On Galvanometry (Excerpt)

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Wilhelm Weber

Editor's Note: An English translation of Wilhelm Weber's posthumous paper "Zur Galvanometrie (Auszug)".¹

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 1 [Web94].

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Chapter 1 On Galvanometry (Excerpt)

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(From the 10th volume of the Abhandlungen der Königl. Gesellschaft der Wissenschaften zu Göttingen — Treatises of the Royal Society of Sciences in Göttingen)^{5,6}

1.1 First Part

The First Part of the present Treatise has the object of precisely determining the resistance of a given *resistance standard* in absolute units.

The determination of a *resistance* according to absolute units is based on the determinations of a *electromotive force* and a *current intensity* according to absolute units; because according to Ohm's law,⁷ the resistance of a circuit is to be equated to the quotient of the electromotive force acting on the circuit, divided by the intensity of the current produced by this force in the circuit.

The *electromotive force* which is exerted by the Earth's magnetic force T on a closed conductor while it is moving can now be determined most accurately by absolute units. If one denotes by S the surface area which encloses the projection of the closed conductor on the normal plane of T, and by dS the change in this surface space in the time element dt as a result of the movement, then the electromotive force exerted by T on the closed conductor in absolute units is

 $^{^{2}}$ [Web94].

³Translated and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

⁴The Notes by Wilhelm Weber are represented by [Note by WW:]; the Notes by H. Weber, the editor of the fourth volume of Weber's *Werke*, are represented by [Note by HW:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

⁵[Note by HW:] This essay is an excerpt from the treatise "On Galvanometry", Wilhelm Weber's Werke, Vol. IV, p. 17, which was probably intended to be published in Poggendorff's Annalen.

⁶[Note by AKTA:] [Web62].

⁷[Note by AKTA:] Georg Simon Ohm (1789-1854). Ohm's law is from 1826: [Ohm26a], [Ohm26c], [Ohm26d], [Ohm26b] and [Ohm27] with French translation in [Ohm60] and English translation in [Ohm66].

$$e = \frac{TdS}{dt} \; .$$

It is not necessary that all parts of the closed conductor take part in the movement; a part of the conductor can remain at rest, if only the movement of the remaining part is such that the value of S can always be precisely determined.

Furthermore, the *intensity of the current produced by such an electromotive force in the closed conductor* would be easily determined in absolute terms using a *tangent galvanometer*⁸ if the effect of the current on the galvanometer were strong enough to be accurately measured. But because this is not the case, the tangent galvanometer, in which the multiplier forms a wide circle around a very small compass needle, must be replaced with a *very sensitive galvanometer*, where the multiplier encloses the compass needle very closely.

Such a sensitive galvanometer can now be constructed in such a way that, as with the tangent galvanometer, in equilibrium, the tangent of the deflection of the compass needle from the meridian is proportional to the current intensity; but the factor by which that tangent must be multiplied in order to give the current intensity according to absolute units, which for the tangent galvanometer has the known value $rT/2\pi$, if r is the radius of the multiplier circle, is unknown for a sensitive galvanometer, where the multiplier turns are very close to the compass needle.

But should this factor also have been determined, which is possible by measuring the damping exerted by the multiplier on the moving compass needle when the circuit is closed;^{9,10,11} it would not be possible to make use of this determination, because the compass needle never reaches the assumed equilibrium under the influence of the current produced by the electromotive force e = TdS/dt, but because of the variability of e, always oscillates.

The *regularity* of these oscillations depends, however, on the concentration of the action

$$= \pi \sqrt{\frac{k\sqrt{\pi^2 + \lambda^2}}{2w\lambda t^3}} \; .$$

Here w now has the meaning of the resistance of the circuit to which the multiplier belongs, in absolute units, the value of which is precisely the task of measuring the resistance, which requires the determination of the current intensity in absolute units. But if this current intensity i can also be represented as a function of w using this factor, then equating the quotient e/i according to Ohm's law with the resistance w leads to an equation, in which w is the only unknown quantity whose value is determined thereby. But after w is found in this way, the above factor is also determined and can then be used to measure all current intensities in the same circuit.

