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Circuit theory in Weber electrodynamics

A K T Assis†‡

Instituto de Física 'Gleb Wataghin', Universidade Estadual de Campinas - Unicamp, 13083-970 Campinas, São Paulo, Brazil

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Abstract. We present a derivation of the equation describing the current flow in a circuit with self-inductance based on Newton's second law plus the Weber force or, alternatively, plus the Lorentz or Liénard–Schwarzschild force. In Weber's approach the self-inductance can be treated as a measure of the effective average inertial mass of the conduction electrons.

Resumo. Apresentamos uma derivação da equação descrevendo o fluxo de corrente num circuito contendo auto-indutância a partir da segunda lei de Newton mais a força de Weber ou, alternativamente, mais a força de Lorentz ou de Liénard–Schwarzschild. No enfoque de Weber a auto-indutância pode ser tratada como uma medida da massa inercial média efetiva dos elétrons de condução.

1. Introduction

The equation describing the flow of current in an electric circuit containing self-inductance L and resistance R in series and subject to an applied electromotive force $V(t)$ satisfies the equation

$$L\frac{\mathrm{d}I}{\mathrm{d}t} + RI = V(t), \tag{1}$$

where $I = \mathrm{d}Q/\mathrm{d}t$ is the current. This equation is identical in form (with I being replaced by $v = \mathrm{d}x/\mathrm{d}t$) to Newton's second law when a mass m is subject to a damping force $-bv$ proportional to its velocity, b being the constant of friction, and to an external applied force $F(t)$:

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + b\frac{\mathrm{d}x}{\mathrm{d}t} = F(t). \tag{2}$$

In this work we show that this identity in form is not a coincidence. As a matter of fact, we show how to derive equation (1) from Newton's second law of motion applied to the conduction electrons. The main difficulty is how to derive the self-inductance of the circuit which is known to have no relation with the electron's mass but only with the geometry of the circuit. To this end we can employ the Weber force ([1]) or, alternatively, the Lorentz force in the Liénard–Schwarzschild form. Essentially we will derive the current equation from a microscopic theory of conduction.

We begin by deriving Ohm's law $V = RI$ or $\vec{J} = g\vec{E}$ for steady currents. Here $\vec{J}$ is the volume current density, g is the conductivity of the wire and $\vec{E}$ is the applied electric field. We consider a thin wire of total length ℓ and uniform cross section of area A with $\sqrt{A} \ll \ell$, made of a homogeneous material. That these two forms of Ohm's law are equivalent to one another is a well known fact [2]. To prove the equivalence we need to utilize $\vec{E} = E\hat{\ell}$ and $\vec{J} = J\hat{\ell}$, where $\hat{\ell}$ is the unit vector parallel to the circuit in each point. We also need to utilize the fact that

$$V = E\ell, \tag{3}$$

$$I = JA = \rho A v, \tag{4}$$

and $R = \ell/gA$. In equation (4) ρ (and $\rho_- = -\rho$) is the positive (and negative) volume charge density of the neutral wire.

Ohm's law is easily derived from Newton's second law $\vec{F} = m\vec{a}$ applied to an electron of mass $m = 9.1\times10^{-31}$ kg, [3]. To this end we only need to suppose an electric force $q\vec{E}$ due to the battery or applied electromotive force EMF and an average frictional force $-b\vec{v}$ due to the collisions of the electron with the lattice of the wire. We then obtain $q\vec{E} - b\vec{v} = m\vec{a}$.

As $q = -e$, with $e = 1.6\times10^{-19}$ C, $\vec{J} = J\hat{\ell} = \rho_-\vec{v} = -\rho\vec{v}$, $\vec{v} = -v\hat{\ell}$ and $\vec{a} = -a\hat{\ell}$ this yields

$$eE = bv + ma. \tag{5}$$

† E-mail address: assis@ifi.unicamp.br
‡ Also Collaborating Professor at the Department of Applied Mathematics, IMECC, State University of Campinas, 13081-970 Campinas, SP, Brazil.

