General Propositions Relating to Attractive and Repulsive Forces Acting in the Inverse Ratio of the Square of the Distance

C. F. Gauss

Editor's Note: An English translation of the Introduction of C. F. Gauss' 1840 paper "Allgemeine Lehrsätze in Beziehung auf die im Verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs- und Abstossungs-kräfte".[1]

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[1] [Gau40] with English translation in [Gau43].

Contents

Chapter 1

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C. F. Gauss[2,3,4]

Nature presents to us many phenomena which we explain by the assumption of forces exerted by the ultimate particles of substances upon each other, acting in inverse proportion to the squares of their distance apart.

Amongst these forces, the first to be noticed is that of universal gravitation, by virtue of which every ponderable molecule μ exercises upon every other such molecule μ', a moving force, which, if we call the distance r, is expressed by $\mu\mu'/r^2$, and tends to produce the approximation of the molecules in the direction of the straight line connecting them.[5]

If, in order to explain magnetic phænomena, we assume two magnetic fluids, one positive and the other negative, two magnetic elements μ, μ', will exert, each on the other, a moving force $\mu\mu'/r^2$, acting along the straight line which joins the two elements, repulsively if μ and μ', are of the same kind of fluid, attractively if they are of different kinds.[6]

The same is true of the mutual action of the particles of electric fluids upon each other.[7]

The linear element ds of a galvanic current exerts in like manner on an element of the magnetic fluid μ (if we allow the latter) a moving force, which is inversely proportional to the square of the distance r; but there is now introduced a new and distinctive circumstance; the direction of the force is not in the connecting straight line, but is perpendicular to the plane passing through μ and the direction of ds; and the intensity of the force depends not

[2] [Gau40] with English translation in [Gau43].

[3] Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[4] The Notes by A. K. T. Assis are represented by [Note by AKTA:].

[5] [Note by AKTA:] This law is due to Isaac Newton (1642-1727) in 1687. See [New34] and [New99]. Portuguese translation in [New90], [New08] and [New10].

[6] [Note by AKTA:] This law is due to Charles Augustin de Coulomb (1736-1806). See [Cou88b] with complete German, Portuguese and English translations in, respectively, [Cou90b], [Ass22] and [AB23].

[7] [Note by AKTA:] This law is also due to Coulomb, [Cou88a] and [Cou88b] with complete German, Portuguese and English translations in, respectively, [Cou90a], [Cou90b], [Ass22] and [AB23].

on the distance alone, but also on the angle which r makes with ds. If this angle be called θ, then $\sin\theta \cdot \mu ds/r^2$ is the measure of the moving force which ds exerts upon μ; and an equal force in the parallel and opposite direction is exerted by μ on the element of the current ds, or on its ponderable carrier.[8]

If we assume with Ampère that the elements ds, ds' of two galvanic currents act attractively or repulsively on each other in the straight line which joins them, then the phænomena require us to consider this force as acting in the inverse ratio of the square of the distance; but as having, at the same time, a somewhat less simple dependence on the direction of the elements of the currents.[9]

We shall restrict ourselves in this treatise to the three first cases, or to those forces which are exerted by one element upon another in the straight line which joins them, and which are therefore simply in the inverse ratio of the square of the distance; although several propositions will be found applicable, with slight alterations, to the other cases also, the more detailed development of which must be reserved for another treatise.

[...]

[8][Note by AKTA:] In German: *dessen ponderabeln Träger*. That is, its ponderable carrier, or material conductor of the current element ds. This law is due to Jean-Baptiste Biot (1774-1862) and Félix Savart (1791-1841). See [Bio21]; [BS24] and [BS85] with English translations in [Far26] and [BS65]; see also [AC15, Chapters 6, 16 and 17].

[9][Note by AKTA:] André-Marie Ampère (1775-1836). Ampère's masterpiece was published in 1826, [Amp26] and [Amp23]. There is a complete Portuguese translation of this work, [Cha09] and [AC11]. Partial English translations can be found at [Amp65] and [Amp69]. Complete and commented English translations can be found in [Amp12] and [AC15]. A huge material on Ampère and his force law between current elements can be found in the homepage *Ampère et l'Histoire de l'Électricité*, http://www.ampere.cnrs.fr and [Blo05], at the homepage of the Friends of André-Marie Ampère, https://saama.fr, and at the homepage of the Ampère Museum, https://amperemusee.fr/en.

Bibliography

[AB23] A. K. T. Assis and L. L. Bucciarelli. *Coulomb's Memoirs on Torsion, Electricity, and Magnetism Translated into English*. Apeiron, Montreal, 2023. Available at www.ifi.unicamp.br/~assis.