 $^{10}[\mbox{Note by AKTA:}]$ The logarithmic decrement is defined as the logarithm of the ratio of any two successive peaks.

¹¹[Note by AKTA:] J. C. F. Gauss (1777-1855) and W. E. Weber utilized the French definition of the period of oscillation t, which is half of the English definition of the period of oscillation T, that is, t = T/2, [Gil71, pp. 154 and 180]. For instance, the period of oscillation for small oscillations of a simple pendulum of length ℓ is $T = 2\pi\sqrt{\ell/g}$, where g is the local free fall acceleration due to the gravity of the Earth, while $t = T/2 = \pi\sqrt{\ell/g}$.

⁸[Note by AKTA:] In German: *Tangentenboussole*. The tangent galvanometer was invented by Johan Jakob Nervander (1805-1848), [Ner33] and [Sih21]. Friedrich Kohlrausch discussed measurement of currents with the tangent galvanometer, [Koh83, Chapters 64 and 65, pp. 188-192].

⁹[Note by WW:] If e denotes the base number of the natural logarithms and e^{λ} : 1 denotes the ratio of two consecutive oscillation arcs of the compass needle under the influence of the *damping* exerted on the compass needle by the multiplier when the circuit is closed, where λ is called the logarithmic decrement and is determined from the observations of the oscillating compass needle; then, if t denotes the oscillation period of the compass needle without damping, and k denotes its moment of inertia, the desired factor is

of the current produced by the electromotive force e = TdS/dt on the compass needle over a time interval in the middle of the oscillation, which is only a very small part of the oscillation period. In order to increase this momentary action as much as possible, the closed conductor is moved during the short period of time in such a way that the value of S either goes from the minimum S_0 to the maximum S^0 or vice versa. Such a movement of the closed conductor is called an *induction surge*,¹² and the sum of the electromotive force exerted by it is

$$\int edt = \pm \left(S^0 - S_0 \right) T \; .$$

According to Ohm's law, if w denotes the unknown constant resistance of the closed conductor, the sum of current produced by it is

$$\int idt = \frac{1}{w} \int edt = \pm \frac{S^0 - S_0}{w} T$$

after which is found the unknown resistance

$$w = \frac{e}{i} = \frac{\int edt}{\int idt} = \pm \frac{(S^0 - S_0)T}{\int idt}$$

If one now denotes the change in the angular velocity¹³ of the compass needle caused by such an induction surge as γ , then with a sensitive, appropriately constructed galvanometer as well as with the tangent galvanometer, γ would be proportional to the current sum $\int i dt$ produced by the induction surge; but the *factor* by which γ must be multiplied in order to give $\int i dt$ according to absolute units, which for the tangent galvanometer has the known value $[r/2\pi] \cdot [k/m]$, is *unknown* for such a sensitive galvanometer, where the multiplier turns are very close to the needle; however, this factor can also be determined by measuring the *damping* exerted by the multiplier on the moving compass needle when the circuit is closed. If, as in the previous Note, the logarithmic decrement resulting from this damping is denoted by λ , then this factor is

$$=\sqrt{rac{k au}{2w\lambda}}$$
,

if τ denotes the oscillation period of the compass needle under the influence of damping, or, if t denotes the oscillation period with the open circuit,

$$= \sqrt{\frac{kt\sqrt{1+\frac{\lambda^2}{\pi^2}}}{2w\lambda}}$$

However, with such a sensitive galvanometer as is required for these experiments, it is of great importance that this factor applies at exactly the same time as the other galvanometer observations necessary for resistance measurement are made. It is therefore particularly important to have such an *arrangement of the induction surges* so that the two quantities γ and λ can be determined at the same time from the observed oscillations of the compass needle.

¹²[Note by AKTA:] In German: *Induktionsstoss*. This expression can be translated as induction surge, induction shock, inductive surge or inductive shock. That is, an induced voltage surge generated by electromagnetic induction, or a short-term induced voltage shock caused by electromagnetic induction.

¹³[Note by AKTA:] In German: *Drehungsgeschwindigkeit*.

