Unipolar Induction

Wilhelm Weber

Editor's Note: An English translation of Wilhelm Weber's 1840 and 1841 papers on "Unipolare Induktion".¹

Third version posted in April 2024 (first version posted in February 2024) at www.ifi. unicamp.br/~assis

¹[Web40] and [Web41b].

Contents

1	Edit	tor's In	ntroduction to Weber's First Paper on Unipolar Induction	5				
	1.1	Farada	ay's Experiment	5				
	1.2	Weber	on Faraday's Experiment	6				
	1.3	Weber	's Experiment	6				
	1.4	On the	e Existence of Magnetic Fluids	8				
	1.5	On the	e Existence of Molecular Currents	8				
	1.6	Unipo	lar Induction and Weber's Electrodynamics	9				
2	Unipolar Induction 13							
	2.1	Genera	al Remarks	14				
		2.1.1	Bipolar and Unipolar Induction	14				
		2.1.2	Method	14				
		2.1.3	Laws	16				
	2.2	Instru	ments	19				
		2.2.1	The Cylindrical Magnets	19				
		2.2.2	The Gears	20				
		2.2.3	Magnetometer and Multiplier	20				
		2.2.4	Connection of the Wire Ends with the Rotating Magnet	21				
		2.2.5	The Inductor Coil	21				
	2.3	Experi	iments	21				
		2.3.1	First Set	22				
		2.3.2	Second Set	25				
		2.3.3	Third Set	25				
		2.3.4	Fourth Set	26				
		2.3.5	Fifth Set	27				
		2.3.6	Sixth Set	27				
		2.3.7	Seventh Set	28				
	2.4	Applic	eations	28				
		2.4.1	Application to Ampère's Electrodynamic Theory of Magnetic Phenom-					
			ena	28				
		2.4.2	Application to the Distribution of Magnetism of Constant Magnets .	30				
		2.4.3	Application to the Distribution of Magnetism in Soft Iron	31				
	2.5	Conclu	usion	31				
3	Edit	tor's Iı	ntroduction to Weber's Second Paper on Unipolar Induction	33				
4	Uni	polar l	Induction	37				

Bibliography

Chapter 1

Editor's Introduction to Weber's First Paper on Unipolar Induction

A. K. T. Assis²

1.1 Faraday's Experiment

Wilhelm Weber (1804-1891) presented in 1839 a paper on unipolar induction which was published in $1840.^3$ We present here the first English translation of this paper.

Michael Faraday (1791-1867) was the first to perform this experiment in 1832.⁴ He placed a copper disk above a cylindrical magnet and connected a galvanometer between the center and edge of the disk through sliding contacts. The magnet and the disk might rotate relative to the ground, while the galvanometer and conducting wires always remained stationary. The copper disk and magnet had the same radii and axis of symmetry, Figure 1.1.



Figure 1.1: Unipolar induction experiment.

²Homepage: www.ifi.unicamp.br/~assis

 $^{^{3}}$ [Web40].

⁴[Far32a, §§ 217 to 230] with Portuguese translation in [Far11] and German translation in [Far32b] and [Far89].

Faraday observed the same electric current flowing through the galvanometer in two situations, namely, (a) when only the disk rotated relative to the ground and (b) when the disk and magnet rotated together relative to the ground. The direction of the current depends on the direction of rotation and also on the orientation of the magnet, that is, if the North pole is above or below the South pole.

In some experiments Faraday removed the disk and considered only a cylindrical magnet, with a galvanometer connected by sliding contacts to the center of the upper face and to a point along the edge of the magnet. When he rotated the conducting magnet relative to the ground, a current was indicated by the galvanometer.

This phenomenon became known as unipolar induction, a name coined by Weber in the paper which is being translated here. Other common names for this experiment are unipolar generator, homopolar induction, homopolar generator and Faraday generator.

1.2 Weber on Faraday's Experiment

Weber seems to have developed the idea of this experiment quite independently from Faraday's earlier work of 1832. He performed the experiment and sent it for publication without being aware of Faraday's publication, as pointed out by Wiederkehr.⁵

When writing the article, Wilhelm Weber appears to have been unaware of Faraday's experiments on unipolar induction. The initially strange fact that Weber suddenly speaks of Faraday and his earlier experiments at the end of his treatise is explained by a letter from Weber to Gauss.

Widerkehr then quoted Weber's letter to Gauss, number 15, from September 06, 1839:

I saw Ettinghausen⁶ in Leipzig on the way to Göttingen. He pointed out to me that the phenomenon which I had considered under the title of unipolar induction had also been noticed by Faraday. I found the passage easily and was able to refer to it since Reimer⁷ was willing to have the last page of this essay reprinted...

1.3 Weber's Experiment

One of Weber's original apparatus related to unipolar induction, his unipolar inductor of 1840, still exists at the Scientific Collections of the Georg-August-University Göttingen in Germany.⁸ It appears in Figure 1.2.

⁵[Wie60, pp. 41-42, footnote 6]. See also [Gau d].

 $^{^{6}\}mathrm{Andreas}$ von Ettingshausen (1796-1878), Austrian physicist.

⁷Karl August Reimer led the Weidmannsche Buchhandlung where Weber's work was originally published. ⁸https://sammlungen.uni-goettingen.de/sammlung/slg_1020/



Figure 1.2: Weber's original instrument. Source: Physicalisches Cabinet, Georg-August-Universität Göttingen / CC BY-SA 4.0; Photo: Sauer Marketing, Gerhard und Maren Sauer.

Figure 1.3 presents the main components of this device.⁹ A brass disk can be fixed at different points along the axis of a magnetized steel cylinder NS. The lower portion of the disk touches a mercury tray or trough. Hand gears allowed the rotation of the magnet around its horizontal NS axis at a known rate. A galvanometer G is connected through conducting wires to the mercury tray and to the center of one extremity of the magnet. When the disk and magnet rotate together relative to the ground, while the galvanometer and conducting wires remain stationary, a current flows through the galvanometer. One of his magnetized cylinders was 26.9 cm long with a diameter of 2.3 cm, while the other was 50.2 cm long with a diameter of 2.05 cm. This experiment is analogous to Faraday's unipolar induction experiment when the disk and magnet rotate together relative to gether relative to the ground, or to Faraday's

experiment in which there was only a magnet which could rotate relative to the ground.



Figure 1.3: A representation of Weber's unipolar inductor.

1.4 On the Existence of Magnetic Fluids

In this paper of 1839 Weber based his explanation of the phenomenon on the existence of magnetic fluids, namely, the northern and southern fluids. Later on he will change completely his point of view, rejecting the existence of these fluids. This was due to his experiments on diamagnetism published in 1852 in his third major memoir on Electrodynamic Measurements. In particular, he discussed this important topic in a Section 22 of his 1852 paper called "On the Existence of Magnetic Fluids".¹⁰ André-Marie Ampère (1775-1826) had already argued against the existence of magnetic poles in his masterpiece.¹¹ In 1852 Weber concluded that the hypothesis of magnetic fluids in the interior of bodies had been refuted, while Ampère's hypothesis of the existence of electric molecular currents in the interior of bodies had been corroborated through diamagnetism. His conclusion runs as follows:¹²

The *diamagnetic* phenomena discovered by Faraday¹³ decide between these two theories in the same way as the phenomena of interference decided between the emission and wave theory in optics. This is the most essential and important meaning associated to this discovery. Thanks to the discovery of diamagnetism the hypothesis of *electric molecular currents in the interior of materials* gets affirmed and the hypothesis of magnetic fluids in the interior of materials gets disproved.

1.5 On the Existence of Molecular Currents

Another aspect which should be emphasized here is that in this paper of 1839 Weber argued, based on the phenomenon of unipolar induction, against the existence of Ampère's molecular currents, see Subsection 2.4.1. Later on he also changed completely his point of view on

¹⁰[Web52a, Section 22] with English translation in [Ass21a, Section 2.22].

¹¹See Section 19 (The Magnetic Poles and Dipoles are Disposable Hypotheses) of [AC11] and [AC15].

¹²[Web52a] with English translation in [Web21b, Section 2.22, pp. 66-68]. See also [Web52b] with English translations in [Web53], [Web66b] and [Web21d, Section 3.1.6].

 $^{^{13}}$ [Far46a] and [Far46b].

this respect. For instance, in his paper of 1852 on diamagnetism he made the following comment:¹⁴

Before in the "Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1839",¹⁵ I tried to justify the conjecture that the phenomenon described by the name "unipolar polarity" could lead to such a decision.¹⁶ However, this is not the case, since there can be given a different explanation for the phenomena described there, as soon as such a connection takes place between the electric fluids moving in the interior of the conductor and the ponderable parts of the conductor, that each force acting on the electric fluids completely or nearly is transferred to the ponderable parts, as I explained in more detail in the "Electrodynamic Measurements" (*Abhandlungen bei Begründung der Königlichen Sächsischen Gesellschaft der Wissenschaften* edited by v. d. F. Jabl. Ges. Art. 19, p. 309).¹⁷

These molecular currents proposed by Ampère became the foundation of many researches which Weber developed in the following years.

