Weber's Electrodynamics

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Maxwell's equations (1861-1873):

Gauss's law:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

There are no magnetic monopoles:

$$\nabla \cdot \vec{B} = 0$$

Faraday's law:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

"Ampère's" circuital law with displacement current:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Lorentz's force (1895):

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Wilhelm Weber (1804 – 1891)

J. C. Maxwell (1831 – 1879)

Professor of physics at Göttingen University working in collaboration with Gauss

Coulomb (1785): $$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$$

Ampère (1822): $$\vec{F} = -\frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{r}}{r^2} f(\alpha, \beta, \gamma)$$

Faraday (1831): $$emf = -M \frac{dI}{dt}$$

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Weber’s hypothesis: $$I d\vec{\ell} \Leftrightarrow q\vec{v}$$

Weber’s force (1846): $$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \left[1 + K_1 v_1 v_2 + K_2 (a_1 - a_2)\right]$$

Weber's force (1846):

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2}\right)$$

$$\dot{r} = \frac{dr}{dt}, \qquad \ddot{r} = \frac{d^2 r}{dt^2}, \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Weber measured *c* in 1856: ***c*** = 3 x 10^8 m/s.

Therefore, connection between electromagnetism and optics before Maxwell!

Properties of Weber's force

- In the static case ($dr/dt = 0$ and $d^2r/dt^2 = 0$) we return to the laws of Coulomb and Gauss.
- Action and reaction: Conservation of linear momentum.
- Force along the straight line connecting the particles: Conservation of angular momentum.
- It can be derived from a velocity dependent potential energy:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r} \left(1 - \frac{\dot{r}^2}{2c^2} \right)$$

- Conservation of energy: $$\frac{d(T+U)}{dt} = 0$$

- Faraday’s law of induction can be deduced from Weber’s force (see Maxwell, *Treatise*, Vol. 2, Chap. 23).
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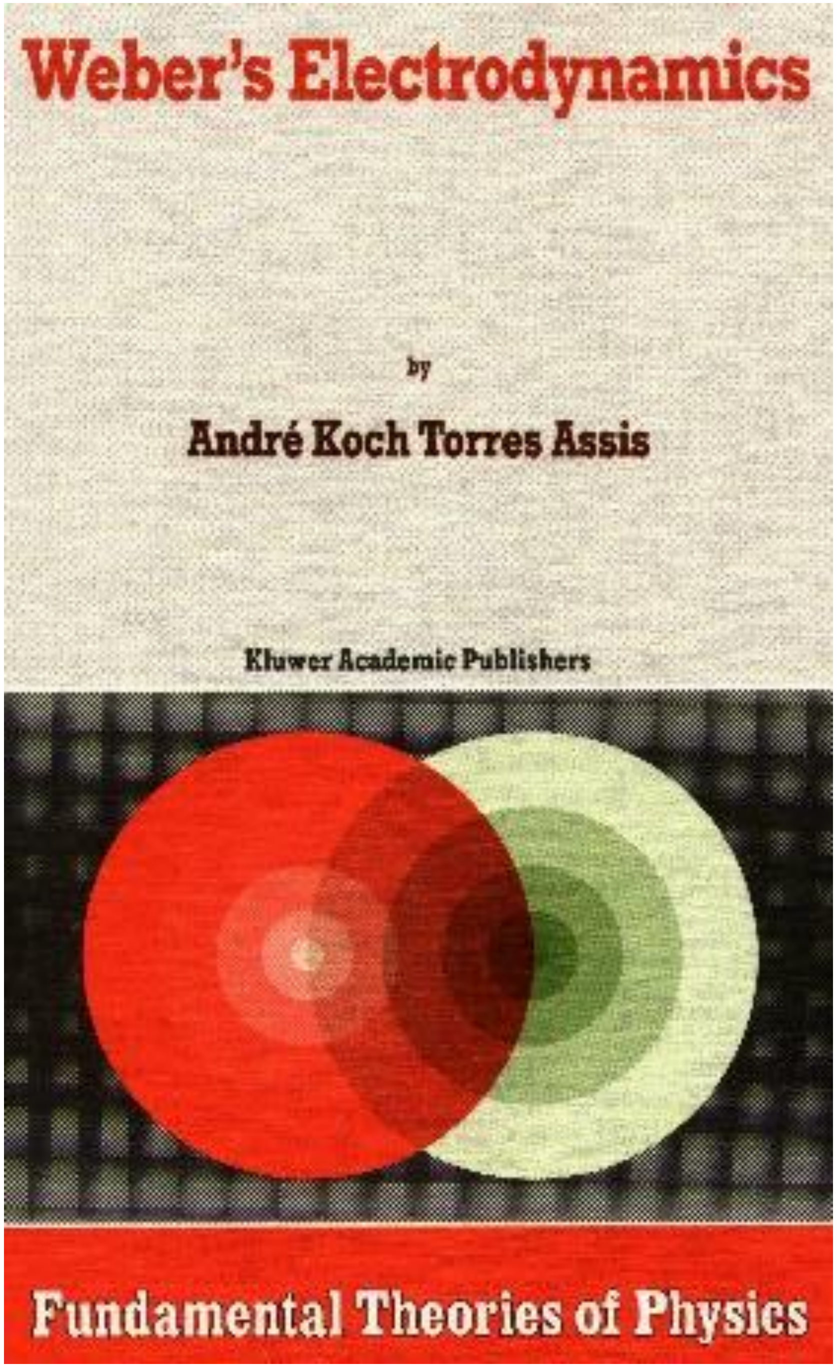
- “Ampère’s” circuital law can be deduced from Weber’s force.
- Weber’s force is completely **relational**. It depends only on r, dr/dt and d^2r/dt^2. It has the same value for all observers and in all systems of reference. It depends only on magnitudes intrinsic to the system of interacting charges. It depends only on the relation between the bodies.



