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Changing the Inertial Mass of a Charged Particle

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We calculate utilizing Weber's law the force on a moving charge exerted by a stationary charged spherical shell surrounding it. We obtain a net force different from zero which is proportional to the acceleration of the test particle relative to the spherical shell. This result can be interpreted by saying that the inertial mass of a test particle should change if it is placed inside a charged spherical shell. We conclude that this modification in the inertial mass is proportional to the electrostatic potential of the charged spherical shell and to the electric charge of the test particle. Then we present some possible experiments which could be performed to test this prediction.

[Weber's law, Weber force, inertial mass, potential dependent mass]

§1. Weber's Electrodynamics

In this paper we propose an experiment to test directly Weber's electrodynamics. Before we discuss this experiment we will present briefly Weber's theory and the reasons why it has been chosen as the basis on which to propose this new experiment.

According to Weber's electrodynamics[1-8] the force exerted by q_2 on q_1 is given by

$$F=\frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1-\frac{\dot{r}^2}{2c^2}+\frac{r\ddot{r}}{c^2}\right), \tag{1}$$

where $\varepsilon_0=8.85\times10^{-12}\,F/m$ is the vacuum permissivity, $r=|r_1-r_2|$ is the distance between the charges, $\hat{r}=(r_1-r_2)/r$ is the unit vector pointing from q_2 to q_1, $\dot{r}=dr/dt$ is the relative radial velocity between the charges, $\ddot{r}=d^2r/dt^2$ is their relative radial acceleration, and c is the ratio between electromagnetic and electrostatic units of charge, which was found experimentally to have the same value as the velocity of light in vacuum ($c=3\times10^8$ m/s).

This force complies with Newton's third law in the strong form. Due to this fact Weber's electrodynamics is compatible with the principles of conservation of linear and angular momentum. Moreover, this force can be derived from a velocity dependent potential energy given by $U=q_1q_2(1-\dot{r}^2/2c^2)/4\pi\varepsilon_o r$. The force is obtained in the usual way by $F=-\hat{r}\,dU/dr$. Due to this fact Weber's electrodynamics is also compatible with the principle of conservation of energy. The force can also be derived from a Lagrangian function given by $L=T-S$, where T is the kinetic energy of the two charges with masses m_1 and m_2 ($T=m_1v_1^2/2+m_2v_2^2/2$) and $S=q_1q_2(1+\dot{r}^2/2c^2)/4\pi\varepsilon_o r$, as has been shown by Weber himself (note the sign change from U to S).

Historically Weber arrived at this expression in order to derive from a single formula Coulomb's force and Ampère's force between current elements, which can be expressed in modern vectorial notation as

$$d^2F=-\frac{\mu_o}{4\pi}I_1I_2\frac{\hat{r}}{r^2}[2(dl_1\cdot dl_2)-3(\hat{r}\cdot dl_1)(\hat{r}\cdot dl_2)]. \tag{2}$$

In this expression d^2F is the force exerted by the neutral current element $I_2\,dl_2$ on $I_1\,dl_1$, and $\mu_o=4\pi\times10^{-7}$ kg m C^{-2} is the vacuum permeability.

One of the reasons for the renewed interest in Weber's electrodynamics recently is connected with the fact that Ampère's force can

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be derived from Weber's theory. In order to understand this fact we need to remember that the only expression which appears in the textbooks nowadays for the force between current elements is Grassmann's force (1845), which can be written as

$$d^2\boldsymbol{F}=I_1 d\boldsymbol{l}_1 \times d\boldsymbol{B}_2 = I_1 d\boldsymbol{l}_1 \times \left[\frac{\mu_o}{4\pi}\frac{I_2 d\boldsymbol{l}_2 \times \hat{r}}{r^2}\right]$$
$$= -\frac{\mu_o}{4\pi}\frac{I_1 I_2}{r^2}[(d\boldsymbol{l}_1 \cdot d\boldsymbol{l}_2)\hat{r} - (d\boldsymbol{l}_1 \cdot \hat{r})d\boldsymbol{l}_2]. \quad (3)$$

In this expression $d^2\boldsymbol{F}$ is the force exerted by the current element $I_2\, d\boldsymbol{l}_2$ on $I_1\, d\boldsymbol{l}_1$ and $d\boldsymbol{B}_2$ is the magnetic field according to Biot-Savart's law.

It is a known fact that the force of a closed circuit on a current element of another circuit is the same according to both expressions, (2) and (3). So for two or more closed circuits we can not distinguish them. On the other hand if we calculate the force on part of a closed circuit due to the remaining circuit it is not yet completely clear if the two expressions agree with one another. In the last ten years many experiments[9-13] have been performed with a single circuit trying to distinguish in the laboratory the two expressions. Although most of these experiments favour Ampère's force against Grassmann's force, this is still an open subject[14-17] and more research is desirable before a definitive conclusion can be drawn.