1.6 Unipolar Induction and Weber's Electrodynamics

Normally when most scientists considered unipolar induction in the last 200 years they concentrated their attention only on the rotations of the disk and magnet around their common axis. Let us represent by ω the clockwise angular rotation of the disk and/or magnet relative to the ground when seen from above, while $-\omega$ represents the anti-clockwise rotation. When we rotate only the disk relative to the ground, we can measure a current I flowing through the galvanometer connected by sliding contacts between the center and periphery of the disk, as represented in Figure 1.1. This current is linearly proportional to ω and to the intensity of the magnet. These facts indicate that the induced current depends on the interaction between the disk and magnet originating from their relative rotation.

If we let the disk stationary in the ground and rotate the magnet in the opposite direction with an angular rotation $-\omega$, most scientists expected the same current I to be measured in the galvanometer. However, the galvanometer measures no current, as shown in Table 1.1

Disk	Magnet	Galvanometer
ω	0	Ι
0	$-\omega$	0

Table 1.1: First apparent paradox.

There is an apparent paradox here. The relative motion between the disk and magnet is the same in both cases, but the measured effect indicated by the current in the galvanometer is completely different for these two cases.

 $^{^{14}}$ [Web52a, footnote 1, p. 536 of Weber's *Werke*] with English translation in [Web21d, footnote 59, page 67].

¹⁵See [Web40, p. 171 of Weber's Werke] and Subsection 2.4.1 with the English translation of Weber's discussion.

¹⁶In 1839 Weber decided in favour of the existence of magnetic fluids and against the existence of molecular electric currents.

 $^{^{17}}$ [Web46, Section 19, p. 134 of Weber's *Werke*] with English translation in [Web21a, Section 5.19, pp. 130-141].

Another apparent paradox originates when the disk and magnet are stationary, or when both of them rotate together relative to the ground. The result of this experiment is indicated in Table 1.2.

Disk	Magnet	Galvanometer
0	0	0
ω	ω	Ι

Table 1.2: Second apparent paradox.

In these two cases there is no relative motion between the disk and magnet. However, the current measured in the galvanometer is very different for these two cases.

The solution of these apparent paradoxes is that in unipolar induction we need to consider not only the disk and magnet, but also the closing circuit (composed of galvanometer and conducting wires connected to the center and periphery of the disk). We are here supposing that the magnetism of the magnet has a much larger intensity than the magnetism of the Earth, so that we can neglect the magnetic influence of the Earth on the outcome of this experiment.

In 1994 a prediction has been made of what would happen in this experiment, if it were possible to rotate the closing circuit relative to the ground.¹⁸ We now have 8 cases to consider, as indicated in Table 1.3.

	Disk	Magnet	Closing circuit	Galvanometer
1	0	0	0	0
2	ω	0	0	Ι
3	0	$-\omega$	0	0
4	0	0	$-\omega$	Ι
5	ω	0	ω	0
6	0	$-\omega$	$-\omega$	Ι
7	ω	ω	0	Ι
8	ω	ω	ω	0

Table 1.3: Prediction based on Weber's electrodynamics.

From Table 1.3 we can see that the opposite of rotating only the disk clockwise is not to rotate the magnet anti-clockwise, but to rotate together the magnet and the closing circuit anti-clockwise, cases 2 and 6. In both cases the same current I should be measured in the galvanometer.

Likewise, the situation when everything is stationary in the ground is not equivalent to rotate together the disk with the magnet, but to rotate together the disk, magnet and closing circuit, cases 1 and 8. No current should be measured in the galvanometer.

Moreover, the situation when we rotate only the magnet is not equivalent to rotate the disk in the opposite direction, but to rotate together the disk and closing circuit in the opposite direction, cases 3 and 5.

¹⁸[AT94].

And finally, the situation in which we rotate together the disk and magnet, should be equivalent to rotating only the closing circuit in the opposite direction, cases 7 and 4.

These predictions have been confirmed by an experiment performed in 2022:¹⁹ Baumgärtel, C., Maher, S. Resolving the paradox of unipolar induction: new experimental evidence on the influence of the test circuit. Sci Rep 12, 16791 (2022). Available at: https://doi.org/10.1038/s41598-022-21155-x.

¹⁹[BM22] and [Bau22].

Chapter 2 Unipolar Induction

Wilhelm Weber^{20,21,22,23,24}

There are two sources responsible for magnetic phenomena, namely *terrestrial magnetism* and bar magnetism²⁵ which are differentiated not because there is a specific difference in the magnetism itself, but because the circumstances under which they act are different and the questions to be answered are different. This distinctness especially shows in the known general magnetic laws (which were found through experiments with bar magnets) often being applied *directly* to the action of bar magnets and give a straight-forward explanation to phenomena stemming therefrom; but being rooted and applied to terrestrial magnetism only *indirectly* through the general theory of terrestrial magnetism, which contains the principles of explanation for all terrestrial-magnetic phenomena. The latter theory was *first* developed by Privy Councillor Gauss²⁶ in the previous Volume of the *Resultate*;²⁷ the theory of bar magnetism is older and, because it is essentially included in the general theory of magnetism, may in some respects be regarded as self-contained and completed for a long time, but this does not prevent the occurrence of individual problems which need to be solved, and through which even new light can be shed on the nature of magnetism. One of such tasks is the core of the present article. The phenomena to be regarded here are *induction phenomena* which consist in general of the excitation of galvanic currents through magnetism in motion. These induction phenomena are split into two categories, where the ones in the *first* category, which shall be titled *bipolar* induction, are sufficiently known and have been shown with

 $^{^{20}}$ [Web40].

²¹Translated by C. Baumgärtel, Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool, L69 3GJ, United Kingdom, ORCID: 0000-0002-0702-0480. Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis. We thank Laurence Hecht, Karin Reich, Klaus Reinsch and Daniel Steil for relevant suggestions.

²²The Notes by E. Riecke, the editor of the second Volume of Weber's *Werke*, are represented by [Note by ER:]; the Notes by C. Baumgärtel are represented by [Note by CB:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:]. The words between square brackets, [], in the middle of the text have been inserted by AKTA in order to clarify the meaning of some sentences.

²³[Note by ER:] See Table VIII, Figures 1-4 [of Weber's Werke].

²⁴[Note by ER:] Resultate aus den Beobachtungen des magnetischen Vereins, 1839, III, pp. 63-90.

²⁵[Note by CB:] It is likely Weber is implying permanent ferromagnetism here.

 $^{^{26}}$ [Note by AKTA:] In German: *Herr Hofrath*. The title by which Weber addressed Gauss can also be translated as Mr. Court Councillor.

²⁷[Note by AKTA:] [Gau39] with English translations in [Gau41] and [GT14].

both bar and terrestrial-magnetism; the ones of the *other* category, in contrast, which shall be titled *unipolar* induction, have been previously unknown and have only been shown to arise from bar magnetism. It is interesting to get to know a case where this is not possible, next to so many existing examples where mostly the same phenomena arise from bar magnetism as well as terrestrial-magnetism (e.g., nearly all electromagnetic and magnetoelectric phenomena). That the reason for this impossibility lies not within magnetism itself, but in external circumstances, (e.g., the Earth is not as good a conductor as the steel of a bar magnet, and not all of Earth's parts are magnetic, — regardless of the size of the Earth preventing experiments to be made) is easily anticipated and proved through testing. — Before we move on to the experiments themselves, which have led to the investigation of unipolar induction, some general remarks shall be made about the nature, the method and the laws of unipolar induction, since this helps the understanding of the experiments and shortens their description.

2.1 General Remarks

2.1.1 Bipolar and Unipolar Induction

The existence of two magnetic fluids is presupposed, one northern and one southern, which exist in the molecules of a magnet in equal amounts, but separate from each other. If such a magnet is set in motion, a galvanic current is induced in a neighbouring conductor following known laws.²⁸ This current must be so designed, that it can be decomposed into two currents, of which one is caused by the motion of the *northern* fluid and the other by the motion of the *southern* fluid. This induction of two currents through the motion of *both* magnetic fluids shall be called *bipolar* induction. But it is also conceivable a kind of induction whereby either only one kind of magnetic fluid moves and the induced current of the other fluid is always zero, or the other fluid induces alternating positive and negative currents whose sum is zero, so that the only remaining current is the one induced by the first fluid. This induction of a current caused by the motion of one magnetic fluid shall be called *unipolar* induction.

