BOOTSTRAP EFFECT IN CLASSICAL ELECTRODYNAMICS

A. K. T. Assis[1] **Marcelo Bueno[2]**

RESUMEN

Se calcula la fuerza en una parte de un circuito producida por el resto del mismo. Para ello se utiliza la Fuerza de Ampère y la fuerza de Grassmann. Al mismo tiempo, considerando una configuración simétrica, realizamos los cálculos utilizando elementos lineales de corriente. Se demuestra que ambas expresiones dan el mismo resultado. Presentamos entonces algunos argumentos generales para demostrar que aún con la Fuerza de Grassmann, un circuito cerrado único de forma arbitraria, no puede ejercer una fuerza resultante sobre el mismo. Además demostramos que la fuerza sobre cualquier conductor rectilíneo, perteneciente a un circuito cerrado de forma arbitraria, debida a la parte restante del circuito, es ortogonal a este conductor y tiene el mismo valor de acuerdo con la fuerza de Ampère y con la fuerza de Grassmann.

ABSTRACT

We calculate the force on part of a circuit due to the remaining circuit using Ampère's force and Grassmann's force. Using a symmetrical configuration we perform the calculations using linear current elements. We show that both expressions give the same result. Then we present some general arguments to show that even with Grassmann's force a single closed circuit of arbitrary form cannot exert a net force on itself. Moreover, we show that the force acting on any straight conductor belonging to a closed circuit of arbitrary form, due to the remaining of this circuit, is orthogonal to this conductor and has the same value according to Ampère's force and to Grassmann's force.

INTRODUCTION

One of the open problems in electrodynamics is to know which one is the correct expression for the force between two current elements, [1]. Ampère obtained an expression for this force as the main result of his experimental work. In modern notation his force can be written as

$$d^2\vec{F}_{21} = -\frac{\mu_o}{4\pi} I_1 I_2 \frac{\hat{r}}{r^2}\left[2\left(d\vec{\ell}_1 \cdot d\vec{\ell}_2\right) - 3\left(\hat{r}\cdot d\vec{\ell}_1\right)\left(\hat{r}\cdot d\vec{\ell}_2\right)\right] \quad (1)$$

In this expression $d^2\vec{F}_{21}$ is the force exerted by a current element $I_2 d\vec{\ell}_2$, located at $\vec{r}_2$, on another current element $I_1 d\vec{\ell}_1$, located at $\vec{r}_1$, $r \equiv \left|\vec{r}_1 - \vec{r}_2\right|$ and $\hat{r} \equiv \left(\vec{r}_1 - \vec{r}_2\right)/r$. This expression follows Newton's action and reaction law in the strong form, for any configuration and orientation of the current elements. Beginning with this force it can be derived the famous circuital law $\oint \vec{B}\cdot d\vec{r} = \mu_o I$.

Ampère's formula does not appear in almost any book of present day physics. Instead of that, we only have Grassmann's force, given by

$$d^2\vec{F}_{21} = I_1 d\vec{\ell}_1 \times d\vec{B}_2 \quad (2)$$

In this expression $d\vec{B}_2$ is the magnetic field as first given by Biot and Savart, namely

$$d\vec{B}_2 = \frac{\mu_o}{4\pi}\frac{I_2}{r^2}\left(d\vec{\ell}_2 \times \hat{r}\right) \quad (3)$$

Performing the double cross product in Eqs. (2) and (3) yields

$$d^2\vec{F}_{21} = -\frac{\mu_o}{4\pi}\frac{I_1 I_2}{r^2}\left[\left(d\vec{\ell}_1 \cdot d\vec{\ell}_2\right)\hat{r} - \left(d\vec{\ell}_1 \cdot \hat{r}\right)d\vec{\ell}_2\right] \quad (4)$$

Changing the symbols 1 to 2 and 2 to 1 yields

$$d^2\vec{F}_{12} = +\frac{\mu_o}{4\pi}\frac{I_1 I_2}{r^2}\left[\left(d\vec{\ell}_1 \cdot d\vec{\ell}_2\right)\hat{r} - \left(d\vec{\ell}_2 \cdot \hat{r}\right)d\vec{\ell}_1\right] \quad (5)$$

[1] Instituto de Física "Gleb Wataghin", Universidade Estadual de Campinas, Unicamp, 13083-970 Campinas, São Paulo, Brasil, e-mail: assis@ifi.unicamp.br, homepage: http://www.ifi.unicamp.br/~assis.

