Wilhelm Weber's Main Works on Electrodynamics Translated into English

Volume III: Measurement of Weber's Constant *c*, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity

Edited by Andre Koch Torres Assis

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Front cover of Volume III: The picture on the cover of Volume III shows Wilhelm Weber around 1876. Source: Friedrich Zöllner, *Principien einer elektrodynamischen Theorie der Materie* (Engelmann, Leipzig, 1876), frontispiece.

Wilhelm Weber's Main Works on Electrodynamics Translated into English

Volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity

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Chapter 1

Introduction to Volume III

A. K. T. Assis[1]

The picture on the cover of Volume 3 shows Weber around 1876. It comes from the frontispiece of a book of 1876 by Friedrich Zöllner (1834-1882) on the principles of an electrodynamic theory of matter.[2]

Volume III contains Weber's main works related to diamagnetism, including his Third major Memoir on Electrodynamic Measurements (1852).

These works are followed by three papers published by Weber and Rudolf Kohlraush in 1855-1857 in which they presented the measurement of Weber's fundamental constant c appearing in his force law. Weber and Kohlrausch's 1857 work is the Fourth major Memoir on Electrodynamic Measurements.

Soon after this measurement, Kirchhoff and Weber succeeded in deducing the complete telegraph equation from Weber's electrodynamics. Their works were published in 1857 and 1864. When the resistance of the wire was negligible, the telegraph equation reduced to the wave equation. The velocity of propagation of an electric wave along the wire was then shown to be independent of the cross section of the wire, of its conductivity and of the density of electricity along the surface of the wire. Its value was equal to the known light velocity in vacuum. This remarkable result of Weber's electrodynamics indicated for the first time in the history of physics a direct and quantitative connection between electrodynamics and optics. This volume contains the English translations of Kirchhoff's two papers of 1857, together with a paper by J. C. Poggendorff emphasizing the independent researches made by Weber and Kirchhoff on this subject in which both scientists arrived simultaneously at similar results. Weber's 1864 work is his Fifth major Memoir on Electrodynamic Measurements. The translation of this paper is also included in this Volume.

[1]Homepage: www.ifi.unicamp.br/~assis

[2][Zöl76]. It also appears, for instance, in [Wie60, p. 208] and [Wie67, p. 155].

Chapter 2

[Weber, 1852b, EM3] Electrodynamic Measurements, Third Memoir, relating specially to Diamagnetism

Wilhelm Weber[3,4,5]

I - Introduction. Concept of Diamagnetic Polarity

Diamagnetism in the few years since its discovery became the topic of various researches. These not only broadened the field but also led to the discovery and examination of several other new natural phenomena. Therefore, the interest on these researches grew continuously. However, the field of *diamagnetism* still needs a *fundamental law*, in order to become comparable to magnetism, electromagnetism, and magnetoelectricity, to which it is closely related. To obtain such a *fundamental law* seemed since its beginning doable, because Faraday[6] managed to find a very simple and general expression concerning the major facts discovered by him, namely the diamagnetic *repulsion* and the *equatorial position* of diamagnetic materials in the vicinity of a strong magnet. Even if his general expression cannot be considered as a fundamental law, it seems to be closely related to one. Faraday namely deduced these diamagnetic actions from the laws of *variable magnets* (iron magnets), by comparing the actions of diamagnetic materials to the ones of magnetized iron for which North magnetism and South magnetism were interchanged. The relation between diamagnetism and magnetism after that is the *law of diamagnetic polarity* found by Faraday.

To make it clear what *magnetic* or *diamagnetic polarity* means, we explain how this notion is used in this paper. It is well-known that Gauss proved,[7] that all actions by which a magnet (or a material which contains galvanic currents) effects other materials, can be deduced from two magnetic fluids, which are distributed on *its surface* in a specific manner.

[3][Web52b] with English translation in [Web21a].

[4]Translated by U. Frauenfelder, urs.frauenfelder@math.uni-augsburg.de. Edited by U. Frauenfelder and A. K. T. Assis.

[5]The Notes by Wilhelm Weber are represented by [Note by WW:]; the Notes by H. Weber, the Editor of Volume 3 of Weber's *Werke*, are represented by [Note by HW:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[6][Note by AKTA:] [Far46a] and [Far46b].

[7][Note by AKTA:] [Gau39] with English translations in [Gau41a] and [GT14].

Gauss called this distribution *the ideal distribution of magnetic fluids*. Hence we refer in this paper by *magnetic* or *diamagnetic polarity* to the state of a material through which it can effect forces to other materials in such a way that these forces can be explained in terms of *the ideal distribution of magnetic fluids*.

Therefore the law of *diamagnetic polarity* implies, that all actions of a diamagnetic material can be explained in terms of *an ideal distribution of the two magnetic fluids* on its surface. Since the law of the *magnetic polarity* requires the same for magnetic materials, it follows under the assumption that there exists really a *diamagnetic polarity* in this sense,

> *diamagnetic materials* do not distinguish themselves essentially from *magnetic* ones in terms of their *actions*, but how they are *generated* and how they *change*.

Namely suppose that before their generation (or transformation) we have an *ideal distribution*, then all the actions are given, independent if it is *magnetism*, *galvanism*, or *diamagnetism* which leads to that *ideal distribution*.

If the law of *diamagnetic polarity* is really universally true, it is not just applicable to the phenomena first discovered by Faraday, namely the interaction of the diamagnetic material with the magnet due to whose influence it became diamagnetic, but to all phenomena a material can effect other materials due to a certain distribution of its magnetic fluids. All these different kinds of phenomena can be classified into *purely magnetic* ones, *electromagnetic* ones, and *magnetoelectric* ones. Therefore it is highly interesting to detect the actual occurrence of these *different modes of effects*. If the *second* effect really existed for diamagnetic materials, it would lead to the fundamental experiment of *electrodiamagnetism*. The *third* effect would lead to the fundamental experiment of *diamagnetoelectricity* (or the diamagnetic induction of electric currents). On the other hand, if not all these effects occurred, this would imply that the law of *diamagnetic polarity* is not universally valid, so that it would loose its importance and theoretical significance.

Concerning the occurrence of these different modes of effects the results of different researchers do not yet agree with each other. This is easily explained, if one takes into account how weak necessarily the later kinds of effects have to be. Therefore it can easily happen that not all researchers can detect them especially since they do not use exactly the same kind of devices. In particular, Faraday did not succeed in convincing himself of the (inducing) effect of diamagnetic materials, despite the fact that he repeated the corresponding experiments with great diligence and care.

How weak for example the effect of a diamagnetic material on a magnetic compass is, can be easily understood by noting that even the forces of a strong electromagnet also in small distance to a diamagnetic material are very weak, although they are proportional to the large forces of electromagnets. If one considers instead of the interaction of a somehow diamagnetic material with a strong electromagnet the interaction of a diamagnetic material with a weak magnetic compass, one easily understands that from this last interaction in the same distance a force occurs which in the same proportion is smaller although the force in the first interaction was already pretty small.

Under these circumstances where one can see *a priori* that the interactions in question, if they exist, are extremely weak, one needs special arrangements to distinguish them from other small actions in order to prove their existence. It does not suffice to improve and refine the observational equipments, but one has to get a deeper understanding of the size of the effects in question which can be observed so that one can be sure that the *observed* ones really corresponds to the thing one was *looking for*. To say it shortly the observation of such small effects needs *quantitative control* to produce results on a sound basis. But such a quantitative control was missing completely so far. In particular, the question about the existence or non-existence of a *diamagnetic induction of electric currents* which is one of the major issues can only be decided by experiment, if the *size of the current*, which has to be induced diamagnetically, can roughly be estimated. Indeed, only after that one can decide about the means needed to check it.

However, in order to achieve such a *quantitative control* of these considerations one has to discuss more carefully the consideration which led to the conjecture of a *magnetic induction of electric currents*. According to this consideration one assumes that all effects of a diamagnetic material can be explained in terms of a certain distribution of the two magnetic fluids on its surface and that on the other hand a diamagnetic material has all effects of the magnetic fluids distributed in this way. It follows from this that one has to associate to each diamagnetic material a *magnetic moment*. Moreover, each kind of diamagnetic action has to be used in order to determine the size of this magnetic moment so that one can predict precisely or to good approximation all kinds of diamagnetic effects. If this consideration is true it opens the way to infer from known diamagnetic phenomena to unknown ones and predict their size so that each experiment which does not have the required accuracy can be discarded immediately. On the other hand each experiment which has the required accuracy but does not give the result or a completely different one can be used to falsify the whole consideration. A serious decision can only be reached in this way.

During the whole paper I tried to follow this way and I believe that the results obtained here leave no doubt, although it is desirable that in the future the quantitative measurements can be carried out with even higher precision. If I had more funding I could have easily obtained better equipments and gotten more precise results, what is definitely desirable, although the main result does not seem to be in doubt.

II - Electrodiamagnetism and Measurement of the Moment of an Electrodiamagnet

2.1 Electromagnet and Electrodiamagnet

In the same way how one distinguishes *usual* iron magnets, i.e., iron magnets whose magnetism is due to the influence of other magnets, from electromagnets, one can distinguish *usual* diamagnets (whose diamagnetism is caused by magnetic influence) from electrodiamagnets. However, between electromagnets and electrodiamagnets there is a huge for the observation important difference. Namely if two equal galvanic currents go around a bar of iron and a bar of bismuth, iron acts by magnetic forces in the distance compared to which the forces of the galvanic current almost vanish, while the diamagnetic forces of bismuth almost vanish compared to the ones of the galvanic current. This is the reason that the *existence of electrodiamagnetism* is difficult to prove. However, this difficulty can be overcome and it even follows from this that the force of an electrodiamagnet is much more suitable to the actual *measurements* than the one of a usual diamagnet. However, for that a special device is needed, in order to get rid of the influence of the galvanic current. Here I *first* want to describe the *device* using that I got the *pure* action of an *electrodiamagnet* so that I could compare the size of its force with the one of an *electromagnet*. *After that* I describe the *results* I obtained in the experiments using that device.

2.2 Electrodiamagnetic Measuring Device

The goal was to observe the effect of an elecrodiamagnet on a magnetic needle one puts in some distance. It was already mentioned before, how small the expected effect of a diamagnetic material on a usual magnetic needle is, especially if this needle is some inches away from the diamagnet. The smaller the expected effect was, the finer methods of observation have to be applied. Therefore a small magnetometer was used, whose needle was 100 millimeters long and carried a mirror in order to be observable according to the method of Gauss using telescope and scale. With this method deflections of the needle of single arc minutes could be measured exactly. The sensitivity of such a needle depends as is well known on the size of the horizontal deflecting force exerted by terrestrial magnetism. If the deflecting force of terrestrial magnetism was not weakened the oscillation period of the needle was 7.687 seconds. To augment the sensitivity the deflecting force was weakened in such a way that the oscillation period increased to 18.45 seconds. This can be achieved in a quite simple way with the help of a strong magnetic bar SN of Figure 2, which one puts with reversed poles in direction of the needle NS in appropriate distance. With the help of a small displacement of this magnetic bar, the sensitivity of the needle could be regulated as one pleases. However, a too high sensitivity puts the precision of the observation in slight danger. Furthermore it turned out that the above mentioned sensitivity was sufficient. It is worth mentioning that the needle was furnished with a damper made of copper which had the effect to reduce the oscillation arcs according to the proportion 3:2 or more precisely the *decrementum logarithmicum*[8] was

[8] [Note by AKTA:] That is, the logarithmic decrement.

$$= 0.178\,87\ .$$

After this description of the magnetic measuring device we now proceed with the presentation of the electrodiamagnet and its deployment. The electromagnet *first* consisted of two equal cylinders made of bismuth whose length was 92 millimeters, whose width was 16 millimeters, and whose combined weight was 343 500 milligrams. They were connected to each other in vertical position at a distance of 100 millimeters, as represented by *aa* in Figure 1.

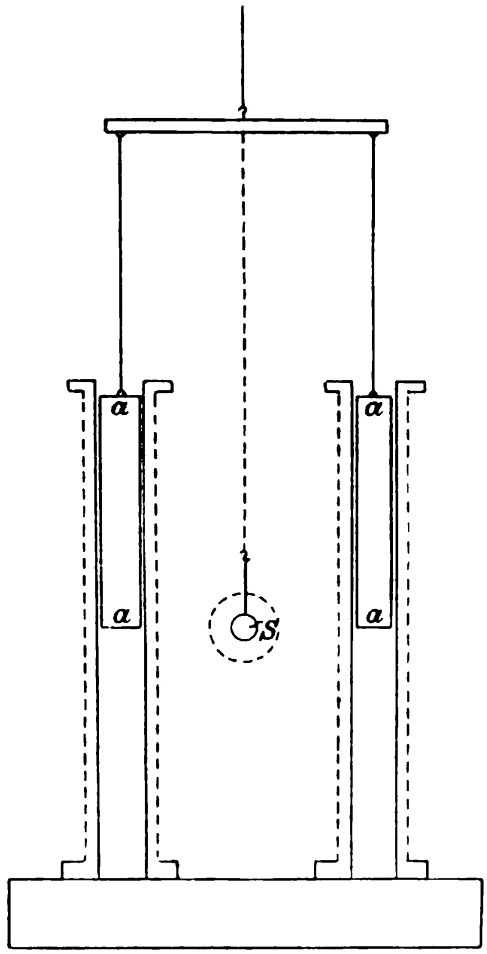


Fig. 1.

Using a simple crank mechanism they could be lifted and lowered. *Secondly* the electromagnet consisted of spiraling copper wires. Each of these spirals had a length of 190 millimeters, an interior diameter of 17 millimeters and consisted of four layers, each layer containing 146 windings. Like columns, they were vertically mounted on a stand at a distance of 100 millimeters and their wires were connected to each other in such a way that a current which went from one to the other passed through them in opposite direction. Both cylinders of bismuth could be lowered simultaneously into these two spirals and were transformed into electrodiamagnets due to the galvanic current. One North pole turned upwards and one North pole turned downwards. To represent the current six Grove's elements[9] were used.

[9][Note by AKTA:] In German: *Grove'schen Bechern*. The Grove voltaic cell or Grove element was named after its inventor, William Robert Grove (1811-1896), [Gro39].

These two spirals were now positioned in such a way that a horizontal plane through the needle bisected them. The southern end S of the needle was floating precisely in the middle between the two spirals. In Figure 2 one can see a horizontal section of the position of the needle NS and of the two spirals around aa. The two cylinders consisting of bismuth were either lowered in the spirals to such an extent that their upper end reached the level of the needle or they were lifted to such an extent that their lower end reached the level of the needle.

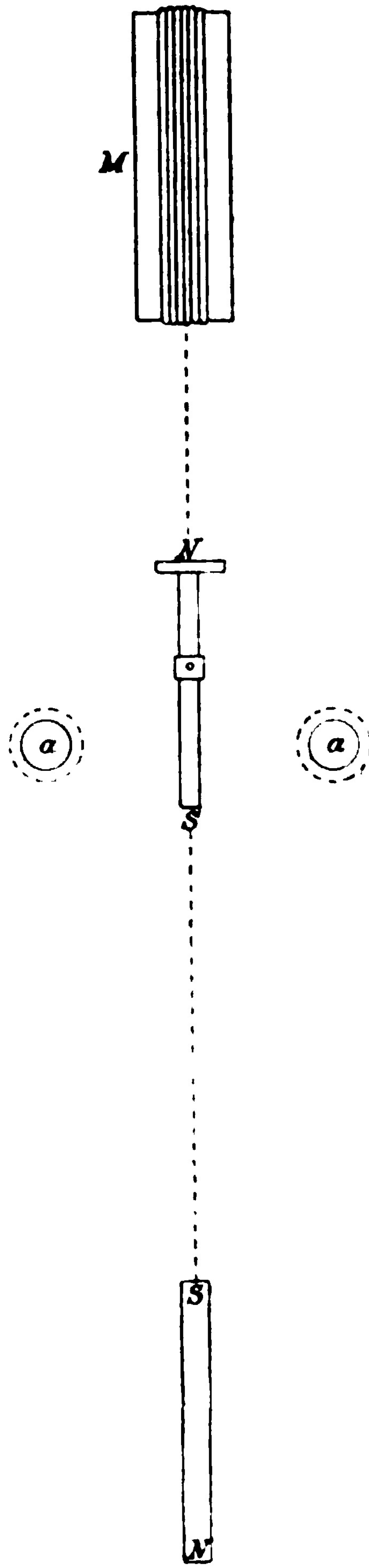


Fig. 2.

The reasons for this deployment are the following. *Firstly* it was important that the galvanic current which went through both spirals did not affect directly the needle despite it was strong and close to the needle and despite the sensitivity of the needle. Due to the symmetric position of the two spirals to the same amount above and below the horizontal plane through the needle, the deflections cancelled. Due to the same distance of the two spirals to the needle and thanks to the opposite direction of their currents, the vertical forces cancelled as well. Otherwise the vertical forces would cause the needle to oscillate. However, since a complete symmetry cannot be achieved in practice a special deployment was needed to compensate the small unavoidable deviations. For this purpose a third wire was used which winded 18 times around a quadrangular frame M and was incorporated into the circuit. This frame had a length of 244 millimeters, a height of 146 millimeters and was erected vertically in the plane of the needle. The same current who went through the two spirals exerted a torque[10] on the needle by passing through the third wire. By moving the frame closer or farther away, the torque could easily be made bigger or smaller until the intended compensation was reached perfectly.

Secondly the two cylinders consisting of bismuth were put alternatively into the lower and the upper position. In the lower position their upper ends influenced the needle more strongly and in the upper position it was their lower ends which had the stronger influence. It was important to achieve this in such a way that *the strength of the diamagnetism changed without inducing through this movement a current in the conductor bismuth.* Here the advantage of a *diamagnet* compared to a *usual* one became manifest. In fact, a *usual* diamagnetic material whose diamagnetism is due to the vicinity of a magnetic pole changes its diamagnetism after each displacement. Moreover, if the material is a conductor, currents are always induced in it. This is quite different for an electrodiamagnet, where the diamagnetic cylinder of bismuth is enclosed by the galvanic spiral. When this spiral winds uniformly and is so long that the cylinder of bismuth has always some distance to the ends of the spiral, the electromagnetic force of the spiral is almost constant in space according to the known laws of electromagnetism. Therefore one can move the cylinder of bismuth inside the spiral without changing its diamagnetism and without inducing galvanic currents in it. Furthermore the material becomes *uniformly* diamagnetized. In the usual case where the diamagnetism is caused by the vicinity of a magnetic pole, such a thing does not happen. The reason is that the parts which are closest to the pole become much stronger than the other ones. This fact prevents all measurements.

If in the set-up described there was no direct influence of the current on the needle and no current was induced in the cylinders of bismuth, the deflection of the needle which one observed had to be a pure effect of the *diamagnetic force* of the bars of bismuth. Moreover, this deflection had, according to the law of *diamagnetic polarity*, to be either *positive* or *negative* depending if the bars of bismuth are in *upper* or *lower position* inside the wire spirals. It follows the lucky circumstance for closer examination that one can increase the deflection by *multiplication*, namely by changing the position of the bars of bismuth always in the moment when the needle reaches the end of its oscillation arc. This is repeated so long until due to the effect of the damper the oscillation arc of the needle during each oscillation decreases in the same amount as it increases due to the diamagnetic effect of the bars of bismuth. The corresponding *limit* can be computed with great accuracy by taking into account the sequence of observed oscillation arcs. If the damping is known, it can be used as a *measure of the strength of the electrodiamagnetism* of the bars of bismuth.

[10][Note by AKTA:] In German: *Drehungsmoment*. It can be translated as "torque", "rotational moment", "rotation moment", or "moment of force".

If one uses instead of the bars of bismuth an iron cylinder of the same length and repeats the same experiments, one can *compare the strength of an electrodiamagnet with the one of an electromagnet*. It is clear that due to the high sensibility of the apparatus one has to weaken the effect of the electromagnet as far as possible by using a very *thin* iron bar. In the following experiments the iron bar was so thin that its weight was only the 59200th part of the weight of the two bars of bismuth. Even in this case its effect was much stronger than the one of the two bars of bismuth together.

Finally, the *third* major point in these experiments is to determine the *direction* of the deflection for every position of the bars of bismuth and to compare it with the direction the deflection had for the iron bars positioned at the same place. Therefore we kept track in the observations of the position of the bars for every oscillation period. The result was always as the following experiments show, that if the bars of iron and the bars of bismuth had the same position, the deflection of the needle was in *opposite direction*. Hence for *electrodiamagnets* the northern and southern magnetic fluid under the same conditions for the currents have to be thought as *opposite* compared to *electromagnets* as is shown by these experiments. The same phenomenon was known for *usual diamagnets* from different effects.

2.3 Experiments and Measurements

The experiments and measurements using the above described devices were made by different people in order to remove the uncertainty a single observer faces with such weak effects. Besides me the following gentlemen kindly agreed to repeat the same measurements at different days, namely Professor Listing, Professor Sartorius von Waltershausen, Dr. von Quintus Icilius and Dr. Riemann.[11] For example instead of the data of my own measurements I provide here all datas of the measurements of Professor Listing, which were carried out with extreme care. I just remark, that my own ones as well as all the others closely agree with the ones of Professor Listing.

Göttingen 1851. June 21.
Observer: Professor Listing.
Galvanic Current of six Grove platin-zinc elements.

[11][Note by AKTA:] Johann Benedict Listing (1808-1882), Wolfgang Sartorius Freiherr von Waltershausen (1809-1876), Ernst Wilhelm Gustav von Quintus Icilius (1824-1885) and Georg Friedrich Bernhard Riemann (1826-1866).

1. Experiments with Both Bars of Bismuth

No. of the oscillation	position of the bars	position of the needle at the beginning and the end of each oscillation	equilibrium position of the needle	oscillation arc of the needle
		500.0		
1.	above			
		467.0	487.6	-40.0
2.	below			
		513.9	488.3	-50.4
3.	above	459.9	488.3	−56.3
4.	below	518.5	489.2	−58.5
5.	above	460.0	487.3	−55.2
6.	below	512.0	489.3	−46.5
7.	above	471.1	484.9	∓ 29.7
8.	above	489.7	487.3	∓ 7.0
9.	below	494.2	489.3	−8.9
10.	above	480.9	488.9	−15.6
11.	below	498.9	482.7	−30.0
12.	above	457.0	483.1	−50.4
13.	below	516.0	487.2	−57.8
14.	above	459.3	484.2	−50.9
15.	below	504.4	487.6	∓ 35.6
16.	below	478.3	483.1	± 12.4
17.	above	476.9	485.6	−14.7
18.	below	504.9	485.7	−36.6
19.	above	459.6	480.6	−42.6
20.	below	499.4	479.6	−39.6
21.	above	460.1	484.1	−46.6
22.	below	513.9	488.2	−51.7
23.	above	464.2	486.8	−45.9
24.	below	506.2	480.0	−50.6
25.	above	446.9	474.1	−55.2
26.	below	498.0	476.4	∓ 44.5
27.	below	460.0	465.6	± 15.5
28.	above	453.1	462.5	−16.8
29.	below	479.8	464.6	−29.8
30.	above	446.9	467.8	−40.3
31.	below	494.6	471.8	−46.0
32.	above	450.4	471.3	−42.2
33.	below	490.5	468.2	−44.0
34.	above	442.6		

2. Experiments with One Bar of Iron

In order to decrease the effect of the iron to the sensitive needle we only used a simple little bar and made two series of measurements where the little bar was first moved in the first spiral back and forth and then in the second one. The little iron bar had the same length as the little bars of bismuth but its weight was just 5.8 milligram, i.e., it was 59200 times lighter than the two little bars of bismuth together. Nevertheless the effect was so strong that the deflection could only be measured in a simple way *without multiplication*.

	First series			
no.	position of the iron bar	elongation of the needle	rest position of the needle	average
		428.1		
			300.4	
		215.2		
1.	below		303.8	302.0
		362.8		
			301.7	
		261.0		
		451.2		
			571.7	
		652.0		
			569.8	
2.	above	515.0		571.0
			571.9	
		609.9		
			570.6	
		544.4		
		435.5		
			298.2	
		206.7		
			301.5	
3.	below	364.7		300.6
			298.6	
		254.6		
			304.0	
		336.9		
		503.2		
			560.1	
4.	above	598.0		560.7
			561.3	
		536.9		

	Second series			
no.	position of the iron bar	elongation of the needle	rest position of the needle	average
1.	above	524.0 590.5 549.3	563.9 565.8	564.9
2.	below	227.4 387.1 275.4 357.9	323.2 320.1 324.9	322.7
3.	above	450.9 661.8 525.3 600.0	577.4 579.9 570.1	575.8
4.	below	217.8 392.2 270.0 349.4	322.4 318.9 317.6	319.6
5.	above	439.7 638.8 495.8 595.0	559.2 553.0 555.3	555.8

It is worth mentioning that the intensity of the current produced by six Grove's elements was measured with a tangent galvanometer[12] whose ring had a diameter of 211 millimeters. The current deflected the compass by an amount of 28° 21' from which the intensity of the current (the horizontal part of the terrestrial magnetic force = 1.8) becomes

$$= 105.5 \cdot \frac{1.8}{2\pi} \cdot \text{tang}\, 28°21' = 16.31 \ .$$

[12][Note by AKTA:] In German: *Sinus-Boussole* and *Tangenten-Boussole*. The tangent galvanometer was invented by Johan Jakob Nervander (1805-1848) and the sine galvanometer by Claude Servais Mathias Pouillet (1790-1868), [Ner33], [Pou37] and [Sih21]. Friedrich Kohlrausch discussed measurement of currents with the tangent and sine galvanometers, [Koh83, Chapters 64 and 65, pp. 188-192].

2.4 Computation of the Experiments

In the Table containing the experiments with the two *bars of bismuth* the positions of the needle observed at the beginning and the end of an oscillation are written in the *third* column. From each three of these consecutively observed positions of the needle there are computed in the *fourth* and *fifth* column the corresponding state of rest and the oscillation arc with respect to the damping. A *positive* sign in front of the oscillation arc means that the needle went in case of the *upper position* of the bars of bismuth from smaller to larger scales, respectively in case of the *lower scale* from larger to smaller ones. The opposite holds for the *negative* sign. After the position of the bars of bismuth was changed several times at the end of each oscillation and the oscillation arc almost reached its limit, a break was produced by keeping the positions of the bars of bismuth during two oscillations unchanged. After that they were changed again after every oscillation. The *negative* oscillation arc was transformed in this way into a *positive* one, which however quickly decreased to zero and very soon became *negative* again. In this way one understood the direction of the deflection caused by the bars of bismuth most clearly. — If one counts the oscillation arcs starting from the one which is closest to zero, one can easily reduce the observed values using the well-known *decrementum logarithmicum* to the *limit* and deduce in this way a more accurate mean value of the limit. In the case at hand the *decrementum logarithmicum* is close to $= \log \frac{3}{2}$ and therefore it suffices to divide the value of the oscillation arc by $\left(1 - \left(\frac{2}{3}\right)^n\right)$ or more precisely since the *decrementum logarithmicum* $= 0.178\,87$ by $(1 - 0.6624^n)$. Using this procedure one obtains the following reduced values.

No.	observed	reduced	average
1.	−40.0	−63.4	
2.	−50.4	−66.6	
3.	−56.3	−67.1	−61.8
4.	−58.5	−65.5	
5.	−55.2	−59.4	
6.	−46.5	−48.8	
11.	−30.0	−47.5	
12.	−50.4	−66.6	−59.8
13.	−57.8	−68.5	
14.	−50.9	−56.8	
19.	−42.6	−67.5	
20.	−39.6	−52.3	
21.	−46.6	−55.5	
22.	−51.7	-57.9	−56.1
23.	−45.9	−49.4	
24.	−50.6	−53.1	
25.	−55.2	−57.0	
30.	−40.3	−63.9	
31.	−46.0	−60.2	−55.8
32.	−42.2	−50.0	
33.	−44.0	−49.3	

Combining all the observations one obtains the following limit

$$x = -58.4\ .$$

The *negative* sign means, that the needle at the *lower position* of the bars of bismuth was driven to a *larger* scale division, while at the *upper position* to a *smaller* one. Moreover, from these experiments carried out according to the *method of multiplication* it follows from the *limit* of the oscillation arcs found to be $= x$ that the *deflection E corresponding to the equilibrium of the needle*

$$E = \frac{x}{2} \cdot \frac{1 - e^{-\lambda}}{1 + e^{-\lambda}} ,$$

according to my rule in the previous paper.[13] Here $\log e^\lambda$ denotes the logarithmic decrement, i.e., $\log e^\lambda = 0.178\,87$. From that the *deflection corresponding to the equilibrium of the needle* follows to be

$$E = -5.93 .$$

From the experiments with the *little iron bar* carried out without multiplication the following equilibria of the needle were obtained alternately for the *upper* and *lower* position:

	first series	second series
above	–	564.9
below	302.0	322.7
above	571.0	575.8
below	300.6	319.6
above	560.7	555.8

From that the values of the deflection E follow immediately:

first series	second series
+134.50	+121.10
+135.20	+126.55
+130.05	+128.10
	+118.10

Hence averaging both columns one obtains for the deflection

$$E' = +128.4 .$$

The *positive* sign means, that the needle at the *lower position* of the iron bars was driven to a *smaller* scale division while at the *upper position* to a *larger* one, i.e., *just opposite as in the case of the bars of bismuth.*

Therefore, the *moment* of the *magnetism* of the little iron bar compared to the moment of the *diamagnetism* of both bars of bismuth behaves as

$$+128.4 : -5.93 ,$$

i.e., the moment of the iron equals 21.7 times the one of bismuth with opposite sign, despite the fact that the mass of the iron was 59 200 times smaller. Hence reducing to equal masses the *diamagnetism of bismuth* becomes 1 285 000 times smaller than the *magnetism of iron.*

From a similar series of experiments carried out by Professor Sartorius von Waltershausen, the *limit*

$$x = -48.2 ,$$

[13][Note by AKTA:] [Web52c, p. 440 of Weber's *Werke*] with English translation in [Web21b].

was obtained, from a third one due to Dr. Quintus Icilius

$$x = -47.3 ,$$

from a fourth one of Dr. Riemann

$$x = -45.0 ,$$

and from the one carried out by me

$$x = -55.8 .$$

The average of all these experiments is therefore

$$x = -50.9 ,$$

$$E = -5.17 ,$$

and therefore the *diamagnetism* of bismuth becomes 1 470 000 times smaller, than the magnetism of iron.

The above experiments allow one to prove the existence of the *electrodiamagnetism* of bismuth. Its derived *size* can only be considered as an approximate one of course. However, such an approximate value is a sufficiently firm base for the following examination of the *diamagnetic induction of galvanic currents*.

2.5 The Most Convenient Device to Observe Diamagnetic Polarity

The previous experiments prove three things:

(i) For the representation of diamagnets as for the representation of magnets, the purely magnetic forces can be replaced by *electromagnetic forces* of galvanic currents.

(ii) In the same way as the *magnetic polarity* of an iron bar magnetized by the same current, the *diamagnetic polarity* of a uniformly diamagnetized bar of bismuth can be observed clearly and for sure with the help of the electromagnetic force of a galvanic spiral in which it is put by observing opposite torques[14] it effects on a magnetic needle depending on the way the bar approaches the needle with one end or with the other.

(iii) Under the circumstances described the torque of a diamagnetic bar of bismuth on a magnetic needle can be determined and compared to the torques of a magnetized iron bar exerted on the same magnetic needle. It follows that the *direction* of the torque is always *opposite*, while the determination of its *magnitude* leads to a *comparison of the magnetic and diamagnetic moments corresponding to each other*.

All these experiments can be carried out with simple means if they are used appropriately. This is even more remarkable by taking into account that the forces under examination are extremely tiny as mentioned in the introduction. Therefore one could think that the

[14][Note by AKTA:] In German: *Entgegengesetztед Drehungskräfte*. This expression can also be translated as “opposite rotational forces” or “opposite rotatory forces”.

observation of clearly recognizable effects of these small forces requires the application of highly sophisticated devices what is in fact not the case. Indeed, a pile of Grove or Bunsen[15] of six to eight elements and some pound of copper wire of appropriate strength are objects needed for many different experiments. Apart from that one just needs in addition a little magnetic needle endowed with a mirror in order to be observed by a telescope (where a sextant telescope is sufficient) as in the case of a magnetometer.

I invented a device in order to make as easy as possible the implementation of these experiments, which are of crucial importance for the justification of the theory of diamagnetism. In particular, I wanted to minimize the pain to install the apparatus. In particular, I recommend it as the most convenient one for the repetition of the experiments. Its essential feature is that instead of two galvanic spirals which were put into vertical position in the experiments described above in Section 2.2, so that one of the poles of a *straight* magnetic needle lay symmetrically between them, the new device only requires a *single* spiral.[16] This single spiral is installed symmetrically in the middle of two poles of a horseshoe-shaped magnetic needle. In Figure 3 the cross section of this spiral is represented by A, which lies symmetrically between the poles N and S of the horseshoe-shaped bent magnetic needle NBS. This magnetic needle is kept by the clip DE, in whose middle C the thread is attached. Figure 4 and Figure 5 illustrate the instrument in a lateral view.[17]

[15][Note by AKTA:] The Bunsen voltaic cell or element was named after its inventor, Robert Wilhelm Eberhard Bunsen (1811-1899). It was a zinc-carbon primary cell.

[16][Note by AKTA:] That is, a single finite solenoid.

[17][Note by AKTA:] Another reproduction of Figures 3, 4 and 5 appear on page 82.

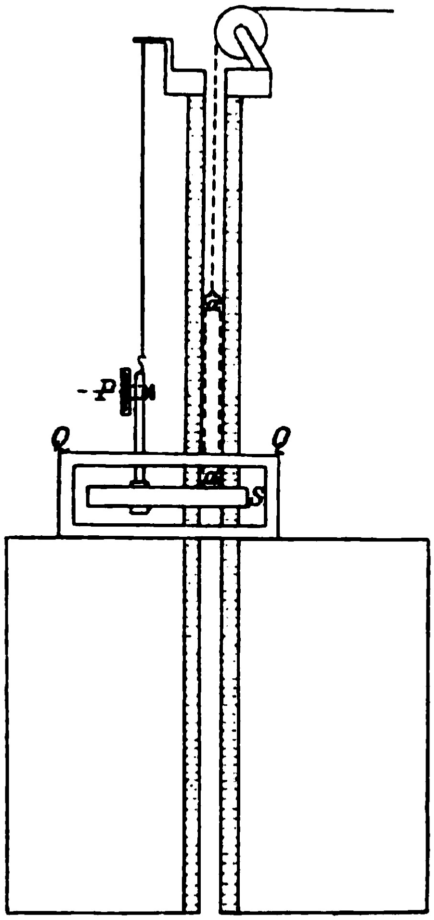


Fig. 4

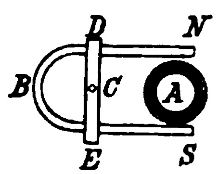


Fig. 3.

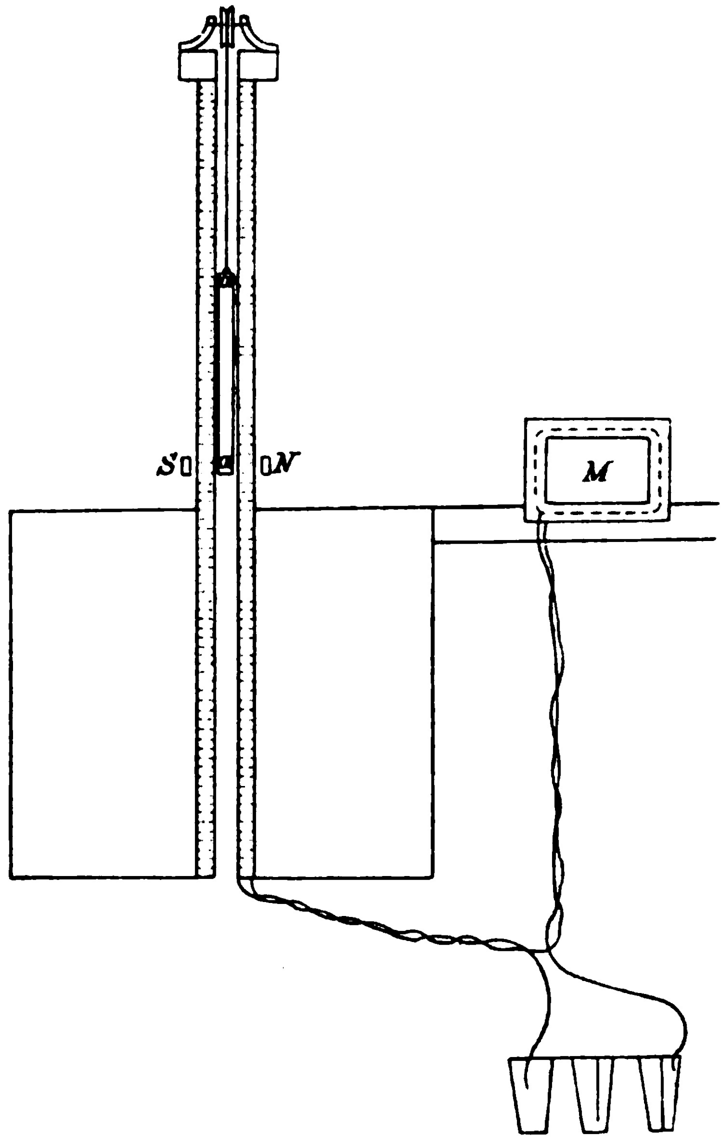


Fig. 5.

It is advantageous to give the spiral a considerable length, for example from 400 to 500 millimeters, which makes it easier to control the mounting of the needle. In particular, one would like to achieve that the spiral is hovering in the horizontal plane which divides the length of the spiral into two equal halves so that the current going through the spiral does not effect any torque on the needle. In case there is a small torque, it can be easily compensated as explained in Section 2.2 by a multiplier M consisting of few windings (see Figure 5). To observe the needle it is necessary to supply it with a mirror P as in Figure 4, in which one observes the mirror image of a remote scale. In addition the magnetic needle is encompassed with a damper QQ as in Figure 4. The bar of bismuth aa is suspended vertically in the spiral with a thread (Figure 4 and Figure 5). It can be lifted or lowered so that either, as represented in Figure 4 and Figure 5, its lower end lies between the poles of the magnetic needle or its upper end. The observations can be carried out in the most convenient way if using coils or a simple crank mechanism the observer himself at the telescope is able to lower or lift the bar of bismuth by lifting or lowering the pedestal. When the current is closed and the magnetic needle at complete rest, if one lifts the bar of bismuth then one observes a small movement of the needle. As soon as the needle attains its largest elongation, the bar of bismuth is lowered again and the magnetic needle moves back with a higher speed. As soon as it attains its largest elongation on this side, the bar of bismuth is lifted again and so on. Between two elongations one notes the position which the bar of bismuth had during the elapsed time. If one interchanges the bar of bismuth with a very thin wire of iron of the same length, one can convince oneself that the deflection of the needle happens in the opposite direction.

III - Diamagnetoelectricity and Measurement of Diamagnetic Induced Electric Currents

2.6 Diamagnetic Induction

The experiments about *diamagnetic induction* are obviously more difficult then the previous experiments on *electrodiamagnetism*, because its observation is more subtle. It requires special techniques to set up the experiments in order to actually reach the goal with limited means. The following experiments show how this is possible. Even if the effects obtained with the help of these means are tiny, they show such an agreement that by taking into account the circumstances they are quite remarkable, if the task at hand is to justify the fact of *diamagnetic induction* and to make sure that one is not deceived by external influences. As we will see the effects can be used for *quantitative* determinations of the *strength* of diamagnetic induction which are applicable to such verifications for which a lesser degree of accuracy is sufficient. Only the desire to give these quantitative determinations the necessary precision for some special examinations will make it necessary in the future to apply more sophisticated instruments. I first describe the diamagnetic inductor and then proceed with the experiments carried out with its help.

2.7 Description of the Diamagnetic Inductor

Here I describe a different diamagnetic inductor then the one with the help of which I found a weak trace of diamagnetic induction (*Berichte* 1847 and Poggendorff's *Annalen* 1848, Vol. 73),[18,19] which however did not have the desirable fineness and accuracy for these experiments. That device was essentially the same which Faraday later used and described in the *Philos. Transact. 1850, P. I.*[20] However, Faraday did not succeed to detect magnetic induction with that device, although he made various different interesting applications with it. The reason for that mixed success probably lies in the finer galvanometric instruments I used. I would have not been able to observe such a diamagnetic induction either, if I had not a galvanometer at my disposal whose needle is observed with mirror and telescope as the magnetometers of Gauss. Nevertheless as well my experiments carried out with that device cannot be considered as sufficient, since the weak effects seem to be combined with other effects from which they hardly can be separated. Moreover, the circumstances do not admit a *quantitative control*. The here described inductor differs from the previous one essentially in the following points.

1. Instead of a usual magnet, an *electromagnet* is used for the induction, whose moment due to the previous examination at least approximately is known. This allows the prediction of the ratio of the inducing effect of the device for a bar of bismuth compared to a bar of iron.

2. The induction is produced by the *mere movement* of the diamagnetic material in a wire spiral at rest. Through this the diamagnetism remains *unchanged* and one avoids the

[18][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 255.

[19][Note by AKTA:] [Web48b], [Web48c, p. 255 of Weber's *Werke*] with English translation in [Web52d] and [Web66c].