2.1.2 Method

Imagine a horizontal circular or annular conductor and move a body containing only northern fluid downwards along the vertical axis, thus a galvanic current will show in the loop,²⁹ the direction of which is opposite to the diurnal motion.³⁰ For uniform velocity [of the northern fluid,] the current increases from zero during the motion from infinite height to the horizontal plane, and decreases similarly back to zero for motion from the the horizontal plane to infinite depth. During this motion the magnitude of the current changes, but never the direction in the conductor. If eventually the body with the northern fluid is moved back from bottom to

²⁸[Note by AKTA:] Weber is referring here to Faraday's law of induction from 1831. See [Far32a] with German translation in [Far32b] and [Far89], and Portuguese translation in [Far11].

²⁹[Note by CB:] In German: *im Ringe*. This expression can be translated as "in the loop" or "in the ring". I decided to translate Weber's use of "Ring" with "loop" because I found "ring" too literal and not conveying the scientific/engineering meaning.

³⁰[Note by AKTA:] In German: dessen Richtung der täglichen Bewegung entgegengesetzt ist. That is, the current will flow counterclockwise when viewed from above.

top, but not in a straight line and rather in a circular motion whose centre is located in the conductor and staying infinitely far away from the same, so that the body does not excite the conductor, the first motion can begin anew and the same current induced in the conductor a second time. In this manner the body containing only northern fluid could continue the same induction arbitrarily, whereby the current's magnitude in the conductor changed, but never the direction. The same would be true for a body only containing southern fluid, but the current's direction would be opposite. In both cases the magnet's path can be shortened drastically, since on all paths where the magnet moves downwards through the conductor and upwards around the loop to return to the initial position, the induction is the same. The essential criterion for a continuous homogeneous induction with a magnet containing only one magnetic fluid is, that this magnet moves downwards through the conductor and upwards and downwards motion or around it for both motions, the direction of the induced current changes and the total effect is zero.

It is simple to apply these laws to the *second* case, where a magnet is responsible for the induction that contains *both* fluids in equal amounts, where both move with the magnet at the same time. The current induced by both fluids at the same time at every instant is the sum of the currents induced by each individual fluid in this instance, which leads to the conclusion that

1. if the magnet is moved back and forth from its original location and position, so that it passes through the loop either not at all (neither downwards nor upwards) or both times (downwards as well as upwards), the total effect is zero, as it vanishes in its parts;

2. if the magnet moves through the conductor only once during this motion (downwards or upwards), the total effect is also zero, because the southern fluid induces an equal but opposite current to the northern one.

It does not follow from this, however, that a continuous homogeneous induction, such as can be produced by a magnet that contains only *one* fluid, is impossible with a magnet containing *both* fluids, rather, there remains a *third* case to be considered, which is not yet included in the previous two, and is possible if *really* magnetic fluids *exist* and are really *spatially separated* from one another in the molecules of the magnet, that is

3. a magnetic molecule is moved in such a way that it does not pass through the loop completely or not at all, but half through it and half remains outside it, e. g. that the half containing *northern* fluid goes down through the loop, upwards around the outside, or vice versa; but the other half containing southern fluid always remains outside. The total effect is then *non*-zero, since one fluid (which moved through the conductor) has induced a current, which has not been nullified, since the other fluid (which did not move through the conductor) has not induced any or only an inhomogeneous current whose combined effect vanishes in total. However, since the conductor as well as the magnetic molecule are *solid* bodies, it appears that this third case is only possible if either one is broken up. Yet, a magnetic molecule cannot be broken in such a way that each part only contained *one* fluid. which would be necessary to move one single fluid through the unbroken conductor, thus the *conductor* has to be broken up, which is easily done: however, it must be noted that the galvanic circuit must not be interrupted while the loop is being broken. The conductor can be broken without interrupting the circuit, if the inseparable magnetic molecule is so designed that the galvanic current can flow right through between both fluids; since that molecule can *conductively connect* both parts of the loop while it is being broken.

It is easy to create a setup which fulfills the conditions of the third case. It is sufficient to

magnetize a steel cylinder in such a way that its magnetic axis coincides with its geometric one, and spin it around this axis. If both ends of a conducting wire then come into contact with the cylinder, one end to the rotational axis at A, Figure 1, the other to the periphery at B, the wire forms a conducting circuit ABCDA, which always remains closed during rotation of the cylinder.³¹



Let ns be a magnetic molecule in the cylinder, where the northern fluid is situated at end n and the southern at s. Let the molecule be so designed that a galvanic current can travel right through. If one imagines the conducting circuit ABCDA containing the point where the centre of the molecule is located, it is easy to see that the northern fluid n is moved downwards through the loop of the circuit and upwards around it with each rotation, if we assume that in the Figure ns is moving downwards during rotation and after half a rotation reaches n's',³² where it moves back upwards. In contrast, the southern fluid s always remains outside the circuit during rotation. These circumstances allow to speculate that a continuous homogeneous current will be excited, whose direction is indicated by the annotated arrow. This speculation is confirmed by experience, as the experiments to be shown later will prove.

After explaining the underlying idea of the experiments to be described, a few theorems shall be developed which have guided the design of individual experiments.

2.1.3 Laws

1. The induction along all paths from the point of contact on the cylindrical surface to the point of contact at the end of the rotational axis is uniform if the magnetic fluids are separated uniformly everywhere.

³¹[Note by AKTA:] In Figure 1 the magnetized cylinder is represented by the horizontal rectangle. Weber considered the rotation of the magnet around its horizontal axis of symmetry. An experiment like this one was first performed by Faraday in 1832, see Chapter 1.

³²[Note by AKTA:] In Figure 1 we should have at the lower portion of the magnet n's' instead of ns.

It is presupposed that all magnetic molecules in the rotating cylinder are equal in strength and equally spaced, as if the cylinder was split into small identical cubes, with magnetic molecules sitting at the ends. The molecules may then form parallel rows to the rotational axis. No matter which path the current takes, it needs to traverse every row of molecules from the surface to the axis, and the probabilistic number of magnetic molecules being cut by the current on its way is proportional to the number n of these rows; furthermore it is directly proportionate to the length l of these molecules and indirectly to their distance a, so that = nl/a. Since all molecules are assumed equal and equally spaced (i.e., l and a constant), it follows that the number of cuttings on all paths is expected to be equal. This statement holds true even for those paths which exceed the rotational axis and cut many more rows of molecules beyond, until eventually reaching the end of the axis; as it appears clear that such a path cuts each row beyond the axis twice, once departing from and once returning towards the axis, each time with equal probability to come across a magnetic molecule. The induction caused by the separation of a particle outbound is nullified on the return, so that with all probability the induction of the whole path is zero.

2. If the galvanic current travels on multiple ways from the surface of the cylinder to the axis at the same time, on all of which the induction is the same, the induction is just as strong as if the current took only one path.

It is known that when *multiple* equally strong galvanic piles are connected with their equal poles in a long circuit (so that all currents emanating from the piles combine immediately after the piles and flow through the long circuit and eventually split immediately before the piles to complete their cycle),³³ then the current in the circuit is just as strong as if the ends of the circuit only made contact with the poles of *one* pile, presupposing that the resistance in the piles is vanishingly small compared to the resistance in the circuit. Applying this theorem to our case, every path through the cylinder can be compared to a path through a pile, from which the present theorem follows, since the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the resistance in the cylinder is vanishingly small compared to the rest of the circuit. From this it follows,

3. The induction is independent of the number of points on the surface of the cylinder being contacted.

4. The induction is independent of the length of the cylinder, whose molecules are equally strong magnetic.

5. The induction is proportional to the cross section of the cylinder under otherwise similar circumstances.

6. When there are different paths through the cylinder, some of which where the induction is larger, some of which where it is smaller, the current will be as strong as if it traversed the latter path alone through the cylinder. This theorem stems from the comparison of our case with a circuit, which is split at the end and connected to multiple unequal piles. Because if such a current division occurs that some parts traverse weaker and some parts stronger piles, the current in the rest of the circuit will be just as strong as if there was no division and the current only traversed the weakest pile, presupposing the resistance of the piles vanishes in comparison to the resistance of the circuit. If one part was led only through a conductor, instead of through a pile, where too the resistance [of the conductor] disappears compared to the resistance of the entire circuit, the galvanic current would cease in the remaining undivided circuit. It is straightforward to apply this to our case. All induction would need to vanish if the surface of the cylinder was connected to the axis by a copper sleeve.