Although we do not find Ampère's force in most textbooks, but only Grassmann's force, it should be remarked that J. C. Maxwell knew both force laws, eqs. (2) and (3), and preferred Ampère's force to Grassmann's one. For instance, in his major and last work, *A Treatise of Electricity and Magnetism*, after presenting Grassmann's force (ref. 4, Vol. 2, article 526, p. 174) and two other force laws of his own, Maxwell made the following comparison of these force laws and that of Ampère, eq. (2): "Of these four different assumptions that of Ampère is undoubtedly the best, since it is the only one which makes the forces on the two elements not only equal and opposite but in the straight line which joins them." (ref. 4, Vol. 2, article 527, p. 174).

Weber suceeded in deriving not only Gauss and Ampère's laws from his expression of the force between two charges, but also Faraday's law of induction (1831). We will not present here the details of this derivation, as it can be found in Maxwell's *Treatise*.[4]

When Weber derived eq. (2) from his force law, eq. (1), he assumed Fechner's hypothesis, namely, that in an usual metallic current the positive and negative charges move in opposite directions with the same velocity. But recently we showed that based only on the neutrality of the current elements and on Weber's force we can still derive Ampère's force, eq. (2), even when the positive ions are fixed in the lattice and only the electrons move generating the current.[18] This overcame the first criticism of Weber's theory.

The second reason for the neglect of Weber's electrodynamics during the first half of this century was Helmholtz criticism of Weber's theory.[4,19] According to him Weber's law could lead to a "negative mass behaviour" of the charges in some situations involving high potentials. A particular consequence of this behaviour would be charges moving at a velocity higher than the light velocity, which has never been observed. The first to overcome Helmholtz criticism has been Phipps[20-21] which proposed a generalization of Weber's potential energy, namely:

$$U_P = \frac{q_1 q_2}{4\pi\varepsilon_o r}\sqrt{1-\frac{\dot{r}^2}{c^2}}$$
$$= \frac{q_1 q_2}{4\pi\varepsilon_o r}\left(1-\frac{\dot{r}^2}{2c^2}-\frac{\dot{r}^4}{8c^4}-\cdots\right). \quad (4)$$

This potential reduces to Weber's expression for low velocities. As it is free of the negative mass behaviour it overcame Helmholtz's criticism of Weber's theory. Two aspects must be observed. The first one is that Phipps' solution is a very recent one, indicating a renewed interest in Weber's theory. The second and most important one is that it indicates clearly what were the limitations of Weber's model, namely, that it should be valid only until the second order in v/c, inclusive.

The third kind of criticism which has been made against Weber's theory is the fact that it is an action-at-a-distance law, like Newton's law of gravity. Two things must be remarked here. The first one is that Moon and Spencer[22) introduced the retarded time ($t-r/c$ instead of t) in Weber's force, and so overcoming this limitation. And recently Wesley[6,7] developed the same kind of idea, but introducing the retarded time in Weber's potentials and fields, instead of directly in the force law. With these two approaches the stigma of being an action-at-a-distance theory is not valid anymore.

The second aspect to be remarked on this respect is that although Weber's theory is simultaneous and instantaneous, the first to derive the wave equation for the propagation of an electric disturbance (a pulse of voltage or current, for instance) in a metallic circuit were Weber and Kirchhoff, in 1856 and 1857. They were the first to obtain the correct equations of the transmission line theory. Both worked with Weber's action-at-a-distance theory coupled with the law of conservation of charges. For a detailed discussion of all these facts see ref. 23–27 and 5, Vol. 2, pp. 523–535.

Apart from these questions of electrodynamics, there are many important and new results which appeared recently dealing with a Weber's force applied to gravitation.[28-30] We will not go into the details here but the main idea is that a generalization of Newton's law of gravitation with terms similar to the generalization of Coulomb's law proposed by Weber yields Mach's principle (the inertial forces like *ma*, the centrifugal and Coriolis forces, being due to a gravitational interaction of any body with the distant universe), the equivalence principle (a derivation of the proportionality between inertial and gravitational masses) and the precession of the perihelion of the planets. Sokol'skii and Sadovnikov have also discussed the fact that a Weber's force law for gravitation models the delay in the propagation of interactions despite being an action-at-a-distance theory.[31]

Despite all these positives remarks regarding Weber's law we have shown that it is only an approximation valid up to second order in v/c, inclusive.[32,33] This means that Weber's electrodynamics should not be applied without modifications to particles moving near the light velocity. In the next section we will analyse a possible experiment to be performed in this regime of low velocities in order to test an essential property of Weber's force, namely, the fact that it depends on the acceleration of the test charge.

