y = +r\cos\psi + ar^2\sin\psi^2 .$$

That will then give, in the mean, for the two particles that were sent from A in the directions that were determined by the angles ψ and $\pi - \psi$:

$$y = ar^2\sin\psi^2 .$$

The mean value over all of the paths that are traversed by the particles that are emitted from A in the direction of the force will then be:

$$\frac{1}{2\pi}\int_0^{\pi/2} 2\pi y\sin\psi d\psi = ar^2\int_0^{\pi/2}\sin\psi^3 d\psi = \frac{2}{3}ar^2 .$$

If that value were equal to r, then the current intensity would be the same as the previously-considered limiting value, namely, it is equal to $n\varepsilon\sigma \cdot [\sqrt{MR^3/T^4}]$. Now, the actual current intensity amounts to only a very small fraction of that, namely, $2ar/3$, which will give that current intensity i^0 as:

$$i^0 = \frac{2}{3} ar \cdot n\varepsilon\sigma \cdot \left[\sqrt{\frac{MR^3}{T^4}} \right] .$$

Finally, in order to determine the coefficient a, it should be pointed out that when γ denotes the accelerating force that acts on the particles that are emitted from A, one will have:

$$\eta = \frac{1}{2} \gamma t^2 = \frac{1}{2} \gamma \cdot \frac{\xi^2}{\alpha^2} = a\xi^2 ,$$

which will give:

$$a = \frac{1}{2} \cdot \frac{\gamma}{\alpha^2} ,$$

and as a result:

$$i^0 = \frac{1}{3} \cdot \frac{\gamma r}{\alpha^2} \cdot n\varepsilon\sigma \cdot \left[\sqrt{\frac{MR^3}{T^4}} \right] .$$

Upon dividing the current intensity i^0, when expressed in *mechanical* units, by $c/2\sqrt{2}$, namely, by the number of electrostatic units that go into a magnetic unit, one will get that same current intensity i, when expressed in *magnetic* units, namely:

$$i = \frac{2\sqrt{2}}{3c} \cdot \frac{\gamma r}{\alpha^2} \cdot n\varepsilon\sigma \cdot \left[\sqrt{\frac{MR}{T^2}} \right] .$$

Now, the *electromotive force* per unit length of the conductor, in *mechanical* units, e^0, is the product of the acceleration γ with the amount of electricity that flows per unit length of the conductor, which is equal to $n\varepsilon/r$, divided by the number of electrostatic units that are flowing in the unit length, which is equal to $n\varepsilon\sigma/r$, so one will have:

$$e^0 = \frac{\gamma}{\sigma} \cdot \left[\sqrt{\frac{M}{RT^2}} \right] .$$

That will then give the electromotive force per unit length of the conductor in *magnetic* units, e, by switching the number of electrostatic units $n\varepsilon\sigma/r$ with the number of magnetic units $n\varepsilon\sigma/r \cdot 2\sqrt{2}/c$, which will make:

$$e = \frac{c}{2\sqrt{2}} \cdot \frac{\gamma}{\sigma} \cdot \left[\sqrt{\frac{MR}{T^4}} \right] .$$

If one now substitutes the value of γ that this gives in the foregoing equation for determining i, then one will get:

$$i = \frac{8}{3c^2} \cdot \frac{er}{\alpha^2} \cdot n\varepsilon\sigma^2 \cdot \left[\sqrt{\frac{MR}{T^2}} \right] .$$

Now, if l denotes the length of the closed conductor, then el will be the total electromotive force that acts upon the closed conductor, and i is the intensity of the current that it generates in *magnetic* units. That will then give the resistance w of the closed conductor:

$$w = \frac{el}{i} = \frac{3c^2}{8} \cdot \frac{\alpha^2}{n\varepsilon\sigma^2} \cdot \frac{l}{r} \cdot \left[\frac{R}{T}\right] ,$$

i.e., a *definition of resistance* that is completely independent of the determination of resistance according to Ohm's law that involves measuring the electromotive force and current intensity.

That will then give a determination of the *conductance*, $= 1/w$, in terms of the causes for the deviation of the particles from the trajectories along which they were ejected. Namely, that explains why the *conductance* must be proportional to:

1. The mass that is found to be in a ballistic motion in the conducting channel.

2. The rate of deviation that is produced by a certain force along a well-defined path in that mass.

In the conducting element r that mass is now $n\varepsilon$, and the deflection velocity caused by a certain force on the path r is *inversely proportional to the square of the ballistic velocity* α^2. The *conductance* $1/w$ must then be proportional to $n\varepsilon/\alpha^2$, and as a result, the *resistance of the conductor* w must be proportional to $\alpha^2/n\varepsilon$. Since that is true for the resistance of the *conducting element* r, that will imply that the *resistance of a conductor of arbitrary length* l will be proportional to $\alpha^3/n\varepsilon \cdot l/r$, which coincides with the formula above, so that resistance is equal to the product of that quantity with the *constant factor* $3c^2/8\sigma^2$.