7. If the cylinder is equally strong magnetic in all parts, two rotations will induce a current

³³[Note by AKTA:] That is, the piles are connected in parallel.

which is equal to the current created by the same cylinder through a *single alternation* in an inductor $coil^{34}$ consisting of a single loop, presupposing that the diameter of the latter is very small compared to the length of the cylinder.

If M is the magnetic moment of the cylinder and L is its length, and if the magnetic fluids are spread across the end face of the cylinder, which is allowed under the previous condition that all particles of the cylinder are equally magnetic, then $\pm \frac{M}{L}$ is the amount of northern or southern fluid situated at one or the other end face. The induced current S by a *single alternation* is then equal to the current induced by *one* fluid $\pm \frac{M}{L}$ if it was moved twice along the same path in the same direction through the inductor loop (presupposing, that the diameter of the later is very small compared to the length of the cylinder), which allows to write

$$S = 2c \cdot \frac{M}{L} \; ,$$

where c is a constant only depending on the resistance of the circuit. If the inductor consists of multiple windings, c would need to be multiplied by the number of windings.

If the cylinder consists of equal and parallel molecules, each of which has a magnetic moment = m, a length = l and whose distance is = a, then the number of molecules is equal to the volume of the cylinder divided by the cubed distance a, or $= \frac{\pi R^2 L}{a^3}$, where R is the radius of the cylinder. The sum of all molecules' moments is equal to the moment M, or

$$\frac{\pi R^2 L}{a^3} \cdot m = M \; .$$

If at one end of each molecule there is $+\frac{m}{l}$ (northern) fluid, at the other end $-\frac{m}{l}$ (southern) fluid: this gives the amount of northern (or southern) fluid which traverses the loop of the circuit during each rotation, and induces a continuous homogeneous current, if $\pm\frac{m}{l}$ is multiplied by the number of molecule rows in the cylinder and by the ratio l/a (which measures the probability that the current cuts a molecule while traversing a molecule row). The amount of induction-causing fluid traversing the loop of the circuit during each rotation is then

$$= \frac{m}{l} \cdot \frac{\pi R^2}{a^2} \cdot \frac{l}{a} = \frac{\pi R^2 m}{a^3}$$

since the number of molecule rows in the cylinder is equal to the cross section πR^2 of the cylinder, divided by the square of the distance *a* between the molecules. According to this the current induced by every rotation is

$$s = c \cdot \frac{\pi R^2 m}{a^3}$$

where c has the same meaning as before. Comparing both currents, one finds

$$S = 2s$$
,

i.e., the current induced by two rotations of the cylinder is equal to the current induced by a *single alternation*, presupposing that the wire of the coil forms only a single loop.

8. If some parts of the cylinder are more strongly magnetized, some more weakly, the current induced by two rotations of the cylinder is weaker than the one by the *single* alternation, presupposing the wire of the coil forms only one loop which is very small compared to the length of the cylinder.

³⁴[Note by AKTA:] In German: *Induktorrolle*.

There is one path among the many which the galvanic current takes through the cylinder, which traverses the most weakly magnetized parts. According to (6.) the current induced through rotation of the cylinder is not stronger than as if the cylinder was only weakly magnetized in all of its parts. In contrast, the current induced by a *single alternation* is increased, even if the magnetism of the cylinder is not amplified in all, but only in individual parts, from which the previous theorem follows by itself.

2.2 Instruments

The instruments used to cause and observe unipolar induction consisted of the following parts: *firstly* two axially magnetized steel cylinders; *secondly* gears with which the cylinders could be rotated around their axes with a known rate; *thirdly* a magnetometer equipped with a multiplier to measure the induced currents; *fourthly* a device for conductively connecting one end of the multiplier wire to the end of the rotational axis, the other end to the cylindrical surface without impairing its rotation; *fifthly* a coil to perform induction experiments described in the previous Volume of the *Resultate*, p. 98 ff.^{35,36} with the same magnet.

2.2.1 The Cylindrical Magnets

Two hardened steel cylinders, one 269 mm long, 23 mm wide, the other 502 mm long, 20.5 mm wide, were fitted with a spike at one end (North end) and equipped with a nut at the other [end]. The latter was attached to a toothed wheel (40 teeth) whose shaft was pointed as depicted in Figure 2. The first steel cylinder was magnetized twice, first weakly, then strongly, so that its magnetic moment was first 65 and then 108 Million according to absolute measure.³⁷ The second cylinder was magnetized to 450 Million.



 $^{^{35}[\}mbox{Note by ER:}]$ Wilhelm Weber's Werke, Vol. II, p. 115.

³⁶[Note by AKTA:] [Web39b, p. 115 of Weber's Werke].

³⁷[Note by AKTA:] Gauss introduced the absolute measure of bar magnetism in 1832, see [Ass21b].

2.2.2 The Gears

The gear was the same as described in the second Volume of the *Resultate* (for 1837) in connection with the induction inclinatorium.^{38,39} Only an additional wheel with 60 teeth was added which was biting into the 40 tooth wheel fixed to the magnet. Each rotation of the crank equalled $8\frac{4}{7}$ of the steel cylinder. To connect the gears with the magnet a rack was used, Figure 3, which consisted of an iron clamp to which the gears were screwed, with a small notch at the end, in which the spike of the wheel fixed to the magnet was inserted, while the spike of the North end of the magnet fit into a similar notch in the second clamp. The clamp was necessary to bring the ends of two large magnets close to the rotating magnet from two opposite sides, as done in some of the tests. The clamps were held by the weight of these magnets. In absence of the magnets the clamps were screwed tight to the table hosting the apparatus.



2.2.3 Magnetometer and Multiplier

The magnetometer used for these tests is the same transportable magnetometer as described in the previous Volume of the *Resultate*.⁴⁰ This smaller magnetometer was preferable over a larger one, because the multiplier wire had a larger number of windings (2000) over a shorter length (roughly 600 metres). Due to the shorter length, the resistance was smaller, and with this the induced current stronger; due to the larger number of windings, the current's intensity was multiplied: both leading to a larger deflection of the magnetometer's needle. To further increase the amplification, a 25 pound magnet-rod was erected about 2 metres South of the magnetometer, with its South-pole facing North. The magnetism of this rod was largely in equilibrium with Earth's magnetic force inside the needle, increasing the

³⁸[Note by ER:] Ibid. p. 77.

³⁹[Note by AKTA:] [Web38, p. 77 of Weber's Works].

⁴⁰[Note by AKTA:] [Web39a] with English translations in [Web41a], [Web66a] and [Web21c].

sensitivity of the magnetometer, having the same effect as an astatic set-up.⁴¹ The needle's period of oscillation⁴² was previously 10 seconds and was increased to about 20 seconds through these means.

2.2.4 Connection of the Wire Ends with the Rotating Magnet

One end of the multiplier wire which was intended to be conductively connected to the rotational axis, was tied to the iron clamp on which the gears were screwed and into which ran the spike, which formed the end of the rotational axis. The other end of the multiplier wire was submerged in a tray of mercury which was placed underneath the rotating magnet. A brass disk was sitting around the centre of the magnet, rotating with it and its lower end running through the tray of mercury.⁴³ This way the magnet's rotation was not obstructed by being connected to both ends of the multiplier wire.

2.2.5 The Inductor Coil

A piece of the same sort of over-woven copper wire⁴⁴ as the multiplier was made of, was wound around a wooden loop of 44 mm diameter with 20 turns. This loop was used as an inductor coil. The resistance was small enough to be negligible compared to the larger resistance of the multiplier; therefore, the currents induced with the same magnet, sometimes by rotation, sometimes by changing this coil, directly measure the magnitude of the induction.

2.3 Experiments

The magnets used for the following tests were, like all magnets, not equally magnetized throughout all their parts, but instead stronger in the middle and weaker towards the ends. Thus, they were not fulfilling the requirements laid out in the previous conditions. Similarly, no magnet can be manufactured that exactly fulfils these requirements. If, therefore, in these experiments, one has to be content with rods which are often very far away from a very uniform magnetization, one cannot expect that the previously-mentioned theorems will be directly and accurately applied to these experiments, and that the strength of the induced currents can be correctly and accurately predetermined from them. The previous laws can and should only be used under these circumstances to get an idea of the intensity of induced currents, or at least estimate the magnitude that can be expected. Only a *limit*

⁴¹[Note by AKTA:] The adjective "astatic" is used in physics with the meaning of something having no tendency to take a definite position or direction. An astatic needle can be a combination of two parallel magnetized needles having equal magnetic moments, but with their poles turned opposite ways, that is, in antiparallel position. The arrangement protects the system from the influence of terrestrial magnetized needle had also been created by Ampère, [Amp21] and [LA98]. An earlier system composed of a single magnetized needle had also been created by Ampère, [Amp20c, p. 198] with Portuguese translation in [CA09, p. 133], [Amp20a, p. 239] and [Amp20b, p. 2], see also [AC15, p. 57].