[20][Note by AKTA:] [Far50].

induction of galvanic currents in bismuth as a conductor. Otherwise these galvanic currents can easily be confused with the diamagnetic induced currents.

The Electrodiamagnet Used for the Induction

The *electrodiamagnet* used for the induction consisted of a bar of bismuth in a long wire spiral, *cccc* of Figure 6 *A* through which a current of eight coal-zinc elements of Bunsen was conducted. The bar of bismuth was 186 millimeters long and weighed 339 300 milligrams. The wire spiral consisted of copper wire spanned with wool and additionally insulated with a capping of gutta-percha. The pure copper wire was 2.3 millimeters thick and the wire consisted of eight layers each having 120 windings. The whole spiral was 383 millimeters long and had 23.9 millimeter interior and 70 millimeter exterior diameter.[21]

[21] [Note by AKTA:] Another reproduction of Figure 6 appear on page 83.

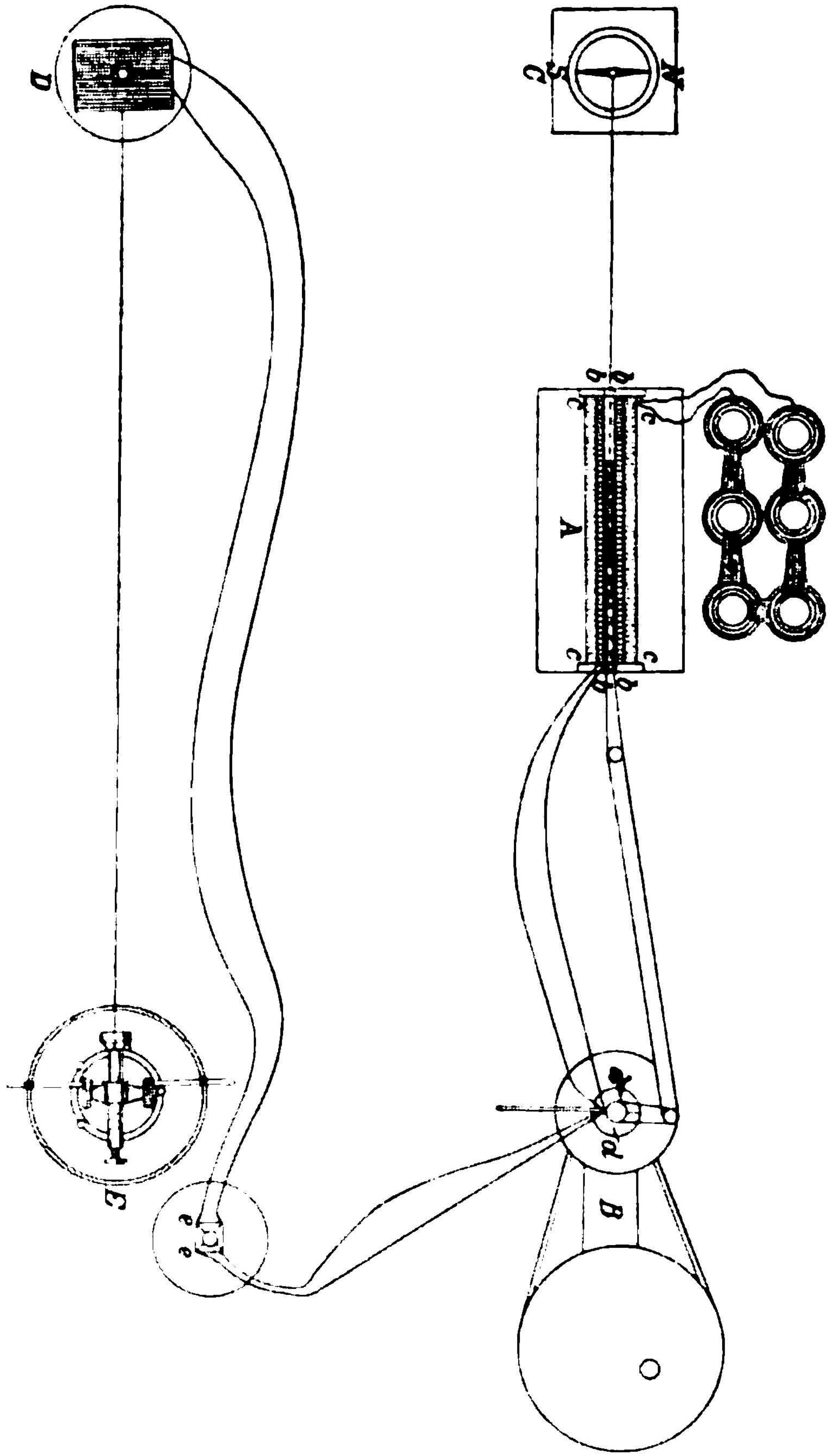


Fig. 6.

The Induction Spiral

The induction spiral *bbbb* of Figure 6 *A* is that spiral in which due to the movement of the *electrodiamagnet* a current is induced. This spiral has to be carefully insulated from the one belonging to the electromagnet through which the current of the galvanic pile flows and has to be connected to the multiplier of the galvanometer in order to observe the induced current. This spiral consisted of a copper wire which was 1 millimeter thick and spanned with silk building three layers each having 294 windings. The length was 383 millimeters, the interior diameter 19, the exterior one 23 millimeters. After it was wrapped with thin gutta-percha for better insulation it was locked tightly in the further tube of the spiral belonging to the electromagnet or more precisely the spiral was wound around it.

The essential point to be noted for this spiral is that it decomposes into two completely symmetric halves. That means that the wire does not uniformly wind in the same direction, rather the spiral decomposes into two halves in which the wire is wound in opposite directions. This is necessary if through the movement of a diamagnetic bar of bismuth or a magnetic iron bar a current has to be induced in this spiral which can be observed with the galvanometer connected to it. Namely if the inducing bar is put in the middle of the spiral and then moved, the induction force in one half of the spiral exerted from its northern end is just opposite to the one exerted from its southern end. The effect of both would cancel out if both halves of the spiral were wound in the same direction. Since they are wound in opposite directions, the induction forces do not cancel each other out but double.

This mechanism necessary for the purpose of induction has another important advantage for the practical implementation. It is clear that the current of the galvanic pile in the spiral of the electrodiamagnet as long as it is *constant* does not exert an inducing force on the induction spiral with respect to that it has a firm, unchanging position. However, due to the slightest *change of its intensity* a current would be induced in the spiral which would be much stronger than the diamagnetic induced current and would disturb the observation of the latter. However, it is obvious that the same mechanism of the induction spiral through which the diamagnetic induction in both halves get doubled as well leads to a cancellation of the induction forces of the current in the galvanic pile so that if the symmetry of both halves is perfect even huge changes of the intensity of the current in the galvanic pile have no influence at all. Moreover, firstly it is very easy to check if this cancellation happens exactly by switching off or commuting the whole current instead of producing small changes. Secondly if it turns out that the cancellation is not perfect, it is easy to make it perfect by winding one end of the induction spiral once or several times around the spiral through which the current of the galvanic pile flows. In this way it is no big problem to free the effects of the diamagnetic induction from all exterior influences.

The Remaining Parts of the Inductor

Concerning the implementation of the remaining parts of the induction device which more or less are left to the taste of the observer I add just the following remarks. In order to move the bar of bismuth in the induction spiral back and forth I connect it with the crank of a wheel, see Figure 6 *B*. Moreover, in order that the induced current when moving the bar of bismuth back has the same direction as when moving the bar of bismuth forward, a *commutator dd* is attached to the wheel, *which turns itself with the wheel* so that after each half turn of the wheel (in the moment, where the bar of bismuth reaches the initial or endpoint of its orbit) the connection of the ends of the wires of the induction spiral with the ones of the multiplier of the galvanometer are interchanged. Therefore the always same direction in which all induced currents through the multiplier of the galvanometer go would deflect the needle always to the same side. In order to enable the observer to produce as well a deflection of the needle to the other side next to the telescope in Figure 6 E a second commutator *ee* is installed, which only from the observer himself is changed. This commutator is referred to as the *auxiliary commutator*. It connects the two wire ends of the multiplier with the two ends of the conductors coming from the *rotating commutator*. By the way one should observe especially the following points. Firstly one tries to intensify the induction more through the acceleration of the turning of the wheel than through the size of the path on which one moves the bar of bismuth back and forth. In the following experiments the bar of bismuth was moved back and forth in a just 58.2 millimeters long path. However, it traversed this path 10.58 times each second. If the path were longer, a part of the bar of bismuth would have approached the end of the spiral through which the current of the galvanic pile went. This would not just change the strength of its diamagnetism but as well induce in it as a conductor a current which produces a secondary induced current in the induction spiral. This has to be avoided if one wants to obtain a pure effect of diamagnetic induction. Secondly the rotating commutator needs special attention, since in it easily a thermomagnetic current is created. Therefore one has to arrange the commutator in such a way that equal metals (brass to brass) rub each other. By this the thermomagnetic currents get just weakened but not avoided completely. The different thermomagnetic currents cancel each other more or less. However, since this cancellation happens in general not completely one has to get rid of their influence by taking it into account. This can be achieved easily if the observer immediately before and after makes the same observations where the rotating commutator is moved without the bar of bismuth. By the way one can arrange the observations as well easily in such a way that the small effects of the thermomagnetic currents alternatively increase and decrease the effects of the diamagnetic induction, which leads to an average value independent of the thermomagnetic current. This is achieved by changing from time to time the direction of the current in the galvanic pile which reverses the diamagnetism in the bar of bismuth. For the galvanometer in Figure 6 D I used as in the case of the electrodiamagnetic measuring device a little magnetometer set up by Gauss which was supplied with a very strong multiplier. The length of the needle was reduced to 30 millimeters. The deflecting force of terrestrial magnetism was reduced as before. The needle also was surrounded by a thick copper ring as a damper. It barely needs to be mentioned that the induction device has to be removed so far from the galvanometer that the current of the galvanic pile used does not influence directly the needle. If there is not enough room to do this, one has to bring the induction device by a special orientation in such a position that its deflecting force on the needle becomes zero or at least very small. Finally, to get a rough estimate of the strength of the current of the galvanic pile itself, a usual compass (Figure 6 C) was installed in an appropriate distance of the spiral through which the current went. In this way the deflection of the compass produced by the current could be used to determine the intensity of the current.

2.8 Experiments

The following experiments as well were not carried out by me alone but Professor Listing, Professor Sartorius von Waltershausen, Dr. Quintus Icilius, and Dr. Riemann participated as in the previous electrodiamagnetic part. As an example I convey here as well the full record of the experiments carried out by Professor Listing with which all the others closely agree.

The inductor was installed in such a way that the vertical plane going through the middle of the galvanometer and through the middle of the wire spiral had an angle of 45 degrees to the magnetic meridian. The axis of that wire spiral was perpendicular to the magnetic meridian. It follows from the laws of electromagnetism confirmed by experience that with this set-up the current does not deflect the needle of the galvanometer. Under these circumstances it was most advantageous to install the compass used to determine the intensity of the current in the direction of the extended axis of the wire spiral through which the current went. This happened in a distance of 708 millimeters from the center on the western side. That current through which the northern end of the compass is deflected *westward* is referred to as *normal current* the one in which the northern part is deflected *eastward* is referred to as *reversed current*. Furthermore, the displacement of the bar of bismuth in the induction device in direction from West to East is called *normal displacement* and in direction from East to West *reversed displacement*. Finally the position the *rotating oscillator* had during the normal displacement of the bar of bismuth is called *normal position* and the one during the reversed displacement is called *reversed position*. A pendulum clock regulated the rotation of the balance wheel and it turned out that the bar of bismuth traversed its path 10.58 times per second. The horizontal distance of the mirror of the magnetic needle from the scale of the galvanometer was 1400 scale divisions. The oscillation period of the galvanometer which for the full deflecting force of terrestrial magnetism was close to 9 seconds was brought to 20.437 seconds through partial cancellation of the force of terrestrial magnetism thanks to the above described method. The logarithmic decrement for the decrease of the oscillation arcs was $= 0.12378$.

The needle of the galvanometer was deflected thanks to diamagnetic induction in the same way when the bar of bismuth moved from West to East as when it moved from East to West, because of the change of the *rotation commutator* in between. This happened without changing the direction of the current in the galvanic pile in the spiral of the electrodiamagnet and the position of the auxiliary commutator. The deflection occurred by moving quickly back and forth in the same way as the one produced by a constant current. However, if the position of the *auxiliary commutator* is changed the deflection of the needle occurs to the opposite side. This implies that in order to get more accurate observations the deflection of the needle can be increased through multiplication by changing the position of the auxiliary commutator always in the moment where the needle attained the end of the oscillation arc, so long, until finally through damping of the needle its oscillation arc is decreased during each oscillation by the same amount as the increase due to the induced current. Therefore between two observed elongations of the needle the by $+$ or $-$ denoted position of the auxiliary commutator was recorded. If the needle at the beginning of the observations was already in swing one started with that position of the auxiliary commutator at which the induced current created a decrease of the present oscillation arc, which than by a continuous change decreased until zero and than started increasing until it attained its limit. When the needle went from smaller to larger scale divisions during the by $+$ designated position of the auxiliary commutator, in the following aggregation of data the $+$ sign was put in front of the oscillation arc, in the opposite case the $-$ sign. The signs of the oscillation arcs turned out to be *opposite* by the *diamagnetic* induction of bismuth compared to the *magnetic* induction of iron. Moreover, the latter oscillation arcs were much bigger, although the bar of iron was much thinner than the bar of bismuth. In fact having the same length the bar of iron weighed 790.86 milligrams where the one of bismuth was 339 300 milligrams. Therefore, to measure the effect of the magnetoelectric induction it was not necessary to move the bar of iron back and forth in the same speed as the bar of bismuth. Instead of that a single translation was sufficient during each swing of the needle in the moment when the swinging needle passed its rest position. The two commutators stayed in their normal position and during each two observations of the elongation one always noted the direction into which the bar of iron was displaced. The direction from West to East was denoted by + and the one from East to West by −, which allowed the comparison to the bar of bismuth. As already mentioned one observed *opposite* effects for the same translations of the bar of iron and the bar of bismuth.

The experiments started by checking 1. if there was an influence of the *thermomagnetic* current and how big it was. For that purpose one started by putting the rotation commutator into motion without moving the bar of bismuth back and forth. The effect was multiplied by changing the auxiliary commutator at each elongation. 2. the *bar of bismuth* was put simultaneously into motion and a bunch of observations were carried out for *normal current*. 3. the same series was done for *reversed current*. 4. the same series again for *normal current*. 5. for *reversed current* and 6. finally again for *normal current*. After that 7. it was checked again if there was an influence of the *thermomagnetic* current and 8. the bar of bismuth was exchanged with the *iron bar* and the induction effect of the latter was measured.

Göttingen 1851. July 13.
Observer: Professor Listing.
Galvanic Current of eight Bunsen coal-zinc elements.

	1. Thermomagnetic current.			
no. of the oscillation	position of the auxiliary commutator	position of the needle at the beginning and end of each oscillation	rest position of the needle	oscillation arc of the needle
		497.0		
1.	+			
		496.2	496.45	−0.5
2.	−			
		496.4	496.35	−0.1
3.	+			
		496.4	496.30	+0.2
4.	−			
		496.0	496.15	+0.3
5.	+			
		496.2		

According to this Table basically no influence of the thermomagnetic current was there.

	2. Induction of the bar of bismuth for *normal current.*				
no. of the oscillation	position of the auxiliary commutator	position of the needle at the beginning and end of each oscillation	rest position of the needle	oscillation arc of the needle	deflection of the compass
		475.3			
1.	−				
		472.8	474.65	+ 3.70	32° 10'
2.	+				
		477.7	475.00	+ 5.40	westward
3.	−				
		471.8	475.20	+ 6.80	
4.	+				
		479.5	475.32	+ 8.35	
5.	−				
		470.5	475.33	+ 9.65	
6.	+				
		480.8	475.52	+10.55	
7.	−				
		470.0	475.70	+11.40	
8.	+				
		482.0	475.87	+12.25	
9.	−				
		469.5	475.85	+12.70	
10.	+				
		482.4	475.90	+13.00	
11.	−				
		469.3			

	3. for *reversed current.*				
no. of the oscillation	position of the auxiliary commutator	position of the needle at the beginning and end of each oscillation	rest position of the needle	oscillation arc of the needle	deflection of the compass
		503.5			
1.	+				
		515.9	511.15	+ 9.50	31° 50'
2.	−				
		509.3	511.13	+ 3.65	eastward
3.	+				
		510.0	510.62	−1.25	
4.	−				
		513.2	510.82	−4.75	
5.	+				
		506.9	510.58	−7.35	
6.	−				
		515.3	510.85	−8.90	
7.	+				
		505.9	510.70	−9.60	
8.	−				
		515.7	510.72	−9.95	
9.	+				
		505.6	510.53	−9.85	
10.	−				
		515.2			

	4. for *normal current*.				
no. of the oscillation	position of the auxiliary commutator	position of the needle at the beginning and end of each oscillation	rest position of the needle	oscillation arc of the needle	deflection of the compass
		480.5			
1.	+				
		471.0	474.57	−7.15	31° 48’
2.	−				
		475.8	474.40	−2.80	westward
3.	+				
		475.0	474.58	+ 0.85	
4.	−				
		472.5	474.40	+ 3.80	
5.	+				
		477.6	474.47	+ 6.25	
6.	−				
		470.2	474.23	+ 8.05	
7.	+				
		478.9	474.27	+ 9.25	
8.	−				
		469.1	474.10	+10.00	
9.	+				
		479.3	473.93	+10.75	
10.	−				
		468.0	473.65	+11.30	
11.	+				
		479.3	473.65	+11.30	
12.	−				
		468.0			

	5. for *reversed current.*				
no. of the oscillation	position of the auxiliary commutator	position of the needle at the beginning and end of each oscillation	rest position of the needle	oscillation arc of the needle	deflection of the compass
		501.5			
1.	+				
		515.0	509.93	+10.15	32° 13’
2.	−				
		508.2	510.35	+ 4.30	eastward
3.	+				
		510.0	510.02	−0.05	
4.	−				
		511.9	510.20	−3.40	
5.	+				
		507.0	509.80	−5.60	
6.	−				
		513.3	509.68	−7.25	
7.	+				
		505.1	509.42	−8.65	
8.	−				
		514.2	509.38	−9.65	
9.	+				
		504.0	509.05	−10.10	
10.	−				
		514.0	508.72	−10.55	
11.	+				
		502.9	508.40	−11.00	
12.	−				
		513.8	508.15	−11.30	
13.	+				
		502.1	507.83	−11.45	
14.	−				
		513.3	567.67	−11.25	
15.	+				
		502.0			

	6. for *normal current.*				
no. of the oscillation	position of the auxiliary commutator	position of the needle at the beginning and end of each oscillation	rest position of the needle	oscillation arc of the needle	deflection of the compass
		486.0			
1.	+				
		461.0	471.20	−20.40	31° 39’
2.	−				
		476.8	470.60	−12.40	westward
3.	+				
		467.8	470.87	−6.15	
4.	−				
		471.1	470.48	−1.25	
5.	+				
		471.9	470.52	+ 2.75	
6.	−				
		467.2	470.08	+ 5.75	
7.	+				
		474.0	470.45	+ 7.10	
8.	−				
		466.6	470.25	+ 7.30	
9.	+				
		473.8	469.92	+ 7.75	
10.	−				
		465.5	469.83	+ 8.90	
11.	+				
		475.0	470.02	+ 9.70	
12.	−				
		465.1	470.13	+10.05	
13.	+				
		575.3	470.17	+10.25	
14.	−				
		465.0	470.08	+10.15	
15.	+				
		475.0	469.95	+10.10	
16.	−				
		464.8			

	7. for *thermomagnetic current.*				
no. of the oscillation	position of the auxiliary commutator	position of the needle at the beginning and end of each oscillation	rest position of the needle	oscillation arc of the needle	deflection of the compass
		486.1			
1.	+				
		486.5	486.30	+ 0.40	
2.	−				
		486.1	486.22	+ 0.25	
3.	+				
		486.2	486.25	−0.10	
4.	−				
		486.5	486.35	−0.30	
5.	+				
		486.2	486.20	0.00	
6.	−				
		485.9	486.25	+ 0.70	
7.	+				
		487.0	486.48	+ 1.05	
8.	−				
		486.0	486.72	+ 1.45	
9.	+				
		487.9	487.05	+ 1.70	
10.	−				
		486.4	487.35	+ 1.90	
11.	+				
		488.7			

	8. induction of the iron bar for *normal current.*				
no. of the oscillation	position of the auxiliary commutator	position of the needle at the beginning and end of each oscillation	rest position of the needle	oscillation arc of the needle	deflection of the compass
		461.0			
1.	+				
		457.2	464.85	−15.30	31° 48’
2.	−				
		484.0	467.17	−33.65	westward
3.	+				
		443.5	466.30	−45.60	
4.	−				
		494.2	466.73	−54.95	
5.	+				
		435.0	466.10	−62.20	
6.	−				
		500.2	466.47	−67.45	
7.	+				
		430.5	466.25	−71.50	
8.	−				
		503.8	466.55	−74.50	
9.	+				
		428.1	466.55	−76.90	
10.	−				
		506.2	466.90	−78.60	
11.	+				
		427.1	467.05	−79.90	
12.	−				
		507.8	467.38	−80.85	
13.	+				
		426.8	467.35	−81.10	
14.	−				
		508.0	467.35	−81.30	
15.	+				
		426.6	467.35	−81.50	
16.	−				
		508.2	467.33	−81.75	
17.	+				
		426.3			

2.9 Computation of the Measurements

If one starts counting the oscillation arcs starting from the one closest to zero, the ones coming closest to the limit can be reduced to the *limit* by dividing the n'th oscillation arc by $(1 - 0.752^n)$ in view of the well-known logarithmic decrement of the decrease of oscillation arcs $= 0.12378$. Hence the following reduced values are obtained for the experiments carried out for bismuth:

	oscillation arc	observed	reduced	average
	8.	+11.40	+13.20	
2.	9.	+12.25	+13.65	+13.60
	10.	+12.70	+13.75	
	11.	+13.00	+13.80	
	8.	−9.60	−14.12	
3.	9.	−9.95	−13.10	−13.08
	10.	−9.85	−12.02	
	9.	+10.00	+13.17	
4.	10.	+10.75	+13.12	+13.06
	11.	+11.30	+13.08	
	12.	+11.30	+12.88	
	10.	−10.10	−12.33	
	11.	−10.55	−12.21	
5.	12.	−11.00	−12.25	−12.16
	13.	−11.30	−12.24	
	14.	−11.45	−12.15	
	15.	−11.25	−11.76	
	11.	+8.90	+10.86	
	12.	+9.70	+11.23	
6.	13.	+10.05	+11.20	+10.95
	14.	+10.25	+11.10	
	15.	+10.15	+10.77	
	16.	+10.10	+10.56	

If one denotes the small influence by x, which the *thermomagnetic* current had on the result of these measurements, one obtains from the values above the *limit* corresponding to the diamagnetic induction alone reduced to *normal current*:

from 2.	$+13.60 + x$	
		+13.34
from 3.	$+13.08 - x$	
		+13.07
from 4.	$+13.06 + x$	
		+12.61
from 5.	$+12.16 - x$	
		+11.555
from 6.	$+10.95 + x$	

Hence on average

$$= +12.644\ .$$

From this *limit* of the oscillation arcs found according to the method of multiplication *for uniform distribution* of the induction pulses on the whole swinging period of the needle, it is now easy to derive the limit value, which would have been obtained by the same method of multiplication if all induction pulses instead of being distributed on the whole oscillation period were concentrated at the *moment*, where the needle passed its rest position. In this way the result obtained for *bismuth* can be compared to the one obtained for *iron*. Namely, by using the well-known logarithmic decrement of the decrease of swinging arcs $0.12378 = \lambda \log e$, where e denotes the unit of the natural logarithm, one finds from the above limit the desired one by multiplication with

$$\frac{\sqrt{\pi^2+\lambda^2}}{1+e^{-\lambda}} \cdot e^{-\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}} = 1.574\,235 \ .$$

Hence the desired limit is

$$+1.574\,235 \cdot 12.644 = +19.905 \ .^{22}$$

The reduction to the *limit* of the experiments carried out with *iron* leads to the following results:

[22][Note by WW:] If there are many induction pulses distributed uniformly on the whole oscillation period, they act as a constant current on the needle. In this case the rule mentioned on pp. 440 and 487, [[Web52c, p. 440 of Weber's *Werke*] with English translation in [Web21b], and [Web52b, p. 487 of Weber's *Werke*] which is equivalent of page 24 of this translation], can be applied to the *limit* x found according to the method of multiplication. According to this rule one has $x = 2E \cdot (1+e^{-\lambda})/(1-e^{-\lambda})$, where E is the deflection corresponding to the equilibrium of the needle in case of a constant current and $\lambda \log e$ denotes the logarithmic decrement of the decrease of oscillation arcs. At this equilibrium position of the needle the deflecting force equals the directive force of the needle, which is given by $\pi^2/T^2 \cdot E$, where T is the oscillation period without the influence of damping. It τ denotes the actual oscillation period taking damping into account, then the velocity the needle obtains is $= \pi^2/T^2 \cdot E\tau$. This happens under the assumption that the current force evenly distributed on the whole oscillation period acts concentrated at one moment. From this velocity one can compute the *limit* of the oscillation arcs, which one approximates according to the method of multiplication in case that the concentrated force always acts on the needle when it passes its rest position. In fact, if one denotes the limit by y, then according to the rule given in the previous article on p. 440, [[Web52c, p. 440 of Weber's *Werke*] with English translation in [Web21b]], by plugging in the value $= \pi^2/T^2 \cdot E\tau$ for the velocity one obtains

$$\frac{\pi^2}{T^2} \cdot E\tau = \frac{y}{2} \cdot \frac{\pi}{T}(1-e^{-\lambda})e^{\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}} \ .$$

Comparing the value of y with the above given value of x leads to the proportion

$$y : x = \frac{\pi\tau}{T} e^{-\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}} : (1+e^{-\lambda}) \ ,$$

where according to the theory of *damping* the quotient τ/T can be replaced by $\sqrt{1+\lambda^2/\pi^2}$.

oscillation arc	observed	reduced	average
8.	−71.50	−84.98	
9.	−74.50	−84.60	
10.	−76.90	−84.47	
11.	−78.60	−84.28	
12.	−79.90	−84.16	
13.	−80.85	−84.04	−83.876
14.	−81.10	−83.50	
15.	−81.30	−83.10	
16.	−81.50	−82.85	
17.	−81.75	−82.78	

From this one obtains for the ratio of the two limits corresponding to the *bar of bismuth* and the *bar of iron*

$$+19.905 : -83.876 \ .$$

Similar experimental series were carried out in the same way by myself, Dr. von Quintus Icilius and Dr. Riemann, where the following ratios were found

$$+18.158 : -83.82 \ ,$$

$$+15.357 : -82.80 \ ,$$

$$+14.890 : -83.45 \ .$$

Averaging all series one obtains the ratio

$$+16.956 : -83.49 \ .$$

Now the intensity of the currents induced from the *bar of bismuth* and the *bar of iron* is directly proportional to these limits und inversely proportional to the *number* of induction pulses during an oscillation, i.e., the number $10.58 \cdot 20.437 = 216.2$ for the *bar of bismuth* and 1 for the *bar of iron.* Hence the electric currents induced from the *diamagnetic bar of bismuth* are according to their direction *opposite* to the ones induced from the *magnetic bar of iron* and the ratio of their intensities is

$$16.956 : 83.49 \cdot 216.2 = 1 : 1064.5 \ ,$$

despite the fact that the bar of bismuth weighed 339 300 milligrams where the bar of iron just weighed 790.86 milligrams. From that one computes that if the bar of bismuth had the same small weight as the bar of iron, the strength of the *diamagnetically* induced current would have been 456 700 times less than that from the bar of iron *magnetically* induced current.

2.10 Comparison of the Two Determinations of the Strength of an Electrodiamagnet from Its Magnetic and Magnetoelectric Effects

After we considered in the previous two Sections the *magnetic* and *magnetoelectric* action of an *electrodiamagnet* individually, we finally compare *quantitatively* the two kinds of action.

It could seem that this comparison can be carried out quite easily by just first expressing the observed *magnetic* action of an electrodiamagnet in terms of the as well observed *magnetic* action of the electromagnet. Then one expresses the observed *magnetoelectric* action of an electrodiamagnet in terms of the as well observed *magnetoelectric* action of the electromagnet. This was already done above and led to the following results

$$1. \quad \frac{\text{magnetic action of the electrodiamagnet}}{\text{magnetic action of the electromagnet}} = \frac{1\,470\,000}{1}$$

$$2. \quad \frac{\text{magnetoelectric action of the electrodiamagnet}}{\text{magnetoelectric action of the electromagnet}} = \frac{1}{456\,700}\ .$$

This simple comparison would only be correct if first *the same electrodiamagnet* used for the representation of the magnetic effects would have been used as well for the representation of the magnetoelectric effects and secondly *the same electromagnet* would have been applied for the representation of both kinds of effects. Finally it would be necessary that the electrodiamagnet as well as the electromagnet acted from a larger *distance* compared to its own size and the one of the material acted on. However, these conditions were not met in the experiments described above and it was impossible to meet them since the representation of the magnetoelectric effects requires the application of quite different devices then the magnetic ones which forced us to make the distances of the materials acting on each other as small as possible.

However if one uses, as was actually the case, *different electrodiamagnets* and *different electromagnets* for the representation of the magnetic and magnetoelectric effects no equality in the mentioned ratios is expected even if they are acting from larger distances. The disparity, namely, that one ratio was about three times larger than the other one, would have been even much larger unless already for the determination of these ratios one took account of the difference of the *masses* of bismuth and iron used for the different electrodiamagnets and electromagnets. By taking into account the inequality of the masses, the coarsest occurring difference was balanced. It is interesting to remark that by taking this into account the above mentioned ratios actually got so close to each other that they can be considered as *quantities of the same order*.

The task at hand is now to detect and determine the other differences which after the difference in mass have the largest influence in order to check how the above ratios change and if they get *closer* to equality.

The reason why this examination is important is that if the used electrodiamagnets and electromagnets were not different at all and acted from a larger distance, the two ratios would have been *quite the same* according to *the laws of diamagnetic polarity discussed in the Introduction.* Since this equality cannot be directly checked in practice, it is important to check at least if one *approximates* this equality the more one takes into account the difference of the electrodiamagnets and electromagnets and the influence the small distance they are acting from has on the *ratio* of their actions. In this way one achieves the same *by approximation* as if one were able to check the claimed equality directly.

The following survey and discussion of all possible differences in question serves this purpose.

In view of the small distance the observed effects refer to, first the *ideal distribution of the magnetic fluids* on the surface of the bar of bismuth compared to the one of the bar of iron should be known more closely. Since this is not the case, it is obvious that such a comparison even if the exactness of the observations were perfect only gives a rough estimate, because the actions effected at small distances have to be put proportional to the *moments*, what strictly speaking is only the case for actions acting at larger distances.

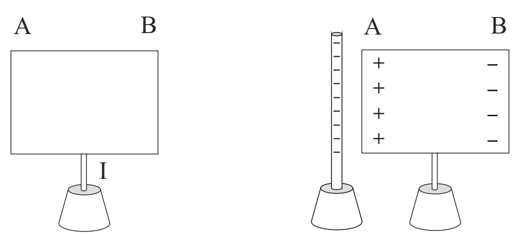
Secondly for the above experiments *two different iron bars* were used, one had a weight of just 5.8 milligrams where the other one was 790.86 milligrams. We cannot assume that the iron of both little bars behaves in magnetic respect quite the same. Therefore the magnetism of both little bars subject to the same galvanic current was compared and indeed for small intensity of this current the ratio of the magnetic moments differed considerably from the ratio of their masses. However, for increasing intensity of the current, this disparity disappeared and the magnetism of both little bars turned out soon to be almost exactly proportional to their masses. It follows that for our experiments where even more intense currents were used, a reduction due to the heterogeneity of iron was not necessary.

Thirdly in the above experiments *different bars of bismuth* were used, namely two smaller ones for the observation of the magnetic effects and a larger one for the magnetoelectric effects. It cannot be supposed that they behave completely the same in diamagnetic respect. Therefore the latter one was divided into two halves which compared to the former two ones almost coincided in terms of length and thickness. Then with both pairs alternately some experiments to compare diamagnetism were carried out from which a not quite insignificant difference turned out. The effect of the first pair compared to the second one was like 1266 : 1000. Hence if from the induction effects of the larger bar according to the two previous Sections the diamagnetic moment of bismuth compared to the magnetic moment of iron turns out to be $= 1/456\,700$, then one obtained for bismuth of the other bar $= 1/360\,740$, which does not decrease the difference of this ratio from the one deduced from magnetic actions but even increases it.

Fourthly one should consider the *difference of the electromagnetic separating force*[23] of

[23][Note by AKTA:] In German: *Elektromagnetischen Scheidungskraft*. This expression can also be translated as "electromagnetic force of separation" or "electromagnetic segregating force".

I present here a simple example of a separating force. Consider a metal plate AB insulated from the ground by a dielectric support I as in Figure (a) of this footnote:



(a) (b)

If a negatively charged straw is placed close to side A of the plate, the charges on the plate become separated as illustrated in Figure (b). Side A of the plate becomes positively electrified, while side B becomes negatively electrified. This polarization of the plate is caused by the electric force of the negatively electrified straw acting on the free electrons of the plate. I presented several interesting experiments on this topic made with simple material, together with many quotes from original sources, in the 2 volumes of the book *The Experimental and Historical Foundations of Electricity* which is available in English, Portuguese, Italian and Russian: [Ass10a], [Ass10b], [Ass15b], [Ass17], [Ass18a], [Ass18b] and [Ass19b].

Another effect of a separating force takes place in electrolysis. The electric forces in general are proportional to the charge q of the test particle on which they are acting. A positively electrified particle with $q > 0$ experiences a force in one direction, while a negatively electrified particle with $q < 0$ will be forced in the opposite direction. If these particles are free to move as in electrolysis, a double current will be produced due the two devices used. This difference can be deduced with sufficient exactness from the designations of these devices and it turned out that the electromagnetic separating force of the inductor was 4.8 times larger than the one of the electrodiamagnetic measuring device.[24] At the same time it follows that in both devices the electromagnetic separating force had such a *strength* that according to the interesting experiments of Müller[25] the magnetic moment of the *little iron bar* could not differ considerably from its maximal value,[26] so that the 4.8 times larger separating force of the inductor did not induce a stronger magnetism in the little iron bar than it obtained from the ordinary force. A different behaviour show the *bars of bismuth*

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to this separating electric force. That is, the positive particles will move in one direction and the negative particles will move in the opposite direction.

[24][Note by WW:] The wire spiral of the electrodiamagnetic measuring device according to Section 2.2 had four layers each consisting of 146 turns and was 190 millimeters long. Its interior diameter was 17 its exterior one 26 millimeters and the intensity of the current was according to Section 2.3 = 16.31. It follows from this that the electromagnetic separating force in its middle is quite close

$$= \frac{4 \cdot 146 \cdot 2\pi \cdot 16.31}{\frac{1}{2} \cdot 190} = 629.9 \ .$$

On the other hand the wire spiral of the inductor according to Section 2.7 had eight layers each consisting of 120 turns and was 383 millimeters long. Its interior diameter was 23.9 its exterior one 70 millimeters and the deflection of a compass laying 708 millimeters to the West was according to the experiments in Section 2.7 around 32° where one has to put the intensity of the horizontal part of terrestrial magnetism = 1.8. From this one can first compute the intensity of the current i and the result is quite close to

$$i = \frac{383}{S} \cdot \frac{1.8 \cdot \tan 32^\circ}{\frac{1}{(708-\frac{1}{2}383)^2} - \frac{1}{(708+\frac{1}{2}383)^2}} \ ,$$

where S denotes the area enclosed by the spiral which was found = 1 793 200 square millimeters, hence $i = 95.6$. The separating force of the spiral in question follows from this very closely $= \frac{8 \cdot 120 \cdot 2\pi \cdot 95.6}{\frac{1}{2} \cdot 383} = 3\,012$. However 3 012 : 629.9 equals in very good approximation 4.8 : 1.

[25][Note by AKTA:] [Mül51b] and [Mül51a].

[26][Note by WW:] A soft iron bar attains a weaker and a stronger magnetism off and on depending on the size of the magnetic or electromagnetic separating force acting on it. Professor Joh. Müller in Freiburg published an interesting examination of the dependence of the magnetism of such iron bars on the strength of the separating forces acting on them in "Berichte über die neuesten Fortschritte der Physik", Braunschweig 1850, p. 494 et seq., [Mül51a]. An interesting point of this publication is that the magnetism of iron bars has been determined for different, even very large, separating forces. From that the remarkable result followed that the magnetism of the iron bar is not at all always proportional to the separating force acting on the iron, but that it approaches a *limit* for increasing separating forces. Müller summarized the results he measured with an electromagnetic spiral in the following formula

$$s = 0.016 \cdot d^{\frac{3}{2}} \cdot \tan \frac{m}{0.001\,08 \cdot d^2} \ ,$$

where, if i denotes the intensity of the current of the electromagnetic spiral in terms of absolute measure (according to page 252 ibid.)

$$i = 66.813 \cdot s \ ,$$

and (according to p. 511) if M denotes the magnetism of the iron bar in the electromagnetic spiral according to absolute measure, then

$$M = 5\,426\,021 \cdot m \ .$$

The iron bars used by Müller were 330 millimeters long (according to p. 502) and laid in a wire spiral which was 300 millimeters long protruding on both sides 15 millimeters. d denotes the thickness of the iron bar. The wire spiral consisted of five layers each having 76 turns. Its interior diameter was 49 millimeters and the thickness of the wire was 2.8 millimeters. Consequently, the strength of *the separating force* the current of one layer of turns whose radius = r exerts on a point in the iron bar laying at a distance = a from the whose diamagnetic moment has to be assumed even for the largest representable separating

spiral is given by the following expression

$$\frac{2\cdot 76}{300}\pi r^2 i\int_{a-150}^{a+150}\frac{dx}{(r^2+x^2)^{\frac{3}{2}}}=\frac{152}{300}\pi i\left\{\frac{a+150}{\sqrt{(a+150)^2+r^2}}-\frac{a-150}{\sqrt{(a-150)^2+r^2}}\right\}.$$

This implies that on average for the whole iron bar the strength of the force is given by

$$\frac{152\cdot\pi i}{300\cdot 330}\int_{-165}^{+165}\left[\frac{a+150}{\sqrt{(a+150)^2+r^2}}-\frac{a-150}{\sqrt{(a-150)^2+r^2}}\right]da$$

$$=\frac{304\cdot\pi i}{99\,000}\left\{\sqrt{315^2+r^2}-\sqrt{15^2+r^2}\right\}.$$

Finally for all five layers

$$\frac{304\cdot\pi i}{99\,000}\cdot\frac{5}{14}\int_{24.5}^{38.5}\left[\sqrt{315^2+r^2}-\sqrt{15^2+r^2}\right]dr=13.562\cdot i\ .$$

This force differs from the *terrestrial magnetic* force only by its strength and can therefore be determined according to the same *absolute measure*, what also happened here. We denote the strength of this force by X, so that

$$X=13.562i\ .$$

Plugging these values into Müller's equation one obtains

$$X=14.498\cdot d^{\frac{3}{2}}\cdot\tan\frac{M}{5860\cdot d^2}\ .$$

This formula is just valid for iron bars of length 330 millimeters. To apply it to bars with a different length the arc $M/(5860\cdot d^2)$ has to be multiplied by 330 and divided by the length ℓ of the bar, hence

$$X=14.498\cdot d^{\frac{3}{2}}\cdot\tan\frac{M}{17.76\cdot d^2\ell}\ .$$

However, Müller himself remarked that the influence of the length taken into account in this way does not completely coincide with experience and has to be checked in more detail. If one applies this rule deduced from the experiments by Müller, to determine the magnetism of the two little bars of iron, which were in the above described spirals of the *electrodiamagnetic measuring device* and the *induction apparatus*, one gets for the *first* little bar $\ell=92$ and in addition for its absolute weight $=5.8$ milligram and its specific weight $=7.78$, from which for its thickness $d=0.1016$. The value of X for this little bar was determined in the previous Note $X=629.9$. Therefore one obtains for this little bar

$$\frac{M}{d^2\ell}=17.75\operatorname{arc\,tang}89^\circ 57'23''=27.886.$$

For the *second* little bar one has $\ell'=186$. In addition its absolute weight $=790.86$ milligrams and its specific weight $=7.78$, so that one finds for its thickness $d'=8342$. The value of X' for this little bar is determined in the previous Note $X'=3\,012$. One obtains

$$\frac{M'}{d'^2\ell'}=17.76\operatorname{arc\,tang}89^\circ 47'23''=27.834.$$

Noting that $d^2\ell$ and $d'^2\ell'$ are proportional to the masses of the two little iron bars, one obtains an almost equal ratio between magnetism and mass of the two little bars, although on the second little bar a 4.8 times larger separating force was acting. A more thorough treatment one finds in Sections 2.24 until 2.26 where as well the doubts expressed by Buff and Zamminer against the experiments by Müller are discussed, [BZ50].

forces as proportionally increasing.[27] Hence if one reduces the result obtained from the induction effects to a 4.8 times weaker separating force in order to make it comparable to the results obtained from the magnetic action, the diamagnetic moment of bismuth has to be assumed to be 4.8 times smaller while the magnetic moment of iron remains unchanged. One then obtains for the former moment compared to the latter one instead of $1/360\,740$ merely $1/4.8 \cdot 360\,740 = 1/1\,731\,560$.