[2] R. José Gonçalves 25/102, Morumbi, 05727-250 São Paulo, SP, Brasil, e-mail: m.a.bueno@bigfoot.com, homepage: http://www.bigfoot.com/~m.a.bueno.

Eqs. (4) and (5) show that we do not always have $d^2\vec{F}_{21} = -d^2\vec{F}_{12}$ with Grassmann's force. This is one of the main differences between the expressions of Ampère and Grassmann. Another difference is that when we have two current elements collinear and parallel to one another, there should exist a repulsion between them according to Ampère's force. On the other hand, no force should be exerted on one another according to Grassmann's force.

Although these two forces are different from one another they do give the same result for the force on a current element of one circuit due to a closed second circuit, [2]. The reason for this remarkable fact is that the difference between Eqs. (1) and (2) is an exact differential, which integrates to zero round any closed circuit.

In recent years controversy has appeared again. The important aspect now is that some physicists are performing experiments to decide the matter and not only discussing philosophical matters or questions of taste, [1]. As the force on one circuit due to another closed circuit is the same with both expressions, the central point has been focused on the question: "What is the force on part of a circuit due to the remaining of this circuit?" The first of these experiments was done in 1982, [3], and dealt with jet propulsion in liquids. Since then, many others have been done dealing with railgun accelerators, [4]; the exploding wire phenomena, [5], [6]; the electromagnetic impulse pendulum, [7], [8]; and with liquid mercury, [9], [11]. Although most of these experimenters seem to favor Ampère's force against Grassmann's force, there is not yet a consensus on this conclusion, [12], [18].

The proof of the equivalence between Ampère's force and Grassmann's force, for the interaction of a closed circuit with a part of itself, has been claimed by some authors in recent years, [19], [23]. With this work we present a new demonstration of the equivalence, trying to overcome the difficulties raised by other authors, [24], [28], against those demonstrations. Moreover, our demonstration is not restricted to the magnetostatic case only, as in [19], [20], since there are some experiments which have been performed using alternate currents, [4], [13].

It has always been very difficult to decide the question even theoretically because when we try to calculate the force on part of a circuit due to the remaining circuit usually the result diverges (the force goes to infinity) with both expressions. To avoid this divergence some people have tried to introduce an explicit finite separation distance between the two parts of a circuit (see citation in [4], p. 183), or introduced a current element of finite size to use finite element analysis in computer calculations, [29]. An unquestionable way of doing these calculations obtaining finite values without arbitrary assumptions is using surface current elements $\vec{K}dA$ or volumetric current elements $\vec{J}dV$, instead of linear current element $Id\vec{\ell}$. The first to calculate explicitly the force between two parts of a circuit which are in contact by this correct but quite involved method, using volumetric current densities was Wesley, [30], [32].

In this paper we utilize a different idea to perform the calculations. We utilize only linear current elements so that the calculations are relatively simple. We calculate the force on part of a circuit due to the remaining circuit in the simplest of all configurations that gives non trivial results. This is shown in the next sections. The analysis of this paper have appeared up to now only in Portuguese, [33].

AMPÈRE'S FORCE

In Fig. 1 we show the circuit utilized for the calculations and the corresponding geometry and labels. We represent the force on part i of a circuit due to another part j by $\vec{F}_{ji}$. The circuit composed of parts 1, 2, ..., 12 is a closed one where flows the constant current I. What we want to know is the force on bridge 1 due to the remaining circuit 2 to 12 (called the support). We can think that bridge 1 is joined to the remaining circuit by liquid mercury cups (or by arc gaps) at both extremities so that the force on it can be measured without interrupting the current. As we are utilizing linear current elements, this calculation is valid only when the diameter d of the wire is much smaller than all other dimensions in the system. In this case this means $d \ll M$, $d \ll L$, $d \ll N$ and $d \ll P$.