⁴²[Note by AKTA:] In German: Schwingunsdauer. Gauss and Weber utilized the old French definition of the period of oscillation t which is half of the English definition of the period of oscillation T, that is, t = T/2, [Gil71, pp. 154 and 180]. For instance, the period of oscillation for small oscillations of a simple pendulum of length ℓ is $T = 2\pi\sqrt{\ell/g}$, where g is the local free fall acceleration due to the gravity of the Earth, while $t = T/2 = \pi\sqrt{\ell/g}$.

 $^{^{43}}$ [Note by AKTA:] See Figure 1.3.

⁴⁴[Note by AKTA:] In German: übersponnenen Kupferdrahtes.

of current intensity is given by these laws to which the induced currents come close but will never reach with an unevenly magnetized cylinder. Thus, the closest aim of the following tests is to check if a current can be generated by the described means at all, more so, if the current intensity is of the expected magnitude as the intensity of a current induced by described alternation, and finally if, as expected according to (8.), that current is exceeded by this one. If answers to those questions can be found as affirmative through the following tests, it shall further be tested to change the external circumstances of the *first* induction in such a way that the induced current is raised towards the given limit, and even surpass the other, previously stronger current. The reason why the induced current is not reaching the previously specified limit, namely that the magnet is weaker towards the ends compared to its *centre*, can be partially or fully alleviated by approaching larger magnets, which will increase the magnetism of the *ends*, while almost not influencing the magnetism in the *centre*. Supposing that magnetism in the *centre* remains completely unaltered by this, and now being the weakest throughout the entire rod instead of previously the strongest, there would result a current which can never be lower than the previously specified limit; transforming the upper bound into a lower bound. It is easy to see that this largely depends on the length and initial magnetism and the magnetic properties of the steel cylinder. With short cylinders the magnetism would not only be amplified at the ends, but also in the centre, increasingly so the weaker the magnet initially was. With long cylinders the magnetism in the centre will be barely affected or not at all at considerable distance from the ends to the centre. From this we can expect that, (1) rotating a short, weakly magnetized cylinder between two fixed magnet-rods to reinforce its ends, will induce a current which *surpasses* the previously specified limit, however, (2) will get the *closer*, the *stronger* the cylinder gets magnetized; (3) if the same cylinder is rotated freely, without the presence of other magnets, the induced current will not reach the specified limit, but get ever closer, the stronger the cylinder gets magnetized; yet even at the highest point of saturation it cannot be reached, even if stronger magnetisation will smooth the unevenness of magnetism of centre and ends, however, it cannot remove the unevenness. (4) Rotating a very long cylinder, even if it is strongly magnetized, it is to be expected that the induced current will never reach the previously specified limit, and can only weakly be alleviated by external magnets slightly reinforcing the rod's ends; since it is to be expected that the area of influence of the latter is not reaching a respectable distance away from the ends and will not suffice to amplify the magnetism of all parts of the magnet so that they would be equal to the centre of the magnet. To verify this, the following sets of experiments are performed.

2.3.1 First Set

Rotation of a short and weakly magnetized cylinder. Its ends were reinforced through the presence of external magnets.

The cylinder was 269 mm long and 23 mm thick, it's magnetic moment according to absolute measure = 65 Million.

60 revolutions in 7 seconds					
Rota	tion for	wards	Rotat	ion bac	kwards
616.3			743.0		
	623.3			736.0	
626.8			732.5		
	622.1			737.0	
619.7			739.2		
	621.9	622.56		736.3	736.54
623.0			734.8		
	623.1			737.5	
623.2			738.8		
	622.4			735.9	
622.0			734.5		
617.0			734.2		
	623.1			736.7	
626.2			738.0		
	622.5			737.0	
620.7			736.5		
	622.2	622.02		737.2	737.12
623.0			737.5		
	621.0			737.2	
620.0			737.0		
	621.3			737.5	
622.0			737.8		

The *first* column shows the observations of maximum and minimum values of magnetometer readings during rotation; the *second* column shows the calculated real value consisting of two observations including damping considerations: the second observation is approximated towards the first by one third of the difference; the *third* column is the mean of the 5 readings in the previous column. Compiling the values of the third column, the differences of the readings, alternating between forwards and backwards rotation, give double the deflection caused by the induced current.

forwards	622.56		
		113.98	
backwards	736.54		
		114.52	114.53
forwards	622.02		
		115.10	
backwards	737.12		

Through the same method, the double of the deflection of 30 rotations in 7 seconds is found as

$$= 56.52$$

which is nearly half of the previous. According to this we can assume on average 57.02 as the single deflection of 60 rotations or double deflection of 30 rotations in 7 seconds, giving

6.652 as single deflection of 1 rotation in 1 second, or 13.304 of 2 rotations in 1 second. For comparison this magnet was also used for the induction experiments described in the previous Volume of the Resultate, p. 98 ff.^{45,46} It should be noted that the period of oscillation of the magnetometer's needle was 20.5 s and the coil had 20 turns. The external magnets had to be removed for these tests. It will suffice to collate the observation of elongations, without specifying the alternation of set-ups described in the place cited.

Elongations	a	b
643.0		
637.0	8.2	
651.2		17.0
654.0	9.2	
642.0		16.0
638.0	9.0	
651.0		15.2
653.2	8.0	
643.0		15.7
637.5	7.8	
650.8		16.7
654.2	8.6	
642.2		15.2
Elongations	a	b
Elongations 639.0	a 8.5	b
Elongations 639.0 650.7	a 8.5 	<i>b</i> 16.5
Elongations 639.0 650.7 655.5	a 8.5 8.7	<i>b</i> 16.5
Elongations 639.0 650.7 655.5 642.0	a 8.5 8.7 	b 16.5 17.7
Elongations 639.0 650.7 655.5 642.0 637.8	a 8.5 8.7 8.5	<i>b</i> 16.5 17.7
Elongations 639.0 650.7 655.5 642.0 637.8 650.5	a 8.5 8.7 8.5 	<i>b</i> 16.5 17.7 16.7
Elongations 639.0 650.7 655.5 642.0 637.8 650.5 654.5	a 8.5 8.7 8.5 8.7	<i>b</i> 16.5 17.7 16.7
Elongations 639.0 650.7 655.5 642.0 637.8 650.5 654.5 641.8	a 8.5 8.7 8.5 8.7 	b 16.5 17.7 16.7 16.3
Elongations 639.0 650.7 655.5 642.0 637.8 650.5 654.5 641.8 638.2	a 8.5 8.7 8.5 8.7 8.2	b 16.5 17.7 16.7 16.3
Elongations 639.0 650.7 655.5 642.0 637.8 650.5 654.5 641.8 638.2 650.0	a 8.5 8.7 8.5 8.7 8.2 	b 16.5 17.7 16.7 16.3 15.3
Elongations 639.0 650.7 655.5 642.0 637.8 650.5 654.5 641.8 638.2 650.0 653.5	a 8.5 8.7 8.5 8.7 8.2 8.0	$ b \\ 16.5 \\ 17.7 \\ 16.7 \\ 16.3 \\ 15.3 $
Elongations 639.0 650.7 655.5 642.0 637.8 650.5 654.5 641.8 638.2 650.0 653.5 642.0	a 8.5 8.7 8.5 8.7 8.2 8.0 	b 16.5 17.7 16.7 16.3 15.3 15.5

On average this yields

$$a = 8.5$$

 $b = 16.15$
 $\frac{a^2 + b^2}{\sqrt{ab}} = 28.44$.

If the last value is multiplied by $\frac{t}{\pi n}$, where t is the oscillation period of the magnetometer's needle (= 20.5 s), n the number of windings in the coil (= 20), the single deflection is found

⁴⁵[Note by ER:] Wilhelm Weber's Werke, Vol. II, p. 115
⁴⁶[Note by AKTA:] [Web39b, p. 115 of Weber's Werke].

which corresponds to 1 winding and 1 alternation in 1 second, = 9.279. If we compare the deflection obtained previously for 2 rotations in 2 seconds = 13.304; it is apparent that the induced current responsible for the deflection is stronger than the one causing the first deflection according to expectation (see the previous point (1)).

2.3.2 Second Set

Rotation of a short, strongly magnetized cylinder.

Its ends were reinforced through the presence of extra magnets.

The cylinder was 269 mm long, 23 mm thick; its magnetic moment according to absolute measure = 108 Million. Because the experiments were carried out just like the previous ones, it suffices to present the results. The double deflection of 60 rotations in 7 seconds was found to be

$$= 152.50$$
,

and of 30 rotations in 7 seconds

= 76.61 .

