

Telegraphy Equation From Weber's Electrodynamics

A. K. T. Assis and J. A. Hernandez

Abstract—We derive the telegraphy equation according to Weber's electrodynamics for signal propagating along a very long bidimensional wire in the shape of a rectangular strip of zero thickness. We also derive this equation for a twin lead composed by two of these parallel very long bidimensional wires facing each other. We compare this result with classical electromagnetism.

Index Terms—Circuit theory, propagation of electromagnetic signals, telegraphy equation, twin leads, Weber's electrodynamics.

I. INTRODUCTION

THE FIRST TO DERIVE the telegraphy equation taking into account the capacitance, self-inductance and resistance of the wire were Weber and Kirchhoff in the year 1857 [1, vol. 1, pp. 144–146, 296–297], [2], [3], [4, pp. 230–232], and [5, Sec. 3.1]. They published independent works but both of them were based on Weber's action at a distance theory. For a wire of negligible resistance, they showed that the signal would propagate at light velocity. Kirchhoff's original papers are [6] and [7], with English translations in, respectively, [8] and [9]. Weber's simultaneous and more thorough work was delayed in publication and appeared only in 1864 [10]. It was only in 1876 that Heaviside succeeded in deriving the same equation based on Maxwell's equations [4, pp. 228–229].

Weber and Kirchhoff considered the propagation of signals in conducting wires of circular cross sections. Our goal here is to extend their works for signals propagating through a long rectangular strip and along a twin lead composed of two of these strips. We will also compare this result based on Weber's electrodynamics with classical electromagnetism. Modern theoretical and experimental discussion of Weber's electrodynamics can be found in [5], [11]–[15], [16, ch. 6], and [17]–[26].

Suppose a charge element dq_2 located at \vec{r}_2 , moving with velocity \vec{v}_2 and acceleration \vec{a}_2 relative to an inertial frame of reference S , and a point charge q_1 located at \vec{r}_1 , moving with velocity \vec{v}_1 and acceleration \vec{a}_1 relative to S . According to Weber's electrodynamics the force exerted by 2 on 1 is given by [5, ch. 3] and [24, ch. 8]

$$d\vec{F} = \frac{q_1 dq_2}{4\pi\epsilon_0 r_{12}^2} \left[1 + \frac{1}{c^2} \left(\vec{v}_{12} \cdot \vec{v}_{12} - \frac{3}{2} (\hat{r}_{12} \cdot \vec{v}_{12})^2 + \vec{r}_{12} \cdot \vec{a}_{12} \right) \right] \quad (1)$$

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The authors are with the Instituto de Física "Gleb Wataghin," Universidade Estadual de Campinas-Unicamp, 13083-970 São Paulo, Brazil (e-mail: assis@ifi.unicamp.br; julioher@ifi.unicamp.br).

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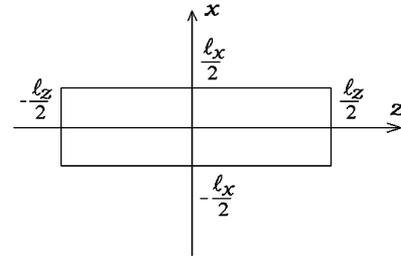


Fig. 1. Geometry of the problem.

where $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$ is the permittivity of vacuum, $c = 3 \times 10^8 \text{ ms}^{-1}$, $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$, $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$, $\vec{a}_{12} = \vec{a}_1 - \vec{a}_2$, $r_{12} = |\vec{r}_{12}|$, and $\hat{r}_{12} = \vec{r}_{12}/r_{12}$ is the unit vector pointing from 2 to 1. This is the basic expression we will employ here, integrating it over the wire.

II. RECTANGULAR STRIP

The first case to be considered is that of a long rectangular strip of length l_z and width l_x in the xz plane centered on the origin of the coordinate system, with the longer side l_z along the z axis, Fig. 1. We suppose air or vacuum outside this bidimensional conductor. As we have a bidimensional problem an element of source charge is given by $dq_2 = \sigma_f(x_2, 0, z_2, t) dx_2 dz_2$, where $\sigma_f(x_2, 0, z_2, t)$ is the surface density of free charges at the position $(x_2, 0, z_2)$ of the strip in time t , and $dx_2 dz_2$ is an element of area. As regards the velocity and acceleration of the source charges we will assume a longitudinal current and propagation, namely $\vec{v}_2(x_2, 0, z_2, t) = v_2(z_2, t)\hat{z}$ and $\vec{a}_2(x_2, 0, z_2, t) = a_2(z_2, t)\hat{z}$. The test charge will be located at (x_1, y_1, z_1, t) , in such a way that its distance to the source charge dq_2 is given by $r_{12} = \sqrt{(x_1 - x_2)^2 + y_1^2 + (z_1 - z_2)^2}$.