The simplest method that achieves this is the *throwback method* given by Gauss,¹⁴ which was described in the "Treatises on electrodynamic measurements, resistance measurements, Appendix C["].^{15,16} According to this, the compass needle is set in such oscillations that a larger elongation a always alternates with a smaller b, where a and b can be observed very precisely. It then emerges

$$\lambda = \log \frac{a}{b} ,$$
$$\gamma = \frac{\pi}{t} \left(\frac{a^2 + b^2}{\sqrt{ab}} \right) \cdot e^{-\frac{\lambda}{\pi} \arctan \frac{\lambda}{\pi}}$$

or more precisely, if one takes into account the value of the logarithmic decrement λ_0 , which remains even when the circuit is open, and sets $\lambda_0 + \lambda = \lambda_1$, $t_0 = t\sqrt{1 + \lambda_0^2/\pi^2}$, where t_0 denotes the oscillation period observed at the logarithmic decrement λ_0 ,

$$\lambda_1 = \log \frac{a}{b} ,$$

$$\gamma = \frac{\sqrt{\pi^2 + \lambda_0^2}}{t_0} \cdot \left(\frac{a^2 + b^2}{\sqrt{ab}}\right) \cdot e^{-\frac{\lambda_1}{\pi} \arctan \frac{\lambda_1}{\pi}}$$

Adding the equations found above, namely, taking λ_0 into account,

$$\int i dt = \gamma \cdot \sqrt{\frac{kt\sqrt{1 + \frac{\lambda_1^2}{\pi^2}}}{2w(\lambda_1 - \lambda_0)}} ,$$
$$w = \frac{(S^0 - S_0)T}{\int i dt} ,$$

this makes it easily to calculate the resistance w from the observed quantities:

$$a, b, \lambda_0, t_0, k, \frac{1}{2}(S^0 - S_0), T.$$

However, as is self-evident, the certainty and accuracy of the results from the observations made according to these regulations depend primarily on the *device* of the galvanometer used for this purpose and the remaining part of the closed circuit that can be moved for the purpose of the induction surges. The solution of this subtler problem of galvanometry, which concerns the most practical arrangement of such a measuring apparatus, therefore forms the main subject of this Part [of the paper].

It is easy to understand that it is not only important to have a very large sensitivity of the galvanometer, which should be so large that a corresponds to a very large number of parts of the scale, so that the value of a from the observations made on the scale is reliably obtained up to a very small fraction of it; but it is also important that b stands in an appropriate relationship to a, so that the value of λ is safely preserved, except for a

¹⁴[Note by AKTA:] In German: Zurückwerfungsmethode. See [Gau38], [Web39] and [WK68, p. 108, Note 13]. ¹⁵[Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 441.

¹⁶[Note by AKTA:] [Web52, p. 441 of Weber's Werke] with English translation in [Web21, Appendix C, p. 404].

very small fraction of it. It is also considered that the accuracy of the observation by the throwback method requires that, since the duration of an induction surge cannot be reduced to less than 1 second, the duration of the oscillation of the compass should be approximately 20 to 30 seconds, and finally that the compass should be fitted with a transverse beam¹⁷ and appropriate weights in order to be able to determine its moment of inertia with great finesse and precision.

If only the sensitivity of the galvanometer were taken into consideration, it would be important to display the smallest and yet precisely observable compass needle, which could be enclosed quite closely by the multiplier, and then to determine the strength of the wire and the cross-section of the multiplier appropriately; but then the *damping* would not be large enough to determine λ exactly.

Since $\lambda = \log(a/b)$, and a can be regarded as determined by the demands made on the sensitivity of the galvanometer, it is clear that in order to obtain the value of λ to the smallest fraction of it with certainty, the following condition must be fulfilled, namely

$$\left(\frac{\lambda db}{d\lambda}\right)^2 = b^2 \left(\log \frac{a}{b}\right)^2 =$$
maximum,

hence $\lambda = 1$ or a/b = e = 2.718...

However, the requirement for such a strong damping can only be met with *stronger magnetism of the compass needle*, which can be easily achieved, without a significant reduction in sensitivity, by increasing the dimensions of the compass needle in proportion to the length and thickness.