On average 76.37 can be assumed as single deflection of 60 rotations or double deflection of 30 rotations in 7 seconds, giving 8.91 as single deflection of 1 rotation in 1 second, or 17.82 for 2 rotations in 1 second.

For comparison the tests with the 20 turn coil were repeated, where the oscillation period of the needle was t = 21.44 s. Which yields

$$a = 14.22$$
$$b = 26.94$$
$$\frac{a^2 + b^2}{\sqrt{ab}} = 47.412$$

If the last value is divided by $\frac{n}{t}\pi = \frac{20}{21.44} \cdot 3.14159...$, the deflection corresponding to 1 rotation and 1 alternation in 1 second is found as

$$= 16.178$$
.

Comparing this to the deflection found previously for 2 rotations in 1 second

$$= 17.82$$
,

it is apparent, that the indued current causing the latter deflection is barely any stronger than the one causing the first deflection, in accordance with expectation (see the previous point (2)).

2.3.3 Third Set

Rotation of a short, strongly magnetized cylinder without the presence of extra magnets.

The cylinder itself was unchanged from the second set.

The double deflection for 60 rotations in 7 seconds was found to be

= 64.33,

for 30 rotations in 7 seconds

= 31.83 .

On average 32.05 can be found as the single deflection for 60 rotations or double deflection for 30 rotations in 7 seconds, giving 3.74 as single deflection for 1 rotation in 1 second, or 7.48 for 2 rotations in 1 second.

If one compares this result with the deflection, which according to the previous series for the same magnet corresponded to 1 turn of the inductor coil and 1 change in 1 second,

= 16.178,

it can be seen that the induced current causing that deflection = 7.48 is in accordance with expectations weaker than the one causing this deflection = 16.178 (see the previous point (3)), however, it is still of the same magnitude, so that according to No. 8 it seems justified to deduce the difference in reading from the considerable unlikeness of magnetism between such a rod's centre and ends.

2.3.4 Fourth Set

Rotation of a long, strongly magnetized cylinder.

Its ends are reinforced through the presence of external magnets.

The cylinder was 502 mm long and 20.5 mm thick, its magnetic moment according to absolute measure = 450 million. The double deflection for 60 rotations in 7 seconds was found to be

$$= 194.22$$
,

for 30 rotations in 7 seconds

= 97.85 .

On average 97.36 can be found as a single deflection for 60 rotations or the double deflection for 30 rotations in 7 seconds, giving 11.36 as single deflection for 1 rotation in 1 second, or 22.72 for 2 rotations in 1 second.

For comparison the induction tests with the 20 turn coil were repeated as well with this magnet. The oscillation period of the needle was t = 22.34 s. Which yields

$$a = 28.76$$

$$b = 57.69$$

$$\frac{a^2 + b^2}{\sqrt{ab}} = 102.01$$
.

If the last value is divided by $\frac{n}{t}\pi = \frac{20}{23.34} \cdot 3.14159...$, the deflection corresponding to 1 rotation and 1 alternation in 1 second is found as

$$= 36.27$$
 .

Comparing this to the deflection found previously for 2 rotations of the cylinder in 1 second

= 22.72,

it is apparent that the induced current causing the latter deflection with this long cylinder even in the presence of amplification of its outermost ends does not equal the current induced through the first method, causing a deflection = 36.27, as was postulated (see the previous point (4)).

2.3.5 Fifth Set

In the experiments described so far, the brass disk running through mercury was always situated on the *centre* of the magnet; however, in the following experiments it was moved to the *end* of the cylinder to confirm that the length of the parallel path which the induced current has to travel inside the magnet of the rotational axis has no influence on the current intensity. The current was first conducted at the end of the rotation axis that was further away from the brass disk and then at the end of the rotation axis closest to the brass disk.

Cylinder and extra magnets remained unchanged from the previous set.

1. Contacting the far end of the rotational axis.

Double deflection for 30 rotations in 7 seconds was found to be

= 57.12 .

2. Contacting the *close* end of the rotational axis.

Double deflection for 30 rotations in 7 seconds was found to be

$$= 59.08$$
 .

Comparing these results it is self-evident that the induced current through the longer path it had to travel parallel to the rotational axis in the cylinder was at least *not* amplified. The difference between these results is too small to justify the conclusion of the opposite assertion.

2.3.6 Sixth Set

Rotation of a long, strongly magnetized cylinder without external magnets.

The cylinder is the same as in the previous two sets; the brass disk running through mercury was situated in the centre of the magnet. The double deflection for 30 rotations in 7 seconds was found as

= 61.70,

yielding 7.20 as single deflection for 1 rotation in 1 second, or 14.40 for 2 rotations in 1 second.

Comparing this result with the deflection, which according to the fourth set for the same magnet corresponded to 1 turn of the inductor coil and 1 alternation in 1 second,

one can see that the induced current, which corresponds to this deflection = 14.40, is much weaker than the one causing this deflection = 36.21, as was also assumed under the prevailing circumstances (see the previous point (4)).

2.3.7 Seventh Set

The experiments of the previous set were repeated by moving the brass disk running through mercury to the end of the cylinder, to confirm the result of the fifth set where no external magnets were used.

1. Contacting the *far* end of the rotational axis.

Double deflection for 30 rotations in 7 seconds was found as

= 20.44 .

2. Contacting the *close* end of the rotational axis.

Double deflection for 30 rotations in 7 seconds was found as

$$= 21.66$$

Comparing these results again shows that the induced current through the longer path travelling parallel to the rotational axis in the first case was *not* amplified.

2.4 Applications

2.4.1 Application to Ampère's Electrodynamic Theory of Magnetic Phenomena

The phenomena of unipolar induction will find close application to Ampère's electrodynamic theory of magnetic phenomena,⁴⁷ or to the question if any physical existence must be ascribed to the two magnetic fluids, or if instead of the fluids the assumption of continuous galvanic currents inside of the magnet is sufficient. To explain unipolar induction the latter assumption does not seem to suffice, while the assumption of the physical existence of two magnetic fluids alone does not seem to give an explanation, still it has led to the investigation of these phenomena.

If one wanted to derive an explanation for phenomena titled with the name of unipolar induction from Ampère's electrodynamic theory of magnetic phenomena, the attempt would fail since according to Ampère galvanic currents can only be resolved into such elements which attract or repel along the straight line joining those elements. It is evident from this that a current element in the plane of a loop cannot be moved vertically by a current in the loop, and conversely, that such a movement of the current element cannot induce a current in the loop. The vital question of unipolar induction seems to consist in the fact that an induction is happening in the moment where the inducing element is present in the plane

⁴⁷[Note by AKTA:] André-Marie Ampère (1775-1836). His masterpiece was published in 1826, [Amp26] and [Amp23]. There is a complete Portuguese translation of this work, [Cha09] and [AC11]. Partial English translations can be found at [Amp65] and [Amp69b]. Complete and commented English translations can be found in [Amp12] and [AC15].

A huge material on Ampère and his force law between current elements can be found in the homepage Ampère et l'Histoire de l'Électricité, [Blo05].

of the loop, because, if in this moment the induction is zero, a transition from positive to negative or vice versa is occurring. The characteristic trait of unipolar induction, however, is rooted in the fact that such a transition never occurs. Thus, it seems futile to search an explanation for unipolar induction in Ampère's electrodynamic theory, at least so long as the decomposition of galvanic currents is limited to such elements that attract or repel along the straight line joining them.

The fruitlessness of this endeavour is made increasingly visible, if the beautiful theorem first proved by Ampère and stated in the previous Volume of the Resultate, p. 51,^{48,49} is considered, with which the magnetic effects of galvanic currents can be defined. With this theorem — that instead of any linear current confining an *arbitrary* surface, a distribution of the magnetic fluids on both sides of that surface in immeasurably small distances from it can be substituted with the intended effect, — it should *first* be regarded that when a linear closed current is given, infinitely many confined surfaces can be thought of; secondly that the current's effects can only be valid for what is also valid of the effects of all magnetic fluids distributed on all those surfaces together: in other words, from this substitution it is not allowed to conclude anything from one surface which cannot also be concluded from each of the other surfaces. Now, imagine the plane of a small circular conductor, through which a continuous galvanic current is running, which is replaced by a magnetic element according to Ampère's hypothesis, perpendicular to the plane, and let AB, Figure 4, be the diameter of the circle; imagining to both sides at immeasurably small distances from the circular plane in AaB and Aa'B the distribution of the northern and southern fluid;⁵⁰ the conductor can be moved in such a way that the magnetism at a is moving through the inductive loop, while a' always stays outside. It is easy to see, however, that this draws a conclusion from the distribution of magnetic fluids in the circular plane, which would not have been concluded from a distribution in any other surface confined by the same circle, which is not allowed to be valid for the effects of the galvanic currents in that circle.