1994
Kluwer
(Springer)

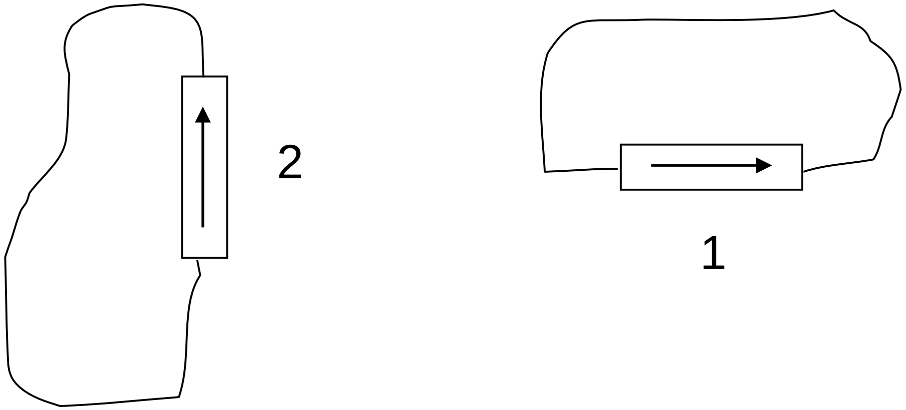
Weber $\rightarrow$ Ampère's force (1822-1826):

$$\vec{F} = -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[2(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - 3(\hat{r}\cdot d\vec{\ell}_1)(\hat{r}\cdot d\vec{\ell}_2)\hat{r}\right]$$

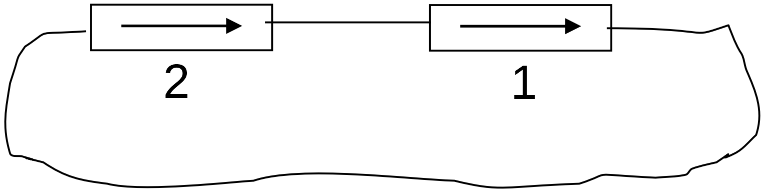
Lorentz $\rightarrow$ Biot-Savart and Grassmann's force (1845):

$$\vec{F} = I d\vec{\ell}_1 \times d\vec{B}_2 = I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi}\frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2}\right)$$

$$= -\frac{\mu_0}{4\pi}\frac{I_1 I_2}{r^2}\left[(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r} - (d\vec{\ell}_1 \cdot \hat{r})d\vec{\ell}_2\right]$$



	$F_{2\text{ in }1}$	$F_{1\text{ in }2}$
A	0	0
G	↑	0



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A	→	←
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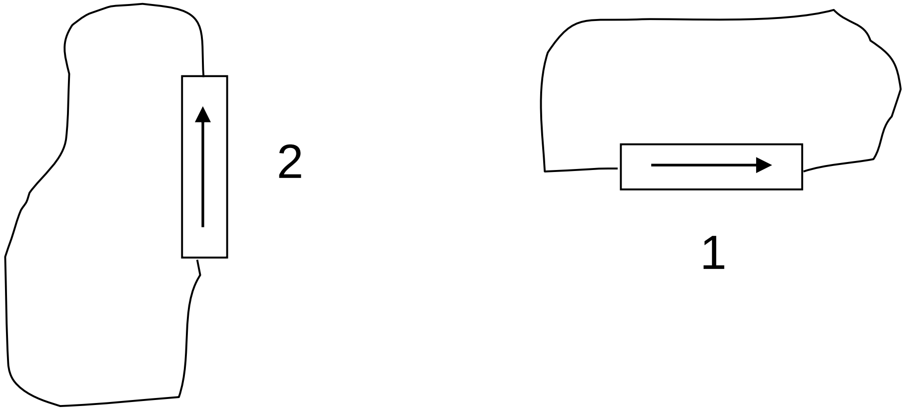