However, with regard to the sensitivity, this increase in the size of the compass needle should not go further than is necessary for the purpose of attenuation; such a strongly magnetic needle would still have a period of oscillation that is far too short. In order to bring this oscillation period to 20 to 30 seconds, it is most expedient to form a *astatic system* by firmly connecting an identical magnet with an oppositely directed axis above the multiplier with the compass needle in the multiplier, and to hang it on an elastic metal wire that is so strong that the oscillation period of the system is thereby established in a prescribed manner.¹⁸

After this, it is essentially only important to establish appropriate rules for the *wire* strength and for the cross-section of the multiplier.

It easily turns out that it is most advantageous if the resistance of the multiplier is made equal to the resistance of the remaining part of the closed circuit, where according to Ohm's laws, the *ratio* l/s of the multiplier wire length l to its cross section s can be considered as given.

Furthermore, with regard to the regular winding of the wire, a rectangular shape of the multiplier cross section may be assumed to be the most expedient, which is determined by the two sides of the rectangle a and b, of which the latter is the horizontal one, which is bisected by the compass needle meridian.

¹⁷[Note by AKTA:] In the original: *virga transversalis*.

¹⁸[Note by AKTA:] The adjective "astatic" is used in physics with the meaning of something having no tendency to take a definite position or direction. An astatic needle can be a combination of two parallel magnetized needles having equal magnetic moments, but with their poles turned opposite ways, that is, in antiparallel position. The arrangement protects the system from the influence of terrestrial magnetized needle had also been created by Ampère, [Amp21] and [LA98]. An earlier system composed of a single magnetized needle had also been created by Ampère, [Amp20c, p. 198] with Portuguese translation in [CA09, p. 133], [Amp20a, p. 239] and [Amp20b, p. 2], see also [AC15, p. 57].

However, the values of a and b should now satisfy the condition that for a given area ab any change in the ratio of a to b would weaken the sensitivity, assuming that the position of the inner rectangle facing the compass needle remains unchanged and only the outer rectangle sides may be moved. This condition equals the mean value of the moment exerted on the compass needle by all current elements on the outer surface of the multiplier, corresponding to the horizontal rectangular side, with the mean value of the moment exerted on the compass needle by all current elements on the side surfaces of the multiplier, corresponding to the vertical rectangular side.

To simplify the equation between a and b resulting from this condition, consider the case where the compass needle occupies only a very small space in the center of the multiplier and the outer and inner surfaces of the multiplier form concentric cylinders around it; if one sets the given radius of the smaller cylinder = 1, the following equation results between aand b, namely:

$$\log \frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}} = \frac{3(1+a)^2-1}{2(1+a)\sqrt{(1+a)^2+b^2}} - \frac{1}{\sqrt{1+b^2}}$$

Finally, you get another condition if, while fulfilling the specified relation between a and b, and with the position of the inner side of the rectangle facing the compass needle remaining unchanged, you let a and b grow at the same time, and calculate the associated growth of the multiplier volume ls = v. We then designate the ratio l/s, given above as constant, by c, after which the wire cross-section $s = \sqrt{v/c}$ is found; this results in the growing number of multiplier turns $2ab/s = 2ab\sqrt{c/v}$. According to this, the magnitude of the moment exerted on the compass needle by a certain current passing through the multiplier can be calculated as a function of the value a or b, and it follows that as a or b increases, this moment also initially grows, but then becomes a maximum, and if a or b continued to grow, it would even decrease again. This also results in the condition of taking the value for a or b for which that moment is a maximum.

If one sticks to the case of a circular multiplier described above to simplify the resulting equation, the following formula results for a:

$$\log \frac{(1+a)\sqrt{(1+a)^2 - 1} + (1+a)^2 + 1}{\sqrt{(1+a)^2 - 1} + 2(1+a)} = \frac{[(1+a)^2 - 1]^{3/2}}{(1+a)[(1+a)^2 + 1]}$$

from which a = 2.0951 follows, and then, according to the previous relations,

$$b = 1.86178$$
 ,
 $v = 100.364$,
 $l = 10.0182 \cdot \sqrt{c}$,

is found. If the radius of the smaller cylinder, which was set = 1, is denoted by ε , one obtains

$$a = 2.0951 \cdot \varepsilon \; ,$$

 $v = 100.364 \cdot \varepsilon^3$, $l = 10.0182 \cdot \sqrt{c\varepsilon^3}$.