 $^{^{48}[\}mathrm{Note}$ by ER:] Gauss' Werke, Vol. V, p. 169.

 $^{^{49}}$ See footnote 27 on page 13, in particular, page 51 of the *Resultate* and page 169 of Vol. V of Gauss' *Werke*.

⁵⁰[Note by AKTA:] In Figure 4 one of the two *a*'s should be replaced by a'.



2.4.2 Application to the Distribution of Magnetism of Constant Magnets

All effects of magnets which are usually observed are effects in external space, from which, as is well known, no definite result can be drawn about the distribution of magnetism in the interior. There are rather countless ways in which internal distributions of magnetism can be assigned, which are all identical in relation to the effects. Among those is one, where no magnetism at all is situated inside the magnet, but all along the surface. Only *one* experiment exists which eluded to the inside distribution and especially showed that the last mentioned kind of distribution, that is, along the surface, does not occur in nature, that is the experiment in which a magnet is *broken apart*.

But we have now gotten to know effects of a magnet through *unipolar induction* which act on the fluids on its interior, being set into flowing motion. It appears as an application from unipolar induction that the inner distribution of magnetism can be investigated without breaking the magnet. While it is impossible to know the distribution fully through this method, it is still very important to obtain only a few new insights with it.

From the point where the conducting wire is touching the cylindrical surface of the magnet to the end of the rotational axis in contact with the conducting wire, a path exists inside the magnet for galvanic current where induction is the *weakest*. If the cylinder is rotated, that path is changing in general and will describe a curved area over a whole rotation bisecting the cylinder into two parts like a cross section. The free magnetism within this area has a ratio to the average of free magnetism within an arbitrary cross section in the first investigated cylinder, according to the *third* set of 7.48:16.178, in the second cylinder according to the *sixth* set of 14.40:36.27. The induced current (which for the *shorter* magnet caused a deflection of = 16.178 scale divisions if repeated every second, for a *longer* magnet = 36.27) by *alternation* of an inductor (consisting of 1 winding) gives a measure of the average amount of free magnetism of all cross sections of the cylinder, while the current induced by 2 *rotations* of the cylinder (for the *shorter* magnet with rotations every 2 seconds caused a deflection of = 7.48 scale divisions and for the *longer* = 14.40) gives according to law (6.), p. 158,⁵¹ a measure of the minimum of free magnetism which is contained in those curved cross sectional areas which can be described by the paths of the galvanic current in the rotating cylinder.

Looking at the results of the *fifth* or *seventh* set, where the current was diverted from the surface of the cylinder not in the middle, as in the other series of tests, but at the end, it is found (as pointed out previously) that the result is almost the same, for the galvanic current having to cross the entire length of the cylinder to reach from its point of entry to its point of exit, as well as not having to cross the length of the cylinder — i.e., in other words, the two minima of free magnetism contained in those cross sections which can be described by the different paths during the cylinder's rotation, taken by the galvanic current from the contacted point on the surface either towards the *close* or the *far* end of the rotational axis are nearly the same, which leads to the conclusion that the galvanic current is only travelling to the rotational axis during its *entrance* and *exit* (which is here the nearly identically strong magnetized ends of the cylinder).

Comparing the results of the *fifth* and *seventh* set and considering that in the former magnetism in the ends (where the galvanic current crosses) was strongly reinforced through the use of external magnets, but not in the latter, the difference which one finds will not be noticeable, namely, that the measured deflection in the first case is nearly three times larger than the latter, or expressed by the ratio 58.10:21.05. It is interesting, however, to note that the former result, i.e., 58.10, is close, but not identical to the one obtained in the sixth set, i.e. 67.10, disregarding the amplification of the cylinder's ends (where the galvanic current crossed) through the use of external magnets, — a proof that this amplification is far from making the magnetism of those ends equal to the magnetism of the *centre* from which the galvanic current was derived in the sixth series of experiments.

Further consideration of this application will need to be kept to a time in the future.

2.4.3 Application to the Distribution of Magnetism in Soft Iron

A special difficulty so far was the investigation of the distribution of magnetism in soft iron. The iron will only form a stronger magnetism if it touches a magnet or is brought at least into very close proximity, where we lack the means to distinguish the effects stemming directly from the iron and stemming from the magnet, all the more so because the latter cannot be regarded as constant because the magnet undergoes a change due to the reaction of the iron. However, unipolar induction now presents such a procedure. When the magnet is stationary and only the iron is rotated, an induction is caused solely by the magnet. Finally, when both are rotated together, it is possible to determine the magnetism of that cross section within the iron where magnetism is weakest (at the end facing away from the magnet).

2.5 Conclusion

It is known that nearly all magneto-electric experiments have electromagnetic counterparts. Thus it can be postulated that for this type of test, which was first performed by Faraday,⁵² such a counterpart exists as well. This is indeed the case. This counterpart will not need to

⁵¹[Note by AKTA:] Page 158 of Weber's Werke, equivalent to page 17 of this English translation.

 $^{^{52}}$ [Note by AKTA:] See footnote 4 on page 5, and Section 1.2.

be performed, since it has already been performed and it has been known for a long time. This counterpart obviously pertains, instead of rotating the magnetic cylinder and inducing a galvanic current in the circuit, driving a galvanic current through the circuit in the opposite direction which causes the magnet to rotate in the same direction as it was previously rotated.⁵³ If one had researched this long-known phenomenon in more detail, this path would have easily led to the herein investigated *unipolar induction*, which to my knowledge has not been done. This long-known experiment seems also to contradict Ampère's hypothesis, that no magnetic fluids but continuous galvanic currents exist inside the magnet; moreover this phenomenon also seems to only be explained by the real existence of two spatially separate magnetic fluids.

W.⁵⁴

 $^{^{53}}$ [Note by AKTA:] This experiment was first performed by Ampère in 1822 and will be discussed in Chapter 3.

⁵⁴[Note by AKTA:] That is, written by Wilhelm Weber.

Chapter 3

Editor's Introduction to Weber's Second Paper on Unipolar Induction

A. K. T. Assis⁵⁵

In his second paper on unipolar induction Weber made some additions and amendments related to his previous publication.⁵⁶

In 1821 Faraday showed that it was possible to rotate the extremity of a straight piece of wire carrying a constant current around the vertical axis of a cylindrical magnet, while the other extremity of the piece of wire was located around the axis of the magnet. He also showed that it was possible to rotate the extremity of a cylindrical magnet around a vertical piece of wire carrying a constant current, while the other extremity of the magnet was located around the axis coinciding with the vertical piece of wire, Figure 3.1.⁵⁷ This phenomenon is usually known as Faraday's motor and should not be confused with Faraday's unipolar induction experiment of 1832.

In 1822 Ampère showed experimentally and theoretically that it was possible to make a cylindrical magnet turn around its axis. To this end he connected a battery between the center of the upper face of the magnet and a point in its edge utilizing conducting contacts. When an electric current was made to flow through the magnet, it rotated around its axis. This is known as Ampère's motor. Alternative names for this device are homopolar motor or the world's simplest motor.⁵⁸ Nowadays it can be easily reproduced utilizing a neodymium's magnet, a nail, a piece of wire and an ordinary battery, Figure 3.2.

In this paper of 1841 Weber compares Ampère's motor with Faraday's unipolar induction experiment. In particular, he considers Faraday's experiment of unipolar induction in which a cylindrical magnet was rotated relative to the ground and this produced an electric current

⁵⁵Homepage: www.ifi.unicamp.br/~assis

⁵⁶[Web40] and [Web41b], both of which are translated here.

⁵⁷[Far22b] and [Far52b] with French translation in [Far21], [Far22a] and [Far52a].

 $^{^{58}}$ [Amp22a] with partial English translation in [Amp69a], [Amp22b] and [Amp22b]. See also [Blo82, pp. 114-115], [GVM01], [GVMA02], [GV02], [AGV07], [Cha09], [AC11], [AC12], [CA13], [AC15, Section 7.2.3: Rotation of a Magnet around Its Axis].



Figure 3.1: Faraday's motor.

indicated by the galvanometer connected by sliding contacts between the center of the upper face of the magnet and a point in its edge.

Most portions of Weber's 1841 paper are identical with his earlier paper published in 1840. When Weber's second paper was reprinted in his Collected Works, the identical parts were not included.



Figure 3.2: Ampère's motor.