[AC11] A. K. T. Assis and J. P. M. d. C. Chaib. *Eletrodinâmica de Ampère: Análise do Significado e da Evolução da Força de Ampère, Juntamente com a Tradução Comentada de Sua Principal Obra sobre Eletrodinâmica*. Editora da Unicamp, Campinas, 2011.

[AC15] A. K. T. Assis and J. P. M. C. Chaib. *Ampère's Electrodynamics — Analysis of the Meaning and Evolution of Ampère's Force between Current Elements, together with a Complete Translation of His Masterpiece: Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience*. Apeiron, Montreal, 2015. Available at www.ifi.unicamp.br/~assis.

[Amp23] A.-M. Ampère. Mémoire sur la théorie mathématique des phénomènes électro-dynamiques uniquement déduite de l'expérience, dans lequel se trouvent réunis les Mémoires que M. Ampère a communiqués à l'Académie royale des Sciences, dans les séances des 4 et 26 décembre 1820, 10 juin 1822, 22 décembre 1823, 12 septembre et 21 novembre 1825. *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, 6:175–387, 1823. Despite this date, the work was only published in 1827.

[Amp26] A.-M. Ampère. *Théorie des Phénomènes Électro-dynamiques, Uniquement Déduite de l'Expérience*. Méquignon-Marvis, Paris, 1826.

[Amp65] A.-M. Ampère. On the Mathematical Theory of Electrodynamic Phenomena, Experimentally Deduced. In R. A. R. Tricker, *Early Electrodynamics — The First Law of Circulation*, pages 155–200, New York, 1965. Pergamon. Partial English translation by O. M. Blunn of Ampère's work "Mémoire sur la théorie mathématique des phénomènes électro-dynamiques uniquement déduite de l'expérience", *Mémoires de l'Académie royale des Sciences de l'Institut de France*, Vol. 6, pp. 175-387 (1823), issued 1827.

[Amp69] A. M. Ampère. The solenoid. Circuits and magnetic shells. In W. F. Magie, editor, *A Source Book in Physics*, pages 456–460, New York, 1969. McGraw-Hill. Extracts from "Théorie des phénomènes électrodynamiques uniquement déduite de l'expérience," Paris, 1826.

[Amp12] A.-M. Ampère. Mathematical Theory of Electrodynamic Phenomena, Uniquely Derived from Experiments. Translated by M. D. Godfrey. Available at https://

archive.org/details/AmpereTheorieEn and https://sites.google.com/site/michaeldgodfrey/physics-information-and-communication, 2012.

[Ass22] A. K. T. Assis. *Tradução Comentada das Principais Obras de Coulomb sobre Eletricidade e Magnetismo*. Apeiron, Montreal, 2022. Available at www.ifi.unicamp.br/~assis.

[Bio21] J. B. Biot. *Précis élémentaire de Physique expérimentale*, volume II. Deterville, Paris, second edition, 1821.

[Blo05] C. Blondel, 2005. Ampère et l'Histoire de l'Électricité. Site of the CNRS which was coordinated by Christine Blondel. Available at www.ampere.cnrs.fr.

[BS24] J. B. Biot and F. Savart. Sur l'aimantation imposée aux métaux par l'électricité en mouvement. In J. B. Biot, *Précis Élémentaire de Physique Expérimentale,* vol. II, troisième édition, pages 704–774. Gauthier-Villars, Paris, 1824.

[BS85] J. B. Biot and F. Savart. Sur l'aimantation imprimée aux métaux par l'électricitè en mouvement, in: *Précis élémentaire de Physique expérimentale*, troisième édition (Paris: Deterville, 1824), Vol. II, pp. 704-774. In J. Joubert, editor, *Collection de Mémoires relatifs a la Physique — Tome II: Mémoires sur l'Électrodynamique*, pages 80–127, Paris, 1885. Gauthier-Villars.

[BS65] J. B. Biot and F. Savart. Magnetization of metals by electricity in motion. In R. A. R. Tricker*, Early Electrodynamics — The First Law of Circulation*, pages 119–139, New York, 1965. Pergamon. Extracts from Biot's *Précis elémentaire de Physique expérimentale*, Vol. II, 3rd edition, pp. 707-723 and 740-746, 1824 (Translated by O. M. Blunn).

[Cha09] J. P. M. d. C. Chaib. *Análise do Significado e da Evolução do Conceito de Força de Ampère, juntamente com a Tradução Comentada de sua Principal Obra sobre Eletrodinâmica*. PhD thesis, University of Campinas — UNICAMP, Campinas, Brazil, 2009. Supervisor: A. K. T. Assis. Available at www.ifi.unicamp.br/~assis and http://repositorio.unicamp.br/Acervo/Detalhe/435449.