The observations given as examples in the treatise finally prove what great accuracy of results can be achieved with instruments whose setup corresponds, even if only approximately, to the given regulations. In order to meet all the regulations exactly, all instruments would have had to be presented from scratch. It seemed sufficient to have shown that the *galvanometric* part of the observations can be carried out so precisely according to these regulations, that it in no way corresponds to the *magnetic* part of the observations for determining the Earth's magnetism T, it follows that the unavoidable uncertainty in determining the absolute resistance resulting from that part of the observations turns out to be even smaller than that resulting from the latter part.

However, the entire measuring apparatus required for the absolute determination of resistance would deserve to be manufactured in the most complete and perfect way for the long term, if it is a definitive determination of a normal standard of resistance, with generally widespread and used standard copies, of which the most precise knowledge of their value in absolute units would be required in order to be able to transfer this knowledge to all other resistances compared with them. It would then be most expedient to set up the measuring apparatus itself in such a way that the closed circuit of it formed the normal standard, because only by repeating the absolute measurement from time to time can full security be achieved that the normal standard really remained unchanged. — However, the achievement of the main purpose of such a statement would depend on the standard copies, in particular on the general distribution and application that they would find, as well as on their guaranteed equality with the normal standard. For the latter purpose, the fineness of the copying methods and the most appropriate addition to the measuring apparatus were discussed.

1.2 Second Part

The second Part of the treatise discusses the possibility of whether absolute resistance measurements could be carried out *in various ways*, according to very different principles.

The resistance is a property of the ponderable body through which the current passes. This property must have its basis in the peculiar nature of the body itself, and should therefore, with complete knowledge of this nature, be directly determinable from this, completely independent of the consideration of all circumstances that do not directly affect the nature of the body, that is, independent *firstly* from the consideration of the variable forces which act on the electrical fluids contained in the body, *secondly* from the consideration of the movements into which these fluids are set by those forces, *thirdly* by considering the effects produced by these movements.

It is only because such a direct determination of resistance from its basis in the nature of the body itself is not possible, owing to lack of knowledge of this nature, that this resistance can only be known *indirectly from experience*, through careful observation of the behavior of bodies (to electricity) under different conditions, and by determining what is *constant* to it.

If the body forms a ring or a closed circuit in which a current i is generated, — an electromotive force e acts on the electrical fluids contained in it, and the current i is created

in the body, — the precise observation of that electromotive force e and this current intensity i shows, even in cases where both have very different values, that every body has a specific and constant value of the *ratio* e/i. — The property of the body that gives it this constant value of e/i is called its *resistance*.

But if the body now forms a closed circuit in which is present the current *i*, the continuation of the current is associated with certain effects, which are called the *electrical work* A,¹⁹ and the precise observation of the current intensity *i* and the electrical work A shows that every body has a certain and constant value of the ratio A/i^2 . The property of the body by virtue of which it has this constant value, could now equally rightly be called its *resistance*, but the question is whether this constant value is *identical* to the previous one.

If this were the case, — which presupposes that the two properties of the same name have the same basis in the nature of the body, — then it would be possible to carry out resistance measurements in two different ways, according to two completely different principles, their agreement between each other would then serve to confirm that both properties were essentially identical. However, the latter method would first require a more detailed discussion of the effects associated with the continuation of the current, which are given the name electrical work.

For these effects of the continuous current can be partly *direct*, partly *indirect*, both of which can be useful for the purpose of measuring resistance, but should not be confused with each other.