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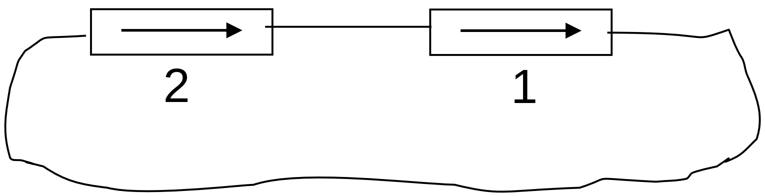
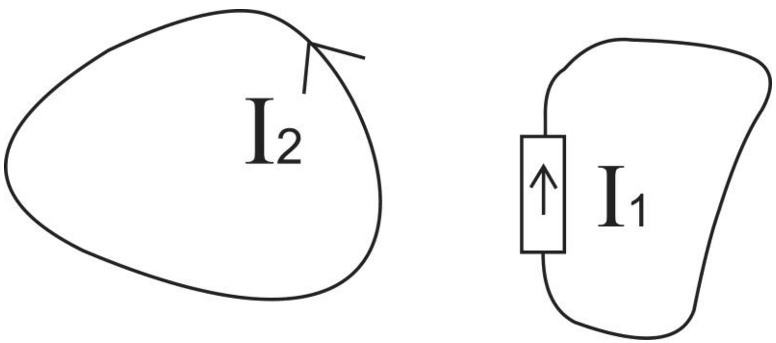


	$F_{2\text{ in }1}$	$F_{1\text{ in }2}$
A	→	←
G	0	0

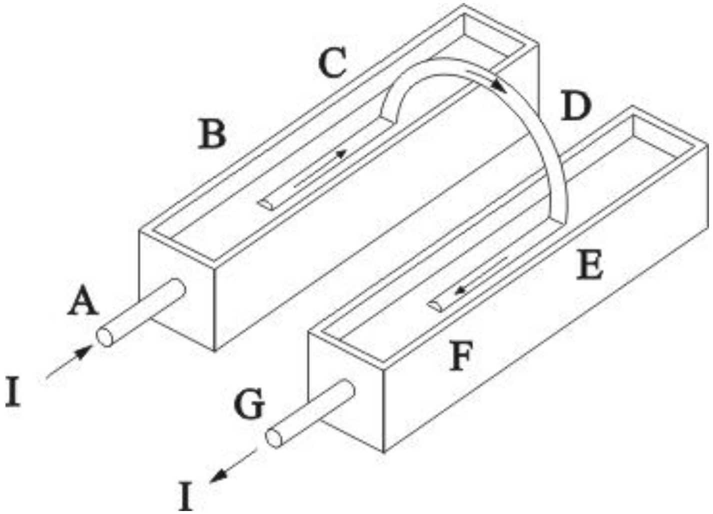


$$F^{A}_{2\,\text{in}\,1} = F^{G}_{2\,\text{in}\,1} =$$

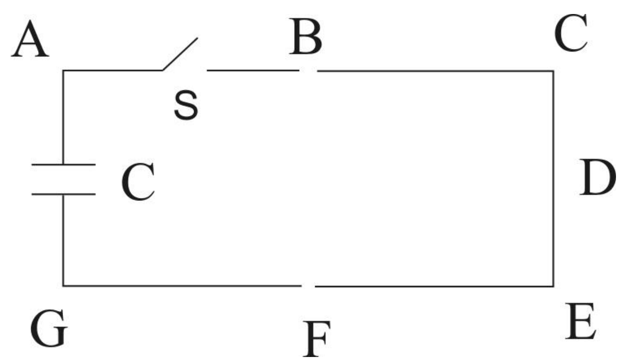
$$I_1 d\vec{\ell}_1 \times \left(\frac{\mu_0}{4\pi} \oint \frac{I_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \right)$$

Ampère X Grassmann:

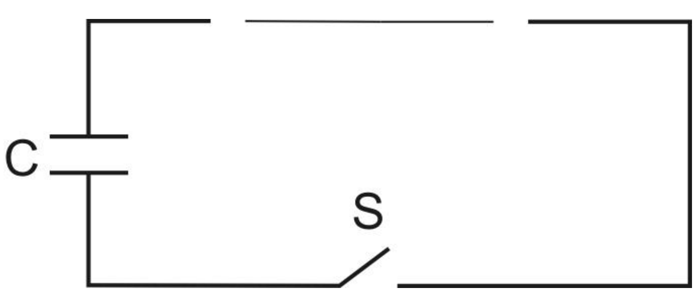
Ampère's bridge:

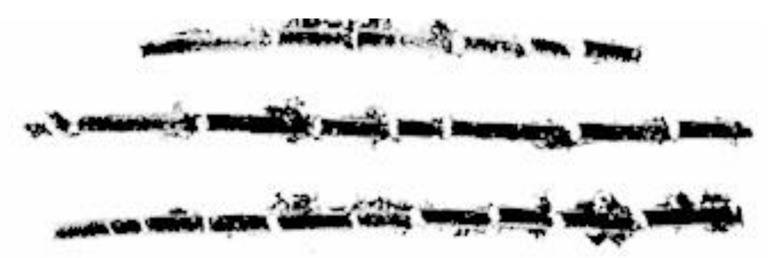


Electromagnetic impulse pendulum:



Exploding wire:





Maxwell (1873) in the *Treatise on Electricity and Magnetism* when comparing the forces between current elements of Ampère (1826), Grassmann (1845) and two other expressions created by Maxwell:

Article 527: “Of these four different assumptions that of Ampère is undoubtedly the best, since it is the only one which makes the forces on the two elements not only equal and opposite but in the straight line which joins them.”

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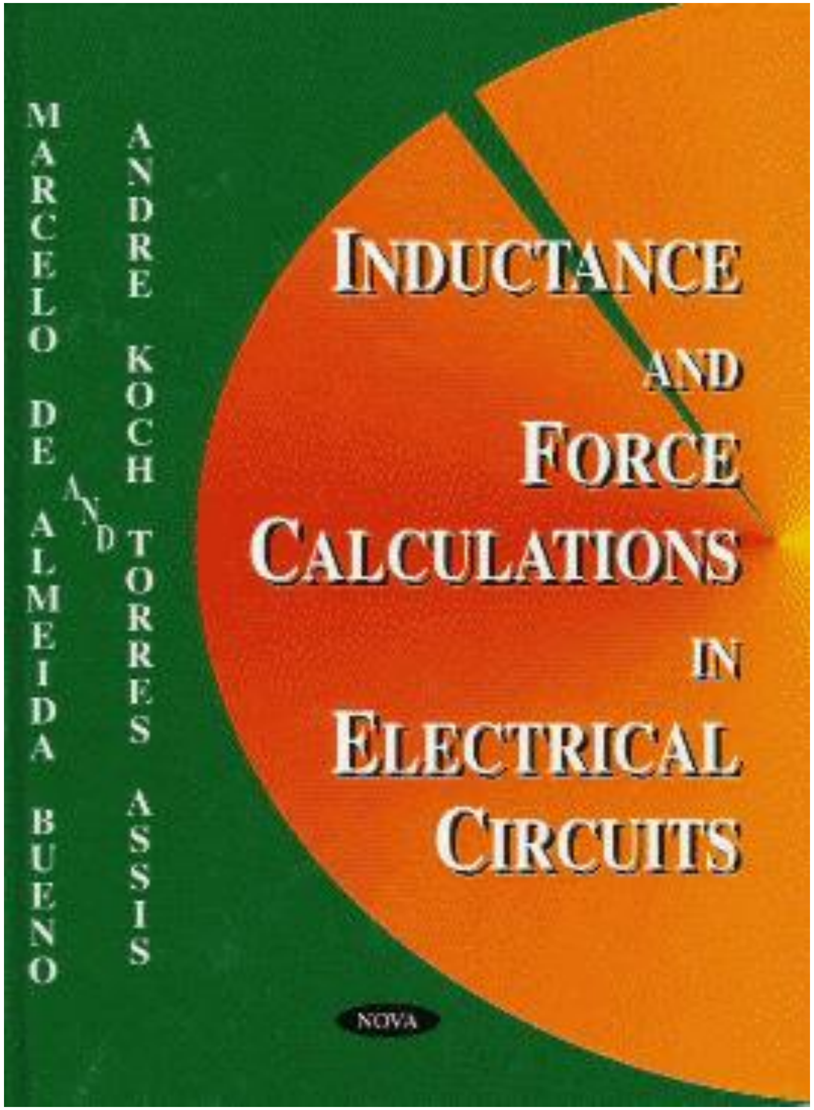
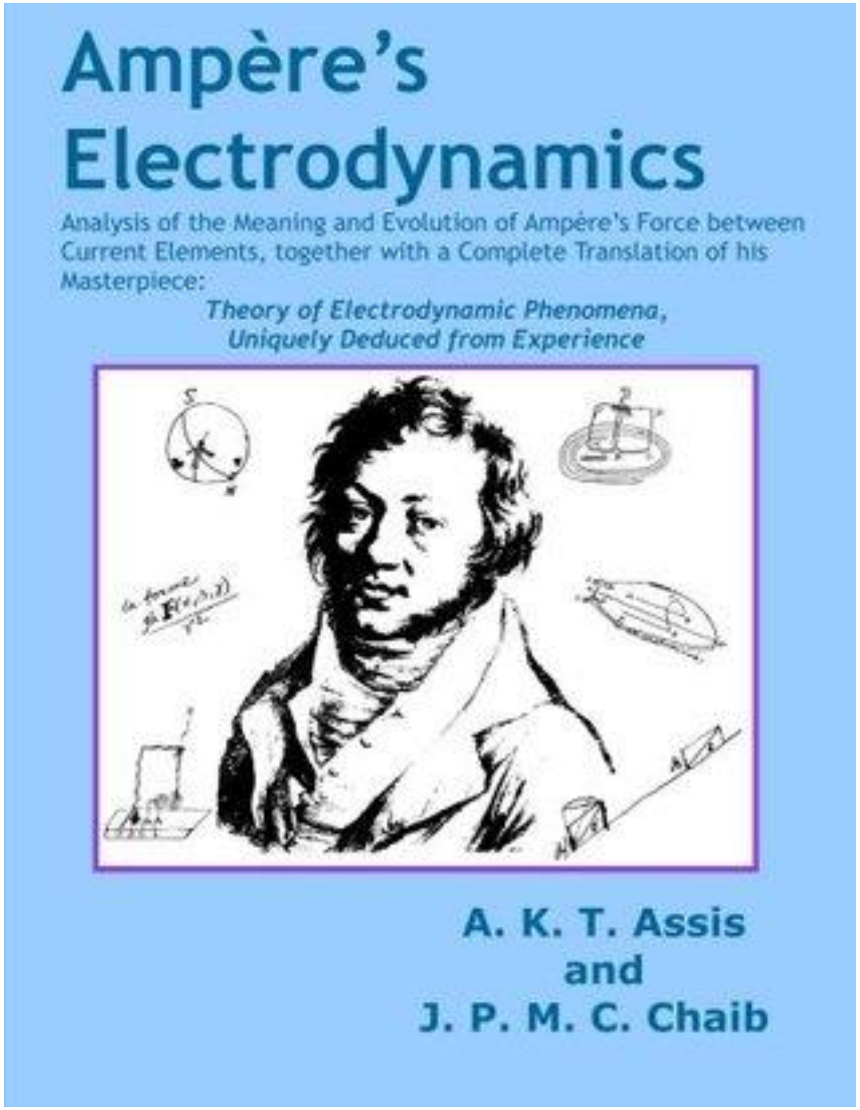
Maxwell’s general assessment of Ampère’s work:

Article 528: “The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science. The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the `Newton of electricity.' It is perfect in form, and unassailable in accuracy, and <u>it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electro-dynamics</u>.”





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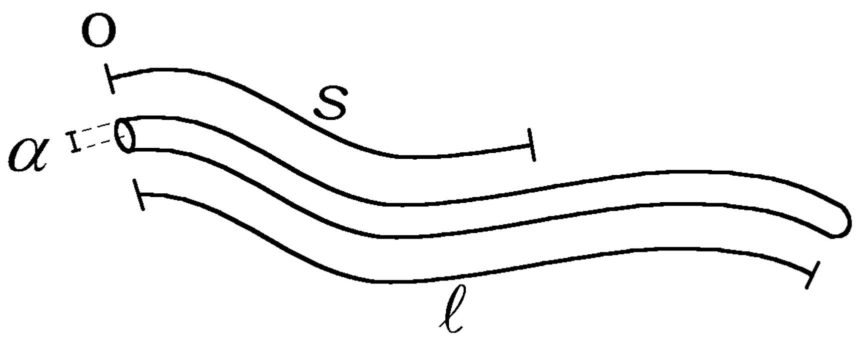
The propagation of electromagnetic signals was first obtained by Weber and Kirchhoff in 1857 utilizing Weber's electrodynamics, before Maxwell. In particular, they obtained the telegraphy equation:

$$\vec{J} = g\vec{E} = -g\left(\nabla\phi + \frac{\partial\vec{A}}{\partial t}\right)$$

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$



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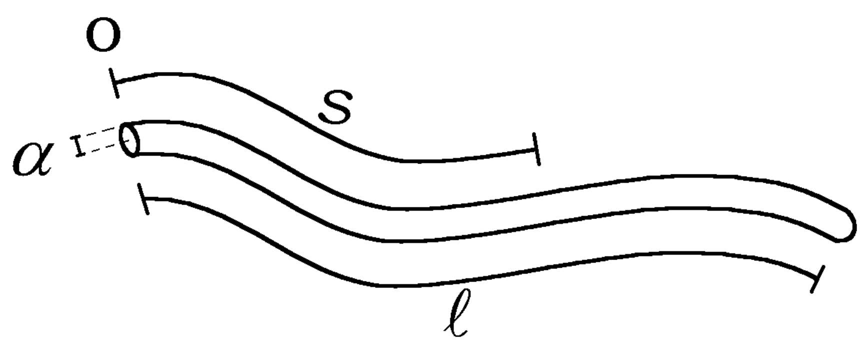


$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = \frac{2\pi\varepsilon_0 R}{\ell \ln \frac{\ell}{\alpha}}\frac{\partial \xi}{\partial t}$$

with $\xi = I, \sigma, \phi, A$

Maxwell introduced the displacement current in “Ampère’s” circutal law in 1864-1873:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

Maxwell introduced the displacement current in “Ampère’s” circutal law in 1864-1873:

$$\nabla \times \vec{B} = \mu \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

However:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Maxwell utilized the constant ***c*** which had been introduced by Weber in 1846.

$$c = 3 \times 10^8 \frac{m}{s}$$

Maxwell knew the value of this constant which had been first measured by Weber in 1856.

$$\frac{\partial^2 \xi}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = 0$$

Maxwell knew that Weber and Kirchhoff, utilizing Weber’s force, had obtained the wave and telegraphy equations in 1857.