This result deduced from the *magnetoelectric* action can now be compared directly to the one found in Section 2.4 according to which the diamagnetic moment of bismuth compared to the magnetic moment of iron was obtained to be

$$= \frac{1}{1\,470\,000} \,.$$

The difference of the two considered ratios which before was 200 percent is reduced to 17 until 18 percent by taking into account the mentioned difference. This approximation of equality has to seem even more satisfactory since the comparison is only rough due to the fact that the mentioned reason of that difference could not be considered. One should also observe that the last mentioned far most influential reason of this difference is capable of a closer consideration, if instead on the above quoted experiments by Müller the analysis is based on the more precise results described in Section 2.24 until Section 2.26. By doing that the ratio $1/1\,470\,000$ is reduced to $1/1\,593\,000$ as explained in Section 2.27 so that only a difference of about 8 percent remains compared to the other ratios.

After this comparison of the ratio of the magnetic and electromagnetic effects of an *electrodiamagnet* with the ratio of the magnetic and magnetoelectric effects of an *electromagnet* the result is confirmed that in the nature of diamagnetism the *electrodiamagnetic* and the *diamagnetoelectric* efficacy is actually justified in the same way as in the nature of magnetism the *electromagnetic* and *magnetoelectric* one. In fact, the diamagnetic effects in their magnitude have the same ratio as the magnetic ones as far as this can be checked. This proves that between *diamagnetic and magnetic efficacy* in manifold aspects *there is no difference*. This gives a proof of the law mentioned in the Introduction of *diamagnetic polarity*.

It only remained to use the results of the above experiments to determine the ratio between the *strength* of diamagnetism of bismuth and the strength of iron magnetism. The previous discussions make it clear that in general one cannot speak of a *definite* ratio between the diamagnetism of bismuth and the magnetism of iron. Indeed, even if one supposes that the bars of bismuth and iron have the same size and form, this ratio heavily depends on the

[27][Note by WW:] There is no known fact which shows a deviation of the law of proportionality of diamagnetism with the magnetic separating force. Instead of that, although measurements are missing, different facts in favour of this law can be mentioned. The most important one and as well in different aspects the most interesting one is the fact discovered and examined more closely by Plücker, according to which *the same magnetic pole depending on the distance induces in the same material* for example charcoal *diamagnetism or magnetism*. The closer examination which Plücker communicated in Poggendorff's *Annalen* 1848, Vol. 73, pp. 616 *et seq.*, [Plü48], proves, that here *the different distance of the magnetic pole has not to be considered directly* but just *indirectly, as a decrease of the force corresponds to a larger distance*. Plücker namely proved that *the magnetism of charcoal is transformed to diamagnetism by the mere increase of the magnetic force acting on the charcoal*. The simplest explanation for this interesting fact is the above mentioned law of proportionality of diamagnetism with the magnetic separating force, as soon as one assumes the law proved by Müller for the magnetism of iron as well for charcoal. Indeed, if the magnetism of charcoal for increasing separating force approximates a *limit* while the diamagnetism of charcoal increases uniformly, it is obvious that diamagnetism finally has to outmatch the magnetism, meaning that the magnetism of charcoal is transformed into diamagnetism.

strength of the magnetic *separating force*. While the diamagnetism is increasing uniformly with increasing separating force, the magnetism approaches a limit. Therefore such a ratio can only be determined under the constraint that the magnetic separating forces are so *small* that the deviation of the magnetism of iron is roughly proportional to these forces. Under this constraint it could be determined the ratio of the *diamagnetism of bismuth* to the *magnetism of iron* using the law of Müller referred to in the footnote 26 of this Section. However, it is advantageous to postpone this determination in order to take into account for the magnetism of iron as well the experiments we get to know in Section 2.25 and Section 2.26 where we add the determination of this ratio.

2.11 The Experiments of Faraday

We do not discuss here the former experiments of Faraday which led him to the assumption which Plücker phrased in the shortest way by saying: "In Bismuth each North pole of a magnet induces a North pole and each South pole a South pole".[28] Plücker says about this assumption that each physicist has to come up with it and that *diamagnetic polarity* is a necessary consequence of it. We restrict ourselves here to these experiments, which Faraday recently carried out to disproof the by him first conjectured *diamagnetic polarity*.

In fact soon after it was realized how important the actual proof of *diamagnetic polarity* is, many and various facts were found and communicated so that this polarity seemed almost to be beyond doubt. In my first article (*Berichte der Königl. Sächs. Gesellschaft der Wissenschaften 1847*, p. 346 and Poggendorff's *Annalen* 1848, Vol. 73, p. 242)[29,30] I stressed in particular the evidence the experiment of Reich has in this aspect.[31] According to this experiment, if North and South pole act from the same side to a piece of bismuth, they repel it in no way with the sum of the forces they are exerting individually, but rather with the difference of these forces. I added other experiments which allowed to recognize *both poles* of a bar of bismuth in a diamagnetic state by the contrast of *attraction* and *repulsion*. Finally, I added the experiments with the device mentioned in Section 2.7 which seemed to detect similar *electromotive forces* exerted from *diamagnetic poles* as well as from *magnetic poles*. Some experiments by Poggendorff, *Annalen* 1848, Vol. 73, pp. 475)[32] followed immediately, which on the one hand served as a confirmation, on the other hand as a supplement. In particular, they provided evidence for the two *diamagnetic poles* by the contrast of the effect which the *galvanic current* is exerting on them. They downright proved that a bar of bismuth in equatorial position would be an actual *transversal magnet*, which turns the line of its North poles to the North pole and the line of its South pole to the South pole of the magnet. Plücker (*Annalen* 1848, Vol. 73, p. 613)[33] found this confirmed by a very smart application based on that, which provided a simple and practically important mean to intensify considerably the diamagnetism of swinging bodies. Plücker himself declared it beyond doubt that the *diamagnetism* consists of a *polar excitement*. Before that he discarded this theory due to the enormous difficulties to justify it. After polarity was confirmed in such a decisive manner he revived the theory. Finally in this article Plücker overcame one of the

[28] [Note by AKTA:] [Plü48, pp. 614-615].
[29] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 255.
[30] [Note by AKTA:] [Web48b], [Web48c] with English translation in [Web52d] and [Web66c].
[31] [Note by AKTA:] [Rei48] and [Rei49].
[32] [Note by AKTA:] [Pog48].
[33] [Note by AKTA:] [Plü48].

most important difficulties mentioned by him, namely the difficulty due to the for many materials observed *magnetic* behaviour in *larger* distance from the magnetic pole and the *diamagnetic* behaviour in *smaller* distance (see the footnote 27 in the previous Section). In view of his closer examination he himself said that[34]

> "the by him not believed, but from a theoretical point of view expected result instead of the former difficulties found a remarkable confirmation of the adopted theory of diamagnetism from Faraday, Reich, Weber, and Poggendorff, to which he now became as well a resolute supporter".

All this confirmations of *diamagnetic polarity* first conjectured by Faraday complemented each other quickly and appeared in the same Volume 73 of Poggendorff's *Annalen*. However, Faraday himself contradicts it in his 23. series of experiments,[35] whose closer consideration is of importance as well for the here described experiments.

In view of the very well deserved authority this great scientist has and the interest his works stir everywhere we can assume that his experiments to disprove *diamagnetic polariy* are well-known. Moreover, there is no doubt on the validity of these experiments in view of Faraday's acknowledged experimental skills. The question is just if and how far these experiments disprove *diamagnetic polarity* as defined here right at the beginning. There are mainly three points to consider. *Firstly*, Faraday did not repeat all experiments carried out to prove diamagnetic polarity. *Secondly*, despite his outstanding skills Faraday restricted himself in the accuracy of the instruments he used. *Thirdly*, Faraday tried to explain in a different way many phenomena which are in the opinion of other physicists due to *diamagnetic polarity*. Therefore it is even not clear if Faraday really contradicts *diamagnetic polarity* in the sense we defined it at the beginning.

Concerning the experiments which are not repeated and considered by Faraday, I first mention that in paragraph 2689 of his article an experiment carried out by me seems to be confused with a one carried out by Reich. Therefore it happened that Faraday completely overlooked the experiment by Reich whose evidence for *diamagnetic polarity* I stressed in particular. According to this experiment North and South pole acting simultaneously from the same side on a piece of bismuth do not repel it with the sum of their individual forces but with their difference. This experiment was carried out by Reich with the most accurate instrument available, namely the torsion balance he used for the classical repetition of the experiments by Cavendish.[36] I can only repeat here what I said in my first article on this experiment, that through it alone it can be deduced with high probability that the reason for the diamagnetic force lies in a moveable imponderable ingredient existing in bismuth which is moved and distributed in different ways when a magnetic pole is approximating it. The simultaneous approximation of two opposite poles from the same side has then namely the effect that the imponderable ingredient neither can assume the one or the other movement or distribution responsible for the appearance of the diamagnetic force, which explains the vanishing of this force. Furthermore one has to mention in this context the experiments carried out by Poggendorff and described in the same volume 73 of his *Annalen* (p. 475–479),[37] through which he obtained by a simple convincing experiment in two ways the same

[34] [Note by AKTA:] [Plü48, p. 618].

[35] [Note by AKTA:] [Far50].

[36] [Note by AKTA:] Henry Cavendish (1731-1810).

[37] [Note by AKTA:] [Pog48].

result without the help of subtle measuring devices. There is no difficulty to repeat the experiments by Poggendorff and many observers carried this out.

Among the devices which allow an even higher degree of fineness and accuracy then the ones used by Faraday are mainly the *magnetometer* and the *galvanometer* arranged according to the instructions of Gauss. I would not have been able at all to carry out my experiments without these instruments. When Faraday repeated these experiments without the help of these instruments it is easily explainable that he was not able to see the very weak effects I observed. Faraday's major concern against my observations described in volume 73 of Poggendorff's *Annalen* is, that I did not mention the by him with great care observed *secondary Volta induction*, which I should have been able to see the more clearly the finer my instruments are. Therefore I mention here, that the above article in Poggendorff's *Annalen* borrowed from the "Berichten der Königl. Sächs. Gesellschaft der Wissenschaften"[38] was just a preliminary note of my work, where the more specialized discussion was postponed to a later article. It should be sufficient to add here that in those experiments I tried to eliminate the influence of the secondary Volta induction as far as possible by a proper combination of the experiments, that it is however not highly preferable at all to remove this influence completely as happened in the experiments described in this article.

Let us briefly summarize which influence the investigation of Faraday had on the question of *diamagnetic polarity* in the sense as defined at the beginning. This influence should be of minor importance. Faraday namely overlooked several experiments by Reich and Poggendorff. Concerning different experiments, namely the ones by Plücker, he just gave an explanation based on different premises, where it is not clear if these premises contradict *diamagnetic polarity* as defined here at the beginning. Finally, related to the doubt Faraday mentions about the validity of the results of my experiments, *firstly* this doubt should be removed by the remark above, *secondly* it has no application to the experiments described in this article.

2.12 The Experiments and the Theory of Feilitzsch

In Section 2.3 and Section 2.4 it was proved that a bar of bismuth in a galvanic spiral as an *electrodiamagnet* exerts on a magnetic needle a torque in *opposite direction* than an iron bar exerts in the same spiral as an *electromagnet*. This contradicts a result of Feilitzsch who, inspired by a different theory, expected a different result and tried to confirm it by experiments (see Poggendorff's *Annalen* 1851, Vol. 82, p. 90–110).[39] Namely he thought that:[40]

> "bismuth inside the electric spiral receives a weaker, but equaly directed polarity, as soft iron."

The reason for this contraction as I believe lies in a very essential difference of the devices used by me and Feilitzsch. Feilitzsch mentioned that[41]

> "the spiral was deployed at a distance of about 200 millimeters on the western side from a small compass suspended on a cocon thread and the needle was brought back to its initial position *by an auxiliary magnet on the eastern side*".

[38] [Note by AKTA:] [Web48b], [Web48c] with English translation in [Web52d] and [Web66c].
[39] [Note by AKTA:] [Fei51].
[40] [Note by AKTA:] [Fei51, p. 103].
[41] [Note by AKTA:] [Fei51, p. 103].

In contrast to that I used two spirals and deployed them symmetrically with respect to the compass so that no *auxiliary magnet* was necessary to bring back the needle to its initial position. The crucial difference of the two arrangements is that in Feilitzsch's case the needle only for a *determined current intensity* lies in the magnetic meridian, but is deflected to either side for each *variation* of the current intensity. On the other hand in my case the variations of the current intensities have no influence on the position at rest of the needle. However, this *independence of the position at rest of the needle from the variations of the current intensity in the spiral* is necessary if the deflection of the needle has to be associated to *the immediate effect of the bar of bismuth on the needle* when the bar of bismuth is put into the spiral. Namely putting the bar of bismuth into the spiral effects a small change of the intensity of the current and this might be in Feilitzsch case the reason for the deflection of the needle. Namely putting the *cold* bar of bismuth into the spiral *heated* by the current leads to a *cooling* of the spiral and therefore an *increase* of the current intensity, which necessarily creates a *deflection of the needle in the direction observed by Feilitzsch*. A long time ago I carried out several experiments according to the same method as Feilitzsch and found similar results. However, a closer examination showed, that the observed force did *not* appear *instantaneously* in the moment the bar of bismuth entered, but rather *gradually*. Also when pulling out the bar the force disappeared *gradually*, what is a sufficient proof that it is not a matter of an *instantaneous* action of the bar of bismuth. One could also increase, decrease, or reverse these influences through a mere cooling or heating of the bar of bismuth. It is likely, that as well the deflections of the needle observed by Feilitzsch are due to the influences of temperature on the intensity of the current.

Concerning the *theory of diamagnetism* which Feilitzsch tried to give in this context, I just want to mention the following. Feilitzsch wants to explain the diamagnetic phenomena from a certain distribution of magnetic fluids, too. However, he assumes that this distribution is due to the *separation of magnetic fluids in the same direction as in iron* and that the only difference is that this separation in an iron bar *decreases* from the middle to the ends, while in the bar of bismuth it *increases*. It follows from this *increase* between the middle and the end of the bar a dispersion of opposite free magnetism as at the end, and if this opposite between the middle and end dispersed free magnetism *were stronger*, than the one at the end, the diamagnetic phenomena could be explained. However, if Feilitzsch examined the conditions more closely which lead to an explanation of the diamagnetic phenomena according to his own presentation, he would have found that this case is only possible, if the magnetic fluids in the middle of the bar are not separated in the same but in opposite directions as at its ends, which contradicts his assumptions. Anyway, one easily sees that it is impossible to explain the diamagnetic phenomena from a distribution of magnetic fluids arising from the *same separation as in iron according to the direction*.

IV - On the Connection Between the Theory of Diamagnetism with the Theory of Magnetism and Electricity

2.13 On the Foundation of a Theory of Diamagnetism

In the first two parts of this paper I tried to establish the law of *diamagnetic polarity* in more generality, mainly by showing that it is valid as well for *electrodiamagnetic* and *diamagnetic* actions. This law alone even if it is general does not establish yet a *theory of diamagnetism*. This is because it only defines diamagnetism in view of its *effects*. However for the foundations of a theory of diamagnetism it is necessary to define it not just in view of its effects but as well in view of its *causes*. Therefore, I will add in this part the necessary complement to the theory on the *causes* of diamagnetism in more generality than what I did in my previous paper.

2.14 On the Way How to Examine the Causes of Diamagnetism

In the theory of magnetism one distinguishes two types of magnets, namely *permanent* ones and *variable* ones. For example a magnet made of glass-hard steel is a permanent one, while a magnet made of soft iron is a variable one. Strictly speaking in reality there is not a strict distinction between permanent and variable magnets, since even the most permanent ones become variable under the influence of strong forces, and in the same way all magnets even the ones made of the most soft iron become permanent under the influence of very small forces. However, since one usually chooses for physical experiments magnets and conditions under which either the permanent or variable aspect of the magnet does not show up, one can assume without loss of generality this simple distinction. For the sequel we point out the following difference between the two kinds of magnets. The permanent ones can only be examined in view of its *effects*, while the variable ones in two ways, namely in terms of its *effects* as well as in terms of its *causes*.

If one tries to apply this distinction to diamagnets, one sees, that *permanent diamagnets* do not exist, or more precisely, that they cannot be distinguished from permanent magnets. Therefore, one only needs to consider *variable diamagnets* and these can be examined in two ways partly by their *effects* and partly by their *causes*.

It is known, that by examining the effect of a magnet on other materials one can obtain the *ideal distribution* of the magnetic fluids on its surface. Gauss has shown that if one knows the *ideal distribution* one can predict all *effects* of the magnet.[42] Many researches take great profit that through its knowledge one does not need any hypothesis about the interior of the material, particularly, if the causes of these effects are unknown and first have to be examined. It is obvious from this that by examining the effects one cannot get further than to the knowledge of the ideal distribution which has to be distinguished necessarily from the true nature of the interior of the magnet. For example, it is not possible by examining the effects to get to know the actual distribution of the magnetic fluids in the magnet or the actual number, strength and position of the electric currents inside.

[42] [Note by AKTA:] See footnote 7 on page 11.

The same is therefore true as well for the effects of a diamagnet. One could get knowledge of the *ideal distribution* of magnetic fluids at the surface of the diamagnet and this replaced the knowledge of its whole true internal state concerning the consideration of all its effects. On the other hand one would not get information about the true internal state of the diamagnet nor the actual nature of diamagnetism nor its generation and transformation. To get a clue of these one must not restrict oneself to the consideration of the *effects* and the *ideal distribution* depending on it, but it is necessary to take into account different points of view which are independent of these effects.

All possible *causes* of *diamagnetism*, as well as of magnetism, can be classified into *internal* and *external* ones. The *external* cause, as the effects, is given through observation. It is the same for magnetism and diamagnetism, namely *a magnetic or electromagnetic separating force having determined size and direction*. Would we know apart from this external cause the internal one in the material itself, then diamagnetism would be determined. Conversely, this opens a way to determine the unknown internal cause if, in addition to the known external cause, the diamagnetism resulting from both is already known from its effects. If one follows the way sketched here and lists as well for iron and bismuth the known magnetic separating forces together with the from the effects deduced ideal distribution, one observes that the same separating force leads to opposite ideal distributions or conversely the same ideal distribution for iron and bismuth gives rise to opposite separating forces. The reason that opposite external causes produce the same effects in iron and bismuth has to be contained in the difference of *internal* causes in iron and bismuth themselves. To determine more closely the *difference of internal causes* in iron and bismuth it is necessary to classify all possible internal causes which can have such effects explainable in terms of an ideal distribution. After that one has to check if among these possible internal causes there are some which can give rise to the above mentioned differences in magnetic and diamagnetic materials.

2.15 Classification of Internal Causes which Can Give Rise to the Given Effects of an Ideal Distribution

One can give *four* essentially different kinds of *internal* causes contained in the materials which can give rise to such effects explainable in terms of an ideal distribution of magnetic fluids.

1. The internal cause of such effects can be due to the existence of two *magnetic* fluids which are more or less *movable independently from their ponderable carrier*.

2. It can be due to the existence of two *magnetic* fluids which are only *movable with the molecules of their ponderable carrier*, i.e., rotating molecular magnets.

3. It can be due to the existence of *permanent molecular currents built from the two electric fluids*, which can be rotated with the molecules.

4. It can be due to the existence of two *movable electric fluids*, which can become a molecular current.

These four here mentioned possible internal causes of the effects due to an ideal distribution at the surface are the only ones which are known and can be examined. The *first* case is the base of the theory of magnetism developed by Coulomb and Poisson.[43] The *third*

[43][Note by AKTA:] Charles Augustin de Coulomb (1736-1806) and Siméon Denis Poisson (1781-1840). See [Cou88a] with partial English translation in [Cou35a], complete English translation in [Cou12] and German translation in [Cou90b]; [Cou88c] with German translation in [Cou90d] and partial English translation in

case is the base of the theory of magnetism using electrodynamics developed by Ampère.[44] The *second* case can be reduced to the third one in view of the theorem due to Ampère that molecular magnets and molecular currents coincide in all their effects if one substitutes the first one for the latter one. It therefore just remains the *fourth* case which was not noticed and discussed before.

2.16 Dependence of the Ideal Distribution on the Magnetic Separating Force According to the Difference of the Four Above Mentioned Possible Internal Causes

For each of these four cases one easily obtains a connection between the type of *ideal distribution* and the *direction of the magnetic separating force* giving rise to the distribution. For the *first* case it follows according to the theory of Poisson, that if one denotes the direction of the magnetic separating force as the *positive* one in which the North pole of a magnetic needle points and if one determines the barycenters of the northern and southern fluid corresponding to the separating force of the corresponding ideal distribution, the former of these two barycenters is displaced in the *positive* direction with respect to the latter one. For the *third* case this connection was developed by Ampère and it follows that it leads to the same dependence of the ideal distribution from the magnetic separating force. It is obvious that the same dependence holds as well for the *second* case since the second case can be deduced from the third one as mentioned above. It therefore remains to discuss just the *fourth* case.

This *fourth* case assumes the existence of *electric fluids* which can become molecular currents. The possibility that such molecular currents develop is based on the assumption that in single molecules or around them there are closed orbits in which the fluids are movable *without resistance*. It follows from this, that only a *current-inducing force*, i.e., a force which acts on the positive and negative fluid in opposite directions, in the direction of this orbit is required to actually move the fluids in this orbit. The theory of magnetoelectricity implies that due to an increasing or decreasing intensity of the magnetic separating force actually an *electromotive force* is given, which acts on the two movable electric fluids in opposite direction and therefore has to induce a current. The *direction of the molecular current* is given by the fundamental law of magnetic induction depending on the increase or decrease of the magnetic separating force. Moreover, the *ideal distribution* is given in its dependence of the molecular currents according to the connection between electrodynamics and the theory of magnetism discovered by Ampère for the third case. It follows from that the connection between the *ideal distribution* and the *increase or decrease of the magnetic separating force* corresponding to the distribution.

[Cou35b]; [Cou88d] with German translation in [Cou90a]; [Cou88b] with German translation in [Cou90c]; [Cou89]; [Cou91]; [Cou93]; [Pot84]; [Gil71b] and [Gil71a]. See also [Poi12a], [Poi12b], [Poi13], [Poi25a], [Poi25b], [Poi22a] and [Poi22b].

[44][Note by AKTA:] André-Marie Ampère (1775-1836). His masterpiece was published in 1826, [Amp26] and [Amp23]. There is a complete Portuguese translation of this work, [Cha09] and [AC11]. Partial English translations can be found at [Amp65] and [Amp69]. Complete and commented English translations can be found in [Amp12] and [AC15].

A huge material on Ampère and his force law between current elements can be found in the homepage *Ampère et l'Histoire de l'Électricité*, [Blo05].

Moreover, it is clear from this that *in each moment* where an increase or decrease of the magnetic separating force occurs, such a molecular current is induced. Therefore the induced currents, if they do not cancel each other, have to be summed up. However, these currents do not disappear by themselves. Indeed, Ampère has shown that one has to associate to electrical molecular currents *permanence*, i.e., that the electric fluids on their circular motions around the ponderable molecules are not subject to such a *resistance* like the electric fluids flowing through a conductor which gives rise to an explanation for the quick disappearance of the electric currents in these conductors. (This *permanence* proved by Ampère for the molecular currents is the reason for the above mentioned theorem that the *possibility* to put electric fluids in a molecular current has as its hypothesis that there exist *closed orbits* in the individual molecules in which fluids can move *without resistance*.) It follows from this that through *continued* increase of the magnetic separating force *in the ideal distribution*, there has to occur a *continued* increase of magnetic fluids as well. It follows from this, that to *each given strength of the magnetic separating force there coincides a certain amount of ideally distributed fluids*. However, this summation only takes place for *molecular currents*, since only for them the movement of electric fluids has *no resistance*. The other currents, which are induced from the same separating force in additional orbits, which however due to the *resistance* they are subject in these orbits disappear quickly, only have magnetic effects on other materials *in the moment they are induced*. These effects immediately vanish as soon as the separating force, which was the reason for the *change*, becomes *constant*. Therefore they are in no determined ratio to the *existing* separating force, what would be necessary, if they should account for the observed magnetic effect for which therefore only *molecular currents* are useful. If one develops this dependence on the *molecular currents* more carefully according to the laws of *magnetic induction*, one finds, that when one denotes this direction as the *positive* one to which the North pole of a magnetic needle points and when one determines the ideal distribution of the barycenters of the northern and southern fluid depending on the separating force, that the former one of these two barycenters is displaced with respect to the latter one in the *negative* direction, i.e., *opposite to the other three cases*.

2.17 Internal Cause of Diamagnetism

This remarkable result can now be applied to justify the theory of diamagnetic phenomena which explains the internal state of a diamagnetic material and the forces responsible for it. Such a justification was not available before. In fact, it does not suffice for such a theory that one is able to represent the diamagnetic state of a material in connection with all its effects by an *ideal distribution* of magnetic fluids on its surface as already argued above. But it is essential to justify as well these *forces* through which that state occurred and, moreover, *on what* these forces act and *according to which laws* they act.

From the compilation and consideration of the possible cases above, through which a state of a material can occur representable by an *ideal distribution* of magnetic fluids, we found *only one [case]* compatible with the *fundamental phenomena* during the *emergence* of diamagnetism. It follows from this, that one can explain the emergence of a diamagnetic state of a material only if one assume that this case *really exists*. Namely the case where the diamagnetic state emerges due to the induced forces which acted on the material and *the electric fluids in the material which move without resistance on circular orbits*. Therefore one assumes that a bar of bismuth consists of molecules which *contain closed orbits* (or canals), *in which the electric fluids can move without resistance*, while in all other orbits they can only move after overcoming a resistance proportional to its velocity. The occurrence of a *pure* diamagnetism not intermingled with magnetism also requires, that the molecules together with those orbits or canals *cannot be rotated*. Otherwise *rotating molecular currents* would emerge leading to a *magnetic state*, if during the rotation their intensity does not change, as proved by Ampère.

2.18 Determination of the Electromagnetic Separating Force in a Galvanic Spiral

According to the presentation given above it is not the *magnetic or electromagnetic separating force* itself which is responsible for the diamagnetic state of a material, but this separating force determines diamagnetism only indirectly as far as *the sum of the electromotive forces* is concerned which before acted on the diamagnetic material and put the electric fluids into motion around the individual molecules. *The strength of the now existing (induced) molecular currents* which is the nature of *diamagnetism* depends on the *sum* of the electromotive forces having acted on the diamagnetic material. In this way the determination of the *intensity* of the *existing* magnetic or electromagnetic separating force is used only *indirectly* to the determination of *diamagnetism* since it gives rise to the *integral value of all changes* to which the magnetic or electromagnetic separating force was subject. To this integral value the sum of the electromotive forces and consequently the strength of the now existing (induced) molecular currents is proportional.

Suppose the wire of a galvanic current spirals uniformly around a cylindrical tube. Denote the *electromagnetic separating force* of the current at the midpoint of the tube in direction of the axis by X. According to the known electromagnetic laws it is given by

$$X = \frac{2\pi n i}{d}$$

where n is the number of windings, i is the intensity of the current, and $2d$ the diagonal of the tube (i.e., when $2a$ is the length of the tube and $2r$ is the diameter, then $d = \sqrt{a^2 + r^2}$).[45] If the intensity of the current i is expressed according to the in the previous paper on *Electrodynamic Measurements* (page 321 of this Volume)[46] determined absolute mass, then in the expression above the *electromagnetic separating force* is given by the same measure, which Gauss used for the determination of the *intensity of terrestrial magnetism*.[47] Strictly

[45][Note by WW:] In fact if r is the radius of a winding, x is the distance of its midpoint from the midpoint of the spiral, $rd\varphi$ the length of a current element and i the current intensity, it is well-known that $ir^2 d\varphi/(r^2 + x^2)^{\frac{3}{2}}$ is the expression for the force due to the current element in the midpoint of the spiral in direction of the axis. It follows from this that the expression of the force due to the whole winding is $2\pi r^2 i/(r^2 + x^2)^{\frac{3}{2}}$, and the expression for n windings of the spiral whose length is $2a$ becomes $2\pi r^2 i \cdot \frac{n}{2a} \int_{-a}^{+a} \frac{dx}{(r^2+x^2)}$, i.e., if one sets $\sqrt{a^2 + r^2} = d$ one obtains $\frac{2\pi n i}{d}$.

[46][Note by AKTA:] [Web52c, p. 321 of Weber's *Werke*] with English translation in [Web21b].

[47][Note by AKTA:] Gauss' work on the intensity of the Earth's magnetic force reduced to absolute measure was announced at the Königlichen Societät der Wissenschaften zu Göttingen in December 1832, [Gau32] with English translation in [Gau33a] and [Gau37a], see also [Rei02, pp. 138-150].

The original paper in Latin was published only in 1841, although a preprint appeared already in 1833 in small edition, [Gau41b] and [Rei19]. Several translations have been published. There are two German versions, one by J. C. Poggendorff in 1833 and another one in 1894 translated by A. Kiel with notes by E.

speaking the stated value of the *electromagnetic separating force* is valid only for the midpoint of the spiral. In most cases this value can be used with sufficient accuracy for a very large part of the space surrounded by the spiral, in particular, if the diameter of the spiral compared to its length is very small. For example if one considers a point on the axis which has the distance b to the midpoint of the spiral one obtains for this point

$$X = \frac{\pi n i}{a}\left[\left(1+\frac{r^2}{(a-b)^2}\right)^{-\frac{1}{2}} + \left(1+\frac{r^2}{(a+b)^2}\right)^{-\frac{1}{2}}\right] ,$$

or if one replaces a by $\sqrt{d^2-r^2}$ and r/d by ρ

$$X = \frac{2\pi n i}{d}\left[1 - \frac{3d^2-b^2}{2(d^2-b^2)^2}\cdot\rho^2 b^2 + \ldots\right] .$$

If the difference of the electromagnetic separating force and its maximal value at the midpoint shall be less than a small fraction m times the maximal value one sets

$$\frac{3d^2-b^2}{2(d^2-b^2)^2}\cdot\rho^2 b^2 = m$$

or

$$\frac{b^2}{d^2} = 1 + \frac{\rho^2}{4m+2\rho^2}\left(1 \pm \sqrt{\frac{16m}{\rho^2}+9}\right) .$$

Hence if the diameter is for example the 40th part of length, then in more than $\frac{7}{8}$ of the whole from the spiral enclosed space the electromagnetic separating force is up to 1 percent constant and in almost $\frac{2}{3}$ of this space it is constant up to $\frac{1}{10}$ percent.

Therefore such spirals can be used to provide in an easy way an arbitrarily long space in which the electromagnetic separating force has an exactly known, arbitrarily big and everywhere constant magnitude. The representation of such a space is however of great importance for many studies and the experiments described in the previous two Sections can serve as examples for this. In fact without using spirals it would have not been possible to carry out these experiments.

Strictly speaking the discussion above deals only with the points laying on the axis of the spiral. However, the result found can easily be extended to the remaining space enclosed by the spiral using a general theorem of Gauss in the "General theory of terrestrial magnetism" (Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1838), article 38.[48,49]

2.19 Determination of Electrodiamagnetism Using the Electromagnetic Separating Force

The integral value of the electromotive force on a circle of radius r for the time needed to move the circle from the perpendicular position with respect to the separating force to a

Dorn; a French version by Arago in 1834; two Russian versions, one by A. N. Drašusov of 1836 and another one by A. N. Krylov in 1952; an Italian version by P. Frisiani in 1837; an English extract was published in 1935, while a complete English translation by S. P. Johnson was published in 2003; and a Portuguese version by A. K. T. Assis in 2003: [Gau33b], [Gau34], [Gau36], [Gau37b], [Gau94], [Gau35], [Gau52], [Gau75], [Gau03] and [Ass03b].

[48] [Note by HW:] Gauss' *Werke*, Volume 5, p. 170.

[49] [Note by AKTA:] See footnote 7 on page 11.

parallel one was determined in the previous paper on *Electrodynamic Measurements* (page 323 of this Volume).[50] For the electromagnetic separating force given by $X = 2\pi ni/d$ one obtains

$$= \pi r^2 X \ .$$

This integral is the *product* of the *intensity* with the *element of time* during which the force with this intensity is acting.

The expression is unchanged if instead of turning the circle by 90° the electromagnetic separating force $X = 2\pi ni/d$ *vanishes*. On the other hand if this separating force is increased from $X = 0$ to $X = 2\pi ni/d$ (by closing the current), then the expression becomes

$$-\pi r^2 X = -\frac{2\pi^2 nr^2 i}{d} \ .$$

The *negative* sign means, that the induced current has such a direction, that the poles of a molecular magnet *equivalent* to it get an opposite orientation than the poles of a compass under the influence of the same force X.

For the determination of the *integral value* of the electromotive force we used the measure of electromotive forces deduced from the absolute measure of *magnetism* as explained in the previous paper, page 321.[51] For the purely *electrodynamic* measure this expression has to be multiplied by a factor $\sqrt{\frac{1}{2}}$, page 361 ibid., hence

$$-\frac{\pi}{\sqrt{2}} \cdot r^2 X = -\frac{\pi^2\sqrt{2} \cdot nr^2 i}{d} \ .$$

According to the previous paper this expression has to be multiplied (page 367 ibid.) by $4/c$ (where c denotes the constant value of the relative velocity for which two electric masses do not influence each other), if one wants to express the electromotive force in terms *of the absolute measure of forces utilized generally in mechanics*, hence

$$-\frac{2\sqrt{2}}{c} \cdot \pi r^2 X = -\frac{4\sqrt{2} \cdot \pi^2 nr^2 i}{cd} \ .$$

This expression gives *the electromotive force for the length of the circular path* under the assumption that in each unit of length of this path the unit of electric fluid is located. One obtains from this *the electromotive force* acting *on each unit of mass of the electric fluid* by division of the circumference of the circle $2\pi r$

$$= -\frac{\sqrt{2}}{c} \cdot rX = -\frac{2\sqrt{2} \cdot \pi nri}{cd} \ ,$$

i.e., the *integral value of the acceleration* for the interval of time in which the electromagnetic separating force grew from $X = 0$ to $X = 2\pi ni/d$, if to each unit of the electric fluid a ponderable unit of mass were attached. If ε denotes the unknown little fraction which expresses the mass belonging to the unit of electric fluid in terms of the ponderable mass measure, we obtain by dividing the above expression by ε the *drift velocity*[52] u produced by

[50][Note by AKTA:] [Web52c, p. 323 of Weber's *Werke*] with English translation in [Web21b].

[51][Note by AKTA:] [Web52c, p. 321 of Weber's *Werke*] with English translation in [Web21b].

[52][Note by AKTA:] In German: *Stromgeschwindigkeit*. This expression can also be translated as *current velocity* of *velocity of the current*. Weber is referring here to the velocity of each electrified particle relative to the matter of the conductor.

the *increase of the electromagnetic separating force*. If one multiplies this expression of the *drift velocity* u by $ae = 4e/c$ (see page 367 ibid.),[53] where e is the amount of the electric fluid expressed in terms of electric measure which is located in each unit of length of the circular path, one obtains the *intensity* of the *induced* circular current according to the measure derived according to purely *electrodynamic* principles (see page 359 ibid.). If one multiplies further this formula by $\sqrt{2}$ one obtains this intensity in terms of the measure according to which a current of intensity $= 1$ acts identically with the *unit of magnetism*[54] according to absolute measure if it circulates around the unit of area, namely

$$-\frac{8e}{c^2\varepsilon} \cdot rX = -\frac{16\pi nrei}{c^2 d\varepsilon} \ .$$

Here i denotes the intensity of the *induced* current according to the same measure.

The *electromagnetic moment* of this induced circular current (molecular current) is found by multiplying the *intensity of the current* stated above by the area πr^2 enclosed by the circular orbit

$$= -\frac{8e}{c^2\varepsilon} \cdot \pi r^3 X = -\frac{16\pi^2 nr^3 ei}{c^2 d\varepsilon} \ .$$

Here one assumes that the *normal* of the plane containing the circular orbit is parallel to the *direction* of the electromagnetic separating force. This can happen for *all* circular orbits only for a *particular* arrangement of the molecules. In case of *bismuth* we do not assume such an arrangement, but instead suppose according to the notion of *homogeneity* that the normals of the circular orbits do not have a prevailing direction. Hence the number of circular orbits whose normals have an angle φ with respect to the direction of the electromagnetic separating force is proportional to $\sin\varphi$. Therefore the *intensity of the current* is proportional to $\cos\varphi$ and the *component parallel to the separating force* to $\cos^2\varphi$. It follows that multiplying the expression above by $\sin\varphi\cos^2\varphi$ one obtains an expression proportional to the part of the *electrodiamagnetic moment* of bismuth coming from all circular currents (molecular currents) whose normals have an angle φ to the direction of the separating force, namely

$$-\frac{8e}{c^2\varepsilon} \cdot \pi r^3 X \cdot \sin\varphi\cos^2\varphi = -\frac{16\pi^2 nr^3 ei}{c^2 d\varepsilon} \cdot \sin\varphi\cos^2\varphi \ .$$

Integrating first this expression from $\varphi = 0$ to $\varphi = \frac{1}{2}\pi$ with respect to $d\varphi$ and multiplying the obtained *integral value* with the number of molecular currents m, one gets the whole *electrodiamagnetic moment* of bismuth expressed by

$$= \frac{8e}{3c^2\varepsilon} \cdot \pi mr^3 X = -\frac{16\pi^2 mnr^3 ei}{3c^2 d\varepsilon} \ .$$

If v denotes the *volume* of bismuth and a the *distance* of the midpoints of its molecular currents whose radius is $= r$, the number of its molecular currents is $m = v/a^3$. Under the assumption that the size of molecular currents is proportional to the supply of molecules, i.e., $a/r = \varkappa$ is constant, the sum of the areas orbited by all molecular currents is $m\pi r^2 = \pi v/\varkappa^3 r$. Substituting this value in the above expression of the electrodiamagnetic moment, one obtains

$$-\frac{8\pi}{3c^2\varepsilon} \cdot \frac{e}{\varkappa^3} \cdot vX = -\frac{16\pi^2 ni}{3c^2 d\varepsilon} \cdot \frac{e}{\varkappa^3} \cdot v \ .$$

[53] [Note by AKTA:] [Web52c, p. 367 of Weber's *Werke*] with English translation in [Web21b].

[54] [Note by AKTA:] A needle with one unit of magnetism has one unit of magnetic moment.

Hence the *electrodiamagnetic moment* of a mass of bismuth is proportional to the *electromagnetic separation moment* X and the volume v and can be found by multiplication of the constant factor $8\pi/3c^2\varepsilon$ extractable from the *general theory of electricity* and the constant factor $-e/\varkappa^3$ depending on the *nature of bismuth.* This last factor one can call the *diamagnetic constant* of bismuth.

2.20 Comparison of the Interaction of Diamagnetic Molecules with the Interaction of Magnetic Molecules

In the previous Section the induction of molecular currents in the circular orbits of molecules was considered *individually* to determine the *electrodiamagnetic moment*, as if on each molecule just the electromotive force acted determined by the existing *electromagnetic separating force*. Strictly speaking this is not the case, but instead in each circular current in addition acted the electromotive forces coming from the *interaction* of diamagnetic molecules, likewise as if on the particles of an iron bar not just the external separating force due to terrestrial magnetism acted but as well the separating forces coming from the interaction of the particles in the bar.

If one wants to take account of this *interaction* although it is so small that its influence is hardly noticeable, it is worthwhile to stress a remarkable contrast which takes place between the interaction of *diamagnetic* and *magnetic* molecules.

Namely, if two *iron particles* lay on a line *parallel* to the direction of the magnetic separating force acting on them and if one denotes by m the *magnetic moment* which was produced by the separating force in each of the iron particles *individually*, the new separating force resulting from the interaction of the particles *increases* the moment m. This new separating force due to the *interaction* of the two particles is expressed according to known laws by $2m/r^3$, when r denotes the distance of the particles. The *total* separating force $(X+2m/r^3)$ produces now in the particle under examination a *larger* moment $= (1+2m/Xr^3)M$. On the other hand if two *particles of bismuth* lay on a line *parallel* to the electromagnetic separating force acting on them, and if one denotes the *diamagnetic moment* corresponding to this separating force by $-\mu$ (the negative sign means that for separating forces acting in the same direction the diamagnetic moment is opposite to the magnetic one), the resulting separating force due to the interaction between the particles is $= -2\mu/r^3$ if r is the distance between the two particles. Therefore to the *total* separating force $= (X-2\mu/r^3)$ corresponds the *decreased* moment $-(1-2\mu/Xr^3)\mu$. Hence the *contrast* is that magnetism for iron particles laying *in the direction of the separating force* gets *intensified* by interaction, while diamagnetism for particles of bismuth laying in this direction gets *weakened* by interaction.

The opposite phenomenon occurs if the particles of iron and bismuth lay on a line *perpendicular* to the direction of the separating force. In this case the magnetism of iron particles gets *weakened* by interaction while diamagnetism of particles of bismuth gets *intensified* by interaction. In fact by using the same notation the *weakened magnetism* of iron particles results in $= +(1-m/Xr^3)m$, while the *intensified diamagnetism* of particles of bismuth results in $= -(1+\mu/Xr^3)\mu$.