We devised this geometry guided by an ingenious idea of Moyssides and Pappas, [34]. As we said in the Introduction, the problems faced by all whom tried to perform calculations using linear current elements were the parts in contact of the circuit. But Moyssides and Pappas circumvented this difficulty bending the bridge's ends. We can explain this technique in Fig. 1. If we calculate the force on part 11 due to the remaining circuit (1 to 10 plus 12) it goes to infinity with linear current elements (this happens with Ampère and Grassmann's forces). But this does not happen with bridge 1 due to the support 2 to 12. If we calculate the force of part 2 on part 1 using Eq. (1) we get an infinite result, $\vec{F}_{21} = -\infty\hat{x}$. But part 12 is symmetrically located relative to part 1 and has the same size so that $\vec{F}_{12,1} = +\infty\hat{x}$. Then $\vec{F}_{21} + \vec{F}_{12,1} = 0$. Although the infinite result we obtained for $\vec{F}_{21}$ may appear artificial, the null result for $\vec{F}_{21} + \vec{F}_{12,1}$ is an exact one arrived at by symmetry considerations as above or by supposing volumetric current elements and integrating for the cross section of the wire. For instance, using a wire with a cross section in the form of a square with side ω, Wesley (we checked his calculations) obtained using Ampère's force and supposing $\omega \ll L$ and $\omega \ll M$, [30], [32]:

$$\vec{F}_{21} = -\frac{\mu_o}{4\pi} I^2 \hat{x} \left(\frac{13}{12} - \frac{\pi}{3} + \frac{2}{3}\ln 2 - \ln\frac{M+L}{M} + \ln\frac{L}{\omega} + O\left(\frac{\omega^3}{L^3}\right) + O\left(\frac{\omega^3}{M^3}\right) \right) = -\vec{F}_{12,1} \tag{6}$$

Again $\vec{F}_{21} + \vec{F}_{12,1} = 0$, although now each part of this sum is finite. What matters is that parts 3 to 11 give the net force on the bridge. As Moyssides and Pappas were more interested in the experiment and did not make the complete calculations for this case, we decided to do it. Performing these calculations with Eq. (1) we obtain the net force on bridge 1 due to the support as given by

$$\vec{F}_{SB} = \frac{\mu_o I^2}{4\pi}\hat{y}\left(\frac{\left[P^2+(L+M)^2\right]^{1/2}}{P} + \frac{\left[P^2+(L+M+N)^2\right]^{1/2}}{P} - \frac{\left[P^2+(M+N)^2\right]^{1/2}}{P} - \frac{(P^2+M^2)^{1/2}}{P} + \ln\frac{L+M}{M} + \frac{P+(P^2+M^2)^{1/2}}{P+\left[P^2+(L+M)^2\right]^{1/2}} + \ln\frac{L+M+N}{M+N} + \ln\frac{P+\left[P^2+(M+N)^2\right]^{1/2}}{P+\left[P^2+(L+M+N)^2\right]^{1/2}} \right) \tag{7}$$

It should be remarked that although the bridge is not symmetrically located in the middle of the side 12 to 4, the resultant force on it has no component along the *x* direction according to Ampère's force. This is an important fact, which will be extended later here.