The first component of Weber's force acting on charge q_1 is the Coulomb force which can be obtained as the gradient of a potential ϕ , $-q_1 \nabla_1 \phi(x_1, y_1, z_1, t)$, where

$$\phi(x_1, y_1, z_1, t) = \frac{1}{4\pi\epsilon_0} \int_{-l_x/2}^{l_x/2} \int_{-l_z/2}^{l_z/2} \frac{\sigma_f(x_2, 0, z_2, t) dx_2 dz_2}{\sqrt{(x_1 - x_2)^2 + y_1^2 + (z_1 - z_2)^2}} \quad (2)$$

Here, $\phi(x_1, y_1, z_1, t)$ is the potential at the location of q_1 due to all source charges dq_2 .

In order to solve this equation, we utilize the approximation

$$l_z \gg l_x \gg r_1 \quad (3)$$

where $r_1 = \sqrt{x_1^2 + y_1^2 + z_1^2}$ is the distance of the test charge to the center of the strip.

We now employ Kirchhoff's main idea in order to solve this integral. For any given r_1 and r_2 , the maximum value of $1/r_{12} = 1/\sqrt{(x_1 - x_2)^2 + y_1^2 + (z_1 - z_2)^2}$ is for $x_1 = x_2$ and $z_1 = z_2$, when $1/r_{12} = 1/|y_1|$. For z_2 far from z_1 , the value of r_{12} will be close to $|z_2|$ due to the approximation (3). In this case, $|z_2|$ will be of the order of ℓ_z , namely $1/r_{12} \approx 1/|z_2| \approx 1/\ell_z$. By approximation (3) we have $1/|y_1| \gg 1/\ell_z$. Kirchhoff's idea is then to remove $\sigma_f(x_2, 0, z_2, t)$ from the integrand by taking its value at the maximum value of $1/r_{12}$, that is, when $z_2 = z_1$ and $x_2 = x_1$. We are then led to the approximate result (see Appendix)

$$\begin{aligned} \phi(x_1, y_1, z_1, t) &\approx \frac{\sigma_f(x_1, 0, z_1, t)}{4\pi\epsilon_0} \int_{-\ell_x/2}^{\ell_x/2} \int_{-\ell_z/2}^{\ell_z/2} dx_2 dz_2 \\ &\quad \times \frac{1}{\sqrt{(x_1 - x_2)^2 + y_1^2 + (z_1 - z_2)^2}} \\ &\approx \frac{\sigma_f(x_1, 0, z_1, t)\ell_x}{2\pi\epsilon_0} \ln \frac{\ell_z}{\ell_x}. \end{aligned} \quad (4)$$

To solve this last integral, we integrated first in z_2 and utilized once more approximation (3).

The longitudinal Coulomb force is then given by (with $\vec{F} = -q_1 \nabla_1 \phi$)

$$F_z = -\frac{q_1 \ell_x}{2\pi\epsilon_0} \frac{\partial \sigma_f}{\partial z_1} \ln \frac{\ell_z}{\ell_x}. \quad (5)$$

The terms proportional to v_2^2/c^2 and to $(\hat{r}_{12} \cdot \vec{v}_2)^2/c^2$ are usually small compared to the Coulomb term and will be neglected here. The reason for this is that $|v_2|$ is of the order of the drifting velocity v_d , which is known to be much smaller than the light velocity c . The terms depending on $\vec{v}_1 \cdot \vec{v}_2$ and $(\hat{r}_{12} \cdot \vec{v}_1)(\hat{r}_{12} \cdot \vec{v}_2)$ give rise to the magnetic force $q_1 \vec{v}_1 \times \vec{B}$ [5, Sec 6.6, 7.4] and [27]–[30]. As the current is in the longitudinal direction \hat{z} , the magnetic field on the strip will be in the y direction. The test charge considered here will be a conduction electron on the strip moving in the \hat{z} direction, so that $q_1 \vec{v}_1 \times \vec{B}$ will be in the x direction. As we are interested only in the longitudinal propagation of the signal along the z direction, we will not consider these terms either.