It follows from this, that to endow a given mass of iron for a given separating force with the strongest magnetism one brings it in the form of a long and thin bar or a *prolate* ellipsoid whose major axis is *parallel* to the direction of the separating force, whereas on the other hand one has to bring a mass of bismuth to endow it with the strongest *diamagnetism* to the form of a plate as thin as possible or in the form of an *oblate* ellipsoid whose *minor* axis is *parallel* to the direction of the separating force. This conclusion could be checked experimentally, but one has to take account that in case of bismuth the influence of the interaction of the particles is very small due to the weakness of the diamagnetism corresponding to a given separating force. However, if one applies the result found to the verification of the theorem first mentioned by Faraday that bismuth under the influence of magnetic separating forces behaves exactly as iron with the only difference that the two magnetic fluids seem to be interchanged,[55] it turns out that this theorem is not completely true. In fact according to Faraday's theorem the prolate elliptic form would be for bismuth as for iron the most favorable one to get the strongest diamagnetism respectively the strongest magnetism, what is not the case. The deduction of these laws of interaction of diamagnetic molecules compared to the interaction of magnetic molecules leads to a simple distinction between magnetic and diamagnetic materials which is the topic of the following Section.

2.21 Distinction of Magnetic and Diamagnetic Materials with the Help of Positive and Negative Values of a Constant

Instead of the not completely accurate distinction between magnetic and diamagnetic materials, where for the same separating force the two magnetic fluids are just interchanged, it is possible to give an alternative correct and equally simple distinction which takes advantage of the difference of the values of a *constant* derived from the nature of each material.

In fact if one considers for simplicity just a *rotationally invariant ellipsoid* of iron or bismuth whose major axis is parallel to the direction of the separating force it was proved by Neumann in Crelle's "Journal für die reine und angewandte Mathematik", volume 37,[56] that *in case of iron* for the given separating force X the magnetic moment of the ellipsoid is given by the expression

$$\frac{kvX}{1+4\pi kS}$$

where v is the volume of the ellipsoid and S is a quantity determined by the ratio of the axes. Namely,

$$S = \sigma(\sigma^2-1)\left\{\frac{1}{2}\log\frac{\sigma+1}{\sigma-1}-\frac{1}{\sigma}\right\}$$

and

$$\sigma = \sqrt{1-\frac{r^2}{\lambda^2}}\ .$$

Here r and $\sqrt{r^2-\lambda^2}$ are the axes of the ellipsoid. The finite number k has for iron a constant value depending on its nature which Neumann denotes as the *magnetic constant* of iron. This value is for iron and *all magnetic* materials necessarily *positive*.

The value of S for an *infinitely long ellipsoid* is $S=0$. Consequently the magnetic moment is

$$= kvX\ ,$$

[55] [Note by AKTA:] [Far46a, § 2429].
[56] [Note by AKTA:] [Neu48b].

hence for $v = 1$ and $X = 1$ the magnetic moment $= k$. Therefore *the magnetic constant* k can be defined as the *limit* which the magnetic moment approaches under the unit of the magnetic separating force, if the ellipsoid of volume one gets more and more prolate. Since the constant k for all magnetic materials is positive, the magnetic moment is positive or negative, depending if the separating force is positive or negative.

For a *ball* one obtains the value $S = \frac{1}{3}$, consequently the magnetic moment is

$$= \frac{kvX}{1 + \frac{4}{3}\pi k} \ .$$

This formula implies, using that k is positive for a piece of iron in form of a ball, there is less magnetism as for a prolate ellipsoid in case the volume is fixed.

For an infinitely thin disklike plate the value of S equals one, consequently the magnetic moment is

$$= \frac{kvX}{1 + 4\pi k} \ .$$

The quantity k can now be used to distinguish different *magnetic* substances. According to the difference of the substances its value can decrease to zero, but, according to *the nature of magnetism* it always remains *positive*. However, one can generalize the applicability of the quantity k as a mean to distinguish substances by not restricting it to magnetic materials but extending it to all materials, by assigning a *negative value* of k which has the physical significance that a material having such a *negative* value of k is not magnetic but *diamagnetic*. Instead of introducing negative values of k we will write for *diamagnetic* materials $-k$. The diamagnetic moment of an ellipsoid of bismuth whose volume $= v$ and on which the electromagnetic separating force X acts parallel to direction of the main axis can therefore be expressed as

$$-\frac{kvX}{1 - 4\pi kS} \ ,$$

where S has the same meaning as above. For infinitely long ellipsoids, where $S = 0$, the diamagnetic moment is

$$= -kvX \ ,$$

for a ball where $S = \frac{1}{3}$ it becomes

$$= -\frac{kvX}{1 - \frac{4}{3}\pi k} \ ,$$

and for an infinitely thin ellipsoid where $S = 1$ it is

$$= -\frac{kvX}{1 - 4\pi k} \ .$$

Hence if one fixes the volume for the *most prolate* form there is the *least* diamagnetism, where for the *most oblate* form there is the *most* diamagnetism, precisely opposite as in the case of magnetism, which was already proved in the previous Section.

However, since $-k$ has a very *small* value even in case of bismuth which is the most diamagnetic one, it follows that the diamagnetism of bismuth always is almost proportional to the product of the volume with the separating force and can be considered as roughly independent of the shape. Therefore the meaning of $-k$ can be directly compared with the one of the *diamagnetic constant* which we discussed at the end of Section 2.19. There as well the diamagnetic moment was expressed as the product of the volume and the separating force with a *constant coefficient* which decomposed into two factors, namely a factor $8\pi/3c^2\varepsilon$ obtained from the *general theory of electricity* and a factor $-e/\varkappa^3$ depending on the *nature of bismuth* which was referred to as the *diamagnetic constant* of bismuth. One easily sees that these two factors are not separated here in $-k$ and that $-k$ has precisely the meaning of the product of these two factors.[57]

2.22 On the Existence of Magnetic Fluids

When a certain class of effects of a material on an other material is such that it can be explained in terms of an *ideal distribution* of magnetic fluids on its surface, one can think of different possibilities for the true causes of all those effects which lay in the *interior* of the material and one can distinguish four different cases, which were mentioned in Section 2.14 and discussed in more detail in the following Sections. Two of these cases assume that there exist *two magnetic fluids* to which in the molecules of the material either a *constant* or *variable* separation is assigned. The other two cases have as hypothesis, that the two according to the theory of electricity existing *electric fluids* are in a certain circular orbit around each of the molecules of the material either in a *constant* or *variable* current. As one easily sees these four different cases are not mutually exclusive at all. Indeed, a part of the magnetic fluids in the molecules can be separated constantly whereas the separation of another part is variable. In the same way a part of the electric current for the circular orbits of each molecule can be constant while the intensity of another part varies. In fact without a *variable part* the *constant* currents cannot exist in view of the many existing electromotive forces. Namely the electric fluids if they are actually freely movable in certain circular orbits around the molecules as is shown by the existence of persistent currents, they need to follow necessarily the impetus of the electromotive forces decomposed according to the direction of the circular orbits. Therefore the first and second case can occur either individually or in combination. The third and fourth case however are in a necessary relation to each other so that either none of these cases or both together have to occur. It follows that the four cases mutually combined can be distinguished into two main cases. Namely, in the first place that two separated or separable *magnetic fluids* exist in the molecules of the material. Secondly that the according to the theory of electricity everywhere existing *electric fluids* are *freely movable* in certain circular orbits around the molecules. These two main cases however can be considered as mutually exclusive as far as the *actual proof of existence* of one case leaves the other as a *superfluous* hypothesis.[58]

For each of the main cases a *theory* can be developed and each of the theories can be split *into two parts*, namely a part where both theories agree in their results and into one where they contradict each other. The same happened in optics concerning the theory of emission and the wave theory which in their results in many aspects *agreed* with each other until the

[57][Note by WW:] We would like to mention that the *magnetic* coefficient k is only *constant* according to the theory of *separable magnetic fluids* (Section 2.15, number 1), but according to the theory of *rotatable molecular magnets* (Section 2.15, number 2) has to be a function of the separating force. On the other hand, the *diamagnetic* coefficient $-k$ according to the theory of diamagnetoelectric induction (Section 2.15, number 4) by its nature is *constant*, as shown in Section 2.19. In Sections 2.23-2.26 we will prove that in connection with *magnetism*, experience is in contradiction to the theory of *separable magnetic fluids* and decides in favor of *rotatable molecular magnets* (or molecular currents Section 2.15, number 3), since the value of k for iron is in reality *not constant*, but depends on the size of the separating force X.

[58][Note by AKTA:] Ampère had already argued against the existence of magnetic poles in his masterpiece, see Section 19 (The Magnetic Poles and Dipoles are Disposable Hypotheses) of [AC11] and [AC15].

discovery of *interference phenomena* led to a closer discussion of that part for which the two theories *contradict* each other in their results. Although until now the two theories based respectively on the existence of *magnetic fluids* and on the existence of *electric molecular currents* agreed admirably in many respects in their results, it is fair to expect here as in optics that finally the discovery of a new class of phenomena leads as well to a closer discussion of that part in which the two theories disagree in their results so that the newly discovered phenomena decide between the two theories.[59,60]

The two optical theories disagreed in their conclusions concerning the *coincidence* of two homogeneous rays of light. According to one theory amplification should always occur while according to the other theory sometimes amplification and sometimes cancellation takes place. The *phenomena of interference* confirmed the results of the *wave theory*. In a similar way the crossroad of our theories can be decided. In fact both agree firstly in all results concerning the phenomena of *permanent* magnets. Secondly they agree as well concerning variable magnets, insofar as each of them leads to a distinction of them *into two classes*, namely into the class of that magnets whose magnetism is due to *the mere orientation of already existing movable molecules* (molecular magnets or molecular currents) and into the class of magnets whose magnetism is due to *the separation and movement of imponderable fluids in molecules at rest* (the separation of magnetic fluids in the molecules or the induction of electric currents in certain circular orbits around the molecules). Finally the two theories agree in their results thirdly concerning the *first* class of variable magnets. However, *they contradict each other in their results concerning the second class*. Namely for this second class *an opposite position of the poles* follows from the two theories. According to *one of the theories* the position of the poles for the second class should *coincide* with the one of the first class, while for the *other theory* the position of the poles for the second class should be *opposite* compared to the first class.

As long as one knew just such variable magnets where the position of the poles (for separating forces pointing in the same direction) *coincided*, both theories explained these magnets and only according to the second theory the assumption was necessary that magnets of the second class do not exist at all or are always connected to magnets of the first class in such a way that the effect of the latter one is always dominating. Since the first theory did not require such a hypothesis, it seemed even to be the preferred theory as long as one just knew magnets with the *same* position of the poles for separating forces pointing in the same direction. As soon as variable magnets (diamagnets) were discovered, where the position of the poles (for separating forces pointing in the same direction) was *opposite*, there was no choice anymore between the two theories. In fact only the second theory could be used since only it explains the formation of two classes of magnets with *opposite* position of the poles for separating forces pointing in the same direction.

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[59] [Note by WW:] Before in the "Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1839", [Wilhelm Weber's *Werke*, Vol. II, p. 171], I tried to justify the conjecture that the phenomenon described by the name "unipolar polarity" could lead to such a decision. However, this is not the case, since there can be given a different explanation for the phenomena described there, as soon as such a connection takes place between the electric fluids moving in the interior of the conductor and the ponderable parts of the conductor, that each force acting on the electric fluids completely or nearly is transferred to the ponderable parts, as I explained in more detail in the "Electrodynamic Measurements" (*Abhandlungen bei Begründung der Königlichen Sächsischen Gesellschaft der Wissenschaften* edited by v. d. F. Jabl. Ges. Art. 19, p. 309), [Wilhelm Weber's *Werke*, Vol. III, p. 134].

[60] [Note by AKTA:] See [Web40, p. 171 of Weber's *Werke*], and [Web46, Section 19, p. 134 of Weber's *Werke*] with English translation in [Web07, Section 19].

The *diamagnetic* phenomena discovered by Faraday decide between these two theories in the same way as the phenomena of interference decided between the emission and wave theory in optics. This is the most essential and important meaning associated to this discovery. Thanks to the discovery of diamagnetism the hypothesis of *electric molecular currents in the interior of materials* gets affirmed and the hypothesis *of magnetic fluids in the interior of materials* gets disproved.

All our hypotheses and notions of materials can always just be applied to a limited range of phenomena and they can be distinguished by the size of their range of applications. We associate *reality* to them as long as we do not know any phenomena outside of the range of their application. In the opposite case we denote them as *ideal*. Even if the *magnetic fluids* have to be treated in the future as *ideal* notions, they nevertheless keep the same importance and meaning they had before as long as one applies them to the range where they are valid. And even if we now associate to *the electric molecular currents* in the interior of materials *reality*, same as to the ether in optics responsible for the propagation of waves, it can happen in the future by further development of science that they have to be transferred to the class of *ideal* notions.

V - On the Dependence of the Magnetic and Diamagnetic Moment on the Size of the Separating Force

2.23 From the Hypothesis of Actually Existing Magnetic Fluids, Based on the Analogy with the Theory of Electricity, and From the Law Given Thereby of the Dependence of the Magnetic Moment on the Magnitude of the Separating Force

The exactness of the result that there do not exist magnetic but just electric fluids for which however in ponderable materials two kinds of orbits exist on which they can move, namely those on which their movement is subject to resistance proportional to their velocity and those were there is no resistance at all (molecular currents), according to the previous discussions is mainly due of the opposite *position of the poles* or their opposite *direction.* In virtue of this consideration one distinguishes between magnetic and diamagnetic materials. However, there is another way how to check the correctness of this result if one examines in addition the *strength* of this separation more closely. In fact there is not such a big difference between the two theories in connection with the *strength* of this separation as in connection with the *direction.* The final decision between the two theories requires the development of these differences which occur in both theories in connection with the *strength* of the ideal separation and checking them with experience.

According to the theory of *actually existing magnetic fluids*, the magnetic moments are *proportional* to the separating forces as mentioned in the footnote at the end of Section 2.21, contradicting experience in view of the experiments by Müller. If on the other hand the theory of *molecular currents* did not lead to such a contradiction with experience, the validity of the latter theory could be shown in this way without reference to the *diamagnetic* phenomena and the *wrong position of the poles* as we did in the previous Sections. However, one has to consider a crucial circumstance which shows that this proof alone just using *magnetic* experiments without reference to the *diamagnetic ones* is not completely decisive. As already discussed in Section 2.14, under the hypothesis of the *actual existence of magnetic fluids* there are *two ways* how magnets *come into existence*, namely by *separation* of magnetic fluids in molecules *at rest* or through *rotation* of molecules in which the magnetic fluids are separated *permanently.* The already mentioned theory developed by Poisson and Neumann explaining that magnetic moments are *proportional* to the separating forces, is only concerned with the laws to determine the magnetism of magnets originating according to the *first* kind. It has to be examined more closely if the same laws without modification can be applied to the determination of the magnetism of magnets of the *second* kind. This is not the case, but for magnets of the *second* kind other laws hold and in fact *the same ones* as for magnets whose magnetism is due to *rotatable molecular currents.* Hence when the laws of the latter magnets coincide with experience, it follows immediately that experience has to coincide as well with the laws of magnets, whose magnetism is due to *rotatable molecules with permanently separated magnetic fluids.* Therefore these laws alone cannot lead to a *general* disproof of the actual existence of magnetic fluids, but just to a disproof of the origin of these magnets by *separation* of magnetic fluids, as assumed in the theory developed by Poisson and Neumann.

But even this *partial* disproof gains a larger meaning by considering the reasons which justified Poisson and Neumann to assume a *separation* of magnetic fluids into molecules at rest and *no rotation* of the molecules with permanently separated magnetic fluids. By examining more closely how the hypothesis of the existence of *magnetic* fluids was proposed one sees easily that it mainly originates by its analogy with the *theory of static electricity*. This analogy consists mainly in the fact that if iron gets magnetized, a similar separation of magnetic fluids takes place in the iron molecules as the one of electric fluids when little conductors get electrified. However, this analogy is completely lost, when the magnetization of iron is not due to a *separation* of magnetic fluids in the iron molecules but due to a *rotation* of the iron molecules themselves. It follows from this that the hypothesis of the existence of two magnetic fluids lose their original foundation based on *analogy with the theory of electricity* by disproving the theory of Poisson and Neumann. Instead of that it has to be considered like a completely new hypothesis. This can be seen as well by the fact that in this case even the name of magnetic *fluids* is not suitable anymore. Indeed, when these substances in the iron molecules are *permanently* separated and always fixed in the same way to the iron particles and are only movable *together* with the iron particles, it does not make sense to talk of a *liquid* state of matter. It is even debatable to consider these substances as separated from iron if they are in reality always fixed to the iron particles. Instead of that it were sufficient to distinguish *two kinds of iron particles*.

The mentioned *partial* disproof also gains a deeper significance in that it destroys each *analogy* one tried to establish before between the hypotheses of *magnetic* and *electric* fluids. This analogy gained a certain likelihood by the hypothesis whose actual value is difficult to determine exactly and therefore can be easily rated too high. In view of the above mentioned *disproof of the separation theory* it disappears completely.

In the same ratio a theory, namely the one built on the *actual existence of magnetic fluids*, loses on likelihood, the other one, namely the one built on *the existence of molecular currents*, gains, in particular, if it can be proved that the *strength* of the magnetic moments of different separating forces behaves precisely as predicted by this theory. The theory so far just checked by the observed *direction* of the separation could be checked and confirmed by observing the *strength* of the separation. It follows from this that this second checking is a crucial supplement and completion of the first one which therefore will be discussed in detail in the following Sections.

2.24 Connection Between the Existence of a Maximal Value of the Magnetic Moment and the Assumption that the Molecules Are Rotatable

Although the assumption of *rotatable molecular magnets* agrees in the determination of the *location of the poles* with the assumption of *separable magnetic fluids* for nonmovable molecules as explained in Section 2.16, the two disagree however in an essential way concerning the law saying that the *strength of the magnetism of a bar of iron* varies according to the *size of the magnetic force* acting on the iron as discussed in the previous Section. It is not difficult to understand that according to the *first* assumption there is a *limit* for the strength of the magnetism which cannot be exceeded and which corresponds to the case where the axes of the molecules attained a *parallel position* by rotation. But such a *limit* for the strength of magnetism *does not exist* according to the *second* assumption building the foundation of the theory due to Coulomb, Poisson and Neumann, since then there exists in the molecules an *inexhaustable* amount of separable magnetic fluids in analogy with the theory of electricity.[61] Even if one wanted to modify this last assumption a bit and assumed that due to the strengthening of the force acting on the iron the *whole* neutral magnetic fluid existing in the molecules gets separated, there still would be a crucial difference between the two assumptions. This difference is that the growth of magnetism for a growing force acting on the iron is subject according to the latter assumption to a quite different law *before* the exhaustion of the neutral magnetic fluid than *after* the exhaustion. Namely until the moment where the last bits of the neutral fluid were separated, the ratio of the strength of the force acting on the iron had to be *constant*. For that reason this ratio is referred to as the *magnetic constant* of iron. However, after this moment this ratio has to decrease rapidly. On the other hand according to the first assumption it follows that this ratio is *always variable* and has to decrease *continuously* from the start to the end according to the same law.

In view of this, one obtains the possibility to decide directly from the *phenomena of iron magnetism* if the magnetization of iron has to be associated *according to the hypothesis of actually existing magnetic fluids* either to a *rotation* of its molecules or the *separation* of the magnetic fluids inside its molecules. In the first case the rotatable molecules can be as well carriers of *molecular currents* as of permanently separated *magnetic fluids*, while in the latter case the existence of magnetic fluids has to be considered as for sure. Indeed, only the *rotation* of molecules but not the *separation* of magnetic fluids in the molecules (due to a given magnetic or electromagnetic separating force) can be a possible substitute for the magnetic fluids due to electric currents.

In view of the above mentioned experiments by Müller one has to consider the *latter* assumption of *separable magnetic fluids* in non-rotatable molecules as disproved. It was only left to check if the *continuous decrease* of the ratio of the strength of the magnetism of iron with respect to the size of the separating force acting on the iron as determined by Müller in his experiments is in agreement with the law derived from a certain *rotatability* of the molecules according to the *first* assumption. It can be left undecided if these molecules are the carriers of separated magnetic fluids or of molecular currents. In the mean time the experiments of Müller were repeated by Buff and Zamminer (*Annalen der Chemie und Pharmacie* of Liebig, Wöhler and Kopp Vol. 75, p. 83).[62] The results found by Müller were not confirmed. Instead of that Buff and Zamminer believe to have proved with their experiments that the ratio of the strength of the magnetism of iron compared to the size of the force acting on the iron is actually *constant* as far as it is possible to check with the means currently at our disposal (here they did not take into account the influence of the force due to coercivity if the iron is not completely soft). This result would only be possible under the assumption of *separable* magnetic fluids in non-rotatable molecules. The assumption of *rotatable molecular magnets* and therefore as well of *rotatable molecular currents* were disproved in this way and the *actual existence of magnetic fluids* would appear to be on a sound foundation.

[61][Note by WW:] Namely according to this assumption the magnetic state of equilibrium is defined that on the surface of all molecular conductors there is a distribution of the two magnetic fluids acting on all points in the interior in such a way that the effect of the external separating forces gets cancelled. It follows easily from this that if one doubles the external separating forces the amount of the magnetic fluid at the surface of all molecules has to be doubled as well, etc.

[62][Note by AKTA:] [BZ50].

It therefore seemed to be mainly necessary to repeat the same experiments once more in order to decide the contradiction at hand. Hence in the following Section, I describe the experiments carried out by me and the special equipments which I made to get a safe result. The results by Müller were confirmed in this way which is in agreement with some experiments made by Joule, already before Müller, reported in *The Annals of Electricity etc. by W. Sturgeon* Vol. V, p. 472.[63]

2.25 Experiments to Prove the Existence of a Maximal Value of the Magnetic Moment

It followed from the experiments carried out by Müller that in case of the same forces acting on iron, the decrease of the ratio between the strength of the iron magnetism and the size of the force acting on iron is smaller for *thin* iron bars than for thick ones. Therefore for the comparison between the experiments carried out by Müller and the ones carried out by Buff and Zamminer it is important to note that the thinnest bar used by Müller had a thickness of only 6 millimeters where the thinnest one used by Buff and Zamminer had a thickness of 9 millimeters. This difference in thickness becomes even more influential since the bar of Müller was 330 millimeters long where the one of Buff and Zamminer only 200 millimeters. I used for the following experiments an even thinner bar than Müller, namely one which had a thickness of 3.6 millimeters, a length of 100.2 millimeters and a weight of 8190 milligrams. It turned out that the magnetism of such a thin bar could be measured with high precision by the displacement at a distance of a little mirror magnetometer. The only difficulty which the use of such a thin bar had was the precise separation of the influences on the magnetometer due to iron magnetism and the ones due to the galvanic current. It is clear that if one uses the same galvanic spiral in order to magnetize thick as well as thin bars as was done by Müller, Buff, and Zamminer, this separation is less precise for thin bars since the effect of the spiral remains the same and therefore is for thin bars comparatively bigger than for thick ones. Therefore for the following experiments a spiral was used which was not wider as needed to put in a thin bar. Even with that I was not satisfied but twisted the end of the spiral wire two times in *opposite direction* around the middle of the spiral in a much bigger circle such that the area enclosed by these two twists coincided with the area enclosed by all twists of the narrow spiral. According to the known laws of electromagnetism it follows from this that the current has no effect on the magnetometer at a distance which can easily be checked and confirmed by experiment. The whole effect observable at the magnetometer is than just due to the magnetism of iron which can be determined by the same acuteness and exactness from the known intensity of terrestrial magnetism as the magnetism of hard steel magnets according to *absolute measure*. In the *Intensitas etc.* Gauss has given a precise instruction how this is done by using deflection experiments.[64]

Moreover, it should be stressed that the spirals used by Müller, Buff, and Zamminer were shorter than the magnetized bars of iron. In Müller's case this difference was small since the iron bars on both ends only protruded 15 millimeters from the spiral. In the case of Buff and Zamminer this difference was much bigger since the ends of the longest and thinnest bars protruded 45 millimeters from the spiral. Moreover, this influence got in addition proportionally increased in the experiments by Buff and Zamminer since the length

[63] [Note by AKTA:] [Jou40].

[64] [Note by AKTA:] See footnote 47 on page 59.

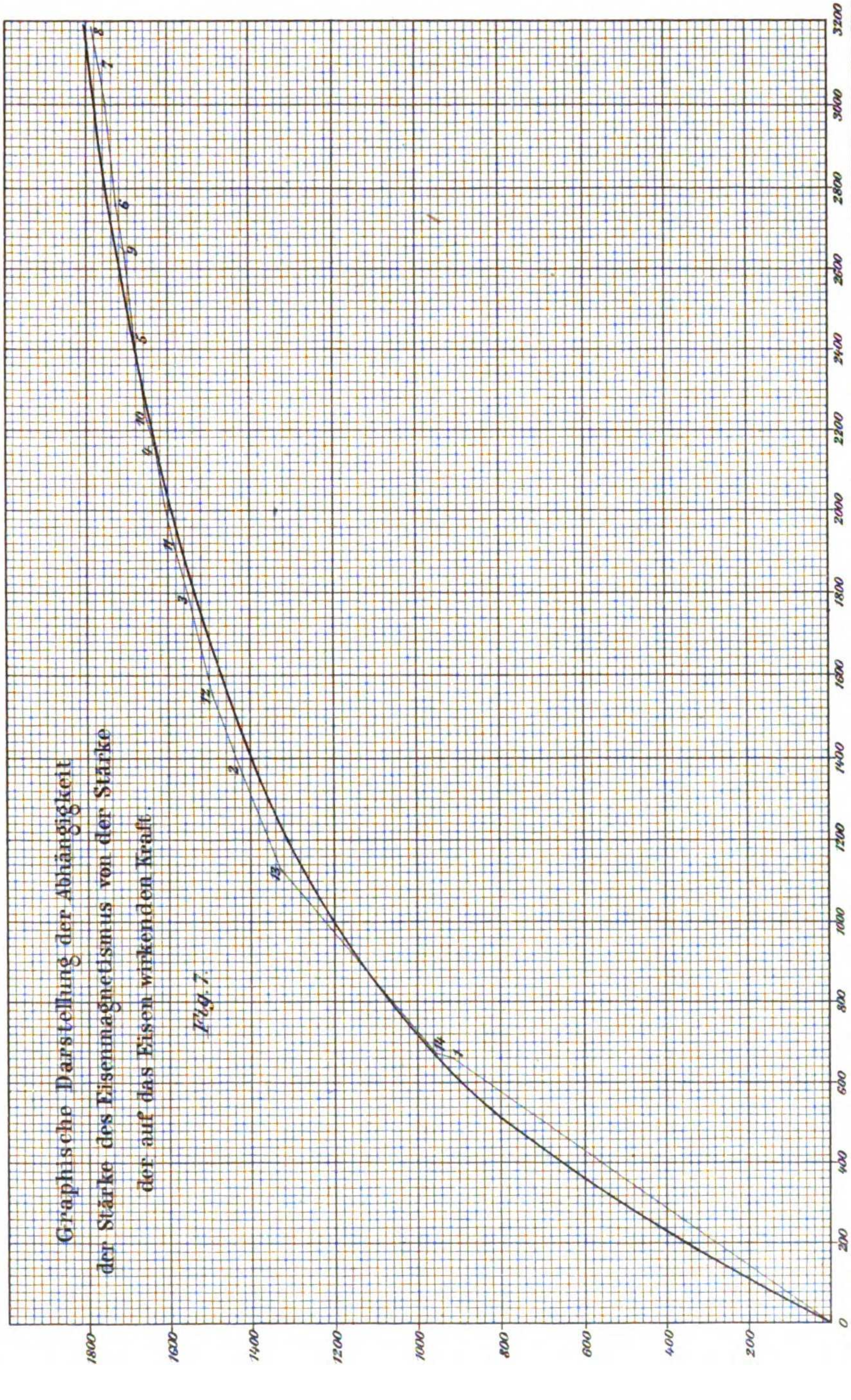
of the part enclosed in the spiral was only 110 millimeters compared to the 300 millimeters in Müller's case. Probably this circumstance is the main reason for the apparent difference of the results the observers obtained. Obviously the effect of the spiral on the iron is strongest in the middle of the spiral but decreasing at its ends and this decrease is exceptionally large outside the spiral. It follows from this that even if by increasing current intensity the effect in the middle part of the bar approached a limit, such an approach could not be felt at all for the parts outside the spiral. For the following experiments a spiral was used which was considerably longer than the iron bar such that according to the laws developed in Section 2.18 the force acting on the ends of the bar does not noticeable differ from the one on the middle. Only through such a set-up one can obtain a reliable result.

I content myself with briefly compiling the results obtained in this way in the following Table. I do not describe the experiments in detail, which does not seem necessary since up to the just mentioned differences they almost coincide with the description given by Müller, Buff, and Zamminer. I only mention that each single measurement is based on changing the direction of the current four times. In this way the closest agreement was obtained showing that the coercivity of iron did not affect the accuracy of the results. It would have been easy to consider the influence of the temperature of the bar of iron by keeping it constant with the help of a water current. However it turned out that the influence of moderate changes of the temperature was so small that to take it into account one needed much more accurate measurements requiring new equipments which one could not obtain immediately. It is not necessary to explain here how to express the magnetism of iron using *absolute* measure which was carried out in the Table according to known rules. The intensity of the current was determined with the help of a tangent galvanometer according to *absolute* measure. The correction already mentioned by Müller to obtain higher precision which depends on the ratio between the length of the needle and the diameter of the galvanic ring was identified precisely and taken into account since this was easy to do. The knowledge of the intensity of the current according to *absolute* measure was further used in order to determine the size of the force acting on the iron according to *absolute* measure through which one expresses *terrestrial magnetism*. This was done using the *number* of windings of the spiral through which the current moved and its *dimensions*. Thanks to this procedure one could compare that force with the known *intensity of the force due to terrestrial magnetism*. In the following Table this force is denoted by X. The iron magnetism M one found was divided by the mass of iron expressed in milligrams $p = 8190$ and the magnetism reduced to unit mass is denoted by m.

No.	X	m
1.	658.9	911.1
2.	1 381.5	1 424.0
3.	1 792.0	1 547.9
4.	2 151.0	1 627.3
5.	2 432.8	1 680.7
6.	2 757.0	1 722.7
7.	3 090.6	1 767.3
8.	3 186.0	1 787.7
9.	2 645.6	1 707.9
10.	2 232.1	1 654.0
11.	1 918.7	1 584.1
12.	1 551.2	1 488.9
13.	1 133.1	1 327.9
14.	670.3	952.0

As one sees, the Table decomposes into two parts, namely one where the size of the force acting on the iron is increasing and one where it is decreasing. In the graphical representation in the Figure 7 one sees that the experiments of the second part which were denoted by no. 8 until no. 14 correspond very well to the experiments of the first part denoted by no. 1 until no. 7.[65]

[65][Note by AKTA:] Text inside Figure 7: Graphical representation of the dependence of the strength of iron magnetism on the strength of the force acting on the iron.


Graphische Darstellung der Abhängigkeit
der Stärke des Eisenmagnetismus von der Stärke
der auf das Eisen wirkenden Kraft.
Fig. 7.
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
3200

For the last experiment of the first part the iron bar attained a higher temperature and one waited before the start of the following experiments until it cooled down again. Nevertheless one sees that both experiments fit well with the other ones proving that the influence of the difference in temperature had to be very small.

From these experiments the result seems to follow that the ratio between the strength of the magnetism of iron and the size of the force acting on iron is *variable*. Therefore it is to be expected that the magnetism of iron approaches a *limit* which it can never exceed. Obviously it is impossible to continue with the experiments so far that this limit can be obtained and determined directly by the observations. Such a direct determination of the limit is however not necessary since it suffices that the *continuous variation* of that ratio is proven. The same experiments were repeated by other observers with the same outcome and I believe that there is no doubt on the obtained results. Mainly the result found by Müller is confirmed in this way.

2.26 The Law of the Dependence of the Magnetic Moment on the Size of the Separating Force According to the Assumption of Rotatable Molecules and Its Comparison with Experiments

It remains to discuss more closely if the variation of the strength of iron magnetism with the size of the forces acting on the iron found by the above experiments coincides with the law deduced from the hypothesis of a certain *rotatability* of the molecules. If this is the case it is clear that one can assume according to Ampère as well that these molecules are the carriers of *molecular currents*. This means that the *emergence* and *transformation* of iron magnetism as well as its *effects* can be explained without the hypothesis of *magnetic fluids* and can be derived merely from the hypothesis of *electric fluids*.

In Figure 8 NS is a molecular magnet rotatable around its midpoint C. ND is the direction to which its magnetic axis is parallel in case of equilibrium when the external force $X = 0$.

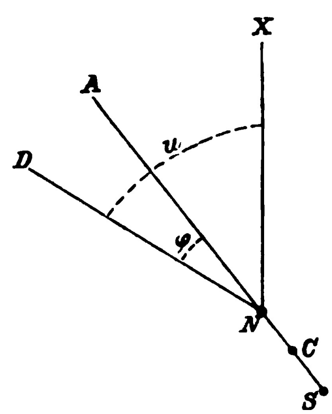


Fig. 8.

The fact that for soft iron the magnetism due to an external force vanishes again as soon as the external force disappears, proves that the molecular magnet whose rotation is responsible for the generated magnetism is driven back in its original position parallel to

ND. The repulsive force due to the interaction of the molecules has to increase according to the deflection $AND = \varphi$ and can be expressed by

$$D \sin \varphi \ ,$$

where D is a constant magnitude referred to as *molecular directive force*.[66] In case in addition to the molecular directive force an external force X is acting whose angle with respect to the direction of the directive force is $XND = u$, the molecular magnet is rotated or deflected by the angle $AND = \varphi$ and for the determination of the new equilibrium position one has the following equation

$$X \sin u \cos \varphi = (D + X \cos u) \sin \varphi$$

or

$$\tan \varphi = \frac{X \sin u}{D + X \cos u} \ .$$

From this deflection φ the *increase* of the *magnetic moment* of the molecule decomposed in the direction of the force X can be determined. Namely if one denotes the whole magnetic moment of the molecule by μ then before deflection its component in direction of the force X was

$$= \mu \cos u \ ,$$

and after deflection

$$= \mu \cos(u - \varphi) \ ,$$

hence the increase x

$$x = \mu(\cos(u - \varphi) - \cos u) \ .$$

Substituting in this formula for φ the value obtained from the above equation $\tan \varphi = X \sin u/(D + X \cos u)$ one obtains

$$x = \mu \left\{ \frac{X + D \cos u}{\sqrt{X^2 + D^2 + 2XD \cos u}} - \cos u \right\} \ .$$

For a system of molecules whose distribution of the axes in the original equilibrium was homogeneous, the number of molecules whose magnetic axis has the angle u with respect to the direction NX of the force X is proportional to $\sin u$. Our task is to determine the magnetic moment y resulting from the rotation of *all* molecules of the system due to the force X.

For this purpose one multiplies the value found above for x by $\sin u du$ and integrates from $u = 0$ until $u = \pi$. This integral value multiplied with the number of molecules n and divided by $\int_0^\pi \sin u du = 2$ gives the *moment* y

$$y = \frac{n}{2} \int_0^\pi x \sin u du \ .$$

[66][Note by AKTA:] In German: *Molekulare Direktionskraft*. Alternative translations: molecular directional force, molecular directing force or molecular force of direction. The concept of "Direktionskraft" (directive force) was introduced by Gauss in 1838, [Gau38, p. 4] with English translation in [Gau41c, p. 254].

Carrying out the integration one obtains for y the following expression[67,68]

$$y = n\mu \frac{X}{\sqrt{X^2 + D^2}} \cdot \frac{X^4 + \frac{7}{6}X^2D^2 + \frac{2}{3}D^4}{X^4 + X^2D^2 + D^4} \ .$$

The force acting on the iron which caused this moment was $= X$. If one denotes by n the number of molecules *in the volume unit*, then the ratio between the moment y and the force X has in the *rotation theory* the same meaning as the magnitude in the *separation theory* which Neumann denoted by k when he determined the magnetic state of an ellipsoid of revolution in Crelle's *Journal für die reine und angewandte Mathematik*, Vol. 37.[69] Substituting the variable value y/X for k in Neumann's computation, it follows that, if n is the number of molecules in the volume or mass unit, that the *magnetism reduced to the volume or mass unit* of iron m is given by the following equation

$$m = \frac{y}{1 + 4\pi S \frac{y}{X}} \text{ for the volume unit },$$

$$m = \frac{y}{1 + 4\pi S \rho \frac{y}{X}} \text{ for the mass unit }.$$

Here ρ denotes the density of iron and S a factor depending on the form, see Section 2.21.

After this the strength of iron magnetism m can be computed from the force X acting on the iron if one knows the constants $n\mu$ and D for iron as well as its density ρ for the reduction to the *unit of mass*. Setting

$$n\mu = 2\,324.68 \ ,$$

$$D = 276.39 \ ,$$

one obtains since the density of iron is $\rho = 7.78$ the following comparison between computation and experiment. Here one has to point out however that to determine the factor S instead of the cylindrical shape of iron an approximating ellipsoidal form was substituted giving $S = 1/249$.

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[67] [Note by HW:] [This value for y is an approximate value, the actual expression is for $X < D$ given by $y = \frac{2}{3} n\mu \frac{X}{D}$ and for $X > D$ given by $y = n\mu \left(1 - \frac{1}{3}\frac{D^2}{X^2}\right)$.

Wilhelm Weber indicated the change in his Note *Verbesserung einer Formel in den elektrodynamische Maassbestimmungen* which appeared in the *Berichte der Königl. Sächs. Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physische Klasse 1852* where he writes:]

> On p. 572, line 22 of the previous article on *Electrodynamic Measurements* in the first volume of the *Abhandlungen der mathematisch-physischen Klasse der Königl. Sächs. Gesellschaft der Wissenschaften* was used for y instead of the accurate expression an approximation. I correct this mistake by pointing out that this has no sensible influence on the numerical values deduced from it. In fact the accurate value for y for all values of X, which are smaller than D is $y = \frac{2}{3} n\mu \frac{X}{D}$, and for all values of X which are larger than D one obtains $y = n\mu \left(1 - \frac{1}{3}\frac{D^2}{X^2}\right)$.

[68] [Note by AKTA:] [Web53d]. See also [Web57].

[69] [Note by AKTA:] See footnote 56 on page 64.

No.	X	m observed	m computed	difference
1.	658.9	911.1	948.4	−37.3
2.	1 381.5	1 424.0	1 387.0	+37.0
3.	1 792.0	1 547.9	1 533.0	+14.9
4.	2 151.0	1 627.3	1 623.5	+3.8
5.	2 432.8	1 680.7	1 685.0	−4.3
6.	2 757.0	1 722.7	1 742.2	−19.5
7.	3 090.6	1 767.3	1 791.2	−23.9
8.	3 186.0	1 787.7	1 803.4	−15.7
9.	2 645.6	1 707.9	1 723.6	−15.7
10.	2 232.1	1 654.0	1 644.8	+9.2
11.	1 918.7	1 584.1	1 568.9	+15.2
12.	1 551.2	1 488.9	1 452.9	+36.0
13.	1 133.1	1 327.9	1 276.8	+51.1
14.	670.3	952.0	957.5	−5.5

Noting that in these experiments for the measurement of the intensity of the currents one used as tangent galvanometer a usual compass only 60 millimeters long where the fractions of a degree could not be observed with certainty, the intensity could easily be found 1 percent too small or too large. Therefore one could not expect a closer agreement between computation and observation as the one found in the Table. In the graphical representation in Figure 7 the computed values are connected by a *thick* line, the observed ones by a *thin* line. It seems that thanks to this there is no doubt on the *rotatability* of the iron molecules. And since one can consider these iron molecules according to Ampère as the carriers of *molecular currents*, a complete accordance of all magnetic phenomena even the ones observed on *variable* magnets with the theory of *molecular currents* is proved. Through this we found an important confirmation of this theory through *magnetic* phenomena to guarantee the explanation given before for *diamagnetic* phenomena.

2.27 Application Made to the Comparison in Section 10

In the previous Section we derived the law to determine the strength of iron magnetism in terms of its dependence on the magnetic and electromagnetic separating force using the theory of rotatable molecules. Its most important application concerns the construction of stronger electromagnets, as actually all electromagnetic instruments, whose action depends on the strength of iron magnetism. Since this application which was stressed by Joule and Müller is not directly related to the topic discussed (diamagnetism), I restrict myself to add just the application of this law on the comparison of the strength of an electrodiamagnet from its *magnetic* and *magnetoelectric* effects, since I referred to this in Section 2.10, page 50.

In fact in Section 2.10 the *magnetism of bismuth* was compared to the *magnetism of iron* in two ways. *First* by examining the *deflection of the needle of a magnet* and *second* by the *electric currents* in a closed conductor induced by the same movement from both materials. From the two comparisons the strength of the *diamagnetism of bismuth* can be determined according to absolute measure as soon as one knows the strength of the *magnetism of iron* according to absolute measure. Hence one just has to apply the law above to the determination of iron magnetism, in order to obtain *two* independent determinations for the diamagnetism of bismuth, which in view of their agreement confirm *the law of diamagnetic polarity*. Although already in Section 2.10 under the conditions there the law derived from the experiments of Müller was applied to the determination of iron magnetism, we remarked, that the result found there is not completely sure and exact at all. Therefore it will give us more certainty and exactness, if we apply the more precisely determined law from the previous Section.