[Cou88a] C. A. Coulomb. Premier mémoire sur l'électricité et le magnétisme. Construction et usage d'une balance électrique, fondée sur la propriété qu'ont les fils de métal, d'avoir une force de réaction de torsion proportionelle à l'angle de torsion. Détermination expérimentale de la loi suivant laquelle les élémens des corps électrisés du même genre d'électricité, se repoussent mutuellement. *Mémoires de l'Académie royale des Sciences. Année 1785*, 88:569–577, 1788. Published in 1788. Reprinted in A. Potier (ed.), *Collection de Mémoires relatifs a la Physique*, Volume 1: *Mémoires de Coulomb*, pp. 107-115 (Gauthiers-Villars, Paris, 1884).

[Cou88b] C. A. Coulomb. Second mémoire sur l'électricité et le magnétisme. Où l'on détermine, suivant quelles loix le fluide magnétique, ainsi que le fluide électrique, agissent, soit par répulsion, soit par attraction. *Mémoires de l'Académie royale des Sciences. Année 1785*, 88:578–611, 1788. Published in 1788. Reprinted in A. Potier (ed.), *Collection de Mémoires relatifs a la Physique*, Volume 1: *Mémoires de Coulomb*, pp. 116-146 (Gauthiers-Villars, Paris, 1884).

[Cou90a] C. A. Coulomb. Erste Abhandlung über die Elektricität und den Magnetismus. In W. König, editor, *Vier Abhandlungen über die Elektricität und den Magnetismus von Coulomb*, pages 3–11. Wilhelm Engelmann, Leipzig, 1890. Uebersetzt und herausgegeben von W. König.

[Cou90b] C. A. Coulomb. Zweite Abhandlung über die Elektricität und den Magnetismus. In W. König, editor, *Vier Abhandlungen über die Elektricität und den Magnetismus von Coulomb*, pages 12–42. Wilhelm Engelmann, Leipzig, 1890. Uebersetzt und herausgegeben von W. König.

[Far26] J. Farrar. *Elements of Electricity, Magnetism, and Electro-magnetism, embracing the Late Discoveries and Improvements, digested into the Form of a Treatise; being the Second Part of a Course of Natural Philosophy.* Cambridge University Press, Cambridge, 1826. This volume, with the exception of the notes, is selected from Biot's *Précis Elémentaire de Physique*, third edition, printed at Paris in 1824, and translated with such alterations as were found necessary in order to adapt it to the English reader.

[Gau40] C. F. Gauss. Allgemeine Lehrsätze in Beziehung auf die im Verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs- und Abstossungs-kräfte. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1839*, volume IV, chapter I, pages 1–51. Weidmannschen Buchhandlung, Leipzig, 1840. Reprinted in Carl Friedrich Gauss *Werke*, Vol. 5, pp. 195-242 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau43] C. F. Gauss. General propositions relating to attractive and repulsive forces acting in the inverse ratio of the square of the distance. In R. Taylor, editor, *Scientific Memoirs,* Vol. 3, pages 153–196, London, 1843. Richard and John E. Taylor.

[New34] I. Newton. *Mathematical Principles of Natural Philosophy.* University of California Press, Berkeley, 1934. Cajori edition.

[New90] I. Newton. *Principia — Princípios Matemáticos de Filosofia Natural.* Nova Stella/Edusp, São Paulo, 1990. Livro I: O Movimento dos Corpos. Portuguese translation by T. Ricci, L. G. Brunet, S. T. Gehring and M. H. C. Célia.

[New99] I. Newton. *The Principia: Mathematical Principles of Natural Philosophy.* University of California Press, Berkeley, 1999. A new translation by I. B. Cohen and A. Whitman, assisted by J. Budenz.

[New08] I. Newton. *Principia — Princípios Matemáticos de Filosofia Natural.* Edusp, São Paulo, 2008. Livro II: O Movimento dos Corpos (em Meios com Resistência). Livro III: O Sistema do Mundo (Tratado Matematicamente). Portuguese translation by A. K. T. Assis.

[New10] I. Newton. *Principia — Princípios Matemáticos de Filosofia Natural.* Folha de São Paulo, São Paulo, 2010. Livro III: O Sistema do Mundo (Tratado Matematicamente). Coleção Folha de São Paulo: Livros que Mudaram o Mundo, Volume 9. Portuguese translation by A. K. T. Assis.