We then need to take into account the acceleration term of Weber's force acting on the test charge q_1 due to the accelerated conduction electrons, which can be written as

$$\begin{aligned} \vec{F} &= -\frac{q_1}{4\pi\epsilon_0 c^2} \int_{-\ell_x/2}^{\ell_x/2} \int_{-\ell_z/2}^{\ell_z/2} \\ &\quad \times \frac{(x_1 - x_2)\hat{x} + y_1\hat{y} + (z_1 - z_2)\hat{z}}{[(x_1 - x_2)^2 + y_1^2 + (z_1 - z_2)^2]^{3/2}} \\ &\quad \times (z_1 - z_2)a_2(z_2, t)\sigma_c dx_2 dz_2. \end{aligned} \quad (6)$$

Here, σ_c is the surface density of conduction electrons (typically one electron per atom in copper conductors). It has a constant value which does not depend on position. This means that it can be removed from the integrand.

By Kirchhoff's approximation method, we can remove $a_2(z_2, t)$ from the integrand taking its value at $x_2 = x_1$ and

$z_2 = z_1$ (that is, at the maximum value of $1/r_{12}$). Considering only the z component of the force we obtain the approximate result (utilizing also that $c^2 = 1/\mu_0\epsilon_0$)

$$\begin{aligned} F_z &\approx -\frac{\mu_0}{4\pi} q_1 \sigma_c a_2(z_1, t) \int_{-\ell_x/2}^{\ell_x/2} \int_{-\ell_z/2}^{\ell_z/2} \\ &\quad \times \frac{(z_1 - z_2)^2 dx_2 dz_2}{[(x_1 - x_2)^2 + y_1^2 + (z_1 - z_2)^2]^{3/2}} \\ &\approx -\frac{\mu_0}{2\pi} q_1 \ell_x \sigma_c a_2 \ln \frac{\ell_z}{\ell_x}. \end{aligned} \quad (7)$$

We now suppose the test charge to be a conduction electron: $q_1 = -e = -1.6 \times 10^{-19}$ C, $\vec{v}_1 = v_2(z_1, t)\hat{z}$, and $\vec{a}_1 = a_2(z_1, t)\hat{z}$. In this case we must also include the frictional force due to its collisions with the lattice. The average value of this force can be represented by $-b\vec{v}_1$, where the coefficient of friction b is given by $b = q_1 \sigma_c \ell_x R/\ell_z$, with R being the resistance of the strip, [31]. This is the necessary value of b in order to obtain Ohm's law for constant current in a bidimensional conductor.

We can now write down the z component of the equation of motion for a conduction electron applying Newton's second law of motion in the longitudinal direction z , namely: $F_z = ma$. Considering the frictional force plus (5) and (7) in Newton's second law of motion, dropping the subscript 1 and writing $q_1 = -e$ yields

$$\frac{e\ell_x}{2\pi\epsilon_0} \frac{\partial \sigma_f}{\partial z} \ln \frac{\ell_z}{\ell_x} + \frac{e\ell_x \mu_0}{2\pi} \sigma_c a \ln \frac{\ell_z}{\ell_x} + e\sigma_c R \frac{\ell_x}{\ell_z} v = ma. \quad (8)$$

Usually $|e\ell_x \mu_0 \sigma_c \ln(\ell_z/\ell_x)/(2\pi)| \gg m$, [31], so that we can neglect the term ma in this equation compared with the term $e\ell_x \mu_0 \sigma_c \ln(\ell_z/\ell_x)a/(2\pi)$. For instance, for a 1-m wire with 1-mm width, we have, with $e = 1.6 \times 10^{-19}$ C and $\sigma_c \approx -1$ Cm⁻² for copper: $|e\ell_x \mu_0 \sigma_c \ln(\ell_z/\ell_x)/(2\pi)| \approx 2 \times 10^{-28}$ kg, which is much greater than the electron mass $m = 9 \times 10^{-31}$ kg. Neglecting for this reason the term ma we can then equate the left-hand side of (8) to zero.