§2. Changing the Inertial Mass of a Charged Particle

We now calculate with Weber's force, eq. (1), the force on a charge q exerted by a surrounding hollow spherical shell of radius R and charge Q uniformly distributed over its surface. We perform the integration following similar procedures utilized in related problems.[29,33] We suppose the spherical shell to be at rest and without rotation in the laboratory. When q is anywhere inside the shell and has any velocity this yields:

$$\boldsymbol{F}_s=\frac{qQ}{12\pi\varepsilon_o c^2 R}\boldsymbol{a}=\frac{q\phi}{3c^2}\boldsymbol{a}. \tag{5}$$

In this expression $\boldsymbol{a}$ is the acceleration of q relative to the center of the spherical shell, and $\phi=Q/4\pi\varepsilon_o R$ is the electrostatic potential anywhere inside the shell, supposing the potential to be zero at infinity. When a particle of inertial mass m and charge q is interacting with N other bodies (with the Earth, with a magnet, with a current-carrying wire, with a spring, etc.) we obtain, utilizing Newton's second law of motion (valid for $v^2 \ll c^2$) and observing that now the charged shell is also exerting a force on q:

$$\sum_{i=1}^{N}\boldsymbol{F}_i=(m-m_w)\boldsymbol{a}, \tag{6}$$

where $\boldsymbol{F}_i$ is the force exerted by the body i on q, and $m_w=qQ/12\pi\varepsilon_o c^2 R=q\phi/3c^2$ is what we call the Weber's inertial mass for this geometry. This shows that we can interpret the result saying that the inertial mass of the test charge should change when it is inside a charged spherical shell.

In order to observe such an effect it is necessary to have an accelerated test charge. So we need to have a resultant force on it due to the other N bodies different from zero.

From eq. (6) many interesting results can be drawn. If q and Q have the same (opposite) sign then the effect of the charged spherical shell is equivalent to a decrease (increase) in the inertial mass of the particle. An important feature of this model is that the change in the inertial mass of the test particle is independent of its velocity. This is a new result not predicted in other theories. Essentially the value of Weber's inertial mass is proportional to the electrostatic potential of the shell.

An order of magnitude for this effect can be easily obtained. The maximum electric field before the corona discharge in atmospheric air is typically 3×10^6 V/m (the breakdown value).[34] The electric field just outside the charged spherical shell is given by $E=Q/4\pi\varepsilon_o R^2$. In order to have an effect equivalent of doubling the inertial mass of an electron, the radius of the shell charged positively having this limiting breakdown field would be approximately 0.5 m. The potential of the shell in this case would be 1.5 MV. This shows that it would be feasible to test the existence of this effect in the laboratory.

In order to perform the experiment the shell should be made of a dielectric material (such as glass). If it is made of metal the test charge when accelerated may induce currents in the shell. Beyond decreasing the energy of the test charge, this effect would disturb the analysis of the experiment.

A possible experiment is to evacuate the sphere (or at least to let it with a low pressure) so that an electron can move through it. The source of electrons can be a heated filament or a radium source. The experiment would be to measure the Larmor radius described by these electrons when they pass near a permanent magnet. According to eq. (6) the radius of the orbit for an electron should be given by (supposing a uniform magnetic field) $r=|(m-m_w)v/eB|$, where $m=9.1\times10^{-31}$ kg, v is the electron velocity, $-e$ its charge, and B is the magnetic field generated by the magnet. We suggest that this experiment should be performed with two variations: with the permanent magnet inside and outside the sphere. This care must be taken because the value of B can change from one situation to another (if the inertial mass of the electrons which constitute the magnet change due to the effect which is being discussed here, their spin can also change). The trajectory of the electron beam can be seen and measured if the charged spherical shell is made of glass and filled with a low pressure gas that can be ionized by the beam.

The same experiment could also be performed analysing the Larmor radius of moving electrons being accelerated by a permanent magnet outside a charged capacitor.[32,33] As we remarked in these works, the geometry of the problem is also relevant for this effect.

In principle photons should not change their frequency ν or mass m_γ ($m_\gamma=h\nu/c^2$, where h is Planck's constant) due to an effect similar to what is being discussed here because they have no net charge. According to eq. (6) the charged shell will not exert any force on them. The same can be said of photons moving inside or outside a charged capacitor.[32,33] Experiments agree with this prediction. Kennedy and Thorndike analysed photons moving through points varying by 5×10^4 V and did not find any effect.[35]

Any other experiment involving a charged particle being accelerated in regions of variable electrostatic potential can be utilized to test this important aspect of Weber's electrodynamics. The relevant feature is that the effect under consideration (the outcome of the experiment) must involve the inertial mass of the charged particle.

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