Main difference between the forces of Weber and Lorentz:

Weber's force depends on the position, velocity and acceleration a_1 of the test body 1:

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2} \right)$$

Lorentz's force depends only on the position and velocity of the test body, but does not depend on its acceleration a_1:

$$\vec{F}_{2\,\mathrm{em}\,1}^{\,\mathrm{Lorentz}} = q_1 \vec{E} + q_1 \vec{v}_1 \times \vec{B}$$

Weber versus Lorentz

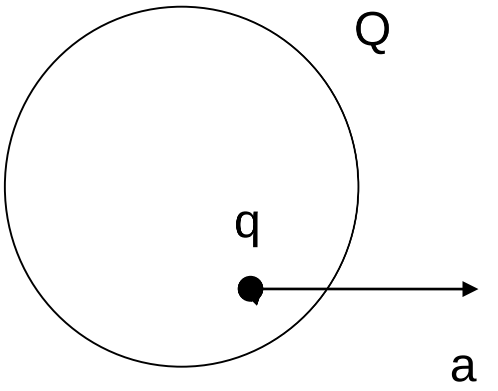
$$\vec{F}_{2\,\mathrm{in}\,1}^{Weber} = \vec{F}(r_1, r_2, v_1, v_2, a_1, a_2) =$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}\left\{1 + \frac{(\vec{v}_1 - \vec{v}_2)\cdot(\vec{v}_1 - \vec{v}_2)}{c^2} - \frac{3\left[\hat{r}\cdot(\vec{v}_1 - \vec{v}_2)\right]^2}{2c^2} + \frac{\vec{r}\cdot(\vec{a}_1 - \vec{a}_2)}{c^2}\right\}$$

$$\vec{F}_{2\,\mathrm{in}\,1}^{\mathrm{Lorentz}} = q_1\vec{E} + q_1\vec{v}_1\times\vec{B} = \vec{F}(r_1, r_2, v_1, v_2, a_2) =$$

$$= q_1\left\{\frac{q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\left[\left(1 + \frac{v_2^2}{2c^2} - \frac{3(\hat{r}\cdot\vec{v}_2)^2}{2c^2} - \frac{\vec{r}\cdot\vec{a}_2}{2c^2}\right)\hat{r} - \frac{r\vec{a}_2}{2c^2}\right]\right\} + q_1\vec{v}_1\times\left\{\frac{q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{\vec{v}_2\times\hat{r}}{c^2}\right\}$$

Force exerted by a charged spherical shell acting on an internal test charge accelerated relative to the shell:



$$\vec{F}^{Lorentz} = 0$$

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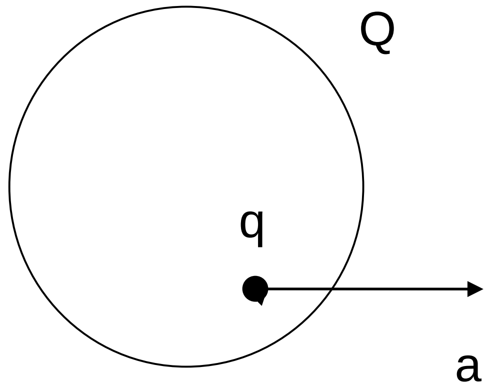
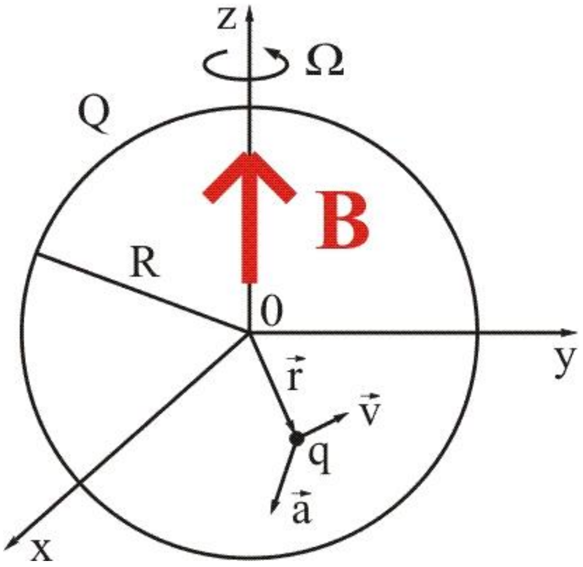


$$\vec{F}^{\,Lorentz} = 0$$

$$\vec{F}^{\,Weber} = \frac{\mu_0 q Q}{12\pi R}\vec{a}$$

According to Weber's electrodynamics, the test charge should behave as if it had an effective inertial mass which depends on the surrounding charges.