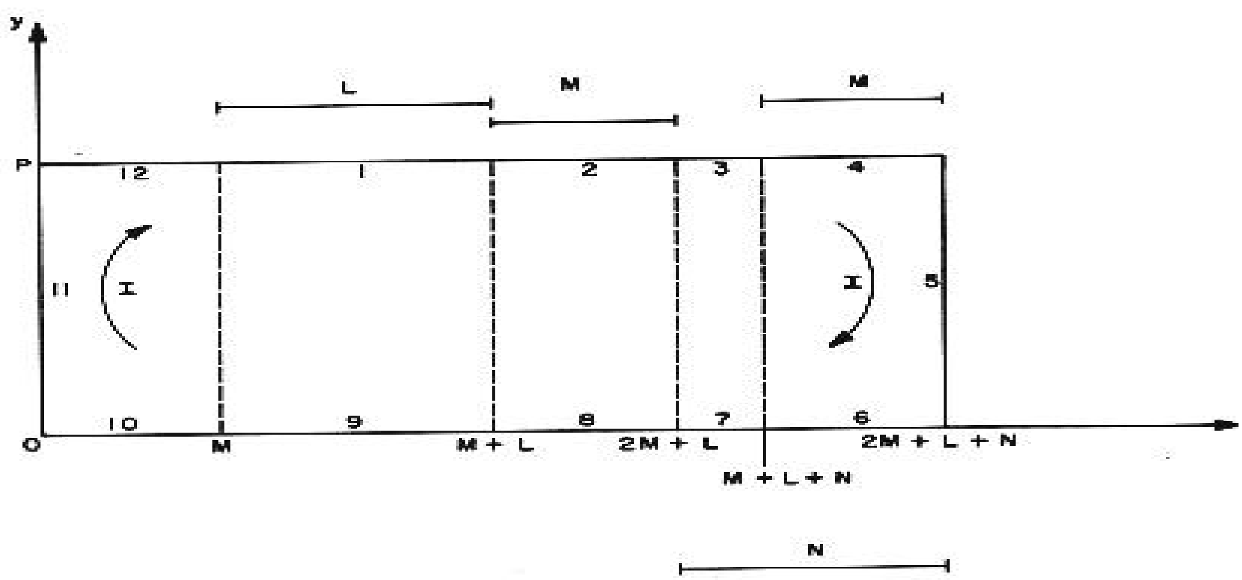

Fig. 1.- Electrical circuit with steady current I. The bridge is represented by 1 and the remaining circuit (The support) by 2 to 12

GRASSMANN'S FORCE

We now utilize Eqs. (2) to (5) in order to calculate the force on the bridge in Fig. 1. The first result is that $\vec{F}_{21} = \vec{F}_{12,1} = 0$. This happens not only integrating with linear current elements but also with surface or volumetric current elements. What matters is that once more parts 3 to 11 give the net force on the bridge because $\vec{F}_{21} + \vec{F}_{12,1} = 0$. Performing the calculation with Eq. (4) we get as our final result for the force on the bridge due to the support exactly Eq. (7).

Our conclusion is that the force on bridge 1 in Fig. 1 is given by expression (7) both for Ampère's force and for Grassmann's force. This is a non-trivial result because we are not integrating in a closed circuit. We now calculate the force on the support and discuss the possibility of a bootstrap effect.

BOOTSTRAP EFFECT

By symmetry considerations we obtain with Ampère's force $\vec{F}_{12} = -\vec{F}_{1,12}$ so that $\vec{F}_{12} + \vec{F}_{1,12} = 0$. Integrating Ampère's force for the force on the remaining circuit due to the bridge we obtain exactly Eq. (7) with an overall reversal of sign. This was expected because Ampère's force always follows Newton's action and reaction law even in differential form, Eq. (1). Adding these two results we obtain that the net force of the circuit on itself is zero according to Ampère's force.

We now make the same calculation using Grassmann's force. From (5) we get $\vec{F}_{12} = \vec{F}_{1,12} = 0$, so that $\vec{F}_{12} + \vec{F}_{1,12} = 0$. Performing the remaining calculation we obtain the force on the support due to the bridge 1 as given by:

$$\vec{F} = -\frac{\mu_o I}{\pi}\left[\hat{x}\left(\ln\frac{M+N}{L+M+N} + \ln\frac{L+M}{M} + \frac{M \quad (P^2 \quad M^2)^{1/2}}{(L+M)+\left[P^2+(L+M)^2\right]} + \ln\frac{L+M+N \; + \left[P^2 + \; L+M+N \;^2\right]}{M \quad N \quad \left[P^2 \quad M \quad N \;^2\right]}\right)\right. \tag{8}$$

$$\left. + \hat{y}\left(\frac{\left[P^2+(L+M+N)^2\right]^{1/2}}{P} - \frac{\left[P^2+(M+N)^2\right]^{1/2}}{P} + \frac{\left[P^2+(L+M)^2\right]^{1/2}}{P} - \frac{(P^2+M^2)^{1/2}}{P}\right)\right]$$

Adding this expression to Eq. (7) yields a result different from zero. Our first impression is that the circuit should exert a net force on itself according to Grassmann's force. Supposing the bridge to be mechanically linked with the remaining circuit this would indicate that the circuit should move in space without being acted on by external forces. This bootstrap effect has never been observed in nature.