We can relate the drifting velocity of the source charges v_2 with the surface current density K utilizing $K(z_2, t) = \sigma_c v_2(z_2, t)$. As the surface density of conduction electrons has a constant value we have $\partial K/\partial t = \sigma_c a_2$. Utilizing this in (8) without the right-hand side, multiplying it by $(2\pi\epsilon_0)/(e\ell_x \ln(\ell_z/\ell_x))$ and utilizing $c^2 = 1/\mu_0\epsilon_0$ yields

$$\frac{\partial \sigma_f}{\partial z} + \frac{1}{c^2} \frac{\partial K}{\partial t} + \frac{2\pi\epsilon_0 R K}{\ell_z \ln(\ell_z/\ell_x)} = 0. \quad (9)$$

There are two unknowns in this equation, σ_f and K . In order to relate them we utilize the equation for the conservation of charges, $\nabla \cdot \vec{J} = -\partial \rho_f/\partial t$. For the case considered here of a current flowing in the z direction over the surface of a strip of length ℓ_z and width ℓ_x this is equivalent to

$$\frac{\partial K}{\partial z} = -\frac{\partial \sigma_f}{\partial t}. \quad (10)$$

Applying $\partial/\partial t$ in (9) and utilizing (10) yields

$$\frac{\partial^2 K}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 K}{\partial t^2} = \frac{2\pi\epsilon_o R}{\ell_z \ln(\ell_z/\ell_x)} \frac{\partial K}{\partial t}. \quad (11)$$

This is the equation of telegraphy, which will also be satisfied by σ_f , by ϕ and by the total current $I = \ell_x K$.

If the resistance of the strip is negligible, Weber's electrodynamics plus Newton's second law of motion predicts a current flow obeying the wave equation. That is, with a signal propagating at light velocity.

III. TWIN LEAD

We now perform the same calculations as above but considering a twin lead composed of two of these strips facing one another and separated by a distance d such that $\ell_z \gg \ell_x \gg d$, with currents flowing longitudinally along the z axis. The parallel planes of the strips are located at $y = d/2$ and at $y = -d/2$.

Beyond the conditions above, we need only two further relations connecting the free surface charge densities and surface current densities in both strips. The most reasonable conditions are that (see [32, pp. 125–130])

$$\sigma_{f,-d/2}(z,t) = -\sigma_{f,d/2}(z,t) = \sigma_f(z,t) \quad (12)$$

and

$$K_{-d/2}(z,t) = -K_{d/2}(z,t) = K(z,t). \quad (13)$$

The approximation utilized here is

$$\ell_z \gg \ell_x \gg x_1, y_1, z_1, d. \quad (14)$$

With these conditions, integrating for both strips as in the previous Section and utilizing Kirchhoff's approximation method yields

$$\phi(y \geq d/2) = \frac{\sigma_f d}{2\epsilon_o} \left(1 - \frac{4y}{\pi\ell_x}\right) \quad (15)$$

$$\phi(-d/2 \leq y \leq d/2) = \frac{\sigma_f y}{\epsilon_o} \left(1 - \frac{2d}{\pi\ell_x}\right) \quad (16)$$

$$\phi(y \leq -d/2) = -\frac{\sigma_f d}{2\epsilon_o} \left(1 + \frac{4y}{\pi\ell_x}\right). \quad (17)$$

The longitudinal Coulomb force given by $F_z = -q_1 \partial\phi/\partial z$ yields

$$F(y \geq d/2) = -\frac{q_1 d}{2\epsilon_o} \left(1 - \frac{4y}{\pi\ell_x}\right) \frac{\partial\sigma_f}{\partial z} \quad (18)$$

$$F(-d/2 \leq y \leq d/2) = -\frac{q_1 y}{\epsilon_o} \left(1 - \frac{2d}{\pi\ell_x}\right) \frac{\partial\sigma_f}{\partial z} \quad (19)$$

$$F(y \leq -d/2) = \frac{q_1 d}{2\epsilon_o} \left(1 + \frac{4y}{\pi\ell_x}\right) \frac{\partial\sigma_f}{\partial z}. \quad (20)$$

The longitudinal component of Weber's force depending on the acceleration yields

$$F(y \leq d/2) = -\frac{q_1 \mu_o d}{2} \left(1 - \frac{4y}{\pi\ell_x}\right) \frac{\partial K}{\partial t} \quad (21)$$

$$F(-d/2 \leq y \leq d/2) = -q_1 \mu_o y \left(1 - \frac{2d}{\pi\ell_x}\right) \frac{\partial K}{\partial t} \quad (22)$$

$$F(y \leq -d/2) = \frac{q_1 \mu_o d}{2} \left(1 + \frac{4y}{\pi\ell_x}\right) \frac{\partial K}{\partial t}. \quad (23)$$

Following the procedure of the previous Section, substituting b by the expression $q_1 \sigma_c R_1 \ell_x / \ell_z$ (where R_1 is the resistance of one strip, [31]) and q_1 by $-e$ yields

$$\frac{ed}{2\epsilon_o} \frac{\partial\sigma_f}{\partial z} + \frac{\mu_o ed}{2} \frac{\partial K}{\partial t} + \frac{eR_1 \ell_x}{\ell_z} K = ma. \quad (24)$$

Utilizing once more (10) and neglecting the right-hand side of (24) yields

$$\frac{\partial^2 K}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 K}{\partial t^2} = \frac{\epsilon_o R \ell_x}{\ell_z d} \frac{\partial K}{\partial t} \quad (25)$$

where $R = 2R_1$ is the resistance of the system of two equal strips.