Force exerted by a spinning charged spherical shell acting on an internal test charge moving relative to the shell:



$$\vec{F}^{\,Lorentz} = q\vec{E} + q\vec{v} \times \vec{B} = q\vec{v} \times \frac{\mu_0 Q\vec{\Omega}}{6\pi\, R}$$

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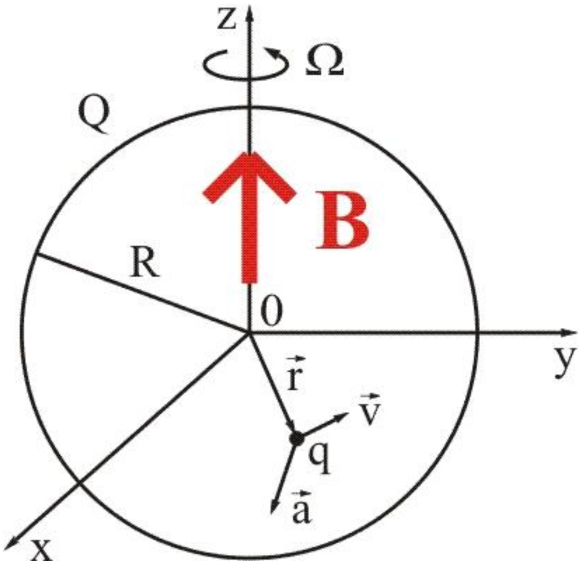

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$$\vec{F}^{\,Weber} = \frac{\mu_0 q Q}{12\pi R}\left[\vec{a} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{r}\right) + 2\vec{v}\times\vec{\Omega}\right]$$

Weber's planetary model of the atom (1870-1880):

$$\frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r^2}\left(1-\frac{\dot{r}^2}{2c^2}+\frac{r\,\ddot{r}}{c^2}\right)\approx q_1 E + m_W a_1 = m_1 a_1$$

$$q_1 E = \left(m_1 - m_W\right) a_1$$ where $$m_W = \frac{q_1 q_2}{4\pi\varepsilon_0 c^2}\frac{1}{r}$$

$$m_1 = m_W$$ when $$r = \frac{\mu_0}{4\pi}\frac{q_1 q_2}{m_1} = r_C$$

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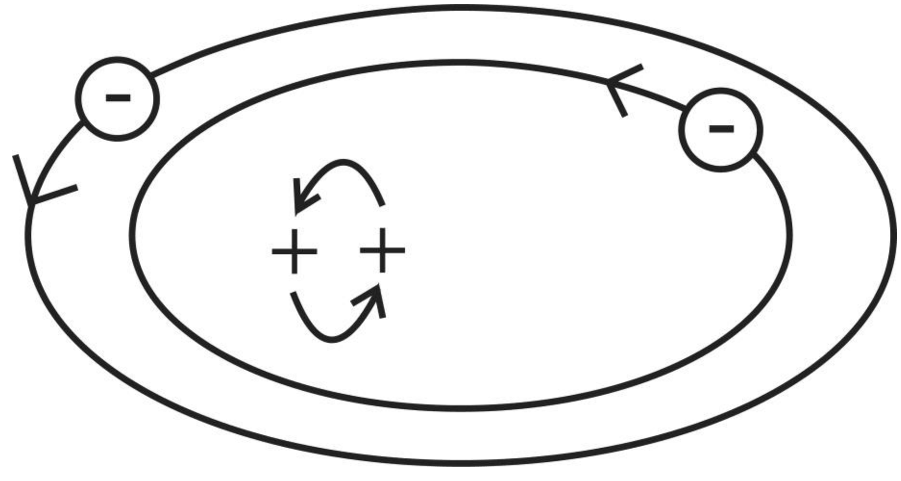
Two positrons atract one another
for distances smaller than: $r_C = 10^{-15}\,m$



Remarkable properties of Weber's model:

- Weber's **prediction** (1870-1880) was made before the discovery of the electron (1897), of Balmer's spectral series (1897) and of Rutherford's scattering experiments (1911)! Bohr's model (1913), on the other hand, was **created (invented)** in order to be compatible with these experimental findings.

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• Weber presented a formula for his critical distance r_c below which two charges of the same sign would attract one another. But he could not calculate its value as the electrons and positrons (1932) were unknown. When we utilize the modern values of the mass and charge of two positrons, we obtain that they will attract each other when **$r_c < 10^{-15}$ m**. Therefore Weber's model gives a **justification** for the known size of the atomic nuclei!