Through careful observation, one now learns to know the *heat generation*²⁰ in the body as an effect associated with the continuation of the current. According to the mechanical theory of heat, however, heat generation is viewed as *work* and it is therefore obvious to recognize the *electrical work* A in this heat generation. The question, however, is whether this thermal effect of the ongoing current is *direct* or *indirect*. Because if it were an indirect effect, there could be other indirect effects besides it, which would have to be taken together in order to obtain the *whole electrical work* A.

For example, if there were a movable magnet nearby, the movement of the magnet would also be an effect associated with the continuation of the current,²¹ which seems to be equally rightly described as electrical work.

An attempt has therefore been made to, *in the first place*, precisely define the effect that is directly and therefore necessarily associated with the duration of the current, which, because it is the original, it deserves to be called *simply the electrical work*, and then *secondly* to particularly research the relationship of every effect of the continuous current that has become known to us from experience to that electrical work.

It is assumed the *molecular constitution of the ponderable body* and the *existence of two electrical fluids* between the body molecules; on the other hand, the presence of two magnetic fluids is ignored and instead, as is known, such a nature of the ponderable molecules is assumed, thanks to which the electric fluids can form *persistent molecular currents* around them, from which all *magnetic* and *diamagnetic* phenomena of the body can be explained.

According to this, the process of a continuous current is that electrical fluid is pulled out of the molecular current of a body molecule, driven to the next body molecule and drawn

¹⁹[Note by AKTA:] In German: *Stromarbeit*. This expression can also be translated as current work.

 $^{^{20}}$ [Note by AKTA:] In German: *Wärmeentwickelung*. This expression can also be translated as heat development.

²¹[Note by AKTA:] Maybe Weber is referring here to the torque exerted by a current carrying wire on a nearby magnet.

into the molecular current there.

The extraction of electrical fluid from a molecular current occurs through *electromotive* force. If such an electromotive force is present, it also continues to act on the drawn-out electrical fluid and *increases its speed* until it re-enters the next molecular current, from which it follows that, with a continuous current, the electric fluid withdrawn from the previous molecular currents enters into all molecular currents at the same time at a greater velocity than it had left them.

According to this, the work directly and necessarily associated with the continuation of the current *i* consists in the *amplification of the molecular currents*, and if we call *A* this work, which can be determined in absolute units, then $A/i^2 = e/i$ is constant for every body, that is, the two principles according to which the resistance is either the ratio of the electromotive force *e* to the current intensity *i* produced thereby, or the ratio of the work *A* done by the current to the square of the intensity of the current *i*, by which it is carried out, are basically completely *identical*.

So all that remains is to determine the ratio of the *heat generation*, as the effect of the ongoing current, to that *immediate electrical work*.

Experience has now shown that the work equivalent of the heat generation associated with the continuation of the current is equal to the immediate electrical work; for it has been shown that the work equivalent of the heat generation associated with the continuation of the current, according to absolute units, divided by the square of the current intensity determined according to absolute units, is equal to the resistance of the body determined according to absolute units by the ratio e/i.

But this leads to the alternative that either the generation of heat itself is nothing other than the direct work of electricity, that is, *amplification of the molecular currents* in the body, or that all immediate electrical work disappears and is replaced by heat generation, perhaps through a still unknown interaction between electrical and heat fluids, every amplification of the molecular currents is converted into a heat generation equivalent to the work.

However, according to the assumed molecular constitution of the ponderable bodies and the *persistence* of the electrical fluids in their molecular current movements, the *latter alternative* is now inadmissible, because then all immediate electrical work, *consisting in amplification of the molecular currents, disappear* and should be replaced by heat development, that is, because then the increased molecular currents *would not be persistent*, as was assumed.

This produces an interesting result, that with the mentioned prerequisites of the molecular constitution of ponderable bodies and of the persistence of molecular currents, as they form the basis of the doctrine of magnetism and diamagnetism, only the former alternative is compatible, namely that the heat generation associated with the permanence of the current is itself nothing other than the immediate electrical work, which is only conceivable, if all heat generation in ponderable bodies consists in the strengthening of the molecular currents of the electrical fluids in these bodies, after which a special thermal fluid would be eliminated in the ponderable bodies, just as was the case with the magnetic fluids under the assumption of persistent molecular currents.