According to the first footnote in Section 2.10, the *diamagnetism* induced in bismuth by an electromagnetic force $X = 629.9$ was compared to the *magnetism* induced in iron by the same force by examining the *torques exerted on a magnetic needle*. Its ratio was found to be

$$1 : 1\,470\,000 \ .$$

Using this ratio, the *diamagnetism* can be determined *according to absolute measure* if one knows the *magnetism of iron according to absolute measure*. According to the previous Section, one has for $X = 629.9$

$$\frac{y}{X} = 3.395\,9 \ .$$

If one substitutes as in the previous Section for the cylindrical shape of the little iron bar, which was 92 millimeters long and 0.1016 millimeters thick, a closely approximating form of an ellipsoid, one obtains according to Neumann

$$S = \frac{1}{138\,780} \ .$$

Using that value one finds by putting $\rho = 7.78$

$$\log m = \log \frac{yT}{X} - \log \left(1 + 4\pi S\rho \frac{y}{X}\right) = 3.329\,19 \ ,$$

hence for *iron magnetism according to absolute measure*

$$m = 2\,134 \ .$$

For this value of *iron magnetism* one obtains according to the ratio quoted above for the *diamagnetism of bismuth according to absolute measure* corresponding to the same force $X = 629.9$

$$= \frac{1}{1\,470\,000} \cdot 2\,134 = \frac{1}{689} \ .$$

Furthermore in footnote 24 in Section 2.10, the *diamagnetism* produced in bismuth by an electromagnetic force $X = 3\,012$ was compared to the *magnetism* produced in iron by the same force *by looking at the intensity of the through their motion induced electric currents in a closed conductor*. Their ratio was found to be $1 : 456\,700$ or after the reduction for bismuth stated in Section 2.10

$$1 : 360\,740 \ .$$

With the help of this ratio the *diamagnetism according to absolute measure* can be determined, when one knows the *iron magnetism according to absolute measure*. According to the previous Section for $X = 3\,012$ one has

$$\frac{y}{X} = 0.771\,33 \ .$$

If one substitutes here as well for the cylindrical form of the little iron bar, which was 186 millimeters long and 0.8342 millimeters thick, a closely approximating form of an ellipsoid, one obtains according to Neumann

$$S = \frac{1}{9\,747}\ ,$$

and therefore, for $\rho = 7.78$,

$$\log m = \log \frac{yT}{X} - \log\left(1 + 4\pi S\rho \frac{y}{X}\right) = 3.362\,74\ ,$$

hence for *iron magnetism according to absolute measure*

$$m = 2\,305.4\ .$$

For this value of *iron magnetism* one obtains from the above mentioned ratio the *diamagnetism of bismuth according to absolute measure* corresponding to the same force $X = 3\,012$

$$= \frac{1}{360\,740} \cdot 2\,305.4 = \frac{1}{156.5}\ .$$

Finally, reducing this strength of diamagnetism obtained for different values of the force X by division by X one obtains according to the *first* comparison (by *magnetic* effects) for the *strength of the diamagnetism of bismuth with respect to the unit of force and the unit of mass according to absolute measure* the value

$$\frac{1}{629.9} \cdot \frac{1}{689} = \frac{1}{434\,000}\ .$$

On the other hand from the latter comparison (by *electric* effects) one obtains

$$\frac{1}{2\,301} \cdot \frac{1}{156.5} = \frac{1}{471\,300}\ .$$ [70]

Averaging one obtains for the *strength of the diamagnetism of bismuth with respect to the unit of force and the unit of mass according to absolute measure* the value

$$= \frac{1}{452\,000}\ .$$

According to the formulas stated in the previous Section, however, one finds a *limit value of the magnetism produced by the unit of force in the unit of mass of iron, according to absolute measure*, the value

$$= 5.6074$$

which is 2 540 000 times bigger as the diamagnetism.

For *small separating forces* and *thin iron bars* for which the magnetism of iron is almost in a *constant* ratio to the diamagnetism of bismuth, it follows that the *diamagnetism of bismuth* is about $2\frac{1}{2}$ *millions* times smaller than the *magnetism of iron*. The larger the separating forces and the thicker the iron bars become, the more the diamagnetism of bismuth is increasing with respect to the magnetism of iron, so that according to the case stated in Section 2.10, it increased up to the 360740th part of the iron magnetism, which is the largest value occurring in the experiments above.

[70][Note by WW:] According to this ratio it follows easily, by assuming the result obtained from the *magnetic* effect of bismuth $= \frac{1}{1\,470\,000}$ found at the beginning of Section 2.10 on page 46, that the result $= \frac{4340}{4713} \cdot \frac{1}{1\,470\,000} = \frac{1}{1\,596\,000}$ deduced from the *magnetoelectric*, has to be put instead of $= \frac{1}{1\,731\,560}$, which was found in Section 2.10 on page 50 based on the experiments by Müller. Incidentally, the more precise result found here has already been mentioned with reference to this Note.

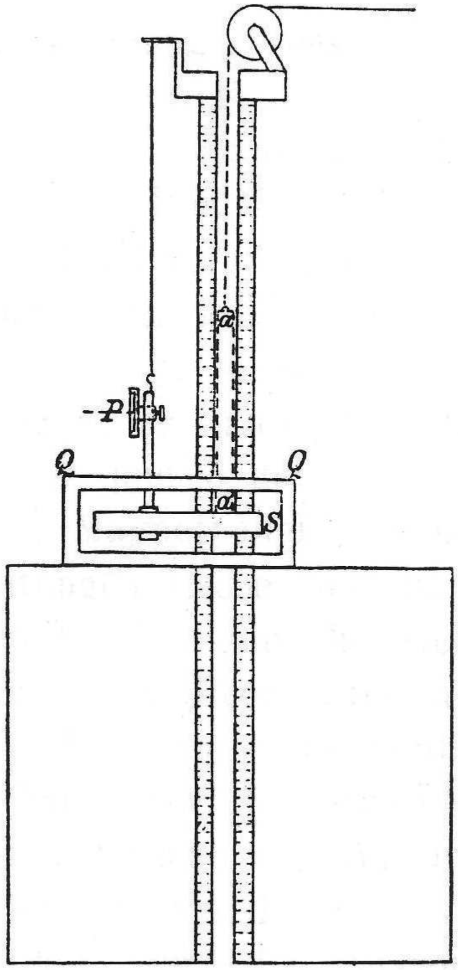


Fig. 4

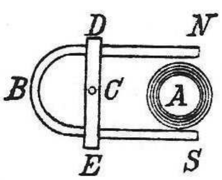


Fig. 3.

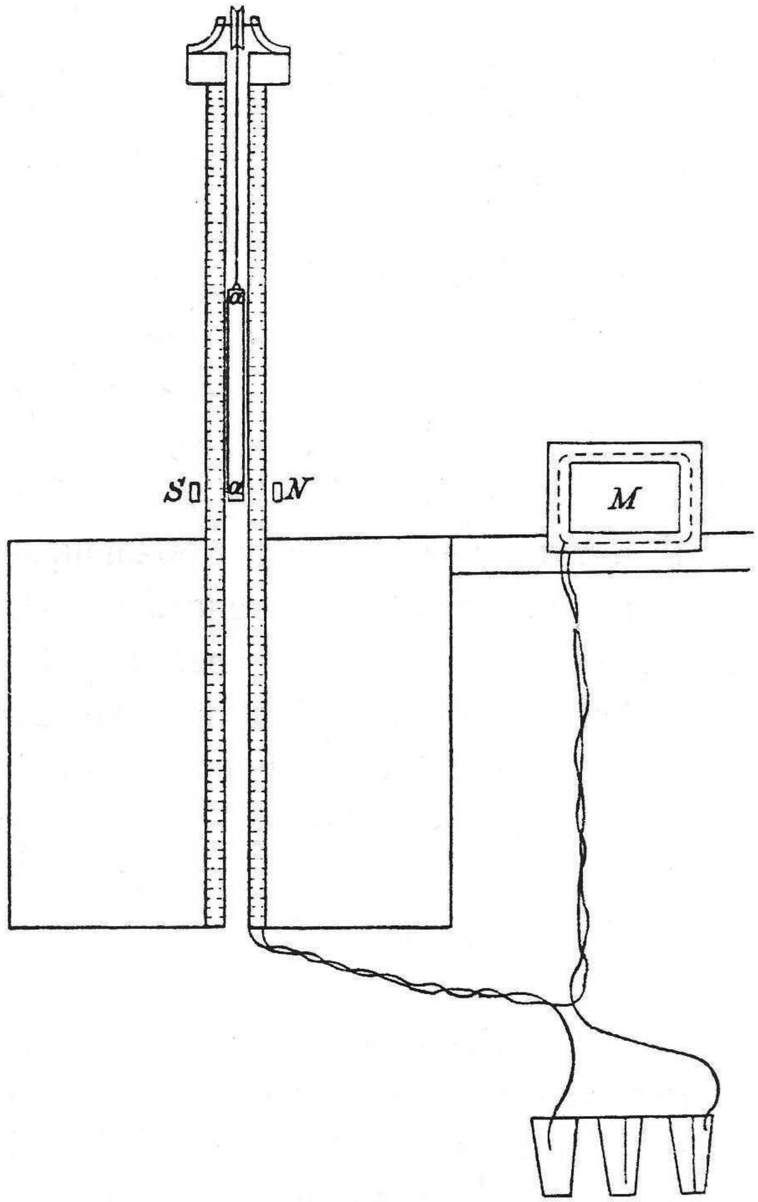
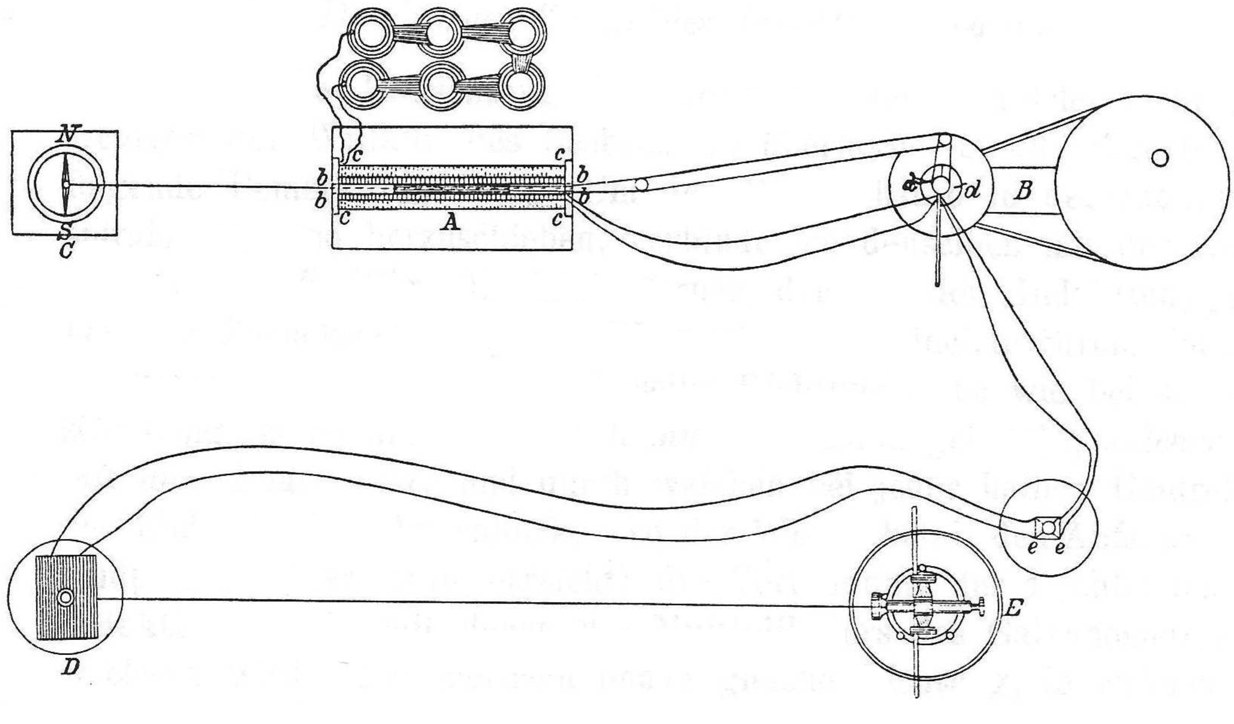


Fig. 5.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Chapter 3

[Weber, 1852c] On the Connexion of Diamagnetism with Magnetism and Electricity

Wilhelm Weber[71,72,73]

> From Poggendorff's *Annalen*, vol. lxxxvii. p. 145. Extracted from the Memoirs of the Royal Society of Sciences of Saxony, p. 483 to 578; also in a separate work, "Memoirs on Electro-dynamic Measurements," by M. Weber. Leipzig: Wiedemann, 1852.[74]

3.1 Theory

In treating of magnetism, a distinction is made between *permanent* and *variable* magnets; we regard, for example, a magnet of hard steel as a permanent magnet, and a magnet of soft iron as a variable one. Were the antithesis between both classes perfect (which, however, is as little the case as that between conductors and insulators in electricity), the magnetism of the permanent magnet could only be investigated through its *effects*, while that of the variable magnet might be investigated through its *causes* as well as through its *effects*. At all events, even though the antithesis be not perfect, the variable magnet is more favourable to a complete examination of the nature of magnetism than the permanent one.

In the same manner, in treating of diamagnetism, it might be attempted to classify *diamagnets* under the two heads *permanent* and *variable*; but then *we should have no mark by which a permanent magnet could be distinguished from a permanent diamagnet*, and thus the classification would lose all practical significance. In the investigation of diamagnetism,

[71] [Web52f] with English translation in [Web53b] and [Web66b].

[72] Wilhelm Weber's Notes are represented by [Note by WW:]; the Notes by H. Weber, the editor of the third volume of Weber's *Werke*, are represented by [Note by HW:]; the Notes by John Tyndall, the editor of the Scientific Memoirs where the English translation of this paper was published, are represented by [Note by JT:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[73] [Note by AKTA:] This work is an abridged version of Weber's Third major Memoir on Electrodynamic Measurements, [Web52b] with English translation in [Web21a], see Chapter 2.

[74] [Note by AKTA:] This text appeared on page 163 of [Web53b] and [Web66b]. It refers to [Web52f] and [Web52a].

therefore, *variable diamagnets* only are to be considered, which permit of examination partly through their causes and partly through their effects.

Now it is known that the investigation of the magnetism of a magnet through its *effects* (produced on other bodies) leads us to the knowledge of the *ideal distribution* of the magnetic fluid on the surface of a magnet, regarding which Gauss has proved,[75] that as far as the explanation of phenomena is concerned, it answers completely to the *true internal condition* of the magnet. In many investigations it is a great advantage to find a way furnished by the ideal distribution towards the simple and complete union of all the observed actions,[76] without the necessity of making any hypothesis regarding the interior of the body; more particularly when the causes of these effects remain unknown, and are still to be investigated. From the fact itself, however, that the knowledge of this ideal distribution, derived from observation, affords a satisfactory and complete view of the phenomena, it evidently follows that from *the observed phenomena alone* we cannot proceed further than to the knowledge of this ideal distribution, which however must of necessity be distinguished from the knowledge of the true internal state of the magnet; or, in other words, that proceeding from the *observed actions*, we are not in a condition to pronounce upon the actual distribution of the magnetic fluid within a magnet, or upon the actual number, strength, and arrangement of the electric currents contained within it.

The same holds good for the actions of a *diamagnet*; from the observation of its actions we might arrive at a knowledge of the ideal distribution of the magnetic fluid on the surface of the diamagnet, and thus find a substitute for the knowledge of its true internal state; but we could thus obtain no information regarding the true internal condition, or the real nature of diamagnetism itself, its *generation* and *modifications*. To come upon the trace of these, we must not limit ourselves to the consideration of the *actions*, and the ideal distribution which depends upon them; but it is necessary to call in the aid of some other consideration which is based upon a foundation independent of these actions.

All the possible causes of diamagnetism (like those of magnetism) may be divided in a general manner into *internal* and *external*. The *external* cause (like the effects) is given by *observation*, it is the same for magnetism and diamagnetism, namely, *a magnetizing or electro-magnetizing force, determinate in magnitude and direction*.[77] If, besides this *external* cause, that which lies *within* the magnet itself were known, by the union of both diamagnetism itself would be completely accounted for; and, inversely, we find a way open to the determination of *the true internal cause*, where, besides the known *external cause*, we become (through its actions) acquainted with the *diamagnetism* which is the resultant of both causes. Following up the way here indicated, and combining *the known magnetizing force of separation* with the *ideal distribution* deduced from the observed actions, for *iron* as well as for *bismuth*, we learn that the same force of magnetization causes opposite ideal distributions in the cases of iron and bismuth; or, inversely, *the same ideal distribution in iron and bismuth corresponds to oppositely directed magnetizing forces*. The reason why opposed external causes produce the same effect in iron and bismuth must be referred *to different internal causes within the iron and the bismuth themselves*. To determine more accurately

[75] [Note by AKTA:] See footnote 7 on page 11.

[76] [Note by AKTA:] *Wirkungen* in German. Alternative translation: effects.

[77] [Note by AKTA:] In German: *eine ihrer Grösse und Richtung nach bestimmte magnetische oder elektromagnetische Scheidungskraft*. An alternative translation might be "a magnetic or electromagnetic separating force, determinate in magnitude and direction". The expression *Scheidungskraft* can be translated as "separating force" or "force of separation", see also footnote 23 on page 47. An example of the action of this force might be the magnetization of a piece of soft iron due to a magnet or due to a current-carrying circuit.

the difference between the internal causes of *iron* and *bismuth*, it is necessary *to classify all possible internal causes* which can produce effects explainable by the *ideal distribution*, and then to ascertain whether, among all that we can reckon, such are embraced as will enable us to render an account of the above *antithesis between magnetic and diamagnetic bodies, subjected to the same external influences*.

3.1.1 Classification of the Internal Causes which may be Assumed as the Sources of the Effects Explainable by an Ideal Distribution

We can adduce *four* essentially different kinds of *internal* causes which are capable of producing effects explainable by an ideal distribution:

1. The internal cause of such effects may be referred to the existence of two magnetic fluids, which are more or less movable *independent of the ponderable matter which carries them.*

2. They may be due to the existence of two *magnetic* fluids, which are only capable of moving *in connexion with their ponderable carriers* (rotatory molecular magnets).

3. They may be due to the existence of *permanent molecular currents* formed by the electric fluids, and which can rotate *with the molecules.*

4. They may be due to the existence of *electric fluids* which can be thrown into *molecular currents.*

These *four* possible internal causes of the actions explainable by an *ideal distribution* on the surface are the only ones which are known, and which can be submitted to examination. The *first* case forms the basis of the magnetic theory of Coulomb and Poisson.[78] The *third* case forms the basis of the theory of Ampère on the connexion of magnetism with electrodynamics.[79] The *second* case may be reduced to the third, inasmuch as Ampère has proved that molecular magnets and molecular currents are alike in all their actions, and hence the latter may be substituted for the former. The *fourth* case, therefore, which has heretofore been unattended to, is the only one that remains to be considered.

For each of these four cases there exists a definite connexion between the character of the *ideal distribution* and the *direction of the magnetizing force of separation* to which it corresponds. Calling that direction along the line of magnetization in which the north pole of a magnetic needle is driven the *positive*, and determining the centre of gravity of the north and south fluids from their *ideal distribution*, then for the *first* case, according to the theory of Poisson, we find that the former of these two centres of gravity, as compared with the latter, is situated in the *positive* direction. For the *third* case this connexion has been developed by Ampère, and it has been found that the same dependence of the *ideal distribution upon the magnetizing force of separation* exists here also. From the reduction of the *second* case to the third, already mentioned, it is evident that the same dependence exists in the second case. Hence, in regard to this dependence, the *fourth* case alone remains open to consideration.

[78][Note by AKTA:] See footnote 43 on page 56.
[79][Note by AKTA:] See footnote 44 on page 57.

This *fourth* case assumes the existence of *electric fluids* which can be thrown into molecular currents. But the *possibility* of being thrown into molecular currents necessitates the inference, that in the single molecules, or around them, *closed paths* exist in which the said fluids can move *without resistance*; from which it follows that a *current-exciting force* (a force which acts upon the positive and negative fluids in opposite directions) in the direction of the path is necessary to cause the fluids *actually to move* along it. Now the facts of magneto-electricity prove that by *the increase or diminution of a magnetizing force*, a *current exciting* force (electromotive) is obtained, which acts upon the electric fluids in opposite directions, and hence *must throw them into current motion*. By the fundamental law of magnetic induction *the direction of this molecular current*[80] is given, *in its dependence on the increase or diminution of the magnetizing force*; and by the connexion of electro-dynamics and magnetism developed by Ampère for the third case, the *ideal distribution* is, in its turn, given in its dependence upon the *molecular current*. We thus obtain, mediately, the connexion between the *ideal distribution and the increase or diminution of the magnetizing force* to which it corresponds.

But it is evident from the above, that *at every moment* when an increase or diminution of the magnetizing force takes place, such *a molecular current* must be generated, and that these currents thus successively excited, if they do not of themselves again disappear, must *sum themselves up*. These currents however *do not* vanish *of themselves*, for Ampère has proved that *persistency* must be ascribed to *molecular currents*; that is, the electric fluids, in their circular motions around the ponderable molecules, suffer no such *resistance* as that encountered when the fluids traverse a ponderable conductor, to which resistance is to be attributed the speedy disappearance of the electric currents in these conductors. (From this *persistency*, which belongs of necessity to the *molecular currents*, it is manifest that the *possibility* of throwing the electric fluids *into molecular currents* is to be referred to the fact, that in the molecules, or around them, *closed paths* exist in which the said fluids *move without resistance*.) From this it follows, that a *continuous increase of the magnetizing force* is accompanied by a *continuous accumulation of the magnetic fluids, according to the ideal distribution*; and hence we infer that *every given strength of the magnetizing force has a definite moment of ideal distribution* corresponding to it. This *summation* however takes place only in the case of *molecular currents*, for in this case alone *the electric fluids move without resistance*. *Other* currents, which are excited by the same force at a greater distance, but which, on account of the resistance experienced, quickly disappear, produce magnetic effects on other bodies only *during the moment of their excitation* (through increased or diminished magnetizing force). These effects immediately vanish as soon as the force has become *constant*, and hence stand in no relation whatever to the magnitude of the *existing magnetizing force*; a relation however must exist if *the effects of variable magnets or diamagnets* are to be accounted for, and hence *molecular currents* alone are available here. Developing *with regard to these molecular currents*, in accordance with the laws of *magnetic induction*, the dependence of the moment of *ideal distribution* upon the magnitude of the *magnetizing force in operation*, we find, that if in the line of magnetization the direction in which the north pole of a magnetic needle is driven be called the *positive*, and the centres of gravity of the north and south fluids, according to the *ideal distribution* dependent on the magnetizing force, be determined, the former of these in comparison with the latter is situated in the

[80][Note by AKTA:] In German: *Molekularströmung*. In [Web53b] and [Web66b] this expression was sometimes translated as "molecular current" and other times as "molecular flux". I utilized "molecular current" is all places.

negative direction, which is *exactly the reverse of what takes place in the other three cases.* This enables us to render an account of the *inner cause of diamagnetism.*

3.1.2 Internal Cause of Diamagnetism

This remarkable result may be applied to the founding of a theory of diamagnetic phenomena, which shall assign an origin to the forces which produce them, a subject hitherto unexplained. For such a theory it is not sufficient that the diamagnetic state of a body may be conveniently represented by an ideal distribution of the magnetic fluids over its surface, but it is essential that it shall render an account of the *forces* which produce the diamagnetic state, and also of the laws according to which these forces act.

From the above statement and consideration of the different possible ways in which a condition representable by an ideal distribution might be developed in a body, one case alone was found from which a law coinciding with the fundamental phenomena of diamagnetism resulted. It follows from this, that an explanation of the development of the diamagnetic state can only be given when this case is regarded as actually existing; according to it the increase of the diamagnetism of a body is proportional to the inducing force acting upon the electric fluids, causing them to move without resistance in definite circular paths around the molecules, and *accelerating* the velocity of their movement in these paths. The diamagnetism of bismuth, for example, is explained by the assumption that the molecules of bismuth contain within them definite paths or canals, in which the electric fluids move without resistance, while in all other paths these fluids can only be set in motion by first overcoming a resistance proportional to their velocity. The generation of pure diamagnetism (unmixed with magnetism) would further necessitate the assumption, that the molecules which contain the above paths or canals are not capable of being rotated; for were the contrary the case, rotatory molecular currents might be generated, of such a strength that a portion of their intensity during the rotation might be regarded as constant, and hence, according to Ampère, would produce the magnetic state as a consequence. Conformably to this assumption, the diamagnetism or electro-diamagnetism of a body can be determined from the magnetizing or electro-magnetizing force exerted upon it.

3.1.3 Determination of the Diamagnetism or Electro-diamagnetism of a Body from the Magnetizing or Electro-magnetizing Force Exerted upon It

The magnetizing or electro-magnetizing force expressed by X[81] exerts upon a circle of the radius r, electromotive forces whose integral value, for the time during which this circle is moved out of a position perpendicular to the direction of the magnetizing force into a position parallel with it, according to Section 11 of the measurements of resistance in my *Electro-dynamic Measurements*,[82,83] is

$$= \pi r^2 X \ .$$

This integral value is the sum of the products of the electromotive force, reduced to an absolute unit in the Section 10 aforesaid,[84] into the element of time during which the force acts with this intensity. The expression of this integral value remains unchanged, if instead of moving the circle through an arc of 90°, the magnetizing force X *disappears*. If, on the contrary, this magnetizing force increases from $X = 0$ to $X = X$ (by closing the circuit), the expression of this integral value is

$$-\pi r^2 X \ ,$$

where the negative sign intimates that the induced circular current has such a direction that the poles of an equivalent molecular magnet are directed in an opposite manner to those of a compass needle under the influence of the force X.

This determination of the integral value of the electromotive force refers to the unit deduced from the absolute measure of magnetism, as established in the place above cited, pages 338 and 339;[85,86] it must be multiplied by $\sqrt{1/2}$ to render it true for the pure electrodynamic unit of electromotive forces given in Section 26 of the work cited;[87] hence

[81][Note by WW:] Every magnetizing force may be compared with terrestrial magnetism, and reduced to the same unit of measure. The electro-magnetizing force of a cylindrical spiral, [Note by AKTA: that is, a finite solenoid] through which a current passes of the intensity i, in accordance with the fundamental laws of electro-magnetism, is expressed by

$$\frac{2\pi n i}{\sqrt{a^2 + r^2}} \ ,$$

where n signifies the number of coils, r the radius, and a the length of the axis. This value is true, in the first place, for the magnetizing force in the middle of the cylinder, and approximates to that due to every other point of the interior space of the cylinder, excepting those which lie near its end, the more closely as the spiral increases in length and diminishes in radius. When, therefore, a bar of bismuth is situated in the centre of such a spiral, nearly equal electro-magnetizing forces are exerted upon all its particles; and hence it can be moved to or fro between certain limits within the spiral without any perceptible change of these forces. Hence such a spiral is particularly suited to experiments in which it is required that the diamagnetism shall remain unchanged. The above expression gives the electro-magnetizing force referred to the same unit as that of magnetizing forces (namely, to the absolute unit used in the determination of terrestrial magnetism), where i denotes the strength of the bar-magnetism, the action of which is equal to the action of the current circulating round a unit of surface.

[82][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 323.

[83][Note by AKTA:] [Web52c, pp. 322-325 of Weber's *Werke*] with English translation in [Web21b].

[84][Note by AKTA:] [Web52c, pp. 321-322 of Weber's *Werke*] with English translation in [Web21b].

[85][Note by HW:] Wilhelm Weber's Werke, Vol. III, p. 321.

[86][Note by AKTA:] [Web52c, pp. 321-322 of Weber's *Werke*] with English translation in [Web21b].

[87][Note by AKTA:] [Web52c, pp. 358-365 of Weber's *Werke*] with English translation in [Web21b].

$$-\frac{\pi}{\sqrt{2}} \cdot r^2 X \ .$$

This expression, multiplied by $4/c$ (where c denotes that constant value of the relative velocity at which *two electric masses exert no influence whatever upon each other*), gives the electromotive force in terms of the absolute unit of measure for all forces, established generally in mechanics (see Section 27 of the work cited);[88] hence

$$-\frac{2\sqrt{2}}{c} \cdot \pi r^2 X \ .$$

This is the value of the electromotive force for the length of the entire circular path, under the assumption that in every unit of length of this path the unit of electric fluid exists. Dividing by the circumference of the circle $2\pi r$, we find the electromotive force exerted upon each unit of electric fluid to be

$$= -\frac{\sqrt{2}}{c} \cdot rX \ .$$

This, according to the principles of mechanics, expresses *the increase of velocity which would be imparted to each ponderable unit of mass if it were connected with the unit of electricity*, in the time during which the magnetizing force increases from $X = 0$ to $X = X$. Let ε denote the unknown small fraction of the mass of the ponderable unit which the unit of electricity forms, then the above value divided by ε gives the *drift velocity* u originated by the given increase of the *magnetizing force*.[89] If this drift velocity u be multiplied by $4e/c$, where e denotes the quantity of electric fluid, referred to the electric unit of measure, which exists in each unit of length of the circular path, we obtain the intensity of the induced circular current according to the pure electro-dynamic unit of measure; and when multiplied by $\sqrt{2}$, we obtain it in terms of that unit according to which a current of the intensity 1, while passing round the unit of area,[90] is equivalent to the unit of magnetism,[91] namely,

$$-\frac{8e}{c^2\varepsilon} \cdot rX \ .$$

The *electromagnetic moment* of this induced circular current (molecular current) is found by multiplying the intensity of the current by the area enclosed by the circular path, and is

$$= -\frac{8e}{c^2\varepsilon} \cdot \pi r^3 X \ .$$

We have here assumed that the normal to the plane of the circular path is parallel to the direction of the magnetizing force, which can only be the case for *all* circular paths by *one*

[88][Note by AKTA:] [Web52c, pp. 365-368 of Weber's *Werke*] with English translation in [Web21b].

[89][Note by AKTA:] In German: *so giebt obiger Werth, mit ε dividirt, die Stromgeschwindigkeit u, welche durch das angegebene Wachsthum der Scheidungskraft hervorgebracht worden ist.* This expression was translated as, [Web52d, p. 171]: "the the above value divided by ε gives the velocity u of the current originated by the given increase of the magnetizing force." I translated *Stromgeschwindigkeit* as "drift velocity", that is, the velocity of the electrified particle relative to the mass of the conductor. The expression *Scheidungskraft* which Tyndall translated as "magnetizing force" might also be translated as "separating force" or "force of separation", see footnotes 23 and 77 on pages 47 and 86.

[90][Note by AKTA:] In German: *Flächeneinheit.* This expression was translated as "element of surface" in [Web53b, 171] and [Web66b]. I replaced it by "unit of area".

[91][Note by AKTA:] See footnote 54 on page 62.

particular arrangement of the molecules. In the case of bismuth we do not assume such an arrangement, but simply, in accordance with the idea of homogeneity, that the normals to the planes of the circular paths have no paramount direction. According to this, the number of circular paths whose normals make an angle φ with the direction of the magnetizing force, must be proportional to $\sin\varphi$. The intensity of the current will then be proportional to $\cos\varphi$, and the component of the moment parallel to the magnetizing force, to $\cos^2\varphi$. If, therefore, we multiply the above value by $\sin\varphi\cos^2\varphi$, we obtain an expression proportional to the contribution of all circular currents (molecular currents) the normals of which form an angle φ with the direction of the magnetizing force to the *electro-diamagnetic moment* of the bismuth, namely,

$$-\frac{8e}{c^2\varepsilon}\cdot\pi r^3X\cdot\sin\varphi\cos^2\varphi\ .$$

Multiplying this by $d\varphi$, and then, further, the integral taken between the limits $\varphi = 0$ and $\varphi = \pi/2$ by the number of molecular currents, we obtain the total electro-diamagnetic moment of the bismuth mass m, when μm denotes the number of molecular currents in the mass,

$$= -\frac{8\pi}{3c^2\varepsilon}\cdot\mu r^3e\cdot mX\ .$$

The electro-diamagnetic moment of a mass of bismuth is therefore proportional to the magnetizing force X and to the mass of the bismuth m, and is found by multiplication with a constant factor $8\pi/3c^2\varepsilon$, taken from the general theory of electricity, and with a constant factor μr^3e dependent on the *nature of the bismuth itself*. This last factor we may call the *diamagnetic constant* of bismuth.

In this determination of the electro-diamagnetic moment, the molecular currents induced in the circular paths have been regarded *singly*, as if on each molecule the electromotive force calculated from the force of magnetization X had alone acted. Strictly speaking, however, this is not the case. In each circular path, on the contrary, electromotive forces, resulting from the action of the molecules upon each other, come into play; just as the particle of an iron bar is not affected by the external magnetizing force, for example, the magnetism of the earth, alone, but also by such forces as result from the reciprocal actions among the iron particles themselves. Although this mutual action of the diamagnetic molecules is so small as scarcely to exert a sensible influence, still a remarkable antithesis between the mutual action of magnetic and diamagnetic molecules deserves consideration here.

3.1.4 Comparison of the Mutual Actions of Diamagnetic and Magnetic Molecules

When two particles of iron are situated in a line parallel to the direction of a magnetizing force X acting upon them, calling the magnetic moment produced by the magnetizing force in each molecule regarded singly m, there results for each particle an additional magnetizing force, due to the action of the other, by which the magnetic moment m is augmented. This new magnetizing force, resulting from the reciprocal action of the particles, is expressed according to known laws by $2m/r^3$, where r denotes the distance between the particles. The *total* magnetizing force $(X + 2m/r^3)$ produces, therefore, in the particle under consideration, the increased moment

$$\left(1 + \frac{2m}{Xr^3}\right) m \ .$$

When, on the contrary, two bismuth particles are situated in a line parallel to the direction of the magnetizing force X, calling the diamagnetic moment corresponding to this force of magnetization $-\mu$ (the negative sign signifies that for similarly directed magnetizing forces the diamagnetic moment is opposed to the magnetic one), then for each particle there results, from the action of the other, a new force $-2\mu/r^3$, where r denotes the distance between the particles; consequently to the *total* force of magnetization

$$\left(X - \frac{2\mu}{r^3}\right)$$

corresponds the *diminished* diamagnetic moment

$$-\left(1 - \frac{2\mu}{Xr^3}\right) \mu \ .$$

Hence the antithesis, *that the magnetism of the two iron particles in the line of magnetization is increased by their reciprocal action; but that, on the contrary, the diamagnetism of the two bismuth particles lying in this direction is diminished by their reciprocal action.*

The result is the reverse when the iron and bismuth particles lie in a line perpendicular to the direction of the magnetizing force; here the magnetism of the particles of iron is weakened by their reciprocal action; the diamagnetism of the bismuth particles, on the contrary, is strengthened through the same cause. We find, in fact, the weakened magnetism of the iron particle

$$= + \left(1 - \frac{m}{Xr^3}\right) m \ ,$$

and the strengthened diamagnetism of the bismuth particle

$$= - \left(1 + \frac{\mu}{Xr^3}\right) \mu \ .$$

From this it follows, that while to impart by a given magnetizing force the strongest magnetism to a given mass of iron, we must convert it into the form of a long thin bar, and set its length parallel to the direction of magnetization; in order to impart the maximum diamagnetism to a given mass of bismuth, we convert it into the thinnest plate possible, and set its thickness parallel to the direction of the magnetizing force. The further development of these laws of the reciprocal action of diamagnetic molecules, compared with that of magnetic molecules, leads finally to a simple distinction of magnetic and diamagnetic substances, which is worthy of more particular examination.

3.1.5 Distinction between Magnetic and Diamagnetic Bodies, through the Positive and Negative Values of a Constant

For the sake of unity, let us limit ourselves to the consideration of an ellipsoid of revolution of iron or of bismuth, the principal axis of which is parallel to the magnetizing force X; for the case of iron, Neumann[92] has proved that the magnetic moment of the ellipsoid is

[92] [Note by AKTA:] See footnote 56 on page 64.

$$= \frac{kvX}{1 + 4\pi kS} ,$$

where v denotes the volume and S a quantity derived from the ratio of the axes of the ellipsoid, namely,[93]

$$S = \sigma \left(\sigma^2 - 1\right) \left\{ \frac{1}{2} \log \frac{\sigma + 1}{\sigma - 1} - \frac{1}{\sigma} \right\} ,$$

$$\sigma = \sqrt{1 - \frac{r^2}{\lambda^2}} ,$$

r and $\sqrt{r^2 - \lambda^2}$ the axes of the ellipsoid. k is supposed here to possess a constant value for iron, and to it Neumann has given the name of the *magnetic constant of iron*; *this constant quantity in the case of iron, as of all other magnetic bodies, is necessarily positive.*

The quantity k serves, therefore, by the different *positive* values which it assumes, as a mark of distinction of the various *magnetic substances*; but the use of the quantity k as a means of distinction may be rendered more general by applying it to all bodies, and permitting it to assume negative values, the physical explanation being attached, that *a body which gives a negative value for k is a diamagnetic body.* (The name *anti-magnetic* or *negative-magnetic* would, therefore, be more suitable to these bodies.) The negative value of k found for a diamagnetic body may be called the *magnetic constant of the diamagnetic body*, or we may call the positive value obtained by changing the sign, the *diamagnetic constant* of the body. Denoting this *always-positive* diamagnetic constant by h, to distinguish it from the likewise *always-positive* magnetic constant k, we obtain, in the same manner as Neumann has determined the magnetic moment of a magnetic ellipsoid, the diamagnetic moment of a diamagnetic ellipsoid,

$$= -\frac{hvX}{1 - 4\pi hS} .$$

Now for an infinitely elongated ellipsoid, for a sphere, and for an infinitely flattened ellipsoid, we obtain successively

$$S = 0 , \quad S = \frac{1}{3} , \quad S = 1 ;$$

hence the corresponding magnetic moments are, successively,

$$+kvX , \quad +\frac{kvX}{1 + \frac{4}{3}\pi k} , \quad +\frac{kvX}{1 + 4\pi k} ;$$

the corresponding diamagnetic moments, on the contrary, are

$$-hvX , \quad -\frac{hvX}{1 - \frac{4}{3}\pi h} , \quad -\frac{hvX}{1 - 4\pi h} .$$

The most lengthened form corresponds, therefore, to the weakest, the most flattened form to the strongest diamagnetism; exactly the reverse of what is true for magnetism, as above proved. As, however, the diamagnetic constant h possesses in all known diamagnetic bodies

[93] [Note by AKTA:] What Weber represents by the symbol "log" in the next equation should be understood as the natural logarithm represented nowadays as "ln".

a value which almost vanishes in comparison with the unit, the diamagnetic moment of all these bodies may, without sensible error, be regarded as independent of their form; it may be set

$$= -hvX \; ;$$

and this expression may be compared with that already obtained for the diamagnetic moment, where the reciprocal actions of the molecules were disregarded. Setting

$$v = \frac{m}{\rho} \; ,$$

where m denotes the mass and ρ the density of the body, we obtain for the diamagnetic moment the expression

$$-\frac{h}{\rho} \cdot mX \; ,$$

instead of the expression found above,

$$-\frac{8\pi}{3c^2\varepsilon} \cdot \mu r^3 e \cdot mX \; .$$

In both methods the diamagnetic moment is represented as the product of the mass m into the magnetizing force X, multiplied with a *constant coefficient*, which in the last expression consists of two factors, namely, the factor $8\pi/3c^2\varepsilon$, to be taken from the general theory of electricity, and the factor $\mu r^3 e$ dependent on the nature of the diamagnetic body; this has been already named the *diamagnetic constant* of the body. These two factors are not separated in h/ρ; [the magnitude] h/ρ, indeed, is nothing else than the product of the above two factors.

The quantity k is here assumed as constant (that is, independent of the strength of the magnetizing force X), because Neumann has proved from the theory of separable magnetic fluids that it must be constant (that is, independent of the strength of the separating force X). The results above stated are, however, independent of this assumption, and retain their validity even should closer examination prove k to be a function of the magnetizing force X. From this examination, however, it will follow of itself, that even if k changes with X, h will nevertheless possess *a constant value* for every diamagnetic body.

By the theory of diamagnetism here developed, it is easy to show that the disputed question, *whether magnetic fluids actually exist*, can be decided.