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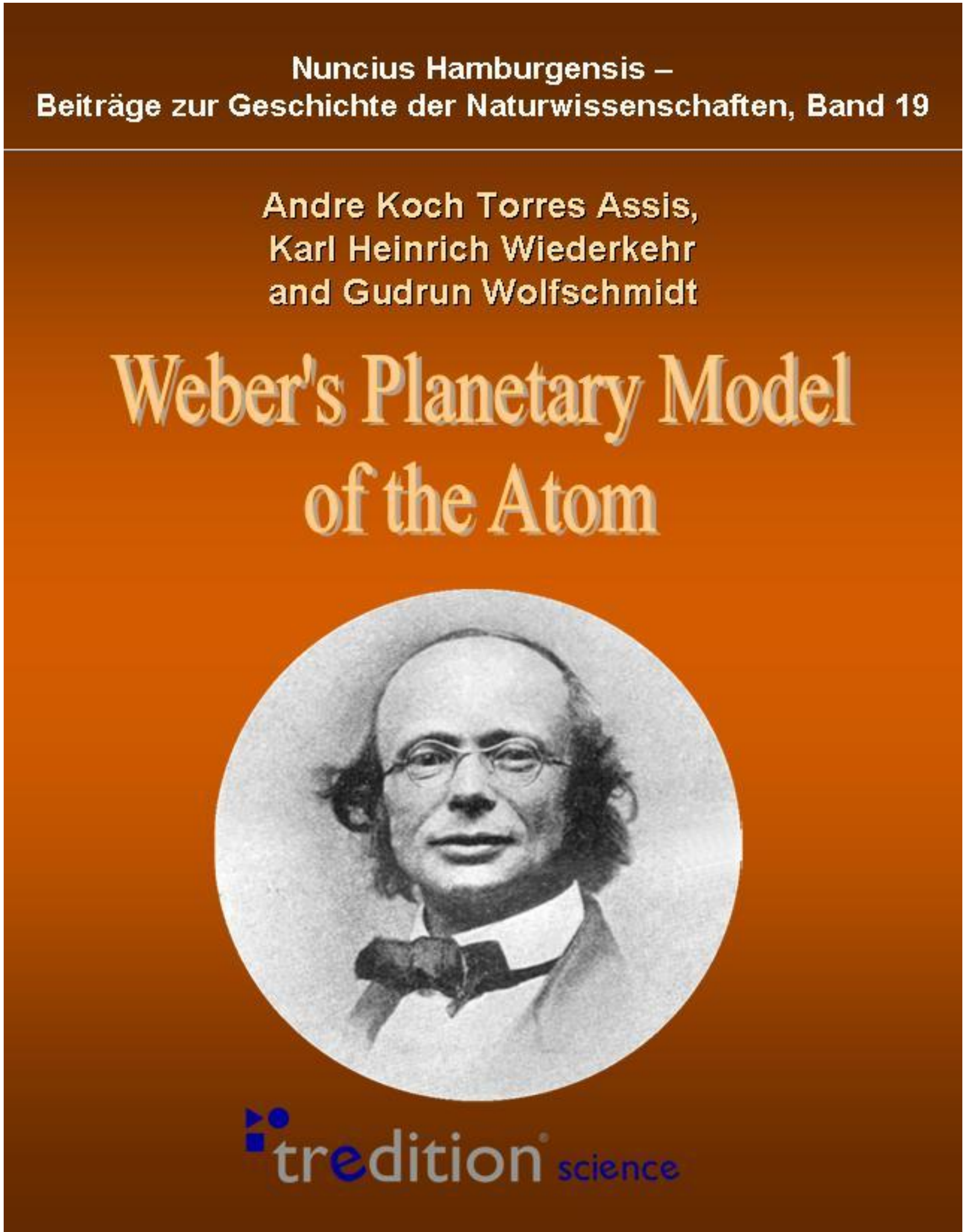
• In modern physics it is necessary to **postulate** the existence of nuclear forces in order to stabilize the positively charged nucleous against Coulomb's repulsive forces. Weber's model, on the other hand, offers an **unification** of electromagnetism with nuclear physics, as the nucleus is held together by purely electrodynamic forces!


Nuncius Hamburgensis –
Beiträge zur Geschichte der Naturwissenschaften, Band 19
Andre Koch Torres Assis,
Karl Heinrich Wiederkehr
and Gudrun Wolfschmidt
Weber's Planetary Model
of the Atom
tredition science

2011

My next project:

To publish an English translation of Weber’s main works on electrodynamics.

I am looking for volunteers to help translate any of the articles.

Conclusion

Weber's electrodynamics is extremely powerful.

In the last few years there has been a renewed interest in Weber's electrodynamics due to novel experiments and new theoretical developments.

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