We recover Ohm's law from equation (5) in steady state ($a = 0$) supposing

$$b = \frac{e\rho}{g} = \frac{e\rho A}{\ell} R. \tag{6}$$

If we were dealing with a current flowing longitudinally over a surface of length ℓ across the uniform side s the same reasoning might be applied by replacing

$$I = Ks = \sigma v s, \tag{7}$$

and

$$b = \frac{e\sigma s}{\ell} R, \tag{8}$$

where $\vec{K} = \sigma_- \vec{v} = -\sigma \vec{v}$ is the surface charge density (σ and $\sigma_- = -\sigma$ being the positive and negative surface charge densities of the neutral wire).

The development presented up to here is not a new one and can be found in many textbooks. However, all microscopic theories of conduction stop here. We might now try to derive equation (1) considering (5) with the acceleration term and a time-dependent electromotive force $V(t)$. From (4) we can write

$$\frac{dI}{dt} = \rho A a. \tag{9}$$

In equation (5) multiplied by ℓ/e this yields, with (3), (4), (6) and (9),

$$V(t) = RI + \frac{m\ell}{e\rho A}\frac{dI}{dt}. \tag{10}$$

This would be equation (1) provided $L = m\ell/(e\rho A)$. However, the self-inductance L is known to be independent of the electron's mass m and to depend only on the geometry of the circuit. Moreover, for a typical copper wire of length $\ell = 1$ m and 1 mm diameter we have $L \approx (\mu_0 \ell/2\pi)\ln(\ell/\sqrt{A}) \approx 10^{-6}$ H, while $m\ell/(e\rho A) \approx 10^{-16}$ H as $\rho \approx 10^{10}$ C m^{-3}. There is apparently a problem with this derivation of the current equation from Newton's second law of motion as the value of L and of $m\ell/(e\rho A)$ differ by ten orders of magnitude. In the next sections we show how to deal with this problem, considering appropriately all the relevant forces acting on the conduction electrons. This is the main contribution of this work.

2. Circuit theory from Weber's force

We now consider the same problem as above but taking into account all forces acting on the conduction electrons. We have already considered the force due to the battery or applied electromotive force $V(t)$ and the frictional force due to the collisions of the electrons with the wire. However, we did not include the electromagnetic force exerted by the positive lattice on our test electron, nor the force exerted by all other conduction electrons on our test electron.

In Weber electrodynamics the force exerted by the infinitesimal charge dq_2 on q_1 is given by ([1], chapter 3):

$$d\vec{F} = \frac{q_1\, dq_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\ddot{r}}{c^2}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2} \times \left[1 + \frac{1}{c^2}\left(\vec{v}_{12}\cdot\vec{v}_{12} - \frac{3}{2}(\hat{r}\cdot\vec{v}_{12})^2 + \vec{r}\cdot\vec{a}_{12}\right)\right], \tag{11}$$

where $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2} is the permittivity of vacuum, $c = 3 \times 10^8$ m s^{-1}, $\vec{r} = \vec{r}_1 - \vec{r}_2$, $\vec{v}_{12} = d\vec{r}/dt = \vec{v}_1 - \vec{v}_2$, $\vec{a}_{12} = d^2\vec{r}/dt^2 = \vec{a}_1 - \vec{a}_2$, $r = |\vec{r}|$, $\dot{r} = dr/dt$, $\ddot{r} = d^2r/dt^2$ and $\hat{r} = \vec{r}/r$ is the unit vector pointing from 2 to 1.

This force is completely relational, depending only on the distance, radial velocity and radial acceleration between the interacting charges.