That definition of the resistance of a conductor proves to be especially interesting due to the fact that it would then follow that when the resistance of a conductor w is constant, in addition to the values of l, n, r, the ratio of the two variables α^2 and ε, namely, α^2/ε, must likewise be constant, i.e., when α^2 and ε have changed into α'^2 and ε', respectively, so one would then need to have:

$$\frac{\alpha^2}{\varepsilon} = \frac{\alpha'^2}{\varepsilon'} \qquad \text{or} \qquad \frac{\alpha^2}{\alpha'^2} = \frac{\varepsilon}{\varepsilon'} .$$

Now, it follows from this that when the resistance w of a conductor is constant and it does not vary with the temperature of the conductor either, the value α^2/ε for that conductor will also be constant, so for such a conductor, according to Section 1.8, the view of thermoelectricity that Kohlrausch presented would be valid. However, since the resistance of metallic conductors varies more or less with their temperature, that would imply that the viewpoint that Kohlrausch presented could be true only approximately, and indeed mostly for metallic conductors whose resistance changes the least with the temperature of the conductor, and it would seem to follow from this that such a conductor would be most suited to the representation of thermomagnetic circuits.

1.10 Distribution of Electricity in Conductors

Electrostatics was founded and developed by Coulomb[56] and Poisson,[57] before the discovery of electromagnetism and electrodynamics, and therefore no account could be taken of these great discoveries by them. Indeed, the law of distribution for the electric fluids that are

[56][Note by AKTA:] Charles Augustin de Coulomb (1736-1806). Coulomb's main works on torsion, electricity and magnetism have now been fully translated and commented in Portuguese and English, [Ass22] and [AB23]. See also [Pot84], [Gil71b] and [Gil71a].

[57][Note by AKTA:] Siméon Denis Poisson (1781-1840). See [Poi12a], [Poi12b] with English translation in [Poi19], [Poi13] and [Poi14].

found in conductors at rest and in equilibrium that is developed in electrostatics, as well as the forces that are exerted by that distribution, have all been found to be in agreement with experiments, to the extent that observation and measurement allows. However, new discoveries, in particular, electromagnetism and electrodynamics, have shown that such a state of equilibrium in electric fluids, as Coulomb and Poisson assumed to exist in conductors, does not actually exist at all, but that all electric fluids in conductors are found to always be in *steady-state motion* around ponderable molecules, from which it would follow that, strictly speaking, the laws of distribution and action of electricity *at rest* that Poisson developed will find no application to the electricity that is found in conductors.

All of the phenomena that were considered in *electrostatics* up to now actually belong to *electrodynamics* accordingly, and it is in the laws of the latter that one must seek the complete explanation for the former. It would then seem that electrostatics, which previously defined the largest and most important part of the study of electricity, would have to experience a complete remodeling. However, one must make the demand of such a remodeling that it must be able to explain the entire sphere of phenomena that was explained by *electrostatics* up to now just as completely and precisely in terms of *electrodynamics*, but that has not happened up to now, nor has any attempt been made to do that yet.

For all of the tendency that one finds, on the one hand, to give the theory of magnetism that Coulomb and Poisson developed simultaneously with electrostatics on the basis of the picture of *magnetic fluids*, it would nonetheless seem, on the other hand, that a certain timidity exists in the face of the consequence that it would imply, namely, the indispensable picture of the existence of steady molecular currents in all magnetic and diamagnetic bodies which would mean that electricity never and nowhere achieves a state of rest and equilibrium. That comes down to saying that up to now any attempt to find an *electrodynamic* explanation for all previously considered phenomena of *electrostatics* has met with great difficulties, namely, due to lack of any assistance on the part of mathematics, which has proved to be powerless to deal with such complicated processes for some time now.

However, on the same grounds, just as little of an attempt has been made at the foundations of *electrostatics* to give a more precise justification for the constitution of the so-called *neutral fluid* and the *process for separating* that fluid in conductors, but one has restricted oneself to a general assumption about the mutual mobility of the two components of the neutral fluid, as well as their mixing, and in that way one has sought to make the development of the law of distribution of electricity independent of a more detailed knowledge of the constitution of the neutral fluid that dissipates everywhere in the interiors of conductors.