The presupposition of an ether distributed in all empty spaces (even between the ponderable body molecules) would remain independent of this; only then would the mediation of heat transfer from one ponderable body to another distant body through this ether, according to the laws of *radiation* and *absorption*, be attributed to an *interaction of the electrical fluids with this ether*, as C. Neumann already tried to justify for the purpose of his theory of the rotation of the polarization plane of light through galvanic magnetic forces.²²

Finally, all other effects that experience has shown to be associated with the duration of the current in a ponderable body, namely all *electromagnetic*, *electrodynamic* and *induction effects* on distant bodies, result more precisely not as effects of the *current duration*; but as effects of the *current decrease* in the body; because even in cases where the current strength is maintained unchanged during such effects, a *decrease in current* takes place in view of these effects, which is only not observed because in these cases, in addition to the electromotive force *necessary* to maintain current in the body, there is another electromotive force which would otherwise produce an *increase in current*, but in these cases it is only used to compensate with that *decrease in current*.

 $^{^{22}}$ [Note by AKTA:] Carl Neumann (1832-1925). See [Neu58] and [Neu63].

Bibliography

- [AC15] A. K. T. Assis and J. P. M. C. Chaib. Ampère's Electrodynamics Analysis of the Meaning and Evolution of Ampère's Force between Current Elements, together with a Complete Translation of His Masterpiece: Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience. Apeiron, Montreal, 2015. Available at www.ifi.unicamp.br/~assis.
- [Amp20a] A.-M. Ampère. Analyse des mémoires lus par M. Ampère a l'Académie des Sciences, dans les séances des 18 et 25 septembre, des 9 et 30 octobre 1820. Annales Générales des Sciences Physiques, 6:238–257, 1820.
- [Amp20b] A.-M. Ampère. Analyse des mémoires lus par M. Ampère a l'Académie des Sciences, dans les séances des 18 et 25 septembre, des 9 et 30 octobre 1820. 20 pages. Reprint of Annales Générales des Sciences Physiques, Vol. 6, pp. 238-257 (1820), 1820.
- [Amp20c] A.-M. Ampère. Suite du Mémoire sur l'Action mutuelle entre deux courans électriques, entre un courant électrique et un aimant ou le globe terrestre, et entre deux aimants. Annales de Chimie et de Physique, 15:170–218, 1820.
- [Amp21] A.-M. Ampère. Suite de la Note sur un Appareil à l'aide duquel on peut vérifier toutes les propriétés des conducteurs de l'électricité voltaïque, découvertes par M. Ampère. Annales de Chimie et de Physique, 18:313–333, 1821.
- [CA09] J. P. M. d. C. Chaib and A. K. T. Assis. Sobre os efeitos das correntes elétricas (segunda parte) — Tradução da primeira obra de Ampère sobre eletrodinâmica. *Revista Brasileira de História da Ciência*, 2:118–145, 2009.
- [Gau38] C. F. Gauss. Anleitung zur Bestimmung der Schwingungsdauer einer Magnetnadel. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1837*, volume II, chapter IV, pages 58– 80. Dieterichschen Buchhandlung, Göttingen, 1838. Reprinted in Carl Friedrich Gauss *Werke*, Vol. 5, pp. 374-394 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).
- [Gil71] C. S. Gillmor. Coulomb and the Evolution of Physics and Engineering in Eighteenth-Century France. Princeton University Press, Princeton, 1971.
- [Koh83] F. Kohlrausch. An Introduction to Physical Measurements with appendices on absolute electrical measurement, etc. J. & A. Churchill, London, 2nd edition, 1883. Translated from the fourth German edition by T. H. Waller and H. R. Procter.