As we are considering neutral conductors, the Coulombian contribution of this force exerted by the positive lattice and by all other conduction electrons on the test electron goes to zero. The typical velocity of the electrons is their drifting velocity, of the order of millimetres per second. This means that we can neglect the velocity terms of equation (11) as they will be of second order in v/c, that is, $v^2/c^2 \approx 10^{-22} \ll 1$. The only relevant terms which will remain are the acceleration terms. For slowly varying currents we can consider that all electrons are accelerated together. This means that on average $\dot{r} = 0$ and $\ddot{r} =$ for any pair of electrons. So the only Weberian force which we still need to consider is the force exerted by the positive stationary lattice on the accelerated test electron. This force is obtained from equation (11) and can be put in the form (utilizing that $1/\varepsilon_0 c^2 = \mu_0$, where $\mu_0 = 4\pi \times 10^{-7}$ kg m C^{-2}):

$$d\vec{F} = \frac{\mu_0 q_1\, dq_2}{4\pi}\frac{\hat{r}}{r}(\hat{r}\cdot\vec{a}_1). \tag{12}$$

We will calculate this force acting on a typical conduction electron in four different situations represented in figures 1–4. The typical or representative test electron on which we will calculate the force will always be considered in the middle of the circuit.

In figure 1 we choose a cylindrical coordinate system with its origin at the centre of the straight wire with radius r and with the z-axis along the length ℓ of the wire. The test electron $q_1 = -e$ is considered to be at the origin, $\vec{r}_1 = 0$, with axial acceleration $\vec{a} = a\hat{z}$. An infinitesimal charge element dq_2 of the lattice is located at $\vec{r}_2 = r_2\cos\varphi_2\hat{x} + r_2\sin\varphi_2\hat{y} + z_2\hat{z}$. Substituting $\rho\, dV_2 = \rho r_2\, d\varphi_2\, dr_2\, dz_2$ for dq_2 in equation (12) and integrating in φ_2 from 0 to 2π, in r_2 from 0 to the radius r of the wire and in z_2 from $-\ell/2$ to $\ell/2$ yields

$$\vec{F} = \frac{\mu_0 e\rho}{4}\left(\ell\sqrt{r^2 + \frac{\ell^2}{4}} - r^2 \ln\frac{\sqrt{r^2 + \ell^2/4} + \ell/2}{\sqrt{r^2 + \ell^2/4} - \ell/2} - \frac{\ell^2}{2}\right)\vec{a}. \tag{13}$$

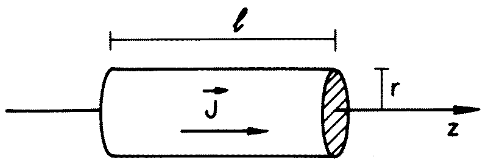

Figure 1. The uniform volume current density $\vec{J} = J\hat{z}$ flowing over the cross section of a cylindrical wire of length ℓ and radius $r \ll \ell$.

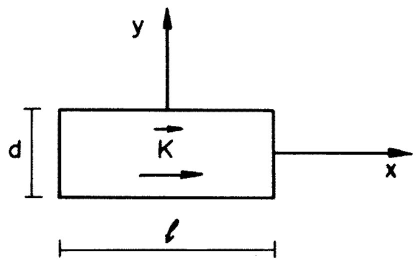

Figure 2. The uniform surface current density $\vec{K} = K\hat{z}$ flowing over the surface of a straight strip of length ℓ and side $d \ll \ell$.

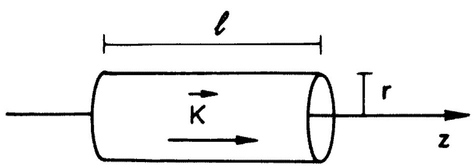

Figure 3. The uniform surface current density $\vec{K} = K\hat{z}$ flowing longitudinally over the surface of a cylindrical conductor of length ℓ and radius $r \ll \ell$.

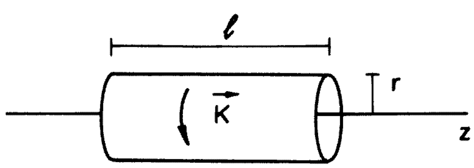

Figure 4. The uniform surface current density $\vec{K} = K\hat{\varphi}$ flowing along the poloidal direction $\hat{\varphi}$ over the surface of a cylindrical conductor of length ℓ and radius $r \ll \ell$.