However, on the basis of the assumption that one makes in electrostatics of the mutual mobility of the two components of a neutral fluid, nothing at all can be determined and established regarding the internal constitution of that fluid itself, and especially nothing regarding whether the two components are at rest and in equilibrium before their separation or whether they are found to be in motion with respect to each other, e.g., a rotating motion, such that the picture of steady molecular currents would still not seem to be excluded completely by the assumption that electrostatics makes of the mutual mobility of the two components of the neutral fluid, in principle. Rather, the picture of steady molecular currents could be regarded as an attempt to explain the assumed mutual mobility of both components. Therefore, electrostatics, as it was developed by Poisson, regardless of the fact that one cares to define it as the study of the distribution of the electricity in conductors when it is *at rest and in equilibrium*, is nonetheless not in direct contradiction with the existence of steady molecular currents of electricity in the interior of conductors. We have the *statics* *of solid bodies*, *hydrostatics*, and *aerostatics*, which also seem to be well-founded, and the same thing applies to them completely. Even when they are defined to be the study of rest and equilibrium for those bodies, that will not, however, contradict the fact that the interiors of those bodies are filled with particles that are not at rest, but are found to be in a state of constant motion, and indeed, large motions. That is because in just the same way that the *magnetic* and *diamagnetic* phenomena in bodies led to continual internal motions (electric molecular currents) in them, similarly, the *thermal phenomena* in those solid, fluid, and gaseous bodies for which statics, hydrostatics, and aerostatics are valid have led to such continual internal motions. That is because at each moment, every ponderable body will possess a temperature that is the effect of the heat that the body contains, and that heat is nothing but the vis viva that the moving particles in the interior of the body possess. However, the measurable magnitude of that vis viva (viz., the mechanical equivalent of heat) has shown that those motions are very large in the interiors of all such bodies.

If positive electric particles move around the ponderable masses that are endowed with negative electric particles in the *interior of conductors*, but do not always remain in the same circular orbit during that motion, but approach the neighboring molecules with an increasing radius until they finally go over to them, and if such transitions from a molecular in the interior of the conductor to all neighboring conductor molecules take place in all directions indifferently, and at the same time, those conductor molecules emit an equal number of particles in all directions to the neighboring molecules, then that would imply, by contrast, that each molecule of the conductor that lies next to its outer surface would need to have an insulator molecule as a neighbor on its outer side, instead of a conductor molecule, and no such emission would take place from it nor would it receive particles that were emitted by other molecules. That implies that when isolated particles have gotten so far from the center of their circular orbits as the radius increases that they go over to the neighboring molecule, when other conductor molecules are still found on the side where they are even found, that process will not even happen when no conductor molecules are present at all on the side where they are found, but only insulator molecules. Every particle will then proceed somewhat further in its circular orbit with increasing radius until it arrives at a side where other conductor molecules are again found in its neighborhood. That will be the case when the resultant of all electric forces is zero at the boundary between the conductor and the insulator.

By contrast, when that resultant is non-zero and points *outward*, it can exert the same influence as a neighboring conductor molecule, namely, it can act in such a way that particles that are also on the side of the bounded insulator can leave the path that was previously followed around the conductor molecule and be fixed at the neighboring insulator particle. Therefore, such eliminated particles of positive electricity can accumulate everywhere on the boundary surface between the conductor and the insulator according to the magnitude and direction of the outward-pointing resultant at each location on it.

If the resultant is non-zero and *inward*-pointing, then its effect on the increase of the particles that are emitted *inward* by the next conductor molecule can only be compensated by the fact that amount of positive electric particles that are found in those conductor molecules has been reduced somewhat, while the amount of negative electric particles that the ponderable mass is endowed with and which orbit around it remains unchanged.

Obviously, the distribution law that Poisson developed, which said that the excess of positive or negative electricity will exist only on the boundary surface between a conductor and an insulator, in any event, will be valid for the excess of positive electricity at some locations on the boundary surface between the conductor and the insulator and for the absence of positive electricity at the other locations that are close to the conductor molecules at the boundary of the insulator (and that lack of positive electricity is equivalent to an excess of negative electricity). For these laws of distribution of electricity on the surface make no difference whether there is a so-called separable, neutral fluid inside the conductor, as Poisson assumes, or whether there are conductor molecules with electricity moving around them, between which a continuous exchange of individual particles takes place. Such conductor molecules shall have just as little influence on the distribution of electricity on the surface as the neutral fluid that Poisson assumed, and conversely, the electricity that is distributed on the outer surface according to Poisson's law will have no effect on the conductor molecules since, according to Poisson, the distribution of electricity on the outer surface will be determined by just the fact that the resultant of the force that is exerted by all of the electricity that is distributed on the outer surface on any point in the interior should be zero, which is a demand that is entirely independent of whether a neutral fluid is found at the location considered in the interior of the conductor or a conductor molecule with electric molecular currents. — The true constitution of bodies and the true processes that depend upon that, although they are also very complicated and can be thought of as being represented by simpler processes only in part, will nonetheless remain the focus and ultimate goal of research, in spite of all obstacles.

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