Chapter 4 Unipolar Induction

Wilhelm Weber^{59,60,61,62}

 $[Excerpts.]^{63,64}$

In a "Note about the interaction of a magnet and a galvanic conductor" contained in Volume 37 of Annales de chimie et de physique,⁶⁵ Ampère gave an account of the rotation of a galvanic conductor around the axis of a magnet as discovered by Faraday, and has tried to connect to this the rotation he discovered of a magnet rotating around its own axis when a galvanic current flows through it.⁶⁶

Ampère first shows that according to the general laws he established, a closed current, which *is not in a fixed connection* with the magnet, cannot rotate it around its axis, nor vice versa (if all the parts of the current conductor are firmly connected to each other) can be rotated by the influence of the magnet.⁶⁷ However, if a portion of the closed current lies within the magnet, he says this part will form a *fixed system* with the magnet, and action and reaction would have to cancel out. The remaining action is only the portion of the

⁵⁹[Web41b].

⁶⁰Translated by C. Baumgärtel, Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool, L69 3GJ, United Kingdom, ORCID: 0000-0002-0702-0480; and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

⁶¹The Notes by Johann Christian Poggendorff, the editor of the Annalen der Physik und Chemie, are represented by [Note by JCP:]; The Notes by E. Riecke, the editor of the second Volume of Weber's Werke, are represented by [Note by ER:]; the Notes by C. Baumgärtel are represented by [Note by CB:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:]. The words between square brackets, [], in the middle of the text have been inserted by AKTA in order to clarify the meaning of some sentences.

⁶²[Note by JCP:] From the "Resultate des magnetischen Vereins" (Volume 4), with some additions and amendments by the author [that is, by W. Weber].

⁶³[Note by ER:] Annalen der Physik und Chemie, Vol. 52, p. 353-386.

⁶⁴[Note by AKTA:] [Web41b].

 $^{^{65}}$ [Note by AKTA:] [Amp28].

⁶⁶[Note by AKTA:] In 1828 Ampère was comparing his own motor of 1822 described by Figure 3.2 with Faraday's motor of 1821 shown in Figure 3.1. Ampère was obviously not discussing Faraday's 1832 experiment of unipolar induction described by Figure 1.1.

⁶⁷[Note by AKTA:] That is, under these conditions, the closed circuit cannot rotate the magnet around the axis of the magnet. Likewise, under these conditions, the magnet can also not rotate the closed circuit around the axis of the closed circuit.

galvanic current which is not fixed to the magnet, and because that current is *not closed*, it will, in general, cause the magnet to rotate around its axis. He notes that it does not matter if the galvanic current flows *through the magnet* or if a portion of the conductor is rigidly connected with the magnet.

These two phenomena which Ampère tries to explain through the *same* reasons, are however, different in nature and each beg their own explanation. The explanation given by him only fits the rotation discovered by Faraday, but is not applicable to the one discovered by himself.

The difference between both phenomena is easily explained with the following.

Let ABCD, Figure 1, be the long cross section of a magnet, abc an insulated galvanic conductor guided through the magnet, the continuation of which outside the magnet is denoted by the dotted line.



In reality, this Figure depicts, as presupposed by Ampère, in the first place, the entire galvanic conductor as a closed curve, in the second place, the portion abc as a fixed system with the magnet. These two conditions which are fulfilled in Figure 1 are no longer valid, when the magnet is rotated around its axis. The insulated conductor abc will remain fixed within the magnet, but its endpoint c will describe a circle around the rotational axis and therefore be separated from the remaining circuit, as shown in Figure 2, at least if the magnet is not surrounded with a conducting belt, which the end c is constantly touching during its rotation. When such a connection is in place, there will be, additionally to the two conductor parts distinguished previously, a third part, that is, the piece of the belt which enables the connection between c and the dotted conductor, which is substantially different from the other two [parts] due to its changing position. This conduction belt was utilised by Faraday through the use of mercury.



With Ampère's experiment, however, where no insulated conductor abc is present in the magnet, but instead the current is moving *freely* through the magnet from a to c, the condition which Ampère presupposes is not applicable, that the portion of the current moving through the magnet is acting exactly similar during rotation of the magnet as a current would, flowing through the insulated conductor *abc*, i.e. forming a *fixed system* with the magnet, and consequently would have to partake in the rotation, but cannot cause it. Magnetism and galvanic current, even if they are present in the same carrier (the steel cylinder) do not form a *fixed system*; since only the magnetism adheres to the steel molecules, and can only move in conjunction with them. The galvanic current, however, does not adhere to the steel molecules, but rather can move freely and independently inside the magnet in all directions. It is apparent from this, that current and magnetism are not allowed to be regarded as rigidly connected bodies rotating with the steel cylinder whose interaction would have to nullify each other, contrary to Ampère's claim. If however, the magnetism is bound to the steel molecules, but not the traversing galvanic current, it follows that the current could move the steel cylinder through magnetism, but not vice versa. This however, removes the reason attributed by Ampère to explain why a magnet rotates when a galvanic current *freely* flows through it, and this phenomenon, which exists without a doubt due to experimental proof, would according to Ampère even seem *impossible*, which is however, not the case if the *physical existence of magnetic fluids* is allowed to be presupposed in the steel molecules, instead of Ampère's hypothetical currents, as is to be demonstrated by the following experiments. The phenomena investigated herein are the *induction phenomena* discovered by Faraday, which form the analogue of the previous electromagnetic phenomena, and which can easily be applied to the latter.⁶⁸

⁶⁸[Note by AKTA:] The paper in the Annalen der Physik und Chemie continues here just as in the original publication in the Resultate aus den Beobachtungen des magnetischen Vereins. However, when this paper of the Annalen der Physik und Chemie was reprinted in Volume 2 of Weber's Werke, the common portion of both papers were not reproduced here, as it had already appeared in the previous reprint of the paper published in the Resultate aus den Beobachtungen des magnetischen Vereins. We are here following this approach of Weber's Werke. The next 3 paragraphs should be compared with the corresponding paragraphs of page 15.

It does not follow from this, however, that a continuous homogeneous induction, such as can be produced by a magnet that contains only *one* fluid, is impossible with a magnet containing *both* fluids, rather, there remains a *third* case to be considered, which is not yet included in the previous two, and is possible if *really* magnetic fluids *exist* and are really *spatially separated* from one another in the molecules of the magnet, that is

3. a magnetic molecule is moved in such a way that it does not pass through the loop completely or not at all, but *half* through it and *half* remains outside it, e. g. that the half containing *northern* fluid goes down through the loop, upwards around the outside, or vice versa; but the other half containing southern fluid always remains outside. The total effect is then *non*-zero, since one fluid (which moved through the conductor) has induced a current, which has not been nullified, since the other fluid (which did not move through the conductor) has not induced any or only an inhomogeneous current whose combined effect vanishes in total. However, since the conductor as well as the magnetic molecule are *solid* bodies, it appears that this third case is only possible if either one is broken up. Yet, a magnetic molecule cannot be broken in such a way that each part only contained one fluid, which would be necessary to move one single fluid through the unbroken conductor, thus the *conductor* has to be broken up, which is easily done: however, it must be noted that while breaking the conductor the galvanic current must not be broken. The conductor can (1) be broken without interrupting the circuit, if the inseparable magnetic molecule is so designed that the galvanic current can flow right through between both fluids; since that molecule can conductively connect both parts of the conductor while it is being broken; (2) the loop can be broken without breaking the circuit if the copper wire forming the loop is wound once around the centre of the magnet before the break, and, after the break, behind the magnet at c, the connection is established at a, Figure 3. Through this connection in a the cut wire ends abc and ab'c can be passed through a and finally their endpoints can be reconnected.



Of theses two methods, the first is to be investigated further here. It is easy to devise a set-up that meets the required conditions.

To conclude this paper some words of explanation about the phenomenon first discovered by Ampère, which was discussed in the introduction, are offered.⁶⁹ It is known that

⁶⁹[Note by AKTA:] This final paragraph of the paper in the Annalen der Physik und Chemie should be compared with the final paragraph of the paper in the Resultate aus den Beobachtungen des magnetischen Vereins, see Section 2.5.

nearly all magneto-electric experiments have electromagnetic counterparts. The herein investigated phenomenon of unipolar induction belongs to the class of magneto-electric experiments, whereas the phenomenon discovered by Ampère as discussed in the introduction belongs to the class of electromagnetic experiments, and further investigation yields that this phenomenon⁷⁰ has to be seen as the counterpart of that,⁷¹ and therefore is to be explained by the same means, so that the counterpart can be demonstrated as a proof of the physical separation of the magnetic fluids. After the well-known reversal, according to which the induction laws are derived from the electro-magnetic ones, the above considerations can easily be applied to this counter-experiment, and it seems superfluous to elaborate further here.

⁷⁰[Note by CB:] Ampère's phenomenon.

⁷¹[Note by CB:] Unipolar induction phenomenon.

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