For $\ell \gg r$ this yields

$$\vec{F} = -\left(\frac{\mu_0 e\rho r^2}{2}\ln\frac{\ell}{r}\right)\vec{a}. \qquad (14)$$

For the situation of figures 2–4 we have a bi-dimensional current flowing over a surface. We then replace in equation (12) $\sigma\,\mathrm{d}s_2$ for $\mathrm{d}q_2$, where $\mathrm{d}s_2$ represents an infinitesimal element of area.

In figure 2 we choose a Cartesian coordinate system with its origin at the centre of the circuit with the x-axis along the greatest side ℓ and with the y-axis along the shortest side d. The test electron is located at $\vec{r}_1 = 0$ with acceleration $\vec{a} = a\hat{z}$. An infinitesimal element of charge of the lattice is located at $\vec{r}_2 = x_2\hat{x} + y_2\hat{y}$. Integrating equation (12) in y_2 from $-d/2$ to $d/2$, and in x_2 from $-\ell/2$ to $\ell/2$ yields, in the approximation $\ell \gg d$,

$$\vec{F} = -\left(\frac{\mu_0 e\sigma d}{2\pi}\ln\frac{\ell}{d}\right)\vec{a}. \qquad (15)$$

In figures 3 and 4 we choose a cylindrical coordinate system with its origin at the centre of the cylinder and with the z-axis along the length ℓ of the cylinder of radius r. The test electron is chosen at $\vec{r}_1 = r\hat{x}$. An element of charge of the lattice $\mathrm{d}q_2 = \rho r_2\,\mathrm{d}\varphi_2\,\mathrm{d}z_2$ is located at $\vec{r}_2 = r\cos\varphi_2\hat{x} + r\sin\varphi_2\hat{y} + z_2\hat{z}$.

For the case of figure 3 the axial acceleration of the conduction electrons is given by $\vec{a} = a\hat{z}$. Integrating equation (12) in φ_2 from 0 to 2π and in z_2 from $-\ell/2$ to $\ell/2$ and utilizing $\ell \gg r$ yields

$$\vec{F} = -\left(\mu_0 e\sigma r\ln\frac{\ell}{r}\right)\vec{a}. \qquad (16)$$

For the case of figure 4 we integrate equation (12) in the same limits and utilize the same approximation. But now the electrons have tangential accelerations given by $\vec{a} = a\hat{\varphi} = -a\sin\varphi\hat{x} + a\cos\varphi\hat{y}$. This means that the tangential acceleration of our test electron located at $\vec{r}_1 = r\hat{x}$, $\varphi = 0$ is then given by $\vec{a} = a\hat{y}$. The final result of the integration is

$$\vec{F} = -\left(\frac{\mu_0}{2}e\sigma r\right)\vec{a}. \qquad (17)$$

We can write equations (14)–(17) as

$$\vec{F} = m_{\mathrm{W}}\vec{a}, \qquad (18)$$

where m_{W} is a Weberian electromagnetic mass having a different value in each geometry. For figure 1 we have $m_{\mathrm{W}} = -(\mu_0 e\rho r^2\ln(\ell/r))/2$, for figure 2 we have $m_{\mathrm{W}} = -(\mu_0 e\sigma d\ln(\ell/d))/(2\pi)$, for figure 3 we have $m_{\mathrm{W}} = -\mu_0 e\sigma r\ln(\ell/r)$ while for figure 4 we have $m_{\mathrm{W}} = -(\mu_0 e\sigma r)/2$.