3.1.6 On the Existence of Magnetic Fluids

When a certain class of actions of one body upon other bodies is so characterized that these actions may be explained by reference to an ideal distribution of magnetic fluids upon the surface; then, for the true interior condition of the body, four different possibilities may be thought of, and thus four different cases distinguished, which have been above stated and discussed. Two of these cases rested on the assumption that two magnetic fluids exist, either in the rotatory molecules of the body, immoveable, but in *constant* separation; or in non-rotatory molecules, moveable, and in *variable* separation. The two other cases, on the contrary, rested on the assumption that the two electric fluids existed either in a definite circuit round each rotatory molecule of the body in *constant* motion, or round each non-rotatory molecule in *variable* motion. These four cases do not by any means reciprocally exclude each other; for it is easy to see that a portion of the magnetic fluids may remain constantly separated in rotatory molecules, while the separation of another portion is variable; and in the same way a portion of the electric flow in given circular paths round rotatory molecules may be constant, while another portion in circular paths round non-rotatory molecules varies in intensity. In the latter respect, indeed, when we consider the numerous electromotive forces present, the existence of a constant flow without a variable portion is perfectly inconceivable; for the electric fluids, if *free to move* in definite paths, as the existence of constant currents proves, must necessarily obey the impulsion of the electromotive forces decomposed in the direction of these paths. Nevertheless the above four cases may be combined pair-wise to two principal cases, each of which, if actually established, would leave the other in the position of a quite superfluous hypothesis, namely, — 1st, that magnetic fluids exist, which with the molecules, or in them, are capable of motion; 2ndly, that the electric fluids, which, according to the doctrine of electricity, are everywhere present, move without resistance in definite circular paths around the molecules.[94]

For each of these two principal cases a theory may be developed, and each of these theories may be divided into two portions, in one of which the results of both theories coincide, and in the other of which they contradict each other; for these theories are similarly circumstanced to the theories of *emission* and of *undulation* in optics, which likewise in many respects were coincident, until the discovery of the phenomena of interference led to the more accurate investigation of those points in which the theories contradicted each other. Now, although the two theories resulting from the assumptions of magnetic fluids and of molecular currents have heretofore exhibited a surprising coincidence in their results, it might, nevertheless, be expected that here, as in optics, *the discovery of a new class of phenomena* would lead to a closer discussion of the points wherein both theories differ. Both theories, indeed, coincide — 1st, in all phenomena which relate to *permanent magnets*; 2ndly, in the circumstance that each permits of a division of variable magnets into two classes, namely, into such as owe their magnetism to the mere arrangement of already existing rotatory molecules (molecular magnets or molecular currents), and into such as owe their magnetism to the excitation of the motion of imponderable fluids in motionless molecules (the separation of magnetic fluids in the molecules, or the excitation of electric currents in definite circular paths around the molecules); 3rdly, both theories agree in their results with regard to the first class of variable magnets. The theories, however, contradict each other in the results which have reference to the second class of variable magnets, for their conclusions *regarding the positions of the poles* are opposed to each other. In accordance with the one, the positions of the poles, in the second class of variable magnets, must be the same as those in the first class; in accordance with the other, the positions of the poles in the second class must be the reverse of those in the first. So long, therefore, as variable magnets in which the positions of the poles (for similarly directed magnetizing forces) were identical were the only ones known, both theories might be applied; but as soon as variable magnets (diamagnets) were discovered, in which the positions of the poles (for similarly directed magnetizing forces) were opposed to each other, no further choice remained between both theories, for the second alone can render an account of the generation of two classes of magnets with poles oppositely situated, the directions of the magnetizing forces being the same.

The diamagnetic phenomena discovered by Faraday[95] serve, therefore, to decide the al-

[94][Note by AKTA:] See also footnote 58 on page 66.
[95][Note by AKTA:] See footnote 6 on page 11.

ternative between both theories, just as the phenomena of interference served to decide the alternative between the theories of emission and of undulation; and this is the most essential and important character that can be ascribed to the discovery of Faraday. Through the discovery of diamagnetism the hypothesis of electric molecular currents in the interior of bodies is corroborated; and the hypothesis of magnetic fluids in the interior of bodies is refuted, — a result which also finds a corroboration in the closer and more direct examination of variable magnetism, namely, in the law according to which the strength of the variable magnetism is determined from the magnitude of the magnetic or electro-magnetic force; this, however, deserves a closer discussion here.

3.1.7 Dependence of the Variable Magnetism upon the Magnitude of the Magnetic or Electro-magnetic Separating Force

According to the foregoing theory of diamagnetism, the diamagnetic moment of a diamagnet is proportional to the magnitude of the magnetic or electro-magnetic separating force. According to the notion heretofore entertained regarding the moveable magnetic fluids within the molecules of iron, the same proportionality holds good for the magnetic moment of a variable magnet. If, however, this notion, together with the hypothesis of magnetic fluids in the interior of bodies, be rejected, and instead of it Ampère's notion, that the molecules of iron are the ponderable bearers of permanent molecular currents, be assumed, from it will follow a different law of dependence between the variable magnetism and the magnitude of the magnetic or electro-magnetic separating force.

In Plate I, Figure 1, let NS be the axis of an unchangeable molecular current, which is capable of rotation around its centre C; when the magnetizing force X is $= 0$, let the position of equilibrium for this axis be parallel to ND.

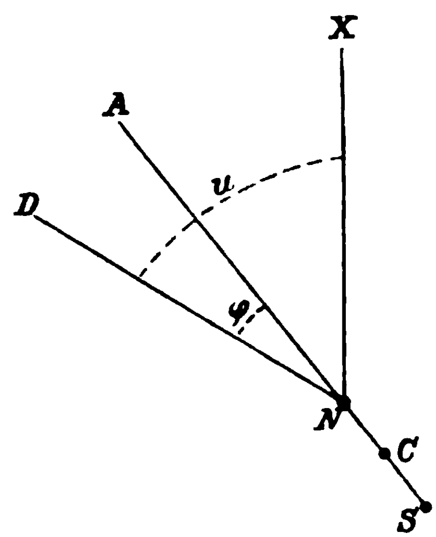

Fig. 1.

The fact that the magnetism excited in soft iron by the magnetizing force disappears again of itself, as soon as the magnetizing force ceases to act, proves that the molecular current, to the rotation of which the excited magnetism is due, recedes of itself to its original position parallel with ND. This force of recession, which is to be referred to the reciprocal actions of the iron particles, must increase with the deflection AND, and may be represented by

$$D \sin \varphi$$

where D is a constant quantity which may be named the *molecular directive force*.[96] If, now, besides this molecular force of direction, the magnetizing force X act upon the molecular current in the direction NX, which encloses the angle $XND = u$ with the line of the directive force, the molecular current will by this be drawn or deflected through the angle $AND = \varphi$, and for the determination of the new position of equilibrium we have the following equation,

$$X \sin u \cos \varphi = (D + X \cos u) \sin \varphi \ ,$$

or

$$\tan \varphi = \frac{X \sin u}{D + X \cos u} \ .$$

From the deflection φ the increase of the magnetic moment of the molecular current, decomposed in the direction of the force X, may be determined. If the total unchangeable magnetic moment of the molecular current be denoted by μ, then it was decomposed in the direction of the force X before the deflection as

$$= \mu \cos u \ ,$$

after the deflection,

$$= \mu \cos(u - \varphi) \ ,$$

hence the required increase x

$$x = \mu \left(\cos(u - \varphi) - \cos u \right) \ .$$

Substituting here for φ the value of it as given by the above equation,

$$\tan \varphi = \frac{X \sin u}{D + X \cos u} \ ,$$

we obtain

$$x = \mu \left\{ \frac{X + D \cos u}{\sqrt{X^2 + D^2 + 2XD \cos u}} - \cos u \right\} \ .$$

For a system of molecular currents whose magnetic axes, in their original positions of equilibrium, point in all directions without distinction, the number of molecular currents whose axes form an angle u with the direction NX of the force X are to be set proportional to $\sin u$. Let it be required to determine the magnetic moment y which results from the rotation of all molecular currents of the system, by the force X.

To this end let the value of x above found be multiplied by $\sin u du$, and let the integral be taken within the limits $u = 0$ and $u = \pi$. This integral value, multiplied by the number n of the molecular currents and divided by

[96][Note by AKTA:] See footnote 66 on page 77.

$$\int_0^\pi \sin u du = 2 \ ,$$

gives the required moment y,

$$y = \frac{n}{2}\int_0^\pi x \sin u du \ .$$

By carrying out the integration we obtain for y the following expression,[97]

$$y = n\mu \frac{X}{\sqrt{X^2 + D^2}} \cdot \frac{X^4 + \frac{7}{6}X^2D^2 + \frac{2}{3}D^4}{X^4 + X^2D^2 + D^4} \ .$$

The force which acted upon the iron, and by which the moment y was generated, was $= X$. Let n denote the number of molecular currents in the *unit of volume*, then the ratio of the moment y to the force X, in the theory of molecular currents, has the same meaning as the magnetic constant which Neumann has denoted by k, in the theory of separable magnetic fluids. Substituting, therefore, for k in the formula of Neumann given above, $kvX/(1+4\pi kS)$, the variable value just found y/X, we obtain the sought magnetic moment of a variable magnet of the form of an ellipsoid of revolution, to which the formula of Neumann refers,

$$= \frac{vy}{1 + 4\pi S \frac{y}{X}} \ ,$$

where S denotes the factor already determined from the ratio of the axes of the ellipsoid.

This result, referring to the dependence of the variable magnetism on the strength of the magnetizing or electro-magnetizing force derived from the view of Ampère in contradistinction to that usually assumed, is actually corroborated by the experiments described by Müller in Poggendorff's *Annalen*,[98,99] 1851, vol. lxxxii. p. 181.

3.2 Experiments

Having in the foregoing pages, for the sake of obtaining a simpler general view, stated, under the title of a theory, the results obtained with regard to the connexion of diamagnetism with magnetism and electricity, we shall, in the present section, give a brief description of the experiments by which the theory is established.

[97][Note by HW:] W. Weber later on improved the following expression for y with the following words (Berichte über die Verhandlungen der Königl. Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physische Klasse, year 1852, p. 164):

> p. 572, line 22 of the last publication on electrodynamic measurements in the first volume of the Abhandlungen der mathematisch-physischen Klasse der Königl. Sächs. Gesellschaft der Wissenschaften is for y, instead of the strict expression, replaced by an approximate value. I improve this oversight, while I notice, that it has no noticeable influence to the derived numerical data. There it follows in fact the precise integral value of y for all values of X, which are smaller than D, $y = \frac{2}{3}n\mu X/D$; for all values of X, which are greater than D, $y = n\mu(1 - \frac{1}{3}D^2/X^2)$.

[98][Note by HW:] Annalen der Physik und Chemie, edited by C. J. Poggendorff.

[99][Note by AKTA:] [Mül51b].

3.2.1 Electro-diamagnetism, and Measurement of the Moment of an Electro-diamagnet

The most convenient arrangement of an electro-diamagnetic apparatus of measurement, for the observation of diamagnetic polarity, consists in a galvanic spiral[100] which is set vertical and symmetrical between the two poles of a magnetic needle bent into the horseshoe form. A (see Figure 2) represents the transverse section of the spiral, which lies symmetrically between the poles N and S of the bent magnetic needle NBS.

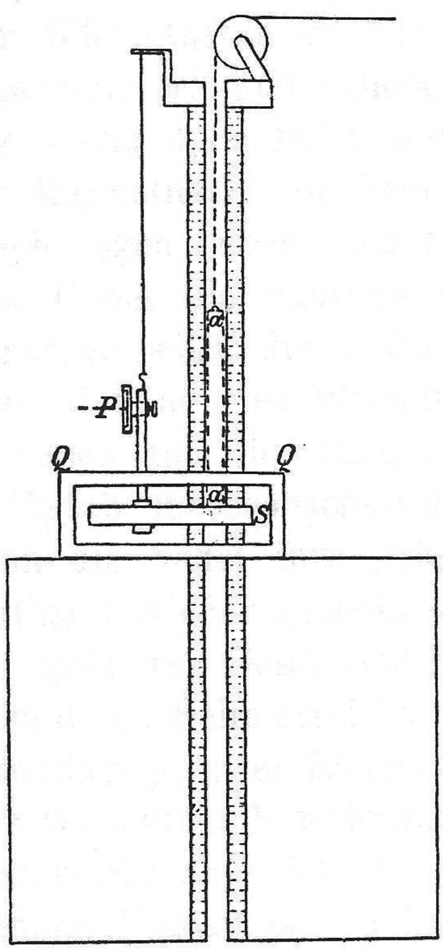

Fig. 3.

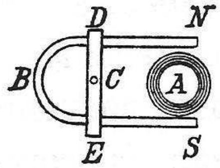

Fig. 2.

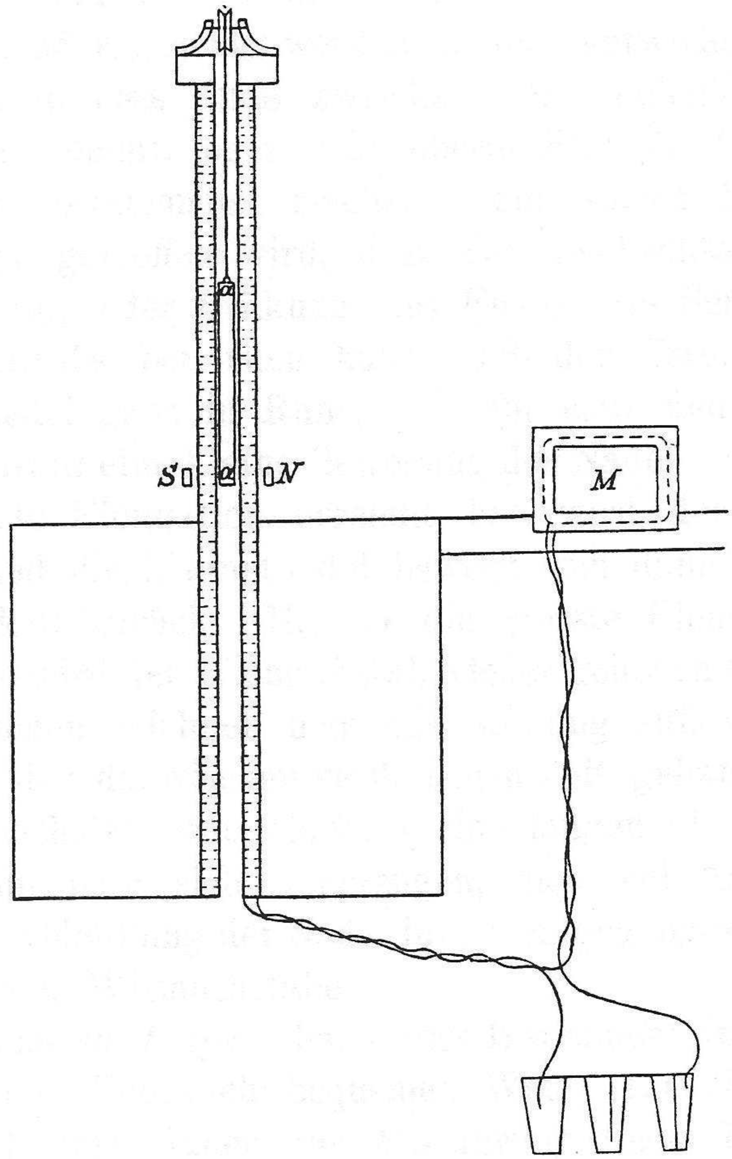

Fig. 4.

This magnetic needle is held by the clamp DE, in the centre of which, C, the suspending fibre is fixed. Figures 3 and 4 represent two side views of the instrument.

It is of advantage to give the spiral a considerable length, say from 400 to 500 millimetres; this renders it easier to regulate the suspension of the needle so that it shall swing in the

[100][Note by AKTA:] That is, a finite solenoid.

horizontal plane which bisects the length of the needle,[101] no moment of rotation being here exerted upon the needle when the current passes through the spiral. If, however, a small moment of rotation should exist, this is easily compensated by a multiplier M (Figure 4) consisting of a few coils, through which the same current is conducted and brought within a suitable distance of the magnetic needle. For the observation of the latter it is necessary to furnish it with a mirror P (Figure 3), and to observe therein, by means of a telescope, the image of a distant scale. The magnetic needle is further surrounded by a damper QQ, (Figure 3). The bismuth bar aa (Figures 3 and 4) is suspended vertically from a fibre within the spiral; it can be raised or sunk, so that either its under or its upper end shall lie between the poles of the magnetic needle, as represented in Figures 3 and 4. The observations are made most conveniently by means of an arrangement of pulleys or levers, which permits the observer himself, while standing beside his telescope, to raise or sink the bar of bismuth. When the circuit is established and the magnetic needle perfectly at rest, the bismuth bar is raised and the consequent small motion of the needle is observed. As soon as the needle has attained its maximum elongation the bismuth bar is again suffered to descend; the magnetic needle then moves back with increased velocity. When the maximum elongation on this side has been attained, the bismuth bar is again raised, and so on. Between every two elongations let the position of the bismuth during the intervening time be noted. If the bismuth bar be exchanged for an iron bar of equal length, but very thin, the experimenter can convince himself that when the positions of the bars are the same, the deflections of the needle produced by the iron and the bismuth are opposite in direction.

M. Leyser[102] of Leipzig has constructed this instrument in the simplest and most convenient manner (for 25 thalers[103] without the telescope); it deserves to be particularly recommended for its applicability to this fundamental experiment on diamagnetic polarity. In connexion with the results of certain experiments made by him and Prof. Hankel,[104] M. Leyser communicates to me as follows:

> "A current of four elements of Grove[105] was made use of, and by means of a multiplier the magnet was retained in its former position. The bismuth was chemically pure, and was so suspended that it could be moved up and down by means of a string, without shaking the magnet in the slightest degree.
>
> Observations of Leyser:

Position of magnet without current	492.0.
With current, bismuth in the middle	493.5,
with current, bismuth above	490.8,
with current, bismuth below	499.8,
with current, bismuth above	491.1,
with current, bismuth in the middle	493.8.

> We here observe, in coincidence with all other experiments executed in a similar manner, that in *drawing up* the bismuth (from the middle to the top) the position

[101][Note by AKTA:] In German: *Nadel*. John Tyndall translated this word as "needle," [Web53b, p. 182]. However, probably Weber was referring to the galvanic spiral or finite solenoid. For this reason Tyndall followed this word "neddle" with the following expression: (spiral? J. T.)

[102][Note by AKTA:] Georg Moritz Ludwig Leyser (1816-1881).

[103][Note by AKTA:] A former German silver coin.

[104][Note by AKTA:] Wilhelm Gottlieb Hankel (1814-1899).

[105][Note by AKTA:] See footnote 9 on page 15.

of rest for the magnet moved to *smaller* numbers, and by permitting it to descend (from the middle to the bottom) the position of rest moved to *higher* numbers. The difference between top and bottom amounts to

$$+8.9 \text{ divisions of the scale.}$$

Without current, bismuth in the middle	492.0,
without current, bismuth above	497.2,
without current, bismuth below	490.2,
without current, bismuth above	498.2,
without current, bismuth in the middle	490.0.

We here observe that *without a current* the action is opposed to that exhibited when a current is present; the difference between top and bottom being

$$-7.5 \text{ divisions of the scale.}$$

Observations of Prof. Hankel:

Position of magnet without current	496.5.
With current, bismuth above	492.1,
with current, bismuth below	500.7,
with current, bismuth above	491.6,
with current, bismuth in the middle	497.7.

The difference between top and bottom amounts therefore to

$$+8.9 \text{ divisions of the scale.}$$

Without current, bismuth in the middle	497.5,
without current, bismuth above	503.5,
without current, bismuth below	498.0,
without current, bismuth above	502.6,
without current, bismuth in the middle	494.8.

The difference between top and bottom amounts here to

$$-5.0.$$

The bismuth bar was then reversed and the same action was exhibited, namely,

Position of magnet without current	500.0.
With current, bismuth above	497.3,
with current, bismuth below	507.1,
with current, bismuth above	498.0.

The difference between top and bottom being

$$+9.4.$$

When, instead of determining the position of rest, the elongations were observed, while the arc of oscillation was multiplied by the alternate elevation and descent of the bar of bismuth, the following results were obtained:

	Elongations	Arc of oscillation
With current, bismuth in the middle		
	500.0	
With current, bismuth above		3.0
	497.0	
With current, bismuth below		16.0
	513.0	
With current, bismuth above		31.5
	481.5	
With current, bismuth below		34.0
	515.5	
With current, bismuth above		39.0
	476.5	
With current, bismuth below		43.8
	520.3	
With current, bismuth above		47.3
	473.0	
With current, bismuth below		49.0
	522.0	
With current, bismuth above		51.0
	471.0	
With current, bismuth below		55.0
	526.0	
With current, bismuth above		57.5
	468.5	

A little bar of iron, suspended instead of the bismuth, when situated above, caused the magnet to move to *higher* numbers, and when below caused it to move to *smaller* numbers; the same was observed when the little bar was reversed. The stand of the instrument must be very heavy; a serpentine stone was found very suitable: the bismuth must be capable of moving freely, and the copper wire pure. In all observations it was found, that applying four of Grove's elements, when the bismuth was *drawn up*, the impulsion amounted to from 8.9 to 9.4 divisions of the scale, the direction being towards smaller numbers, while a fine iron wire under the same conditions caused the magnet to move towards *higher* numbers; further, that the bismuth *without a current* acted as an iron wire, and caused a motion through 5 to 7.5 divisions of the scale. When the latter action is taken into account, *the mean diamagnetic impulsion of the bismuth* by the application of four of Grove's elements is found $= 15.4$ divisions of the scale. By multiplication, the arc of oscillation could be increased to 57.5 divisions, and retained at this magnitude, inasmuch as the action of the copper damper which surrounded the magnet held the diamagnetic action in equilibrium."

The following series of experiments was made with an apparatus somewhat different from that just described; a particular description is however unnecessary, as the difference exercises no particular influence.

Experiments with Bismuth					
No. of oscillation	Position of bismuth during the oscillation	Position of needle at the commencement or end of each oscillation	Arc of oscillation of the needle	Reduced arc of oscillation	Mean value
		500.0			
1	above		−40.0	−63.4	
		467.0			
2	below		−50.4	−66.6	
		513.9			
3	above		−56.3	−67.1	
		459.9			−61.8
4	below		−58.5	−65.5	
		518.5			
5	above		−55.2	−59.4	
		460.0			
6	below		−46.5	−48.8	
		512.0			
7			∓29.7		
	above	471.1			
8			±7.0		
		489.7			
9	below		−8.9		
		494.2			
10	above		−15.6		
		480.9			
11	below		−30.0	−47.5	
		498.9			
12	above		−50.4	−66.6	
		457.0			−59.8
13	below		−57.8	−68.5	
		516.0			
14	above		−50.9	−56.8	
		459.3			
15			∓35.6		
	below	504.4			
16			±12.4		
		478.3			
17	above		−14.7		
		476.9			
18	below		−36.6		
		504.9			
19	above		−42.6	−67.5	
		459.6			
20	below		−39.6	−52.3	
		499.4			
21	above		−46.6	−55.5	
		460.1			
22	below		−51.7	−57.9	−56.1
		513.9			
23	above		−45.9	−49.4	
		464.2			
24	below		−50.6	−53.1	
		506.2			
25	above		−55.2	−57.0	

Experiments with Bismuth					
No. of oscillation	Position of bismuth during the oscillation	Position of needle at the commencement or end of each oscillation	Arc of oscillation of the needle	Reduced arc of oscillation	Mean value
		446.9			
26			∓ 44.5		
	below	498.0			
27			± 15.5		
		460.0			
28	above		-16.8		
		453.1			
29	below		-29.8		
		479.8			
30	above		-40.3	-63.9	
		446.9			
31	below		-46.0	-60.2	
		494.6			-55.8
32	above		-42.2	-50.0	
		450.4			
33	below		-44.0	-49.3	
		490.5			
34	above				
		442.6			

In the *third* column of this Table the positions of the needle observed at the beginning and end of each oscillation are noted; in the fourth column the corresponding arcs of oscillation (the mean of every two successive ones). A *positive* sign before the arc of oscillation denotes that when the position of the bismuth was *above*, the needle proceeded from smaller to larger numbers, or when the bismuth was *below*, from larger to smaller numbers; the reverse applies to the *negative* sign. After the position of the bismuth had been several times regularly changed at the end of each oscillation, and the limit of the arc of oscillation nearly attained, an interruption was effected by permitting the bismuth to remain unmoved during two oscillations, and then again regularly changed as before. The *negative* arc of oscillation was thereby suddenly converted into a *positive* one, which, however, soon diminished to zero, and very soon afterwards passed over into a *negative*; the deflection caused by the bismuth (in its upper and lower positions) was here most clearly exhibited. When the arcs of oscillation are counted from that which is nearest to zero, the arc nearest to the limit may, by means of the known *decrementum logarithmicum*,[106] be easily reduced to the *limit-value*, and thus a more accurate mean value for the latter may be found. For this purpose we have only to divide the observed value of the nth arc of oscillation in the above experiments, where the *decrementum logarithmicum* was nearly $= \log \frac{3}{2}$ by $(1 - (2/3)^n)$. We thus obtain the reduced values exhibited in the *fifth* column, and the mean values derived from the latter in the *sixth* column. From all the observations taken together, we find the limit-value to be

[106][Note by AKTA:] That is, the logarithmic decrement.

$$x = -58.4 \ .$$

From this limit-value of the arc of oscillation, the deflection E corresponding to the needle's position of equilibrium can be deduced, namely $E = -5.93$; or taking the mean of several series of experiments made by different observers,

$$E = -5.17 \ ,$$

while for a bar of iron of 59200 times less weight, the same value, determined by similar experiments, was found to be

$$E' = +128.4 \ .$$

From this we learn, by reduction to the same weight, that the diamagnetism of the bismuth is 1 470 000 times less than the magnetism of the iron. The result, however, is only true for a particular form of the iron bar and for a definite strength of the magnetizing force, namely $X = 629.9$, a number obtained from the measured strength of current and from the coils of the electro-magnetic spiral.

3.2.2 Diamagneto-electricity and Measurement of Electric Currents Diamagnetically Induced

The apparatus for diamagnetic induction to be first described is so arranged that the induction is excited solely by *the motion of the diamagnetic body*, while the spiral *remains at rest*, and the diamagnetism of the body *remains unchanged*; by this means the formation of galvanic currents in the bismuth *as a conductor* is avoided, and with it a secondary inductive action, easy to be mistaken for the diamagnetic induction. The practical construction of such an apparatus consists in the application of a galvanic spiral, through whose electro-magnetizing force a bar of bismuth placed in its centre may, as remarked above, be uniformly diamagnetized and moved to and fro within certain limits, without suffering any change as to the strength of its diamagnetism.

3.2.3 The Electro-diamagnet made use of for the Diamagnetic Induction

The electro-diamagnet made use of for diamagnetic induction consisted of a bar of bismuth, a long wire spiral, *Acccc*, Figure 5, Plate I, through which a current from eight of Bunsen's couples[107] was conducted. The bismuth bar was 186 millimetres in length, and weighed 339 300 milligrams. The spiral consisted of copper wire covered with wool, and afterwards coated with gutta percha; the pure copper wire was 2.3 millimetres thick, and formed eight layers one above the other, each of which was composed of 120 coils. The whole spiral was 383 millimetres in length, and had an interior diameter of 23.9 millimetres, and an exterior diameter of 70 millimetres.

[107][Note by AKTA:] In German: *Bunsen'schen Kohlenzinkbechern*. See also footnote 15 on page 26.

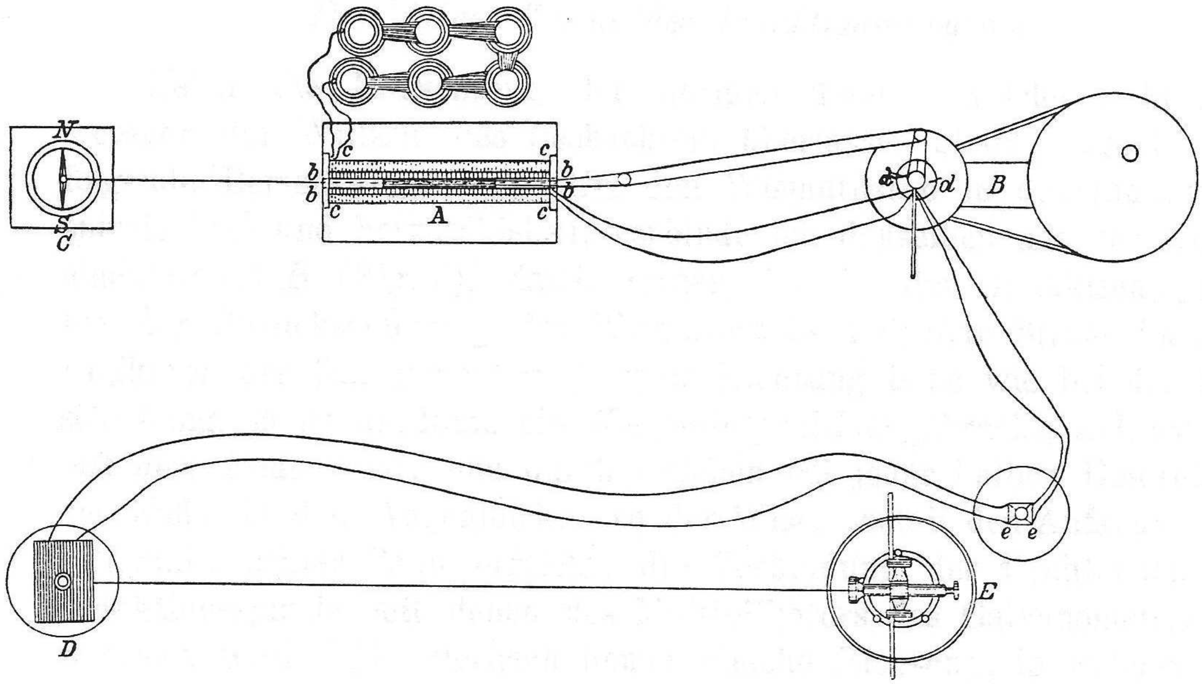

Fig. 5.

3.2.4 The Induction-spiral

The induction-spiral, *Abbbb*, Figure 5, Plate I, is that in which a current is to be induced *by the motion of the electro-diamagnet*. This spiral must be carefully insulated from that which belongs to the electro-diamagnet, through which the current passes from the galvanic battery, and connected with a multiplier, by which the induced current may be observed. This spiral consisted of copper wire overspun with silk, and 1 millimetre thick, forming three layers, each of which contained 294 coils; the length of the spiral was 383 millimetres, the interior diameter was 19, and the exterior 23 millimetres. Surrounded with gutta percha for the sake of better insulation, it was inclosed in the wider tube of the spiral of the electro-diamagnet, or rather the latter spiral was coiled round it.

The most essential point relating to this spiral is, that its length must be divided into two perfectly symmetrical and symmetrically coiled halves. The wire is not continuously coiled throughout the entire length in the same direction, but the spiral divides itself into two equal portions, which are coiled in opposite directions. This is necessary, if through the motion of a diamagnetic bar of bismuth, or a magnetic one of iron, a current is to be induced in the spiral; for when this inducing bar is placed in the centre of the spiral and moved there, the force of induction exerted by its north end in one half of the spiral is exactly opposed to that exerted by its south end in the other half, and the action of both would destroy each other if the two halves of the spiral were coiled in the same direction. Through the opposed coiling, the inductive forces, instead of destroying each other, are caused to exert a twofold action.

This necessary arrangement, for the purposes of induction, presents an important advantage as regards the practical carrying out of the experiments. It is manifest that the current of the galvanic battery, as long as it is *constant* in the spiral of the electro-diamagnet, can exert no inductive action on the induction-spiral; the slightest alteration of its intensity, however, would be sufficient to induce in the spiral a current much stronger than that diamagnetically induced, and which therefore would prevent the observation of the latter. It is, however, manifest that the same arrangement of the induction-spiral by which we have secured that the diamagnetic induction in both halves shall be a twofold power, also effects the mutual destruction of the currents excited in the two halves of the induction-spiral by the galvanic current; so that if the symmetry of the two halves be perfect, the greatest changes of intensity on the part of the current can produce no effect. To this we may add, — 1st, that it is very easy to prove if this compensation actually exists, by effecting, not small changes of intensity, but by interrupting or commutating the entire current; 2nd, that if it should appear that this compensation is not complete, it is very easily rendered so by winding an end of the induction-spiral once or oftener round the end of the spiral of the electro-diamagnet. In this way it is easy to rescue the diamagnetic induction from every foreign influence.

3.2.5 The Remaining Portions of the Induction-apparatus

With regard to the arrangement of the remaining portions, which are more or less left to the discretion of the observer, I would make the following remarks. To move the bar of bismuth to and fro in the induction-spiral, I connect it with a crank, attached to the wheel B, Figure 5, Plate I. To cause the current excited in the induction-spiral by the forward motion of the bismuth to have the same direction through the multiplier as that excited by its backward motion, a commutator dd is attached to the wheel, which moves with the latter, and by which the connexion of the ends of the spiral with the wire of the multiplier is reversed at the end of every semi-revolution (at the moment when the bismuth attains the commencement or end of its path). The uniform direction of all induced currents through the multiplier would be followed by a uniform deflection of the needle towards the same side. To enable the observer to produce a deflection towards the other side, a second commutator, ee, Figure 5, is placed beside the telescope E, which can be changed by the observer himself, and which may be called the *subsidiary commutator*. The following must be particularly attended to, — 1st, that the induction must be increased by increasing the velocity of the wheel, rather than by lengthening the path traversed by the bismuth; 2nd, that no thermo-magnetic current shall be generated at the rotating commutator; it must be so arranged that metals of the same kind only rub against each other. The influence of such currents, when very feeble, may be readily eliminated by suitably combining the observations. Finally, to obtain an approximate idea of the strength of the galvanic current, an ordinary compass is placed at a proper distance from the spiral of the electro-diamagnet, so that the deflection produced by the current passing through the spiral may be conveniently measured. The experiments were carried out as follows:

1st. The direction of the current being *normal*, the commutator was caused to rotate, and the bar of bismuth at the same time set in motion to and fro within the induction-spiral. At each elongation of the galvanometric needle, the observer changed the subsidiary commutator, until the arc of oscillation thus multiplied approached its limit-value; 2nd, the same series of experiments was then made with the direction of the current *reversed*; 3rd, with the current again in the *normal* direction; 4th, with the *reversed* current; and finally, 5th, once more with the *normal* current; 6th, the bismuth bar was exchanged for a thin *bar of iron*, and its induction measured in the same manner, with the current *normally* directed.

1. Induction of the *Bismuth Bar with a Normal* Current

No. of oscillation	Position of subsidiary commutator	Position of needle at beginning and end of each oscillation	Arc of oscillation of the needle	Reduced arc of oscillation	Mean value	Deflection of compass
		475.3				
1	−					
		472.8	+ 3.70			
2	+					
		477.7	+ 5.40			
3	−					
		471.8	+ 6.80			
4	+					
		479.5	+ 8.35			
5	−					
		470.5	+ 9.65			
6	+					
		480.8	+ 10.55			
7	−					
		470.0	+ 11.40	+ 13.20		
8	+					
		482.0	+ 12.25	+ 13.65		
9	−				+ 13.60	30° 10′ W
		469.5	+ 12.70	+ 13.75		
10	+					
		482.4	+ 13.00	+ 13.80		
11	−					
		469.3				

2. With a *Reversed* Current						
No. of oscillation	Position of subsidiary commutator	Position of needle at beginning and end of each oscillation	Arc of oscillation of the needle	Reduced arc of oscillation	Mean value	Deflection of compass
		503.5				
1	+					
		515.9	+ 9.50			
2	−					
		509.3	+ 3.65			
3	+					
		510.0	−1.25			
4	−					
		513.2	−4.75			
5	+					
		506.9	−7.35			
6	−					
		515.3	−8.90			
7	+					
		505.9	−9.60	−14.12		
8	−					
		515.7	−9.95	−13.10	−13.08	31° 50′ E
9	+					
		505.6	−9.85	−12.02		
10	−					
		515.2				

3. With a *Normal* Current						
No. of oscillation	Position of subsidiary commutator	Position of needle at beginning and end of each oscillation	Arc of oscillation of the needle	Reduced arc of oscillation	Mean value	Deflection of compass
		480.5				
1	+					
		471.0	−7.15			
2	−					
		475.8	−2.80			
3	+					
		475.0	+ 0.85			
4	−					
		472.5	+ 3.80			
5	+					
		477.6	+ 6.25			
6	−					
		470.2	+ 8.05			
7	+					
		478.9	+ 9.25			
8	−					
		469.1	+ 10.00	+ 13.17		
9	+					
		479.3	+ 10.75	+ 13.12		
10	−				+ 13.06	31° 48′ W
		468.0	+ 11.30	+ 13.08		
11	+					
		479.3	+ 11.30	+ 12.88		
12	−					
		468.0				

4. With a *Reversed* Current

No. of oscillation	Position of subsidiary commutator	Position of needle at beginning and end of each oscillation	Arc of oscillation of the needle	Reduced arc of oscillation	Mean value	Deflection of compass
		501.5				
1	+					
		515.0	+ 10.15			
2	−					
		508.2	+ 4.30			
3	+					
		510.0	−0.05			
4	−					
		511.9	−3.40			
5	+					
		507.0	−5.60			
6	−					
		513.3	−7.25			
7	+					
		505.1	−8.65			
8	−					
		514.2	−9.65			
9	+					
		504.0	−10.10	−12.33		
10	−					
		514.0	−10.55	−12.21		
11	+					
		502.9	−11.00	−12.25		
12	−				−12.16	32° 13′ E
		513.8	−11.30	−12.24		
13	+					
		502.1	−11.45	−12.15		
14	−					
		513.3	−11.25	−11.76		
15	+					
		502.0				

5. With a *Normal* Current

No. of oscillation	Position of subsidiary commutator	Position of needle at beginning and end of each oscillation	Arc of oscillation of the needle	Reduced arc of oscillation	Mean value	Deflection of compass
		486.0				
1	+					
		461.0	−20.40			
2	−					
		476.8	−12.40			
3	+					
		467.8	−6.15			
4	−					
		471.1	−1.25			
5	+					
		471.9	+ 2.75			
6	−					
		467.2	+ 5.75			
7	+					
		474.0	+ 7.10			
8	−					
		466.6	+ 7.30			
9	+					
		473.8	+ 7.75			
10	−					
		465.5	+ 8.90	+ 10.86		
11	+					
		475.0	+ 9.70	+ 11.23		
12	−					
		465.1	+ 10.05	+ 11.20		
13	+				+ 10.95	30° 39′ W
		475.3	+ 10.25	+ 11.10		
14	−					
		465.0	+ 10.15	+ 10.77		
15	+					
		475.0	+ 10.10	+ 10.56		
16	−					
		464.8				

6. Induction of the *Iron Bar with a Normal* Current						
No. of oscillation	Position of subsidiary commutator	Position of needle at beginning and end of each oscillation	Arc of oscillation of the needle	Reduced arc of oscillation	Mean value	Deflection of compass
		461.0				
1	+					
		457.2	−15.30			
2	−					
		484.0	−33.65			
3	+					
		443.5	−45.60			
4	−					
		494.2	−54.95			
5	+					
		435.0	−62.20			
6	−					
		500.2	−67.45			
7	+					
		430.5	−71.50	−84.98		
8	−					
		503.8	−74.50	−84.60		
9	+					
		428.1	−76.90	−84.47		
10	−					
		506.2	−78.60	−84.28		
11	+					
		427.1	−79.90	−84.16		
12	−				−83.876	31° 48′ W
		507.8	−80.85	−84.04		
13	+					
		426.8	−81.10	−83.50		
14	−					
		508.0	−81.30	−83.10		
15	+					
		426.6	−81.50	−82.85		
16	−					
		508.2	−81.75	−82.78		
17	+					
		426.3				

If we denote the very trifling influence exerted by the thermomagnetic current upon the result by x, x^I, x^{II}, x^{III}, x^{IV}, and neglect the still smaller differences $x - x^I$, $x^I - x^{II}$, $x^{II} - x^{III}$, $x^{III} - x^{IV}$, we obtain the following results for the limit-value corresponding to the *diamagnetic induction* alone, reduced to the *normal* direction:

from	1	$+13.60 + x$		
			$+ 13.34$	
	2	$+13.08 - x^I$		
			$+ 13.07$	
	3	$+13.06 + x^{II}$		Mean + 12.644.
			$+ 12.61$	
	4	$+12.16 - x^{III}$		
			$+ 11.555$	
	5	$+10.95 + x^{IV}$		

From this limit-value of the arc of oscillation produced by the uniform distribution of the inductive shocks[108] (by the motion of the bismuth to and fro) over the entire time of oscillation, it is easy to deduce the limit-value which would correspond to a concentration of all the inductive shocks during the oscillation, into a single moment. The value of the arc of oscillation above found, $= +12.644$, must for this purpose be multiplied by $\pi/2$, or more accurately, taking the influence of the damper into account, with 1.574235, by which we obtain the sought limit-value

$$= +19.905 \; .$$

For the bar of iron (where all the inductive shocks were thus concentrated), the corresponding limit-value is found to be

$$= -83.876 \; .$$

From a great number of similar experiments, executed by various observers, we find the ratio of the limit-value for bismuth to that for iron to be

$$+16.956 : -83.49 \; .$$

Now the intensity of the currents induced by bismuth and by iron is *directly proportional* to these limit-values, and *inversely proportional* to the number of inductive shocks during a time of oscillation (that is, as

$$1 : 216.2 \; ,$$

because in the experiments with the bismuth bar 216.2 shocks, and with the iron bar only 1 shock, took place during a time of oscillation). Hence *the currents induced by the diamagnetic bar of bismuth were opposed in direction to those induced by the magnetic bar of iron, and their intensities were in the ratio of*[109]

$$16.956 : 83.49 \cdot 216.2 = 1 : 1064.5 \; ,$$

although the bismuth bar weighed 339 300 milligrammes, and the iron bar only 790.86. From this we find by calculation, that a bar of bismuth of the same weight as a bar of iron would

[108][Note by AKTA:] In German: *Induktionsstösse*. It can also be translated as inductive kicks, blows or hits.