But this is a wrong conclusion. The main point to take notice is that we only calculated the force on the bridge due to the support and the force on the support due to the bridge. We did not calculate the force on the bridge due to itself or the force on the support due to itself. This is what we make now.

The force on bridge 1 (Fig. 1) due to itself is zero according to Ampère's force or Grassmann's force. This is easily seen or calculated.

The force on the support due to itself is zero according to Ampère's force. This follows straight away from the fact that $d^2\vec{F}_{21} = -d^2\vec{F}_{12}$ for any orientation of the current elements. In order to calculate the force of the support on itself with Grassmann's force we use once more a consideration of symmetry. The force on parts 10, 11 and 12 due to themselves is $-\alpha\hat{x}$, where $\alpha>0$. The value of α is equal to infinity if we utilize linear current elements. On the other hand, α equals to a finite value if we utilize surface or volumetric current elements and integrate also in the cross section of the wire. And if this cross section goes to zero, $\alpha \to \infty$. On the other hand parts 4, 5 and 6 are symmetrically located relative to the other end and have the same size so that the net force of these parts (4, 5 and 6) on themselves is $+\alpha\hat{x}$. In this way we have $\vec{F}_{10,11,12 \text{ on } 10,11,12} + \vec{F}_{456 \text{ on } 456} = 0$. All the other calculations can be performed now using only linear current elements and linear integrations. The final result using Eqs. (4) or (5) is that the net force of the support (parts 2 to 12) on itself is given by

$$\vec{F}_{SS} = +\frac{\mu_o I^2}{4\pi}\left[\left(\ln\frac{M+N}{L+M+N} + \ln\frac{M+L}{M}\right.\right.$$

$$+\ln\frac{M+(P^2+M^2)^{1/2}}{(L+M)+\left[P^2+(L+M)^2\right]^{1/2}}$$

$$\left.+\ln\frac{(L+M+N)+\left[P^2+(L+M+N)^2\right]^{1/2}}{(M+N)+\left[P^2+(M+N)^2\right]^{1/2}}\right) \qquad (9)$$

$$+\hat{y}\left(\ln\frac{M+N}{L+M+N}+\ln\frac{M}{M+L}+\ln\frac{P+\left[P^2+(L+M)^2\right]^{1/2}}{P+(P^2+M^2)^{1/2}}\right.$$

$$\left.\left.+\ln\frac{P+\left[P^2+(L+M+N)^2\right]^{1/2}}{P+\left[P^2+(M+N)^2\right]^{1/2}}\right)\right]$$

Adding Eq. (9) to Eq. (8) we obtain exactly Eq. (7) with an overall change of sign. This means that even with Grassmann's force in this single circuit the resultant force on the support $(\vec{F}_{BS}+\vec{F}_{SS})$ is minus the resultant force on the bridge $(\vec{F}_{SB}+\vec{F}_{BB})$. Moreover, the resultant force on each one of these parts according to Grassmann's force is the same as according to Ampère's force. As we will show in the next section, this is a general result valid in any circuit with any geometry, and not only in this rectangular one. Our conclusion is that the resultant force of this closed circuit on itself is zero according to both expressions, so that even with Grassmann's force there is no bootstrap effect in this configuration of a closed circuit. This happens due to the fact that there is bootstrap effect in part of a circuit interacting with itself according to Grassmann's force, see Eq. (9).

The main non trivial result of this section can be summarized as follows: If we divide a closed circuit in two parts *A* and *B* and want to know the resultant force on part *A* according to Grassmann's expression, we need to calculate not only $\vec{F}_{BA}$ but also $\vec{F}_{AA}$. This is an extremely important fact neglected by most authors.