Newton's second law taking into account an applied electric field $\vec{E}(t)$ (which will give rise to the applied EMF $V(t)$), the frictional force $-b\vec{v}$ (which will give rise to Ohm's resistive term RI) and the Weber force in the form of equations (12) and (18) yields $q\vec{E}(t) - b\vec{v} + m_{\mathrm{W}}\vec{a} = m\vec{a}$. Considering $q = -e$ and all vectorial terms parallel at each point to the unit direction $\hat{\ell}$ of the wire ($\vec{E} = E\hat{\ell}$, $\vec{v} = -v\hat{\ell}$ and $\vec{a} = -a\hat{\ell}$) yields an analogous equation (5), namely

$$eE(t) = bv + (m - m_{\mathrm{W}})a = bv + m_{\mathrm{eff}}a, \qquad (19)$$

where $m_{\mathrm{eff}} = m - m_{\mathrm{W}}$ is the effective inertial mass of the electron. For typical copper wires like those of figures 1–4 with $\ell \approx 1$ m and $r \approx 1$ mm or $d \approx 1$ mm we have $|m_{\mathrm{W}}| \approx 10^{-20}$ kg. As $m = 9 \times 10^{-31}$ kg we have $|m_{\mathrm{W}}| \gg m$, so that $m_{\mathrm{eff}} \approx -m_{\mathrm{W}}$. This means that we can neglect the usual mass of the electron in this problem. Multiplying this equation by ℓ/e and utilizing equations (3)–(9) yields (10) but now with m_{eff} replacing m:

$$V(t) = RI + \frac{m_{\text{eff}}\ell}{e\rho A}\frac{\mathrm{d}I}{\mathrm{d}t}. \tag{20}$$

However, this is indeed equation (1) observing that the self-inductance of the circuits of figures 1 and 3 with $\ell \gg r$ is given approximately by ([4], p 35):

$$L \approx \frac{\mu_0}{2\pi}\ell \ln\frac{\ell}{r}. \tag{21}$$

For figure 2 with $\ell \gg d$ we have ([5])

$$L \approx \frac{\mu_0}{2\pi}\ell \ln\frac{\ell}{d}. \tag{22}$$

while for figure 4 with $\ell \gg r$ we have

$$L \approx \frac{\mu_0 \pi r^2}{\ell}. \tag{23}$$

That is, we derived the equation describing the flow of current in a RL circuit from Newton's second law of motion together with the Weber force. The self-inductance L has been shown to be directly proportional to the Weberian or effective inertial mass of the electron, namely

$$m_{\text{eff}} = e\rho\frac{A}{\ell}L, \tag{24}$$

for a circuit of length ℓ and area of cross section $A \ll \ell^2$. For figure 1 we had $A = \pi r^2$.

For a current flowing along the length ℓ of a surface with side s we obtained

$$m_{\text{eff}} = e\sigma\frac{s}{\ell}L. \tag{25}$$

For figure 2 we had $s = d$, for figure 3 we had $s = 2\pi r$ while for figure 4 the tangential current flowed over the length $2\pi r$ crossing the side $s = \ell$. We can then say that the self-inductance of a circuit can be treated as a measure of the average effective inertial mass of the conduction electrons.

3. Circuit theory from the Liénard–Schwarzschild force

We now perform the same calculations as above but utilizing the Lorentz force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. This force does not depend on the acceleration of the test charge q. From the analysis in the previous section we might imagine that we would not be able to derive the self-induction due to this fact (we could only derive it from Weber electrodynamics utilizing the fact that the Weber force depends on the acceleration of the test charge). As a matter of fact we will see that even with the Lorentz force we will derive the same term. In the Lorentz force $\vec{v}$ is the velocity of the test charge q relative to an inertial frame of reference, $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields generated by the source charges. These fields can be written in terms of the retarded scalar potential ϕ and the retarded vector potential $\vec{A}$. We can also utilize the Liénard–Wiechert potentials. Expanding the retarded time $t_{\text{r}} = t - r/c$ around the present time t, including radiation effects and relativistic corrections, yields the force exerted by the infinitesimal charge element $\mathrm{d}q_2$ localized at $\vec{r}_2(t)$ on the point charge q_1 localized at $\vec{r}_1(t)$ as ([6–8]):

$$\mathrm{d}\vec{F} = q_1\left\{\frac{\mathrm{d}q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\left[\hat{r}\left(1 + \frac{\vec{v}_2\cdot\vec{v}_2}{2c^2} - \frac{(\hat{r}\cdot\vec{v}_2)^2}{c^2} - \frac{\vec{r}\cdot\vec{a}}{2c^2}\right) - \frac{r\vec{a}_2}{2c^2}\right]\right\} + q_1\vec{v}_1 \times \left\{\frac{\mathrm{d}q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{\vec{v}_2\times\hat{r}}{c^2}\right\}. \tag{26}$$

This force is the correct relativistic expression valid up to second order in v/c. All terms on the right-hand side of equation (26) are to be calculated and measured at the present time t.