[109][Note by AKTA:] That is, as

$$16.956 : (83.49 \times 216.2) = 1 : 1064.5 = 9.394 \times 10^{-4} \; .$$

induce a current 456 700 times weaker than the latter. This result is only true however for a definite form of the bar of iron, and for a definite amount of magnetizing force, namely $X = 3\,012$, which is determined from the measured strength of the current, and the number of coils of the electro-magnetic spiral.

3.2.6 On the Dependence of the Strength of the Magnetism of Changeable Magnets upon the Amount of the Magnetizing or Electromagnetizing Force

From the theory of diamagnetism, in connexion with that of magnetism, discussed in the foregoing pages, it follows, that the assumption of two magnetic fluids capable of free motion in the molecules of iron, hitherto made in the case of magnetism, (and from which the proportionality of the magnetism to the magnetizing force follows as a consequence) is not admissible, and that hence the hypothesis of Ampère, according to which the molecules are the rotatory bearers of permanent molecular currents, and no proportionality between the magnetism and the magnetizing force exists, must be set in its place. This consequence of the theory admits of being brought to the test of experiment, and with reference to it the experiments of Müller have been already adduced. Other experiments have however been made, particularly by Buff and Zamminer, which have led to different results.[110] Before repeating these experiments it will be first necessary to state the conditions upon which a certain decision of the point depends.

From Müller's experiments, it appeared that the divergence of the magnetism of iron from a proportionality with the magnetizing force exhibited itself at much smaller intensities of the latter in the case of *thin* bars than when *thick* bars were made use of. In the comparison of Müller's experiments with those of Buff and Zamminer, we must remember that the thinnest bar made use of by Müller was only 6 millimetres thick, while the thinnest of those used by Buff and Zamminer was 9 millimetres thick; and this difference was rendered more influential by the relation of the length to the thickness; Müller's thin bar was 330 millimetres long, while that of Buff and Zamminer was only 200 millimetres. In the following experiments a still thinner bar than that of Müller was made use of, namely, one 3.66 millimetres in thickness, 100.2 in length, and 8190 milligrammes in weight. The magnetism of such a thin bar may be measured with great exactness by the deflection which it is able to produce upon a small magnetometer, placed at a distance, and observed by means of a telescope and mirror. The only difficulty here is the proper separation of the action of the magnetized iron from that of the galvanic current. It is manifest that when the same galvanic spiral is applied to the magnetization of thick and thin bars, as has been the case with Müller, Buff and Zamminer, the above separation admits of less accuracy in the case of thin bars; for here, inasmuch as the action of the galvanic spiral remains the same, its comparative influence will be greater than when thick bars are applied. In the following experiments, therefore, a spiral was made use of which tightly embraced the thin bar; and besides this, it was arranged that together with these narrow convolutions, each spiral formed two greater convolutions, which were traversed by the current in an opposite direction, and which embraced an area equal to that embraced by all the narrow convolutions taken together. In accordance with the known laws of electro-magnetism, we could here have no immediate action exerted by the current upon the magnetometer, a conclusion capable of easy proof. The entire action exhibited

[110][Note by AKTA:] [BZ50].

by the magnetometer was therefore due to the *magnetism* of the *iron* alone, and may be determined with equal sharpness and exactitude as that of permanent magnets, according to the directions given by Gauss in the *Intensitas, etc.*[111] It is especially to be mentioned that the spirals made use of by Müller, Buff and Zamminer were shorter than the bars of iron which they were used to magnetize. With Müller this difference was but trifling, for the iron bar projected only 15 millimetres beyond the ends of the spiral. With Buff and Zamminer it was however much greater, for here the ends of the largest and thinnest bar projected 45 millimetres beyond the two ends of the spiral. The injurious influence of this was increased by the circumstance that the length embraced within the spiral amounted to only 110 millimetres, whereas in the case of Müller the enclosed length was 300 millimetres. This circumstance is probably the chief cause of the divergence in the results arrived at by these observers; for it is manifest that the action of the spiral upon the iron is greatest at the centre, decreasing towards the ends, and that this decrease must, beyond the limits of the spiral, be exceedingly speedy. Hence, although the action produced by the galvanic current upon the central portions of the bar may have nearly attained its limit-value, it by no means follows that this is the case with the portions without the spiral. To effect this approximation at all points of the iron bar simultaneously, in the following experiments a spiral was made use of which was considerably longer than the bar of iron, so that the force exerted by the spiral (whose diameter was very small in comparison to its length) upon the ends of the bar did not differ sensibly from that exerted upon its centre, by which precaution alone secure results could be obtained.

Without entering upon the details of these experiments, I will here limit myself to a tabular statement of the results obtained in this way, and merely remark that each determination is the result of four changes of current in the spiral, the most exact coincidence being in all cases exhibited, a proof that the coercive force of the iron did not operate to the prejudice of the measurements. The reduction, according to known rules, of the magnetism of the iron to an absolute unit, requires no further explanation. The strength of the current, measured by means of a tangent galvanometer,[112] is also reduced to an absolute unit, and the correction depending upon the ratio of the length of the needle to the diameter of the ring is taken into account. From the strength of the current thus determined, the number of convolutions and the dimensions of the spiral, the magnetizing force operating upon the iron was finally calculated, in terms of the same unit as that to which the terrestrial magnetic force is referred; it appears in the *second* column of the following Table under X. The magnetism of the iron M, divided by the mass of the iron $p = 8190$, and thus reduced to the unit of mass, is exhibited in the *third* column under m.

[111] [Note by AKTA:] See footnote 47 on page 59.

[112] [Note by AKTA:] See footnote 12 on page 22.

No.	X	m
1	658.9	911.1
2	1 381.5	1 424.0
3	1 792.0	1 547.9
4	2 151.0	1 627.3
5	2 432.8	1 680.7
6	2 757.0	1 722.7
7	3 090.6	1 767.3
8	3 186.0	1 787.7
9	2 645.6	1 707.9
10	2 232.1	1 654.0
11	1 918.7	1 584.1
12	1 551.2	1 488.9
13	1 133.1	1 327.9
14	670.3	952.0

By these experiments the result obtained by Müller is confirmed, and it only remains to ascertain whether the change in the strength of the magnetism of the iron effected by the action of different forces of magnetization, here exhibited, coincides with the law deduced at the end of the first Section, from the assumption of a *definite capacity of rotation on the part of the molecules*. If this be the case, it evidently follows that we can assume, with Ampère, that these molecules are the bearers of molecular currents, by which assumption the *generation and changes of the magnetism of iron are rendered quite independent of the idea of magnetic fluids, and are reduced to the assumption of moveable electric fluids alone.*

According to the assumption already made (at the end of the first Section) of a definite capacity of rotation on the part of the molecules, the character of every body, in magnetical respects, is determined by two distinctive marks, — 1st, *by the product of the magnetic moment of a molecule* (in the direction of its magnetic axis) *into the number of molecules of the body*; 2ndly, *by the constant to which we have given the name of the molecular directive force.*[113] The product is denoted above by $n\mu$, the constant by D. Setting in the case of iron,

$$n\mu = 2324.68 ,$$

$$D = 276.39 ,$$

where the number of molecules n is referred to the unit of mass, we obtain from the formulae given at the end of the first Section, reference being made to the unit of mass instead of to the unit of volume, the density of iron being taken at 7.78, and the numerical factor S referring to a bar 100.2 millimetres long and 3.66 millimetres thick, that is, $S = 1/249$ — the following contemporaneous values of X and m:

[113][Note by AKTA:] See footnote 66 on page 77.

X	m
658.9	948.4
1 381.5	1 387.0
1 792.0	1 533.0
2 151.0	1 623.5
2 432.8	1 685.0
2 757.0	1 742.2
3 090.6	1 791.2
3 186.0	1 803.4
2 645.6	1 723.6
2 232.1	1 644.8
1 918.7	1 568.9
1 551.2	1 452.9
1 133.1	1 276.8
670.3	957.5

It will be observed that these calculated values differ but little from those contained in the foregoing Table, and which were derived from observation.

According to the same formulae, we obtain for the *iron* bar with which, by a comparison of the *magnetic actions*, m was found to be 1 470 000 times greater than its value for bismuth, X being $= 629.9$,

$$m = 2\,134\ ;$$

hence for *bismuth*, where $X = 629.9$,

$$m = \frac{2\,134}{1\,470\,000} = \frac{1}{689}\ .$$

For the *iron* bar, on the contrary, with which, by a comparison of the *inductive actions*, m was found to be 360 740 greater than for bismuth, X being $= 3\,012$, we have

$$m = 2\,305.4\ ;$$

hence for *bismuth*, where $X = 3\,012$,

$$m = \frac{2\,305.4}{360\,740} = \frac{1}{156.5}\ .$$

According to this it appears that when the magnetizing force is increased 4.8 times, the diamagnetism of the bismuth increases 4.4 times; that is, *nearly proportional*, although one determination is founded on a comparison of the *magnetic actions*, the other upon a comparison of the inductive actions. We thus find the proposition confirmed, that

> *the relation of the inductive actions to the magnetic ones is the same in the case of diamagnetic bismuth as in that of magnetic iron.*

Reducing diamagnetism to the same absolute unit as magnetism, we obtain finally *the strength of the diamagnetism of 1 milligram of bismuth*, operated upon by the magnetizing force $X = 1$,

$$= \frac{1}{452\,000}\ ,$$

and the strength *of the magnetism of 1 milligram of iron*, under the influence of the magnetizing force $X = 1$,

$$= 5.607\,4 \ ,$$

that is, *the magnetism of a thin bar of iron exceeds the diamagnetism of an equal mass of bismuth, when small and equal magnetizing forces operate upon both, about* $2\frac{1}{2}$ *million times.* For thicker bars and greater magnetizing forces this number is found to be somewhat smaller.

[J. T.][114]

[114][Note by AKTA:] That is, John Tyndall (1820-1893), translator.

Chapter 4

[Weber, 1855a] On the Theory of Diamagnetism. Letter from Professor Weber to Prof. Tyndall

Letter from Professor Weber to Prof. Tyndall[115,116,117]

Göttingen, Sept. 25, 1855.

My Dear Sir,

Accept my best thanks for your kind communication of the 3rd of September; I am gratified to learn that the apparatus executed by M. Leyser[118] in Leipzig for the demonstration of diamagnetic polarity has so completely fulfilled your expectations. This intelligence is all the more agreeable to me, inasmuch as before the apparatus was sent away, it was not in my power to go to Leipzig and test the instrument myself.

It gave me great pleasure to learn that Mr. Faraday[119] and M. De la Rive[120] have had an opportunity of witnessing the experiments, and of convincing themselves as to the facts of the case.

It was also of peculiar interest to me to learn that you had succeeded in establishing the polarity of the self-same heavy glass with which Faraday first discovered diamagnetism.[121] This is the best proof that these experiments do not depend upon the conductive power of bismuth for electricity.

I have read with great interest your memoir "On the Diamagnetic Force," &c. contained in the Philosophical Transactions, vol. cxlv.[122] It has been your care to separate the *fact* of diamagnetic polarity from the *theory*, and to place the former beyond the region of doubt. Allow me, with reference to this subject, to direct your attention to a passage at page 39

[115] [Web55a].

[116] The Notes by A. K. T. Assis are represented by [Note by AKTA:].

[117] John Tyndall (1820-1893). For his works on diamagnetism see [Jac15].

[118] [Note by AKTA:] See footnote 102 on page 101.

[119] [Note by AKTA:] Michael Faraday (1791-1867).

[120] [Note by AKTA:] Auguste Arthur de la Rive (1801-1873).

[121] [Note by AKTA:] See footnote 6 on page 11.

[122] [Note by AKTA:] See [Tyn55b], [Tyn55a], [Tyn55c] and [Tyn88, pp. 111-192].

of your memoir, which you adduce as a conclusion from my theory; the passage runs as follows:[123]

> "The magnetism of two iron particles in the line of magnetization is increased by their reciprocal action; but, on the contrary, the diamagnetism of two bismuth particles lying in this direction is diminished by their reciprocal action."

This proposition is by no means a necessary assumption of my theory, but is rather a direct consequence of diamagnetic polarity, if the facts be such as both you and I affirm them to be. What, therefore, you have adduced against the above conclusion must be regarded as an argument against diamagnetic polarity itself. The *diamagnetic reciprocal action* of the bismuth particles in the line of magnetization is necessarily opposed to the *action of the exciting magnetic force*. The latter must be enfeebled, because the diamagnetic is opposed to the *magnetic reciprocal action* of iron particles which lie in the line of magnetization, through which latter it is known the action of the exciting magnetic force is increased. Hence also the *modification* produced in bismuth by magnetic excitement, whatever it may be, must be weakened, because the force of excitation is diminished.

(I believe, however, that this argument against diamagnetic polarity may also be surmounted. The phænomenon which you have observed must be referred to other circumstances, also connected with the compression of the bismuth. For the diamagnetic reciprocal action is, as I have shown, much too weak to produce an effect which could be compared in point of magnitude with the reciprocal action produced in the case of iron.)

I take this opportunity of adding a few remarks for the purpose of setting my theory of diamagnetic polarity in a more correct light.

My theory assumes: — 1, that the fact of diamagnetic polarity is granted; 2, that in regard to magnetic phaenomena, Poisson's theory of two magnetic fluids,[124] and Ampère's theory of molecular currents,[125] are equally admissible. Whoever denies the first fact, or rejects the theory of Ampère, cannot, I am ready to confess, accept my theory.

But supposing that you do not reject Ampère's theory of permanent molecular currents, but are disposed to enter upon the inner connexion and true significance of the theory, you will easily recognize that it is by no means *an arbitrary assumption of mine*, that in bismuth molecular currents are excited, when the exciting magnetic force is augmented or diminished; but that the excitation of such molecular currents is *a necessary conclusion from the theory of Ampère*, which conclusion Ampère himself could not make, because the laws of voltaic induction, discovered by Faraday,[126] were unknown to him. In all cases where molecular currents *exist*, by increase or diminution of the magnetic exciting force *molecular currents must be excited*, which either add their action to, or subtract it from, the action of those already present.

Finally, permit me to make a few remarks on the following words of your memoir:[127]

[123][Note by AKTA:] [Tyn55b, p. 39]. Tyndall was quoting from Weber's work of 1852, [Web52f, p. 158 of the *Annalen der Physik und Chemie* and p. 565 of Weber's *Werke*] with English translation in [Web53b, p. 173] and [Web66b, p. 173]. It is an abridged version of Weber's Third major Memoir on Electrodynamic measurements, [Web52b, p. 531 of Weber's *Werke*] with English translation in [Web21a, p. 64]. See, in particular, pages 63 and 93 on Subsection 3.1.4 and on Section 2.20, respectively.

[124][Note by AKTA:] See footnote 43 on page 56.

[125][Note by AKTA:] See footnote 44 on page 57.

[126][Note by AKTA:] [Far32a] with German translation in [Far32b] and [Far89], and with a Portuguese translation in [Far11].

[127][Note by AKTA:] [Tyn55b, p. 39].

> "To carry out the assumption here made, M. Weber is obliged to suppose that the molecules of diamagnetic bodies are surrounded by channels, in which the induced currents, once excited, continue to flow without resistance."

The assumption of channels which surround the molecules, and in which the electric fluids move without resistance, is an assumption contained in the theory of Ampère, and is by no means added by me for the purpose of explaining diamagnetic polarity. *A permanent molecular current without such a channel involves a manifest contradiction*, according to the law of Ohm.[128]

I may further observe, that I do not wonder that you regard a theory which is built upon the assumption of such channels, as "so extremely artificial that you imagine the general conviction of its truth cannot be very strong." In a certain sense I quite agree with you, but I only wish to convince you that this objection applies really to the theory of Ampère, and only applies to mine in so far as it is built upon the former. (You may perhaps find less ground for objecting to the speciality of such an assumption, if you separate the simple fundamental conception, which recommends itself particularly by a certain analogy of the molecules to the heavenly bodies in space, from those additions which Ampère was forced to make, in order to apply the mathematical methods at his command, and to make the subject one of strict calculation. He was necessitated to reduce the case to that of *linear* currents, which necessarily demand channel-shaped bounds, if every possibility of a lateral outspreading is to be avoided.)

To place my theory of diamagnetic polarity in a truer light, I am anxious also to convince you that this theory is by no means based upon new assumptions (hypotheses); but that it only rests upon such conclusions as may be drawn from the theory of Ampère, when the laws of voltaic induction discovered by Faraday, and the laws of electric currents by Ohm, are suitably connected with it. I affirm, that, even if Faraday had not discovered diamagnetism, by the combination of Ampère's theory with Faraday's laws of voltaic induction, and Ohm's laws of the electric current, as shown in my memoir, the said discovery might possibly have been made.

In respect, however, to the artificiality of the theory of Ampère, I hope that mathematical methods may be found whereby the limitation before mentioned to the case of linear currents may be set aside, and with it the objection against channel-form beds. All our molecular theories are still very artificial. I for my part find less to object to in this respect in the theory of Ampère than in other artificialities of our molecular theories; and for this reason, that in Ampère's case the nature of the artificiality is placed clearly in view, and hence also a way opened towards its removal.

To Mr. Faraday I beg of you to present my sincerest respect.

Believe me, dear Sir,

Most sincerely yours,

Wilhelm Weber.

Professor Tyndall.

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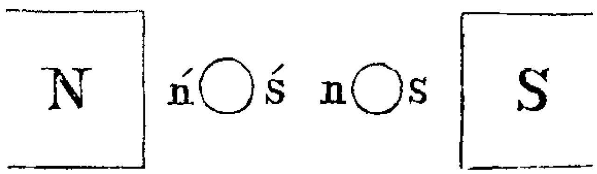
[128][Note by AKTA:] Georg Simon Ohm (1789-1854). Ohm's law is from 1826: [Ohm26a], [Ohm26c], [Ohm26d], [Ohm26b] and [Ohm27] with French translation in [Ohm60] and English translation in [Ohm66].

The foregoing letter possesses more than a private interest, and I have therefore laid it before the readers of the Philosophical Magazine. On one point in it only would I ask permission to make a remark, and that is the proposition, that the diminution of the excitement of a row of bismuth particles in the line of magnetization by their reciprocal action is "*a direct consequence of diamagnetic polarity.*" M. Weber (I believe) founds this proposition on the following considerations: — Let a series of bismuth particles lie in the axial line between the magnetic poles N and S: the polarity excited in these particles by the direct action of the poles will be that shown in the figure, being the reverse of that of iron particles under the same circumstances.



But as the end n of the right-hand particle tends to excite a magnetism like its own in the end s' of the left-hand particle, and *vice versa*, this action is opposed to that of the magnet, and hence the magnetism of such a row of particles is enfeebled by their reciprocal action.

Now it appears to me that there is more assumed in this ingenious argument than experiment at present can bear out. There are no experimental grounds for the assumption, that what we call the north pole of a bismuth particle exerts upon a second bismuth particle precisely the same action that the north pole of an iron particle would exert. Magnetized iron repels bismuth; but whatever the *fact* may be, the *conclusion* is scarcely warranted, that *therefore* magnetized bismuth will repel bismuth. Supposing it were asserted that magnetized iron attracts iron and repels bismuth, while magnetized bismuth attracts bismuth and repels iron, would there be anything essentially impossible, self-contradictory, or absurd involved in the assertion? I think not. And yet if even the possible correctness of such an assertion be granted, the proposition above referred to becomes untenable. It will be observed that it is against a conclusion rather than a fact that I contend. With regard to the fact, I should be sorry to express a positive opinion; for this is a subject on which I am at present seeking instruction, which may lead me either to M. Weber's view or the opposite. Be that as it may, the result cannot materially affect the respect I entertain for every opinion emanating from my distinguished correspondent on this and all other scientific subjects.

J. T.

Chapter 5

[Weber, 1855b] *Foreword* to the Submission of the Treatise: *Electrodynamic Measurements, specially Attributing Mechanical Units to Measures of Current Intensity*

Wilhelm Weber[129,130,131]

Session on 20 October 1855.

I am submitting the aforementioned treatise, which was written by myself and Professor Kohlrausch[132] in Marburg, to the Royal Scientific Society. It consists of a continuation of three treatises that were submitted previously, and which appeared under the same general title.[133]

As was developed already in the first treatise, the general law of electrical action and the fundamental laws that can be derived from it for various branches of the theory of electricity (with the exception of the fundamental law of electrostatics) include a *constant* whose numerical value, when expressed in terms of known units, has great importance for the whole theory of electricity, both theoretically and practically. That is because although one can make numerous applications of those laws to the determination of ratios or quotients in which that constant cancels in the denominator and the numerator while having no knowledge of the value of that constant, nonetheless there will be many other applications of the above laws that are not possible without determining the values of the constants that are included

[129] [Web55b] with English translation in [Web21c]. Related to [KW57] with English translation in [KW21], see Chapter 7.

[130] Translated by D. H. Delphenich, http://www.neo-classical-physics.info/index.html and e-mail: feedback@neo-classical-physics.info. Edited by A. K. T. Assis.

[131] The Notes by A. K. T. Assis are represented by [Note by AKTA:].

[132] [Note by AKTA:] Rudolf Hermann Arndt Kohlrausch (1809-1858).

[133] [Note by AKTA:] [Web46] with partial French translation in [Web87] and English translation in [Web07]; [Web52c] with English translation in [Web21b]; and [Web52b] with English translation in [Web21a].

in them. Up to now, the determination of that value has been lacking, and the closing of that gap in the determination of electrodynamic measurements is the next objective of the present treatise.

The simplest path to achieving that goal is to appeal to the measurements of current intensity in absolute units, which was discussed in detail in the Second treatise,[134] namely, the ones that are based upon the magnetic or electrodynamic current effects, to which the electrolytic current effects can be easily reduced with the help of corresponding observations. That is because the current intensity, when expressed in terms of either units, is nothing but the amount of positive electricity that flows through each cross-section of the conductor in one second in the direction of the current, multiplied by either $\sqrt{8}$ or 4, and divided by that *constant*, which explains the fact that when only that amount of electricity can be measured, the measurement of the current intensity in one of the two absolute units will lead to the determination of the value of that *constant*. However, that total positive electricity that flows through the cross-section of the conductor in one second in the direction of the current was referred to as the *mechanical measure* of the current intensity in the Second treatise, from which, it emerged that the goal of that treatise — namely, determining the value of the *constant* — would be achieved when one succeeded in *reducing the measurements of the current intensity that are obtained from both measurements to mechanical measures*.

However, the total amount of electricity that flows through the cross-section of a conductor in a certain time interval cannot be measured while it is flowing. It must then be previously measured while it is found in a state of rest. One must then previously collect a certain amount of electricity that one would like to have flow through a conducting wire — e.g., in a Leyden jar — and one must then seek to measure it while it is found in a state of rest according to *electrostatic* principles, and one must then determine the *intensity* and *duration* of the current that is produced when that same amount of electricity flows (from the Leyden jar to the Earth, for instance) through a conducting wire *in absolute units*.

Now, as far as that initial measurement of the total electricity that is collected in the Leyden jar is concerned, the *electrostatic* principles that must be applied to that measurement are indeed known, in general, but many difficulties have been found in regard to applying those principles to the electricity that has been collected in a Leyden jar. Coulomb, whom we have to thank for those principles, made applications to only very small amounts of electricity with which the small spheres of his electrical [torsion] balance were charged.[135] The solution of those difficulties was then the main problem that needed to be addressed as the goal of this treatise. Several of those difficulties were eliminated by Professor Kohlrausch in earlier investigations, and that fact was what led to the ambition for us to combine the work that we had done, and it was only by such combined efforts that we could hope to achieve satisfactory results.

As far as concerns the measurement of the *intensity* and *duration* of the electricity that is stored in a Leyden jar from the current that is created by its discharge into the Earth, it becomes clear that neither the measurement of that *intensity* nor that *duration* can be performed directly, because the intensity is not constant and the duration of the current that is created in that way is immeasurably small. The only thing that can be measured precisely is the so-called *integral value* of the current that is created — i.e., the sum of the products of each time element dt into the intensity i (expressed in absolute units) of the current that is present in that time element ($= \int idt$), as calculated from the beginning to the end of the

[134][Note by AKTA:] [Web52c] with English translation in [Web21b].

[135][Note by AKTA:] See footnote 43 on page 56.

current. However, that integral value is nothing but the total amount of electricity in the Leyden jar that is discharged into the Earth, multiplied by either $\sqrt{8}$ or 4 and divided by the desired *constant*. Meanwhile, it must be observed in all of this that not all of the electricity that is stored in the Leyden jar flows from the jar to the Earth, but only half of it, while an equal amount of negative electricity will simultaneously flow from the Earth to the jar, and that will neutralize the other half of the electricity that is collected in it.

We must avoid going into the details of those measurements in this brief report, and therefore refer to the treatise itself[136] for the electrostatic measurement of the amount of electricity stored in a Leyden jar, as well as all things concerned with the electrodynamic measurement of the integral value of the current that is created when it is discharged into the Earth. It might suffice here to briefly cite the results of those measurements.

The measurement of the total amount of electricity E stored in a Leyden jar for five different charges gave the following results:

No.	E
1.	35 786 000
2.	41 618 000
3.	49 313 000
4.	44 007 000
5.	49 276 000

The meaning of the numbers that are quoted under E is as follows: For the *first* charge, an amount of positive electricity was stored in the jar such that if it had been concentrated into a point then an equal amount of electricity that had been concentrated into a point at a distance of 1 millimeter from it would repel it with a force that equals the weight of $(35\,786\,000)^2 \cdot 1/g$ milligrams, where g denotes the acceleration of ponderable bodies due to gravity: i.e., $g = 9\,811$ millimeters/(second)^2. The measurement of the integral value $\int idt$ of the current that is created by removing the electricity E that is stored in the Leyden jar in terms of the absolute units that are based upon magnetic current effects gave the following results in those five cases:

No.	$\int idt$
1.	0.000 119 4
2.	0.000 130 0
3.	0.000 156 8
4.	0.000 148 0
5.	0.000 158 9

However, from the above, when one observes that only half of the *positive* amount of electricity E flows from the Leyden jar to the Earth, because the other half will be *negative* electricity that flows from the Earth to the jar in the opposite direction, which will neutralize the latter, one will have the quotient $E/\int idt = c\sqrt{2}$, in which c denotes the desired constant. As a result, one gets the five following mutually-independent determinations of the value of the unknown constant c from the five values of E above and the associated values of $\int idt$:

[136] [Note by AKTA:] [KW57] with English translation in [KW21], see Chapter 7.

No.	c
1.	$423\,870 \cdot 10^6$
2.	$452\,750 \cdot 10^6$
3.	$444\,760 \cdot 10^6$
4.	$420\,510 \cdot 10^6$
5.	$438\,560 \cdot 10^6$

The mean of those five measurements yields the value of the *constant* $c = 436\,090 \cdot 10^6$.

The meaning of the *constant* c is that of a well-defined velocity, and indeed the velocity with which two electric masses must approach or separate from each other if neither attraction nor repulsion is to exist between them. Here, the velocity c is expressed by the number of millimeters that will be traversed in one second at that velocity. With 7408 meters per mile, that velocity is calculated to be 58 868 miles per second.

Finally, with the value of that *constant*, all of the current intensity measurements that are quoted in absolute units (whether they are based upon magnetic, electrodynamic, or electrolytic current effects) can be easily reduced to *mechanical units*, which might imply, e.g., that a positive amount of electricity of $16\frac{4}{9}$ billion mass units and an equal amount of negative electricity would be required in order to decompose 1 milligram of water. If that positive amount of electricity were in a cloud and that negative electricity were concentrated into a location on the surface of the Earth along a perpendicular beneath the cloud then that would imply an attraction of the cloud to the Earth that would be equal to the weight of 27 545 kilograms, or almost 551 hundredweights, when the two are separated by 1000 meters.

The *second* part of the treatise is concerned with applications that can be made, in part when one extends the laws that were developed in the previous treatises by the determination that is thus obtained, and in part when one seeks to employ the newly-obtained determination as the foundation for new investigations.

In all laws into which the *constant* c enters, it always appears as the denominator of the velocity with which the bodies actually move relative to each other or as the denominator of the velocity with which the bodies would move relative to each in the course of a unit time (viz., second) when the acceleration that is present continues throughout that time interval. It is therefore of practical interest that all actual velocities that we know — even those of the planets — can be considered to be vanishingly small compared to the velocity c. That is because the only velocity that is known to us, which comes close to the speed c, — namely, the speed of the propagation of light — is not an actual velocity with which bodies can move relative to each other. That yields some interesting applications: e.g., that one can also adapt the extension that was made to the law of electrostatics to the law of gravitation, since the change in the gravitational force that it would imply would vanish entirely for all phenomena in which one might observe it. The fact that for electricity the change in the *electrostatic force* (which corresponds to the gravitational force between ponderable bodies) does not vanish everywhere when one adds the aforementioned extension is based merely upon the total cancellations of the electrostatic forces that takes place under the neutralization of positive and negative electricity. Where no such neutralization takes place, but only free electricity is present, a consideration of the electrostatic force will always suffice for the effects of that free electricity, because its change in accordance with that extension can be likewise regarded as totally vanishing, which has great practical significance for the consideration of free electricity in closed circuits.

Finally, an important principle for new investigations is that all forces that act upon an electric mass will also act immediately upon the ponderable masses that they pertain to and from which they can be separated only by overcoming the resistance of the ponderable bodies. Now, if those electrical forces can be determined by *mechanical measures* then one will learn about the molecular forces that mediate the *mechanical* and *chemical* effects of electricity on ponderable bodies in that way. That will show how the research into the galvanic resistance of water can be employed to obtain a precise insight into and understanding of the *chemical affinity* of oxygen and hydrogen in water. For instance, if one could rigidly couple all oxygen particles to each other in the water in a mixture of water and sulfuric acid of specific weight 1.25 that occupies a column of arbitrary cross-section and one millimeter in height and move them to one side by means of a tensed string, while all of the hydrogen particles were coupled rigidly to each other and moved to the opposite side by means of a tensed string then the tensions in those two strings would amount to 1478 hundredweights if the systems of oxygen particles and hydrogen particles in 1 second were to move so far from each other that the component of 1 milligram of water would be free at the two ends of the water column.

Most likely, similar applications can also be made in relation to the mechanical effects of electricity when it jumps from one conductor to another while small ponderable particles are torn from the one conductor by the electricity. If one were to possess a more precise knowledge of all the relationships that come into play essentially in that phenomenon then it would probably be possible to determine the mass of the neutral electric fluid that exists in ponderable bodies, and under certain likely assumptions, it already seems that one can deduce an exceptionally large magnitude for that mass. However, the larger that mass becomes, the smaller the velocity would be with which it moves in galvanic currents,[137] and in that way, it already seems that one can assume with great likelihood that the actual velocity with which electric masses displace in closed circuits is rather small, and in no way to be confused with the large velocity with which galvanic currents propagate in closed circuits, which is what Wheatstone sought to measure.[138]

[137] [Note by AKTA:] Weber is referring here to the drift velocity, that is, to the velocity of the electrified particles relative to the matter of the conductor.

[138] [Note by AKTA:] Charles Wheatstone (1802-1875), see [Whe34]. In 1857 Weber and Kirchhoff deduced independently from one another, although both works were based on Weber's force of 1846, that an electric wave propagates along a wire of negligible resistance with light velocity, see Chapters 8, 9, 10, 18 and 19.

Chapter 6

[Weber and Kohlrausch, 1856] On the Amount of Electricity which Flows Through the Cross-section of the Circuit in Galvanic Currents

Wilhelm Weber and Rudolf Kohlrausch[139,140]

Prefatory Note by Rudolf Kohlrausch

The Editor desired for the *Annalen*[141] a report on work carried out jointly by Prof. Weber and myself, whose results were presented in a more fundamental and conclusive way by Prof. Weber in Vol. 5 of the Treatises of the Royal Saxon Scientific Society in Leipzig, under the title *Elektrodynamische Maassbestimmungen, insbesondere Zurückführung der Stromintensitätsmessungen auf mechanisches Maass* (Leipzig, S. Hirzel, 1856).[142] Herewith I give a short precis.

[139] [WK56]. The English version presented in this book is based on the translation by the late Susan P. Johnson, [WK03], see also [Joh97]. It was edited by Laurence Hecht and A. K. T. Assis. A Portuguese translation was published in 2008, [WK08].

[140] Wilhelm Weber and Rudolf Kohlrausch's Notes are represented by [Note by WK:]; the Notes by H. Weber, the editor of the third volume of Weber's *Werke*, are represented by [Note by HW:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[141] [Note by AKTA:] Kohlrausch is referring to Johann Christian Poggendorff (1796-1877). He was the Editor of the *Annalen der Physik und Chemie* from 1824 to 1876. The modern *Annalen der Physik* is the successor to Poggendorff's *Annalen*.

[142] [Note by AKTA:] [KW57] with English translation in [KW21], see Chapter 7.

6.1 Problem

The comparison of the effects of a closed galvanic circuit with the effects of the discharge-current of a collection of free electricity, has led to the assumption, that these effects proceed from a *movement of electricity* in the circuit. We imagine that in the bodies constituting the circuit, their *neutral* electricity is in motion, in the manner, that their entire positive component pushes around in the one direction in closed, continuous circles, the negative in the opposite direction. The fact that an accumulation of electricity never occurs by means of this motion, requires the assumption, that the same amount of electricity flows through each cross-section in the same time-interval.

It has been found suitable to make the *magnitude of the flow*, the so-called *current intensity*, proportional to the amount of electricity which goes through the cross-section of the circuit in the same time-interval. If, therefore, a certain current intensity is to be expressed by a *number*, it must be stated, which current intensity is to serve as the measure, i.e., which magnitude of flow will be designated as 1.

Here it would be simplest, as in general regarding such flows, to designate as 1 that magnitude of flow which arises, when in the time-unit the unit of fluid goes through the cross-section, thus defining the measure of current intensity from its *cause*. The unit of electrical fluid is determined in electrostatics by means of the *force*, with which the free electricities act on each other at a distance. If one imagines two equal amounts of electricity of the same kind concentrated at two points, whose distance is the unit of length, and if the force with which they act on each other repulsively, is equal to the *unit of force*, then the amount of electricity found in each of the two points is the *measure* or the *unit* of free electricity.

In so doing, *that force* is assumed as the unit of force, *through which the unit of mass is accelerated around the unit of length during the unit of time.* According to the principles of mechanics, by establishing the units of length, time, and mass, the measure for the force is therefore given, and by joining to the latter the measure for free electricity, we have at the same time a measure for the current intensity.

This measure, which will be called the *mechanical measure* of current intensity, thus *sets as the unit, the intensity of those currents which arise when, in the unit of time, the unit of free positive electricity flows in the one direction, an equal amount of negative electricity in the opposite direction, through that cross-section of the circuit.*

Now, according to this measure, we can not carry out the measurement of an existing current, for we know neither the amount of neutral electrical fluid which is present in the cubic unit of the conductor, nor the velocity, with which the two electricities displace themselves [sich verschieben] in the current. We can only compare the intensity of the currents by means of the *effects* which they produce.

One of these effects is, e.g., the *decomposition of water.* Sufficient grounds converge, to make the current intensity proportional to the amount of water, which is decomposed in the same time-interval. Accordingly, *that current intensity will be designated as 1, at which the mass-unit of water is decomposed in the time-unit*, thus, e.g., if seconds and milligrams are taken as the measure of time and mass, that current intensity, at which in one second one milligram of water is decomposed. *This measure of current intensity is called the electrolytic measure.*

The natural question now arises, how this electrolytic measure of current intensity is related to the previously established mechanical measure, thus the question, how many (electrostatically or mechanically measured) positive units of electricity flow through the cross-section in one second, if a milligram of water is decomposed in this interval of time.

Another effect of the current is the *rotational moment*[143] it exerts on a magnetic needle, and which we likewise assume to be proportional to the current intensity, conditions being otherwise equal. If a current intensity is to be measured by means of this kind of effect, then the *conditions* must be established, under which the rotational moment is to be observed. One could designate as 1 that current intensity which under *arbitrarily* established spatial conditions exerts an *arbitrarily* established rotational moment on an arbitrarily chosen magnet. When, then, under *the same* conditions, an m-fold large rotational moment is observed, the current intensity prevailing in this case would have to be designated as m. Precisely the impracticability of such an arbitrary measure, however, has led to the *absolute* measure, and thus in this case the electromagnetic measure of current intensity is to be joined to the absolute measure for magnetism. This occurs by means of the following specification of *normal conditions* for the observation of the magnetic effects of a current:

The current goes through a circular conductor, which circumscribes the unit of area, and acts on a magnet, which possesses the unit of magnetism,[144] *at an arbitrary but large distance* $= R$*; the center of the magnet lies in the plane of the conductor, and its magnetic axis is directed toward the center of the circular conductor.*

The rotational moment D, exerted by the current on the magnet, expressed according to mechanical measure, is, under these conditions, different according to the difference in the current intensity, and also according to the difference in the distance R; the product R^3D *depends, however, simply on the current intensity*, and is hence, under these conditions, the *measurable* effect of the current, namely, that effect by means of which the current intensity is to be measured, according to which one therefore obtains as *magnetic measure of current intensity* the intensity of that current, for which $R^3D = 1$.

The electromagnetic laws state, that this measure of current intensity is also the intensity of that current which, if it circumscribes a plane of the size of the unit of area, everywhere exerts at a distance the actions of a magnet located at the center of that plane, which possesses the unit of magnetism and whose magnetic axis is perpendicular to the plane; — or also, that it is the intensity of that current, by which a *tangent galvanometer*[145] *with simple rings of radius* $= R$ is kept in equilibrium, given a deflection from the magnetic meridian

$$\varphi = \arctan \frac{2\pi}{RT}$$

if T denotes the horizontal intensity of the terrestrial magnetism.

Here, too, arises the natural question about the relation of the *mechanical* measure of current intensity to this *magnetic* measure, thus the question, how many times the electrostatic unit of the amount of electricity must go through the cross-section of the circuit during one second, in order to elicit that current intensity, of which the just-specified deflection, φ, is effected by the needle of a tangent galvanometer.

The same question repeats itself in considering a *third* measure of current intensity, which is derived from the electrodynamic effects of the current, and is therefore called the *electrodynamic measure* of the current intensity.

The three measures drawn from the *effect* of the currents have already been compared

[143][Note by AKTA:] See footnote 10 on page 18.
[144][Note by AKTA:] See footnote 54 on page 62.
[145][Note by AKTA:] See footnote 12 on page 22.

with one another. It is known that the magnetic measure is $\sqrt{2}$ larger than the electrodynamic, but $106\frac{2}{3}$ times smaller than the electrolytic,[146] and for that reason, in order to solve the question of how these three measures relate to *mechanical* measure, it is merely necessary to compare the later with one of the others.

This was the goal of the work undertaken, which goal was to be attained through the solution of the following problem:

> *Given a constant current, by which a tangent galvanometer with a simple multiplier circle of radius* $= R^{mm}$ *is kept in equilibrium at a deflection* $\varphi = \arctan 2\pi/RT$, *if* T *is the intensity of the horizontal terrestrial magnetism affecting the compass: it should be determined how the amount of electricity, which flows in such a current in one second through the cross-section of the conductor, relates to the amount of electricity on each of two equally charged (infinitesimally) small balls, which repel one another at a distance of 1 millimeter with the unit of force. The unit of force is taken as that force, which imparts 1 millimeter velocity to the mass of 1 milligram in 1 second.*

6.2 Solution of This Problem

If an amount E of free electricity is collected at an insulated conductor and allowed (by inserting a column of water)[147] to flow to earth through a multiplier, the magnetic needle will be deflected. The magnitude of the first deflection depends, given the same multiplier and the same needle, solely on the amount of discharged electricity, since the discharge time is so short, compared with the oscillation period of the needle, that the effect must be considered as an impulse.

If a constant current is put through a multiplier for a similarly short time, the needle receives a similar impulse, and in this case as well, the magnitude of the first deflection depends *solely on the amount of electricity* which moves through the cross-section of the multiplier wire during the duration of the current.

Now, if in the same multiplier, *exactly the same* deflection were to occur, the one time, when the known amount of free electricity E was discharged, the other time, when one let a *constant current* act briefly, then, as can be proven, the amount of positive electricity, which flows during this short time-interval in the constant current, in the direction of this current, through the cross-section, equals $E/2$.

Accordingly, the problem posed requires the solution of the following two problems:

> *a) measuring the collected amount* E *of free electricity with the given electrostatic measure, and observing the deflection of the magnetic needle when the electricity is discharged;*
>
> *b) determining the small time-interval* τ*, during which a constant current of intensity* $= 1$ *(according to magnetic measure) has to flow through the multiplier of the same galvanometer, in order to impart to the needle the same deflection.*

[146] [Note by AKTA:] See [Web41b, p. 17 of Weber's *Werke*] and [Web42] with English translation in [Web20a]; [WK56, p. 600 of Weber's *Werke*] with English translation in [WK03, p. 290] and Portuguese translation in [WK08, p. 96]; [KW57, pp. 614, 649 and 650 of Weber's *Werke*] with English translation in [KW21, p. 8]; [Web62, p. 88 of Weber's *Werke*]; and [Web64, p. 165 of Weber's *Werke*] with English translation in [Web21d].

[147] [Note by AKTA:] That is, by inserting a conducting column of water.