GENERAL EQUIVALENCE

We first prove that the resultant force on any straight conductor (with the diameter small compared to its length) of an arbitrary closed circuit has the same value according to Ampère's force and to Grassmann's force. Then we prove that the resultant force (being finite or infinite) is orthogonal to the conductor. The proofs of these facts are given in Figs. 2A, 2B and 2C. In Fig. 2A is represented a closed circuit *C* with arbitrary form. In Fig. 2B is represented a square circuit of sides $3|d\vec{\ell}|$ and a circuit *C´* which is similar to *C* in most points, except those near the square circuit. The element *ab* of length $|d\vec{\ell}|$ is in the middle of the lower side of the square. There is a distance *d* between the sides of the square and the equivalent pieces of the circuit *C´*. The force on the current element *ab* in Fig. 2B is given by the force of the open square circuit *befa* on it (which has the same value according to Grassmann's force and to Ampère's force, as we showed in the earlier sections) plus the force of the circuit *C´* on it (again this has the same value according to both laws because now *C´* is a closed circuit, [2]). Although the resultant force on *ab* can depend on the value of *d*, the fact that this resultant force has the same value according to both laws does not depend on the value of *d*, and so this will remain valid when $d\to 0$. In this situation ($d\to 0$) the force of the open square on *ab* plus the force of the circuit *C´* on *ab* is equivalent to the force of the open circuit *bca* (Fig. 2A) on *ab* as can be seen in Fig. 2C, which is equivalent to Fig. 2A. And this was what we wanted to prove.

From Fig. 2B we can also prove an important fact. By considerations of symmetry (which were confirmed by our calculations in Sections 3 and 4) the force on the element *ab* due to the open square *befa* is orthogonal to this element. But we know that the force of the closed circuit *C´* on *ab* is also orthogonal to *ab*. We know this from two reasons. *(I)* Ampère himself proved this fact experimentally, [1]. *(II)* The force of a closed circuit on an element of another circuit has the same value according to Ampère's force and to Grassmann's force, [2]. But Grassmann's force can be expressed as Eq. (2). For this reason we see straight away that the force of a closed circuit 2 in an element $I_1 d\vec{\ell}_1$ is orthogonal to $d\vec{\ell}_1$. As this last fact is independent of the distance *d* (Fig. 2B) it will remain valid when $d\to 0$. But when $d\to 0$ we recover the situation of Fig. 2A, which was what we wanted to prove.

As there is no bootstrap effect with Ampère's force since it follows Newton's action and reaction law, we conclude with this proof that the same will happen with Grassmann's force when considering a single closed circuit.