As with the Weber force, the Coulombian component of this force can be neglected due to the charge neutrality of the wire. The components of this force depending on the velocity can also be neglected as they are of second order in v/c. The only relevant terms which will remain are the acceleration terms of the form

$$\mathrm{d}\vec{F} = -\frac{\mu_0 q_1\,\mathrm{d}q_2}{8\pi}\frac{1}{r}[(\hat{r}\cdot\vec{a}_2)\hat{r} + \vec{a}_2]. \tag{27}$$

We perform the same calculations as above in the situations of figures 1–4. As the positive ions are fixed in the lattice, they do not have any acceleration, $\vec{a}_{2+} = 0$. This means that their net force (27) on the test electron is zero, in contrast to what happened with the Weber force. We only need to compute the force exerted on the test electron by all other conduction electrons. With the Weber force this was zero because $\dot{r} = 0$ and $\ddot{r} = 0$ for any two electrons, but the Lorentz force depends only on the acceleration of the test charge $\vec{a}_2$ and not on $\vec{a}_1 - \vec{a}_2$. This means that the acceleration of all other conduction electrons will yield a net force on the test electron according to the Lorentz force even when this test electron is being accelerated together with all other electrons, as we are considering here for slowly varying currents.

For figures 1–4 we will replace in equation (27) $\rho_-\,\mathrm{d}V_2 = -\rho\,\mathrm{d}V_2$ or $\sigma_-\,\mathrm{d}s_2 = -\sigma\,\mathrm{d}s_2$ for $\mathrm{d}q_2$, where $\mathrm{d}V_2$ and $\mathrm{d}s_2$ are volume or area elements, respectively. For figures 1–3 we have $\vec{a}_2 = a_2\hat{z}$, while for figure 4 $\vec{a}_2 = a_2\hat{\varphi}_2 = -a_2\sin\varphi_2\hat{x} + a_2\cos\varphi_2\hat{y}$. Utilizing the same coordinate systems as in the previous section and integrating for the same limits, equation (27) yields, for figure 1

$$\vec{F} = -\frac{\mu_0}{2}e\rho r^2\vec{a}_2\left(\ln\frac{\sqrt{r^2+\ell^2/4}+\ell/2}{r}\right). \tag{28}$$

For $\ell \gg r$ this yields

$$\vec{F} = -\left(\frac{\mu_0}{2}e\rho r^2\ln\frac{\ell}{r}\right)\vec{a}_2. \tag{29}$$

For figures 2 and 3 we get (with $\ell \gg d$ and $\ell \gg r$, respectively):

$$\vec{F} = -\left(\frac{\mu_0 e\sigma d}{2\pi}\ln\frac{\ell}{d}\right)\vec{a}_2, \tag{30}$$

$$\vec{F} = -\left(\mu_0 e\sigma r \ln\frac{\ell}{r}\right)\vec{a}_2. \tag{31}$$

For the case of figure 4 with $\ell \gg r$ we get:

$$\vec{F} = -\left(\frac{\mu_0}{2} e\sigma r\right) a_2\hat{y}. \tag{32}$$

It should be observed that the acceleration which appears in equations (29)–(32) is that of the source electrons and not that of the test electron. But for the slowly varying currents being considered here, the test electron is supposed to have the same magnitude of acceleration as the source ones. In the situations of figures 1–3 the acceleration of the test electron is $\vec{a} = a_2\hat{z}$. In figure 4 its tangential acceleration is $\vec{a} = a_2\hat{y}$, as $\varphi_1 = 0$ because we are supposing $\vec{r}_1 = r\hat{x}$. We can then write equations (29)–(32) as equation (18).