- 16
- [LA98] E. K. Lauridsen and N. Abrahamsen. The history of astatic magnet systems and suspensions. *Centaurus*, 40:135–169, 1998.
- [Ner33] J. J. Nervander. Mémoire sur un Galvanomètre à châssis cylindrique par lequel on obtient immédiatement et sans calcul la mesure de l'intensité du courant électrique qui produit la déviation de l'aiguille aimantée. Annales de Chimie et de Physique, 55:156–184, 1833.
- [Neu58] C. Neumann. Explicare tentatur quomodo fiat ut lucis planum polarisationis per vires electricas vel magneticas declinetur. Halis Saxonum, 1858.
- [Neu63] C. Neumann. Die magnetische Drehung der Polarisationsebene des Lichts Versuch einer mathematischen Theorie. Verlag des Buchhandlung des Waisenhauses, Halle, 1863.
- [Ohm26a] G. S. Ohm. Bestimmung des Gesetzes, nach welchem Metalle die Kontakt-Elektrizität leiten, nebst einem Entwurfe zu einer Theorie des Voltaschen Apparates und des Schweiggerschen Multiplikators. Journal für Chemie und Physik, 46:137–166, 1826. Reprinted in Ostwald's Klassiker der exakten Wissenschaften, Nr. 244, C. Piel (ed.), (Akademische Verlagsgesellschaft, Leipzig, 1938), pp. 8-29.
- [Ohm26b] G. S. Ohm. Ein Nachtrag zu dem vorstehenden Aufsatz. Annalen der Physik und Chemie, 7:117–118, 1826.
- [Ohm26c] G. S. Ohm. Versuch einer Theorie der durch galvanische Kräfte hervorgebrachten elektroskopischen Erscheinungen. Annalen der Physik und Chemie, 6:459–469, 1826.
- [Ohm26d] G. S. Ohm. Versuch einer Theorie der durch galvanische Kräfte hervorgebrachten elektroskopischen Erscheinungen (Beschluss). Annalen der Physik und Chemie, 7:45–54, 1826.
- [Ohm27] G. S. Ohm. *Die Galvanische Kette, mathematisch bearbeitet.* T. H. Riemann, Berlin, 1827.
- [Ohm60] G. S. Ohm. *Théorie Mathématique des Courants Électriques*. L. Hachette et Co., Paris, 1860. Traduction, préface et notes de J.-M. Gaugain.
- [Ohm66] G. S. Ohm. The galvanic circuit investigated mathematically. In R. Taylor, editor, Scientific Memoirs, Vol. 2, pages 401–506, New York, 1966. Johnson Reprint Corporation. English translation by W. Francis.
- [Sih21] A. Sihvola. Johan Jacob Nervander and the quantification of electric current. *IEEE Antennas & Propagation Magazine*, 63:123–128, 2021. Doi: 10.1109/MAP.2020.3039803.
- [Web39] W. Weber. Der Induktor zum Magnetometer. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1838*, volume III, chapter IV, pages 86–101. Weidmannschen Buchhandlung, Leipzig, 1839. Reprinted in *Wilhelm Weber's Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 105-118.

- [Web52] W. Weber. Elektrodynamische Maassbestimmungen insbesondere Widerstandsmessungen. Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physischen Classe, 1:199–381, 1852. Reprinted in Wilhelm Weber's Werke, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 301-471.
- [Web62] W. Weber. Zur Galvanometrie. Abhandlungen der Königlich Gesellschaft der Wissenschaften zu Göttingen, mathematische Klasse, 10:3–96, 1862. Reprinted in Wilhelm Weber's Werke, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), pp. 17-96.
- [Web94] W. Weber. Zur Galvanometrie (Auszug). In H. Weber, editor, Wilhelm Weber's *Werke*, Vol. 4, pages 526–538, Berlin, 1894. Springer.
- [Web21] W. Weber. Electrodynamic measurements, second memoir, relating specially to measures of resistance. In A. K. T. Assis, editor, Wilhelm Weber's Main Works on Electrodynamics Translated into English, volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, pages 291–441, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.
- [WK68] W. Weber and R. Kohlrausch. Über die Einführung absoluter elektrischer Maße. In S. Balke, H. Gericke, W. Hartner, G. Kerstein, F. Klemm, A. Portmann, H. Schimank, and K. Vogel, editors, Ostwalds Klassiker der exakten Wissenschaften, new series, Vol. 5. Friedrich-Vieweg & Sohn, Braunschweig, 1968. Commented by F. Kohlrausch and K. H. Wiederkehr.