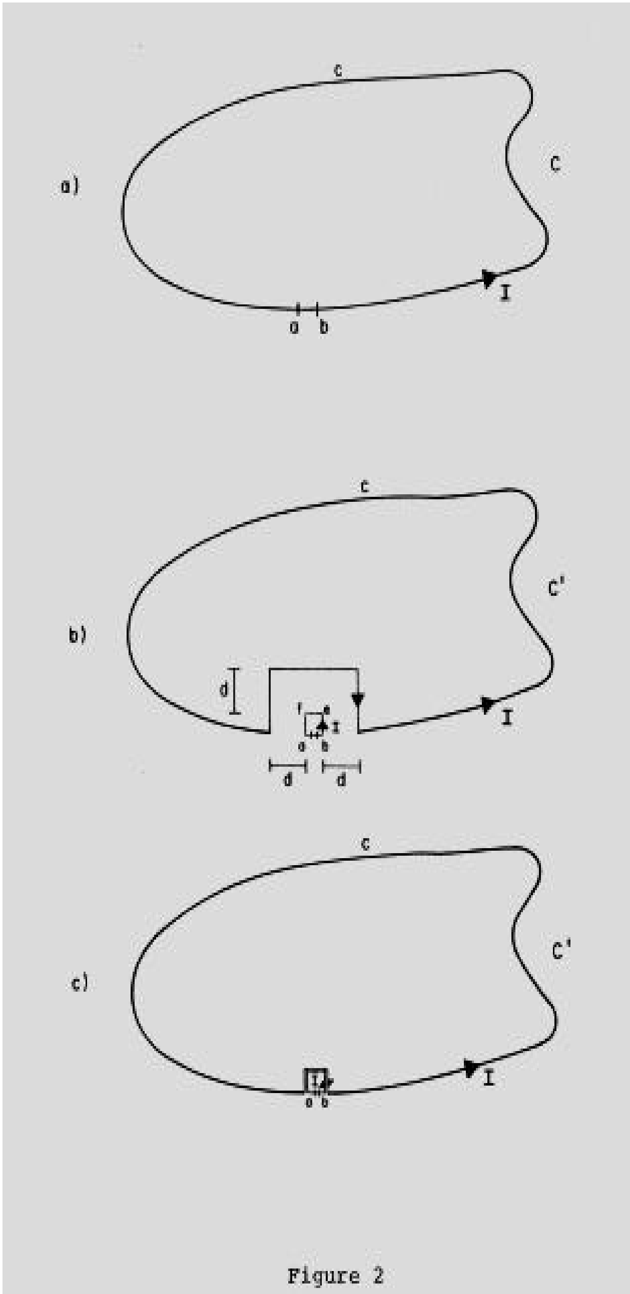

Fig. 2 - (A) A closed circuit *C* of arbitrary form where flows the current *I*. The straight conductor $Id\vec{\ell}$ is represented by the segment *ab*. (B) An infinitesimal square circuit *abefa* of side $3\left|d\vec{\ell}\right|$ and a closed circuit *C´*, in both of them flowing in the same direction a current *I*. The conductor *ab* of length $\left|d\vec{\ell}\right|$ is in the middle of the lower side of the square. (C) Situation when $d \to 0$ and $C' \to C$ except near *ab*. This is equivalent to Fig. 2A.

CONCLUSION

Wesley calculated the force on parts 10 to 12 (Fig. 1) due to parts 1 to 9 and vice-versa, using volumetric integrations, [30-32]. He obtained finite but different results for these two forces using Grassmann's force. He thought he had found a bootstrap effect with these calculations. But he did not calculate the force of parts 10 to 12 on themselves nor of parts 1 to 9 on themselves. This shows that his calculations are correct (we checked them) but incomplete because these forces needed to be taken into account as we showed here.

As regards the experiments, what we can say is that the repulsive force between two parts of a single closed circuit, which some physicists are measuring, can be equally accounted by Ampère's force or by Grassmann's force. For instance, the repulsive force acting on Ampère's bridge in Fig. 1 (force along the negative *x* axis acting on parts 10 to 12) is due, according to both laws, to the force acting on piece 11. As pieces 10 and 12 are mechanically linked to piece 11, they will move together to the left if there are mercury cups (or arc gaps) between 12 and 1, and between 9 and 10. Even according to Ampère's force the resultant force acting on piece 12 due to the whole circuit is along the positive *y*-axis, and this is balanced by an opposite resultant force acting on 10. Even in Ampère's bridge both expressions agree with one another. So, these experiments do not seem to be decisive either.

There are however other experiments which have shown conclusively the existence of longitudinal forces (or at least tensions and compressions) in closed current carrying circuits: exploding wires, [4, 6, 35, 36] and railgun recoil, [4, 37-39], for instance. These experiments cannot be explained by Grassmann's expression (2), which can never predict forces parallel to the current. In this work, we have shown that Ampère's expression (1) is not responsible for these effects either. We have no doubts about the existence of these longitudinal forces, but their quantitative explanation is as yet unknown for us.

Recently Robson and Sethian have performed an experiment and have found that there is no longitudinal forces in closed current carrying circuits, [14]. The geometry of their experiment can be simplified to that of our Fig. 1. In the experiment they utilized arc gaps of equal lengths separating piece 1 from 2 and 12. Then, they have obtained the non-existence of longitudinal interaction between piece 1 and the whole circuit. Their important null experiment is a confirmation of the findings of this paper.

REFERENCES

[1] A. K. T. Assis; "Weber's Electrodynamics", Dordrecht, Kluwer, 1994, Chap. 4. ISBN: 0-7923-3137-0.

[2] R. A. R. Tricker; "Early Electrodynamics - The First Law of Circulation", New York, Pergamon, 1965, p. 55.