The same equation is obtained for σ_f , for ϕ and for the total current $I = \ell_x K$.

IV. CLASSICAL ELECTROMAGNETISM

Equations (11) and (25) can be put in the form

$$\frac{\partial^2 K}{\partial z^2} - \frac{LC}{\ell_z^2} \frac{\partial^2 K}{\partial t^2} = \frac{RC}{\ell_z^2} \frac{\partial K}{\partial t} \quad (26)$$

where R , L , and C are the resistance, self-inductance, and capacitance of the system. This can be seen observing that the self-inductance and capacitance of a single strip of length ℓ_z and width ℓ_x , with $\ell_z \gg \ell_x$, are given by, respectively, $L = (\mu_o/2\pi)\ell_z \ln(\ell_z/\ell_x)$ and $C = 2\pi\epsilon_o\ell_z/\ln(\ell_z/\ell_x)$. In the case of the twin-lead considered above, the values of these quantities are $L = \mu_o\ell_z d/\ell_x$ and $C = \epsilon_o\ell_x\ell_z/d$.

Equation (26) is the equation of telegraphy obtained in classical electromagnetism, [4, p. 229] and [33, p. 318]. We then conclude that as regards the telegraphy equation for long straight strips or for twin leads composed of these strips we obtain the same result with classical electromagnetism and Weber's electrodynamics.

APPENDIX

Herein, we calculate the integral that appears in (4) as shown in the equation at the bottom of the page, where we used the

$$\begin{aligned} \int_{-\ell_x/2}^{\ell_x/2} \int_{-\ell_z/2}^{\ell_z/2} \frac{dx_2 dz_2}{\sqrt{(x_1 - x_2)^2 + y_1^2 + (z_1 - z_2)^2}} &= \int_{-\ell_x/2}^{\ell_x/2} \ln \frac{(\frac{\ell_x}{2} - z_1) + \sqrt{(x_1 - x_2)^2 + y_1^2 + (\frac{\ell_x}{2} - z_1)^2}}{-(\frac{\ell_x}{2} + z_1) + \sqrt{(x_1 - x_2)^2 + y_1^2 + (\frac{\ell_x}{2} + z_1)^2}} dx_2 \\ &\approx \int_{-\ell_x/2}^{\ell_x/2} \ln \frac{\ell_z^2}{(x_1 - x_2)^2 + y_1^2} dx_2 \end{aligned}$$

approximation given by (3). Now we calculate the last integral, yielding

$$\begin{aligned} & \int_{-\ell_x/2}^{\ell_x/2} \ln \frac{\ell_z^2}{(x_1 - x_2)^2 + y_1^2} dx_2 \\ &= -2\ell_x - 2|y_1| \left(\arctan \frac{\ell_x/2 - x_1}{|y_1|} + \arctan \frac{\ell_x + x_1}{|y_1|} \right) \\ &+ \frac{\ell_x}{2} \left[\ln \frac{\ell_z^2}{(\ell_x - x_1)^2 + y_1^2} + \ln \frac{\ell_z^2}{(\ell_x + x_1)^2 + y_1^2} \right] \\ &+ x_1 \ln \frac{(\ell_x/2 - x_1)^2 + y_1^2}{(\ell_x/2 + x_1)^2 + y_1^2} \approx 2\ell_x \ln \frac{2\ell_z}{\ell_x} \quad (27) \end{aligned}$$

where again we used the approximation (3). The integral can then be given by

$$\begin{aligned} & \int_{-\ell_x/2}^{\ell_x/2} \int_{-\ell_z/2}^{\ell_z/2} \frac{dx_2 dz_2}{\sqrt{(x_1 - x_2)^2 + y_1^2 + (z_1 - z_2)^2}} \\ & \approx 2\ell_x \ln \frac{2\ell_z}{\ell_x} \approx 2\ell_x \ln \frac{\ell_z}{\ell_x}. \quad (28) \end{aligned}$$

This is a good approximation for $\ell_z > 1000\ell_x$.

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