This means that we will also derive (1) from the Lorentz or Liénard–Schwarzschild force.

4. Discussion and conclusions

We have shown before that in Weber electrodynamics a test charge behaves as having an effective inertial mass depending on its electrostatic potential energy [9, 10]. Let us discuss this relation for the cases discussed in this paper. We consider the electrostatic potential energy $\mathrm{d}U$ between point charge q_1 and an infinitesimal charge element $\mathrm{d}q_2$ separated by a distance r as given by

$$\mathrm{d}U = \frac{q_1\,\mathrm{d}q_2}{4\pi\varepsilon_0}\frac{1}{r}. \tag{33}$$

Considering $q_1 = -e = -1.6\times10^{-19}$ C, $\mathrm{d}q_2 = \rho\,\mathrm{d}V_2$ or $\mathrm{d}q_2 = \sigma\,\mathrm{d}s_2$ being an element of positive charge in the lattice and integrating this energy for the test electron interacting with the stationary lattice with the previous approximations and locations of the test electron yields

$$U = m_W c^2, \tag{34}$$

for the situations of figures 1–3 and

$$U = 2\ln\frac{\ell}{r} m_W c^2, \tag{35}$$

for the situation of figure 4, with the appropriate Weberian mass obtained above for each figure.

We can then say once more that the order of magnitude of the Weberian mass m_W is usually U/c^2. Comparing figures 3 and 4 we find that, for the same electron, when we try to accelerate it in the radial direction we get $m_W = Uc^2$, while when we try to accelerate it tangengially in the poloidal direction we find $m_W = U/(c^2 2\ln(\ell/r))$. This anisotropy in the effective inertial mass of a test charge had already been noticed before [1, 10]. That is, m_W behaves like a tensor and not like a scalar quantity.

We were able to derive equation (1) with the Weber and Lorentz forces, although with a completely different interpretation. According to Weber electrodynamics, when we accelerate electrons relative to the lattice there is no average force of one electron on any other. However, the positive lattice exerts a force on the conduction electrons opposing their acceleration, see equation (18), remembering that in the situations analysed in this work we obtained $m_W < 0$. That is, when an external applied EMF tries to accelerate the conduction electrons in one direction the positive lattice tries to hold them according to Weber electrodynamics. With the Lorentz or Liénard–Schwarzschild force the explanation for the same effect is completely different. Now according to equation (27) the positive lattice exerts no force on the conduction electrons (no matter their acceleration) because the lattice is considered at rest in the laboratory, which means $\vec{a}_{2+} = 0$. On the other hand, all other accelerated conduction electrons will exert a force on our test electron (no matter what its acceleration $\vec{a}_1$) in the opposite direction of their own acceleration, as also given by equation (18). That is, suppose we have two electrons 1 and 2 along the x-axis with $x_1 < x_2$. When we accelerate 2 to the right with an acceleration a_2 due to an external force, q_2 will exert an electric force on q_1 pointing to the left and proportional to a_2, no matter the acceleration of q_1. According to Liénard–Schwarzschild's expression, this electric force will act on q_1 even when $a_1 = a_2$.

That the Weber and Liénard–Schwarzschild forces are different from one another is easily seen by comparing equations (11) and (26), or (12) and (27). Even the integrated result is different, as can be seen by comparing equations (13) and (28). However, the average approximate result obtained here is essentially the same for both models, namely, equation (18).

For the situations analysed in this work these two completely opposite explanations yield the same final result, equation (1). This means that they cannot be distinguished here. Possible experimental tests of a force law depending on the acceleration of the test charge have been published elsewhere, [11, 12]. Only after performing experiments of this kind can we decide which one of these explanations for the self-inductance L is the most suitable one.

In any case, the Weberian interpretation presented here for the self-inductance as a measure of the effective inertial mass of the conduction electrons by $L = \ell m_{\mathrm{eff}}/(e\rho A)$ offers a new insight to the microscopic theory of conduction.

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