[3] P. Graneau, Nature, Vol. 295, p. 311, 1982.

[4] P. Graneau; "Ampere-Neumann Electrodynamics of Metals", Nonantum, Hadronic Press, 1985.

[5] J. Nasilowski; Phys. Lett. A, Vol. 111, p. 315, 1985.

[6] P. Graneau, Fortschr. Phys., Vol. 34, p. 457, 1986.

[7] P. T. Pappas, Nuovo Cimento B, Vol. 76, p. 189, 1983.

[8] P. Graneau and P. N. Graneau, Nuovo Cimento D, Vol. 7, p. 31, 1986.

[9] T. E. Phipps and T. E. Phipps Jr., Phys. Lett. A, Vol. 146, p. 6, 1990.

[10] T. E. Phipps Jr., Phys. Essays, Vol. 3, p. 198, 1990.

[11] J. P. Wesley, Found. Phys. Lett., Vol. 7, p. 95, 1994.

[12] P. G. Moyssides, IEEE Trans. Magn., Vol. 25, p. 4313, 1989.

[13] V. Peoglos, J. Phys. D, Vol. 21, p. 1055, 1988.

[14] A. E. Robson and J. D. Sethian, Am. J. Phys., Vol. 60, p. 1111, 1992.

[15] T. Ivezic, Phys. Rev. A, Vol. 44, p. 2682, 1991.

[16] A. K. Singal, Phys. Rev. E, Vol. 48, p. 4138, 1993.

[17] T. Ivezic, Phys. Rev. E, Vol. 48, p. 4140, 1993.

[18] A. K. T. Assis and Marcelo Bueno, IEEE Trans. Mag., Vol. 32, p. 431, 1996.

[19] D. C. Jolly, Phys. Lett. A, Vol. 107, p. 231, 1985.

[20] J. G. Ternan, J. Appl. Phys., Vol. 57, p. 1743, 1985.

[21] C. Christodoulides, J. Phys. A, Vol. 20, p. 2037, 1987.

[22] C. Christodoulides, Am. J. Phys., Vol. 56, p. 357, 1988.

[23] C. Christodoulides, Phys. Lett. A, Vol. 141, p. 383, 1989.

[24] P. Graneau, Phys. Lett. A, Vol. 107, p. 235, 1985.

[25] P. Graneau, J. Appl. Phys., Vol. 58, p. 3638, 1985.

[26] P. Cornille, J. Phys. A, Vol. 22, p. 4075, 1989.

[27] P. T. Pappas, Phys. Essays, Vol. 3, p. 15, 1990.

[28] P. Graneau, Found. Phys. Lett., Vol. 6, p. 491, 1993.

[29] P. Graneau and P. N. Graneau, Appl. Phys. Lett., Vol. 46, p. 468, 1985.

[30] J. P. Wesley, In "Progress in Space - Time Physics 1987", edited by J. P. Wesley, Blumberg, Benjamin Wesley Publisher, 1987, p. 170.

[31] J. P. Wesley, Spec. Sci. Technol., Vol. 10, p. 47, 1987.

[32]J. P. Wesley, Found Phys. Lett., Vol. 3, p. 443, 1990.

[33] Marcelo Bueno and A. K. T. Assis; "Cálculo de Indutância e de Força em Circuitos Elétricos", Florianópolis/Maringá, Editora da Universidade Federal de Santa Catarina/Editora da Universidade Estadual de Maringá, 1998, ISBN: 85-328-0119-6.

[34] P. G. Moyssides and P. T. Pappas, J. Appl. Phys., Vol. 59, p. 19, 1986.

[35] P. Graneau, Phys. Lett. A, Vol. 97, p. 253, 1983.

[36] P. Graneau, IEEE Trans. Magn., Vol. 20, p. 444, 1984.

[37] P. Graneau, J. Appl. Phys., Vol. 53, p. 6648, 1982.

[38] P. Graneau, J. Appl. Phys., Vol. 62, p. 3006, 1987.

[39] P. Graneau and D. S. Thompson, Phys. Lett. A, Vol. 145, p. 396, 1990.