If next we multiply $E/2$ by the number which shows how often τ is contained in the second, then the number $E/2\tau$ expresses the amount of positive electricity, which, in a current whose intensity = 1 according to magnetic measure, passes through the cross-section of the conductor in the direction of the positive current in 1 second.

Problem a is treated in the following way.

First, with the help of the sine-electrometer,[148] the conditions are determined with greater precision, in which the charge of a small Leyden jar is divided between the jar itself and an approximately 13-inch ball coated with tin foil, which was suspended, by a good insulator, away from the walls of the room, so that from the amount of electricity flowing on the ball,[149] as soon as it was able to be measured, the amount remaining in the little jar could also be calculated down to a fraction of a percent.

The observation consisted of the following:

The jar was charged, the large ball put in contact with its knob; three seconds later, the charge remaining in the jar was discharged through a multiplier[150] consisting of 5635 windings, by the insertion of two long tubes filled with water, and the first deflection φ of the magnetic needle, which was equipped with a mirror in the manner of the magnetometer, was observed. At the same time, the large ball was now put in contact with the approximately 1-inch fixed ball of a torsion balance[151] constructed on a very large scale. This fixed ball, brought to the torsion balance, shared half of its received charge with[152] the moveable ball, which made it possible to measure the torsion which was required, to a decreasing extent over time, in order to maintain the two balls at a fully determinate, pre-ascertained distance.

From the torsion coefficients of the wire, found in the manner well known from oscillations experiments, and the precisely determined dimensions, the amount of electricity occurring at each moment in the torsion balance could be measured in the required absolute measure, taking into consideration the non-uniform distribution of electricity in the two balls (which consideration was advisable because of the not insignificant size of the balls compared with the distance between them). The observed decrease in torsion also yielded the loss of electricity, so that it was possible, by means of this consideration, to state how large these amounts would be, if they could already have been in the torsion balance at the moment at which the large ball was charged by the Leyden jar.

This amount [of electricity] went from the large ball to the fixed ball. From the precisely measured diameter of these balls, the proportion of the distribution of electricity between them could be determined (according to Plana's work),[153] so that, by means of the measurement in the torsion balance, without further ado, it was known what amount of electricity remained in the Leyden jar after charging the large ball, and what amount was discharged

[148][Note by AKTA:] In German: *Sinus-Elektrometers*. Instrument created by R. Kohlrausch, see [Koh53].

[149][Note by AKTA:] In German: *aus der auf die Kugel übergegangenen Elektricitätsmenge*. Alternative translation: from the amount of electricity passed to the ball.

[150][Note by WK:] The mean diameter of the windings was 266 *mm*; the almost 2/3-mile-long wire, very well coated with silk, was previously drawn through collodion along its entire length, while the sides of the casing were strongly coated with sealing wax. A powerful copper damper moderated the oscillations.

[151][Note by WK:] The frame of the torsion balance, in whose center the balls were located, was in the shape of a parallelepiped 1.16 meters long, 0.81 meters wide, and 1.44 meters high. The long shellac pole [Stange], to which the moveable ball was affixed by means of a shellac-arm, allowed the observation of the position of the ball under a mirror, and then dipped into a container of oil, by means of which the oscillations were very quickly halted.

[152][Note by AKTA:] In German: *halbirte ihre empfangene Ladung mit*. Alternative translation: gave half its received charge to.

[153][Note by AKTA:] Giovanni Antonio Amedeo Plana (1781-1864), [Pla45] and [Pla54].

3 seconds later by the multiplier. Only one small correction was still required on account of the loss of available charge, which occurred during these 3 seconds from leakage into the air and through residue formation.

In the following Table are assembled the results of five successive experiments. The column headed E contains the amounts of discharged electricity, the column headed s the corresponding deflections of the magnetic needle in scale units, and the column headed φ the same deflections, but in arcs for radius = 1.

Number	E	s	φ
1	36 060 000	73.5	0.005 708 7
2	41 940 000	80.0	0.006 213 6
3	49 700 000	96.5	0.007 495 2
4	44 350 000	91.1	0.007 075 7
5	49 660 000	97.8	0.007 596 2

Problem b requires knowing the time-intervals τ, during which a current of that intensity denoted 1 in magnetic current measure, must flow through the same multiplier, in order to elicit the deflections φ observed in the five experiments.

The rotational moment, which is exerted by the just-designated currents on a magnetic needle, which is parallel to the windings of the multiplier, is developed in the "second part of the *Electrodynamische Maassbestimmungen* of W. Weber".[154] This rotational moment is proportional to the magnetic moment of the needle and the number of windings, but moreover is a function of the dimensions of the multiplier and the distribution of magnetic fluids in the needle, for which it suffices, to determine the distance of the centers of gravity of the two magnetic fluids, which, in lieu of the actual distribution of magnetism, can be thought of as distributed on the surface of the needle. The needle always remaining small compared with the diameter of the multiplier, for this distance a value derived from the size of the needle could be posited with sufficient reliability, so that the designated rotational moment D contains only the magnetic moment of the needle as an unknown.

If this rotational moment acts during a time-interval τ, which is very short compared with the oscillation period t of the needle, then the angular velocity imparted to the needle is expressed by

$$\frac{D}{K}\tau ,$$

where K signifies the moment of inertia.[155] The relationship between this angular velocity and the first deflection φ then leads to an equation between τ and φ,

$$\tau = \varphi \cdot A ,$$

in which A consists of magnitudes to be truly rigorously measured, thus signifies known constants, namely $A = 0.020\,915$ for the second as measure of time.

Thus, if it is asked how long a time-interval τ a constant current of magnetic current intensity = 1 has to flow through the multiplier, in order to elicit the above-cited five observed deflections, one need only insert their values for τ into this equation. In this way the values in seconds result as

[154][Note by AKTA:] [Web52c, footnotes on pages 360-365 and p. 454 of Weber's *Werke*] with English translation in [Web21b].

[155][Note by AKTA:] In German: *Trägheitsmoment*.

Number	τ
1	0.000 119 4
2	0.000 130 0
3	0.000 156 8
4	0.000 148 0
5	0.000 158 9

If we now divide $E/2$ in the five experiments by the pertinent τ, we obtain

Number	$E/2\tau$
1	$151\,000 \cdot 10^6$
2	$161\,300 \cdot 10^6$
3	$158\,500 \cdot 10^6$
4	$149\,800 \cdot 10^6$
5	$156\,250 \cdot 10^6$

thus as a mean,

$$\frac{E}{2\tau} = 155\,370 \cdot 10^6 \ .$$

The *mechanical measure of the current intensity* is thus related

to the *magnetic [measure of current intensity]* as $1 : 155\,370 \cdot 10^6$,
to the *electrodynamic* as $1 : 109\,860 \cdot 10^6 \left(= 1 : 155\,370 \cdot 10^6 \cdot \sqrt{\frac{1}{2}}\right)$,
to the *electrolytic* as $1 : 16\,573 \cdot 10^9 \left(= 1 : 155\,370 \cdot 10^6 \cdot 106\frac{2}{3}\right)$.

6.3 Applications

Among the applications, which can be made by reducing the ordinary measure for current intensity to mechanical measure, the most important is the determination of the constants which appear in the fundamental electrical law, encompassing electrostatics, electrodynamics, and induction. According to this fundamental law, the effect of the amount of electricity e on the amount e' at distance r with relative velocity dr/dt and relative acceleration d^2r/dt^2 equals[156]

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right] \ ,$$

and the constant c represents that relative velocity, which the electrical masses e and e' have and must retain, if they are not to act on each other any longer at all.

In the preceding Section, the ratio of the magnetic measure to the mechanical measure was found to be

[156][Note by AKTA:] The next equation should be understood as:

$$\frac{ee'}{r^2}\left\{1 - \frac{1}{c^2}\left[\left(\frac{dr}{dt}\right)^2 - 2r\frac{d^2r}{dt^2}\right]\right\} \ .$$

$$= 155\,370 \cdot 10^6 : 1 \; ;$$

in the Second treatise on *Electrodynamic Measurements*, the same proportion was found[157]

$$= c\sqrt{2} : 4 \; ;$$

the equalization of these proportions results in

$$c = 439\,450 \cdot 10^6$$

units of length, namely, millimeters, thus a velocity of 59 320 miles per second.

The insertion of the values of c into the foregoing fundamental electrical law makes it possible to grasp, why the electrodynamic effect of electrical masses, namely

$$\frac{ee'}{r^2}\frac{1}{c^2}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right) \; ,$$

compared with the electrostatic

$$\frac{ee'}{r^2}$$

always seems infinitesimally small, so that in general the former only remains significant, when, as in galvanic currents, the electrostatic forces completely cancel each other in virtue of the neutralization of the positive and negative electricity.

Of the remaining applications, only the application to electrolysis will be briefly described here.

It was stated above, that in a current, which decomposes 1 milligram of water in 1 second,

$$106\frac{2}{3} \cdot 155\,370 \cdot 10^6$$

positive units of electricity go in the direction of the positive current in that second through the cross-section of the circuit, and the same amount of negative electricity in the opposite direction.

The fact that in electrolysis, ponderable masses are moved, that this motion is elicited by electrical forces, which only act on electricity, not directly on the water, leads to the conception, that in the atom of water, the *hydrogen atom* possesses free positive electricity, the *oxygen atom* free negative electricity. Many reasons converge, why we do not want to think of an electrical motion in water without electrolysis, and why we assume that water is not in a state of allow electricity to flow through it in the manner of a conductor. Therefore, if we see in the one electrode just as much positive electricity coming from the water, as is delivered to the other electrode during the same time-interval by the current, then this positive electricity which manifests itself is that which belonged to the separated hydrogen particles.

If we take this standpoint, so that we thus link the entire electrical motion in electrolytes to the motion of the ponderable atoms, then it additionally emerges from the numbers obtained above, that the hydrogen atoms in 1 milligram of water possess

$$106\frac{2}{3} \cdot 155\,370 \cdot 10^6$$

[157] [Note by AKTA:] [Web52c, p. 367 of Weber's *Werke*] with English translation in [Web21b].

units of free positive electricity, the oxygen atoms an equal amount of negative electricity.

From this it follows, secondly, that these amounts of electricity together signify the *minimum of neutral electricity*, which is contained in a milligram of water. Namely, if the atoms of water were still to possess neutral electricity beyond their free electricity, then the mass of neutral electricity in a milligram of water would be still greater.

Under the foregoing assumptions, we are also in a position to state the force with which the totality of the hydrogen particles of a mass of water is acted upon in the one direction, the totality of the oxygen particles in the opposite direction.

Imagine, for example, a cylindrical tube of 10/9 square millimeter cross-section, which is to serve as a decomposition cell, filled with a mixture of water and sulphuric acid of specific gravity 1.25, which thus contains in each 1-millimeter segment a milligram of water. Through Horsford,[158] we know the proportional relation of the specific resistance of this mixture to that of silver, and through Lenz,[159] the proportional relation of the resistance of silver to that of copper. In the treatises of the Königliche Gesellschaft der Wissenschaften in Göttingen (Vol. 5, "Ueber die Anwendung der magnetischen Induction auf Messung der Inklination mit dem Magnetometer"),[160,161] the resistance of copper is determined according to the absolute measure of the magnetic system. This makes it possible to additionally state, in *absolute magnetic measure*, the resistance which the water (under the influence of the admixed sulphuric acid) exerts in a 1-mm long segment of that cylindrical decomposition cell. This resistance, multiplied by the current intensity, the latter being expressed in magnetic measure, yields the electromotive force in relation to this small cell, likewise in the *magnetic* system of measure. However, the *magnetic* measure of the electromotive force is as many times smaller than the *mechanical*, as the magnetic measure of the current intensity is greater than the mechanical, and since this latter proportion is now known, that electromotive force calculated in magnetic measure can be transformed into mechanical measure simply by division by $155\,370 \cdot 10^6$. The number which results, then signifies the *difference between the two forces*, of which in the direction of the current, the one acts to move *each single unit* of the free positive electricity in the hydrogen particles, the other to move *each single unit* of the free negative electricity in the oxygen particles, and therefore, in order to obtain the *entire force at work*, this number must still be multiplied by the total of units of the free positive or negative electricity, which is contained in the 1 millimeter-long water column, that is, in 1 milligram of water, namely, by

$$106\frac{2}{3} \cdot 155\,370 \cdot 10^6 \ .$$

If one carries out the calculation and presupposes that current intensity, at which 1 milligram of water is decomposed in 1 second, then one obtains a force difference

$$= 2 \cdot \left(106\frac{2}{3}\right)^2 \cdot 127\,476 \cdot 10^6 \ ,$$

in which the unit of force is that force, which imparts to the unit of mass of 1 milligram a velocity of 1 millimeter in 1 second. Thus, if one divides by the intensity of gravity = 9811, one obtains this force difference, expressed in weight[162]

[158][Note by AKTA:] Eben Norton Horsford (1818-1893), see [Hor47] and [Sto88].

[159][Note by AKTA:] Heinrich Friedrich Emil Lenz (1804-1865), see [Len38, p. 119].

[160][Note by HW:] Wilhelm Weber's Werke, Vol. 2, p. 319.

[161][Note by AKTA:] [Web53e, p. 319 of Weber's *Werke*], [Web53a] and [Web53c].

[162][Note by AKTA:] The German Zentner since metrification is defined as 50 kg.

$$= 2 \cdot 147\,830 \cdot 10^6 \text{ milligrams} = 2 \cdot 147\,830 \text{ kg} = 2 \cdot 2956 \text{ centner}$$

under the influence of gravity.

This result can be expressed in the following way: *If all hydrogen particles in 1 milligram of water were linked in a 1 millimeter-long string, and all oxygen particles in another string, then both strings would have to be stretched in opposite directions with the weight of 2956 centners, in order to produce a decomposition of the water at a rate such that 1 milligram of water would be decomposed in 1 second.*

One easily convinces oneself, that this stretching remains the same for a cell of 1 mm length but a different cross-section, but that it must be proportional to the length of the cell, and also proportional to the current intensity, that is, to the velocity of the electrolytic separation.

If, in the wet cell described above, we now see a pressure on the totality of hydrogen particles of the weight of 2956 centners, and if no acceleration of motion occurs, which motion must, however, amount to 1759 million miles per second, but rather the hydrogen continues with the constant velocity of 1/2 millimeter per second, then we are compelled to assume, that a force would be acting counter to the decomposition of the water, a force which increases with the velocity of the decomposition, so that in general, only that velocity of decomposition remains, at which the *force of resistance* is equal to the electromotive force, so that its effect on the totality of hydrogen particles in the milligram of water in the foregoing case likewise would equal the weight of 2956 centners. Namely, in that case, the ponderable particles would uniformly flow forth with the velocity attained.

It is natural, to seek the basis for this force of resistance in the *chemical forces of affinity*. Even though the concept of "*chemical affinity*" remains too indeterminate, for us to be able to derive from it, how the forces proceeding from this affinity increase with the *velocity of the separation*, nevertheless, it is interesting to see what enormous forces enter into operation in a chemical decomposition, as are easily elicited by electrolysis.

Chapter 7

[Kohlrausch and Weber, 1857, EM4] Electrodynamic Measurements, Fourth Memoir, specially Attributing Mechanical Units to Measures of Current Intensity

Rudolf Kohlrausch and Wilhelm Weber[163,164,165]

7.1 Measure of Current Intensity Based on Observed Magnetic, Electrodynamic and Electrolytic Effects

The intensity of an electric current can be usually determined by observing either its *magnetic*, *electrodynamic*, or finally, its *electrolytic* effects. However, those effects can be observed under very different situations, and it is the task of the observer to choose those situations in such a manner as to give his observations the greatest completeness, while appealing to the *electromagnetic*, *electrodynamic*, and *electrolytic laws*, which can *reduce* the effects that are observed in the various situations to each other. That is because it is only by *reducing the observations under the same conditions* that one can achieve a *comparison of the current intensities*. Now, one calls those *common conditions*, to which all observations that were made under different circumstances should be reduced, *normal conditions*, and *the unit of current intensity* will be established by defining those normal conditions according the following rule:

163[KW57] with English translation in [KW21]. See also [WK68].

164Translated by D. H. Delphenich, feedback@neo-classical-physics.info and http://www.neo-classical-physics.info/index.html. Edited by A. K. T. Assis.

165The Notes by Kohlrausch and Weber are represented by [Note by KW:]; the Notes by H. Weber, the Editor of Volume 3 of Weber's *Werke*, are represented by [Note by HW:]; D. H. Delphenich's Notes are represented by [Note by DHD:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

The unit of current intensity is the intensity of the current that will produce one unit of measurable effect *under normal conditions*.

The normal conditions for the observation of the *magnetic effects* of a current are the following: *The current goes through a circular conductor that encircles a unit area and acts upon a magnet that possesses one unit of magnetism*[166] *at an arbitrary, but large, distance equal to R. The center of the magnet lies in the plane of the conductor, and its magnetic axis points towards the center of the circular conductor*. Under those conditions, the *rotational moment D* that is exerted on the magnet by the current will vary with current intensity, as well as with distance R. However, the product R^3D depends upon merely the current intensity, and is therefore *the measurable effect* of the current under those conditions, from which one will then get the *unit of the current intensity* from the intensity of the current whose measurable effect under the conditions that were just described will be:

$$R^3D = 1 \ .$$

That unit of current intensity, which is then obtained from *electromagnetic laws*, is at the same time also the intensity of that current that flows around a planar region of size one unit of area, producing the same effect everywhere at a distance as a magnet that is found at the center of that region that possesses one unit of magnetism and whose magnetic axis is perpendicular to the plane. Alternatively, it is also the intensity of the current *that will equilibrate a tangent galvanometer*[167] *with a single multiplier loop of radius equal to R* when it deviates from the magnetic meridian by:

$$\varphi = \arctan \frac{2\pi}{RT} \ ,$$

if T denotes the horizontal component of the Earth's magnetism.

The normal conditions for the observation of the *electrodynamic effects* of a current are as follows: *The same current goes through two circular conductors, each of which encircles a unit area and lie at an arbitrary, but large, distance equal to R from each other: The line of intersection of the two perpendicular planes of the circles bisects the first circular conductor*. — Under those conditions, the *rotational moment D* that the current in the *first* conductor exerts upon the current that flows in the *second* conductor will vary with the current intensity, as well as with the distance R. However, the product R^3D depends upon merely the current intensity and is therefore the *measurable effect* of the current under those conditions, from which one will then get the *unit of the current intensity* from the intensity of the current whose measurable effect will be:

$$R^3D = 1$$

under those conditions.

The normal conditions for the observation of *electrolytic effects* are the following ones: *The current goes through water during a time interval T that can be measured with arbitrary precision without suffering any change in intensity*. — Under those conditions, the mass of water M that is decomposed by the current, when expressed per assumed unit of mass (e.g., milligrams), will vary with current intensity, as well as with the time interval T (expressed

[166][Note by AKTA:] See footnote 54 on page 62.
[167][Note by AKTA:] See footnote 12 on page 22.

in seconds). However, the quotient M/T will depend upon merely the current intensity, and is therefore the *measurable effect* of the current under those conditions, from which one will then get the *unit of the current intensity* from the current whose measurable effect is:

$$\frac{M}{T} = 1$$

under those conditions.

All that remains for one to be able to compare the intensities of all currents whose *magnetic*, *electrodynamic*, or *electrolytic* effects were observed is to relate the three units that were given under the aforementioned normal conditions to each other.

One infers the relationship between the *first two units* from the *fundamental electrodynamic law*, which include the laws of magnetism and electromagnetism, as Ampère exhibited them,[168] namely, as was proved before in the Second treatise on *Electrodynamic Measurements*, p. 261,[169,170] one infers that the *first* unit relates to the *second* one like:[171]

[168][Note by AKTA:] See footnote 44 on page 57.

[169][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 360.

[170][Note by AKTA:] [Web52c, p. 360 of Weber's *Werke*] with English translation in [Web21b].

[171][Note by KW:] It is therefore interesting to point out that one can exhibit a complete identity between those two units when one, under the aforementioned *normal conditions*, defines the *electrodynamic effect* to be the rotational moment that the current in the *second* circle exerts upon the current in the *first* one, instead of the rotational moment that the *first* one exerts upon the *second*. The reason why that is not done is found merely in the fact that the expression that Ampère gave for the force of repulsion between two current elements would remain unchanged, so if α and α' are the lengths of both elements, i and i' are the current intensities, r is the distance between them, ε is the angle between α and α', ϑ is the angle between α and r, and ϑ' is the angle between α' and the extension of r, then that force will be represented by:

$$-\frac{\alpha\alpha'}{r^2} ii' \left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) ,$$

or

$$\frac{1}{2} \cdot \frac{\alpha\alpha'}{r^2} ii' \left(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon\right) .$$

However, all that generally follows from Ampère's *fundamental law of electrodynamics* is that that force is *proportional* to that expression, and therefore when one leaves the measure of the current intensity undetermined, the force itself will be represented by the product of that expression with an arbitrary constant, and so by:

$$-C \cdot \frac{\alpha\alpha'}{r^2} ii' \left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) ,$$

or by

$$D \cdot \frac{\alpha\alpha'}{r^2} ii' \left(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon\right) ,$$

in which C or D refer to the aforementioned constant. Now, Ampère assigned the value $C = 1$ to the constant C or the value $D = 1/2$ to the constant D in order to establish a well-defined unit for the current intensity, and in that way, he obtained the aforementioned expression for the force of repulsion between two current elements:

$$-\frac{\alpha\alpha'}{r^2} ii' \left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) = \frac{1}{2} \cdot \frac{\alpha\alpha'}{r^2} ii' \left(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon\right) ,$$

which reduces to:

$$-\frac{\alpha\alpha'}{r^2} ii'$$

$$\sqrt{2} : 1 \ .$$

The *third* unit will imply the reduction to the *first*, and therefore also the *second* one immediately, by simultaneous observations of the *magnetic* and *electrolytic* effects that are produced by one and the same current. Namely, one will find upon comparing the reduced observations, under the aforementioned normal conditions, that the *third* unit of current intensity, or the intensity of the current that will decompose 1 milligram of water in 1 second, is $106\frac{2}{3}$ times larger than the *first* unit,[172] or than the intensity of the current that, when it flows around a plane of size one unit area, will produce the same effects everywhere at great distances as a magnet at the center of that planar region that possesses one unit of magnetism and whose magnetic axis is perpendicular to the plane. See "Resultate aus der Beobachtungen des magnetischen Vereins in Jahre 1840," p. 96,[173,174] and Casselmann "Über die galvanische Kohlenzinkkette. Marburg 1844," p. 70.[175]

7.2 Mechanical Measure of Current Intensity Based on the Following Causes — Drift Velocity and Electricity Content of the Conductor

However, the intensity of an electric current can be determined not only by its *effects*, but also by its *origins*. Nonetheless, the deepest roots of an electric current lie in the mass of neutral electric fluid that is contained in a closed conductor, and in the velocity with which its two components, namely, the masses of the *positive* and *negative* fluids, move simultaneously in opposite directions. On the basis of this *origin*, *the unit of the current intensity* will be established from the following measures:

> The unit of the current intensity is the intensity of the current that is *produced* by such a velocity for the two electric fluids, for which the mass of each fluid that flows

for two parallel current elements that are perpendicular to r and for which $\varepsilon = 0$ and $\vartheta = \vartheta' = 90^o$. However, for the sake of agreement with the electromagnetic measurements, it would be preferable to set $D = 1$ or $C = 2$, which would then make the expression for the force of repulsion between two current elements equal to:

$$\frac{\alpha\alpha'}{r^2} ii' \left(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon\right) = -2\frac{\alpha\alpha'}{r^2} ii' \left(\cos\varepsilon - \frac{3}{2}\cos\vartheta\cos\vartheta'\right) \ ,$$

and for two current elements that coincide with r, [that is, for two current elements parallel to one another and pointing along the direction r,] for which $\vartheta = \vartheta' = \varepsilon = 0$, that will reduce to:

$$\frac{\alpha\alpha'}{r^2} ii' \ .$$

The cited *change in the normal conditions for the electrodynamic current effects* will agree with that, and in that way, one will arrive at a complete identity between the *electrodynamic* unit of the current intensity and the *magnetic* one.

[172][Note by AKTA:] See footnote 146 on page 134.

[173][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 17.

[174][Note by AKTA:] [Web41b, p. 96 of the *Resultate*] and [Web42, p. 17 of Weber's *Werke*], with English translation in [Web20a].

[175][Note by AKTA:] [Cas43, p. 70].

through the cross-section of the conductor divided by the time during which it flows through it is equal to 1.

This unit is the *mechanical unit of the current intensity*, and the problem that is being addressed in this treatise is to reduce the units that were described in the previous Section to this unit, which is most simply based in the essence of the current, and will therefore have an advantage over the other measures for the fundamental determination of that current intensity.

I - Reducing the Magnetic, Electrodynamic, and Electrolytic Units for the Current Intensity to Mechanical Units

7.3 Lack of Electrostatics Measurement of an Accumulated Amount of Electricity to Be Set in Motion

Up to now, no attempt has been made to determine current intensities from a *mechanical unit*, and even less, to *reduce* the current intensities that were determined from the other units to the latter. One merely knows that the *amount of electricity* that flows in the form of weak currents through the cross-section of the closed circuit, which can be produced by the most humble galvanic processes, must also be very large for a very brief time, since the most powerful electrification machine (whose conductor is coupled with the site of friction by a conducting wire) will give a much weaker current than a single galvanic element that is closed by a conducting wire of very large resistance.

The lack of any way of determining current intensities by *mechanical measurements* is based upon the difficulties that one finds in their implementation, while the determination of current intensities in the other aforementioned units is very easy to do, and thus allows for a much higher degree of precision. The last units will always be the first choice to be applied *in practice*, and one will essentially deal with the fact that a current intensity that is known in one of the latter units can only *once* be measured as precisely as possible in order to ascertain the ratio of the magnitude of the *mechanical unit* to that of the latter unit, and in that way, to find oneself in a position to *reduce* all of the determinations that were made in those units *to mechanical units*.

For such a measurement, one lacks, above all, any knowledge of the *amount of electricity* in a closed conductor that carries current, or rather, since that knowledge while the current is flowing can not be obtained, [one lacks] a knowledge of the *amount of electricity that is transferred by the current, and which is found to have been accumulated previously* — e.g., *in a Leyden jar*. In order to do that, one possesses excellent means and methods for measuring electricity that go back to Coulomb,[176] but which are never used to measure the electricity that is collected in a charged Leyden jar.[177,178]

The question of the *amount of electricity* that is found to be collected in a Leyden jar is

[176] [Note by AKTA:] See footnote 43 on page 56.

[177] [Note by KW:] In the *Annalen der Chemie und Physik*, Vol. 86, p. 33, Buff found, with the help of his tangent galvanometer and long conducting wires, that the amount of electricity that would be sufficient to liberate 1 milligram of hydrogen from 9 milligrams of water electrically, when one possesses the means to condense it, was to charge a battery of 45,480 Leyden jars that were each 480 millimeters in height and 160 millimeters in diameter up to a spark gap of 100 millimeter. Buff's determination is the best and most precise one that exists, but it still does not suffice to determine the *amount of electricity* that is included in that jar, for which, from *mechanical principles*, one requires a knowledge of the *force of repulsion* that this *amount of electricity*, when concentrated into a point, would exert upon an equal amount of electricity that is also concentrated into a point at a large distance from the latter. However, knowledge of that *force of repulsion* is still lacking, and up to now, no attempts have been made to measure such forces by the various means and methods that were given by Coulomb and others, or even to gain a better knowledge of it.

[178] [Note by AKTA:] As a matter of fact, this paper was published in the *Annalen der Chemie und Pharmacie* edited by J. F. v. Liebig (1803-1873), F. Wöhler (1800-1882) and H. F. M. Kopp (1817-1892) and not in the *Annalen der Chemie und Physik* edited by J. C. Poggendorff (1824-1876): [Buf53, p. 33].

often raised: Once it has been answered, and the *amount of electricity* has been determined by the *forces* that it might exert, it is in no way merely a question of curiosity, but is linked with important determinations that are presently still lacking in the theory of electricity, and might lead the way to interesting investigations.[179]

This question in regard to the amount of electricity in a Leyden jar has a special relationship to *electrodynamic units* that deserves a closer look, in any event. A fundamental law of electrical action was presented in the first part of this series of articles on determinations of units that simultaneously included *electrostatics*, *electrodynamics*, and *induction*.[180] According to that fundamental law, the force that the electrical mass e exerts upon the electrical mass e' at a distance r is not merely a function of that *distance*, but at the same time, a function of the relative state of motion of the two electrical masses that is given by the *relative velocity* dr/dt and acceleration d^2r/dt^2 with which they pass to the distance r. That fundamental law of the electrical action is:

$$\frac{ee'}{r^2}\left[1-\frac{1}{c^2}\left(\frac{dr^2}{dt^2}-2r\frac{d^2r}{dt^2}\right)\right]$$

in which *the constant* c means *the relative velocity at which the electrical masses would exert no effect at all on each other, as long as it remained unchanged*. In the second article in this series,[181] it was then shown how the determination of the value of that *constant* c might provide the possibility of reducing not merely the units of the electromotive forces, but also the units of the current intensities, to the *units of mechanics*, and that in itself will give the relationship by which the *constant* c will allow one to determine the *amount of electricity* that passes through a cross-section of the conductor in a unit of time in terms of the unit of measurement of current intensity that is based in the magnetic and electrodynamic effects of the current. Conversely, the knowledge of that *amount of electricity* that is acquired in other ways would also lead to a determination of the value of the *constant* c, which is brought to our attention by the fundamental law above. The determination of such a *constant* of nature is a topic that is especially appropriate for a finer measurement. In the foregoing case, that determination can be reduced to the following problem.

[179][Note by KW:] When one observes that most of the applications of the laws of nature depend upon determining the values of certain constants, that fact is based upon the determination of the unknown *constants in the theory of electricity*, which depend upon answering the question above. — Moreover, it is very likely that determining the electricity required to *decompose water* by the forces that it might exert could be used to investigate the forces that are active in the decomposition of water, and that in the same way, a determination of the amount of electricity by which a wire would be made to *glow* within a certain period of time from the *forces* that it might exert would lead to a deeper insight into the forces at work during the generation of heat, etc. Some of those applications will be discussed in more detail in Part Two.

[180][Note by AKTA:] [Web46] with partial French translation in [Web87] and a complete English translation in [Web07].

[181][Note by AKTA:] [Web52c] with English translation in [Web21b].

7.4 Problem. To Determine Electrostatically the Amount of Electricity Which Flows Through the Cross-Section of the Conductor in 1 Second According to the Magnetic Measure of Current Intensity

Determine the *amount of electricity* that passes through the cross-section of a conductor in unit of time for a current whose intensity has a unit of measurement that is based in its *magnetic* or *electrodynamic* or *electrolytic* effects, and indeed, that amount of electricity shall be determined *from the magnitude of the fundamental electrostatic force that it exerts*; or, more especially:

> *Let a constant current be given,*[182] *under which a tangent galvanometer with a simple multiplier circle of radius equal to R would take on a deflection of $\varphi = \arctan 2\pi/RT$ in equilibrium, where T means the intensity of the horizontal component of the Earth's magnetism that directs the compass. It should be determined how the amount of electricity that flows through the cross-section of the conductor in one second under such a current, relates itself to the amount of electricity that is contained on each of two small equally-charged balls that repel each other with a unit force at a unit distance. In that way, the unit of force shall be taken to be the force that would accelerate a mass of one milligram to one unit of velocity in one second.*

From our previous determination, the given *current* is one that will exert entirely the same effects at a distance as a magnet that possesses one unit of magnetic moment when it flows around a planar region of magnitude one unit of area; i.e., the current whose strength is ordinarily chosen the be the unit of the strength of all other currents by observing it with the tangent galvanometer, and the *amount of electricity* that is present on each of the small balls is the amount that one is accustomed to assign as the basis for the unit of measurement for electrostatic measurements with the Coulomb torsion balance.[183]

7.5 Plan for Solving the Problem — Electrostatic Measurement of the Amount of Electricity Accumulated in a Leyden Jar — Electromagnetic Measurement of the Electricity Generated by the Discharge of the Jar

If the *amount of electricity E that is collected on an isolated conductor* is discharged to Earth through the multiplier of a galvanometer, then it will exert a rotational moment on the magnetic needle of the galvanometer as it flows through it. Now, if one also extends the *discharge time* as much as needed by inserting a column of water into the path of the current

[182] [Note by AKTA:] Current of intensity $= i$.
[183] [Note by AKTA:] See footnote 43 on page 56.

in order for no spark to jump between the windings of the multiplier, then that *discharge time* will still define only an extremely small fraction of the *period of oscillation* of the magnetic needle, such that the part of the path that the needle covers during that *discharge time* (that is during the action of the discharge current) will be vanishingly small in comparison to the entire path of the needle; i.e., in comparison to the magnitude of the *elongation* that the needle attains over the course of *one-half an oscillation period*. The effect of the discharge current can then be considered to be an *impulse* that the needle would experience in its rest position, after which the *angular velocity* that the needle acquired could be calculated from known laws of oscillation by *observing the initial elongation of the needle after the discharge* at the moment of the impulse itself.

Furthermore, everything behaves exactly like an *induction impulse*, and also insofar as the nature of the discharge current is entirely indifferent to whether it consists of many separate partial discharges that rapidly follow each other or whether it is continuous with an intensity that decreases to zero rapidly according to some law. *The angular velocity that the needle acquires in that way will always depend upon the amount of electricity E.*[184]

We can give the needle of the galvanometer a similar *impulse* by means of a *constant current* when we let the current act for only a very short time, and indeed the initial elongation will be the same whether the current has an intensity i during the time t or with the greater intensity ni during the shorter time t/n . Namely, if the duration t of the current is very small compared to the period of oscillation of the needle, then the *angular velocity* will always be found to be the same.[185] However, precisely the same *amount of electricity* will flow through the cross-section of the conductor in time t with an intensity of i that flows through it in time t/n with an intensity of ni.

Hence, when we impart an *impulse* to the needle by a *constant current of short duration*, the *angular velocity* of the needle (and as a result, its *elongation* as well) will also depend *solely upon the amount of electricity that has moved through the cross-section of the multiplier during the duration of the current* in this case.

Now, if we have discharged a known amount E of positive electricity through the same multiplier *in one case* and produced *the same elongation of the magnetic needle* by means of a constant current of very short duration *in the other case*, then we can conclude that *the positive amount of electricity x that flows through the cross-section of the conductor during the short duration of the constant current is:*

$$x = \frac{1}{2}E ,$$

which is a result of whose validity one can easily convince oneself, and which one might have to envision in terms of the processes that take place inside of the conductor during the

[184][Note by KW:] One finds that fact confirmed in all experiments. As is shown in the experiment in Appendix II (among other things), the elongation is not only proportional to the *amount of* discharged *electricity*, but it is also independent of the *discharge time*, within wide limits; because it does not matter how long or short the water column that one inserts is, provided the windings of the multiplier are not jumped over or the discharge time is extended in such a way that the effect of the discharge current will still continue when the needle has already moved noticeably from the rest position.

[185][Note by KW:] As long as the direction of its magnetic axis deviates only slightly from the plane of the multiplier windings, the *acceleration* that a needle whose magnetic moment is M and whose moment of inertia is K will acquire from a *constant current* of intensity i will be equal to AMi/K, where A means a constant that depends upon the dimensions of the multiplier and the distribution of magnetism in the needle. It follows from this that the *angular velocity* that it acquires during time t will be equal to $AMit/K$, whose value will remain unchanged when i is replaced with ni and at the same time t is replaced with t/n.

discharge.

If one would like to assume of the discharge, e.g., that *all* of the accumulated *positive* amount of electricity E flows through the *entire* multiplier in only the direction of the Earth, or that an *equal* amount of *negative* electricity flows in the opposite direction from the Earth, then the magnetic effect of such a *discharge current* would be precisely the same as the effect of a current for which only *one-half* of that positive amount of electricity flowed through the cross-section of the conductor in the given direction, but at the same time, an equal negative amount of electricity flowed in the opposite direction, which is a process that is assumed to take place at *constant current*. — However, should one be of the opposite opinion, namely, that absolutely none of the amount of electricity E that is collected in the isolated conductor (and just as little of the amount that is found in the Earth) will flow through the total windings of the multipliers, but that it will merely *give rise to* a double current in the wire that will include masses of neutral fluids that are large enough that a very small change in those masses will suffice to supply the isolated conductor with so much negative electricity that the positive electricity E that is collected in it will be neutralized, then one would also arrive at the same result in that way, since the whole discharge wire could be divided into a very large number of small pieces such that the amount of electricity $+\frac{1}{2}E$ would flow from each piece into the *following* one, while $-\frac{1}{2}E$ would flow into the *preceding* one, and as a result, an amount of electricity $+\frac{1}{2}E$ would flow from the last piece into the Earth, which would replace the first piece of the wire with the isolated conductor, while the amount of electricity $-\frac{1}{2}E$ would flow out of the first piece into the isolated conductor and neutralize the electricity that remains in it, but which will replace the last piece of the wire with the Earth. Finally, if one were also required to assume that somewhat more than one-half of the positive amount of electricity E went from the isolated conductor to the wire, while somewhat less than $-\frac{1}{2}E$ of negative electricity went in the opposite direction from the wire to the isolated conductor, then nothing would change in the result, since the magnetic effect will be determined by the sum of the two moving charges.

The impulse that the needle feels when the *accumulated amount of electricity E discharges* through the multiplier will be *just the same* as when a *constant current* goes through the multiplier during a time interval τ such that precisely *one-half* of E goes through the cross-section in the direction of the current *as positive electricity* and just as much goes in the opposite direction *as negative electricity*, assuming that the time interval τ represents only a very small part of the period of oscillation of the needle.

The solution to the problem will then emerge from taking the following *two steps*:

1. Measure the amount of electricity E in the given electrostatic units and observe the elongation of the magnetic needle of a galvanometer under its discharge.

2. Determine the small time τ during which a constant current of intensity equal to 1 (in magnetic units) must go through the multiplier of that galvanometer in order for the needle to acquire the same elongation.

If one then multiplies $\frac{1}{2}E$ by the number that shows how often t is included in one second, then $E/2\tau$ will express the *amount of positive electricity* that passes through the cross-section of the conductor during one second in the direction of a current whose intensity is equal to 1 in magnetic units. In other words:

$$\frac{1}{2\tau} \cdot E : 1$$

is the ratio of the *amount of positive electricity* that passes through that cross-section, to the one whose unit is based upon measuring the accumulated amount of electricity E in the isolated conductor, namely, the amount that must be found on each of two small balls for them to repel each other with a force equal to 1 at a distance equal to 1.

As far as the *second* step is concerned, the determination of τ requires no special experiment, since the value of τ can be determined by calculation from the number and dimensions of the windings of the multiplier, the elongation of the tangent galvanometer that is observed under the discharge, and the intensity of Earth magnetism much more precisely than would be possible by direct experiment, as one will see in Section 7.13.

However, the *first* step, which is concerned with determining the amount of electricity E, requires a combination of several experiments, which shall be described in Sections 7.6 up to 7.12. Namely, it is important that, *first of all*, a still-unknown, but greater, amount of electricity is split into two parts in a *previously*-determined ratio, *and then* that the *greater* part E *is discharged* through the tangent galvanometer in order to observe its magnetic effect, but *finally*, the *smaller* part is measured by the electric force that it exerts upon the Coulomb torsion balance in order for the discharged part E to also get measured by the same measurement.

A *Leyden jar* whose external coating is connected to the Earth in a well-conducting way seems to be most suitable as a vessel for that amount of electricity whose part E should not be insignificant if its discharge were to produce a precisely-measurable effect on the needle of the tangent galvanometer. Hence (Section 6), that would next require the *ratio* by which the *positive charge in that jar* is divided between it and *a large isolated ball*, the latter of which contacts the knob on the jar. The ratio $n : 1$, by which the charge in the jar *before* contact with the large ball to its charge *afterwards* is determined with the help of the *sine electrometer*,[186] which will yield the ratio $1 : (n-1)$ of the amount of electricity E to the amount that goes over to the ball.

After this *ratio* has been determined precisely by several repetitions, the measurement of the amount of electricity that would go over to the large ball after such a division would be continued, to which end, the large ball, likewise after a charge that results from contact with the Leyden jar, would itself once more contact the 1 inch large *fixed ball* of a *Coulomb torsion balance* that is equipped with a large measuring scale. As Poisson and Plana have shown,[187] the ratio by which the electricity is divided between those two balls can be calculated from the ratio of their radii. That will be done in Section 7.8, from which, the charge that the *large ball* has received from the Leyden jar can be found from the amount of electricity e that is transferred to the *fixed ball of the torsion balance*, and then also the amount that remains in the Leyden jar, which will be employed to find the *discharge current* whose *magnetic effect* is to be observed.

The amount of electricity e would be measured after the *fixed ball* of the Coulomb torsion balance in which it is contained contacts the equally large *moving ball*, and in that way, e would be divided equally between those two balls. Namely (Section 7), from observing the gradual decrease in the torsion that would be necessary in order to keep the two balls at a *well-defined distance* from each other, the torsion would then be calculated that would, on first glance, be required if the charge in it were likewise to be able to go from the large ball through the Leyden jar, the fixed ball through the large one, and the moving one though the fixed ball while one observes the torsion. — In Section 7.9, one will find the calculation of the

[186][Note by AKTA:] See [Koh53].
[187][Note by AKTA:] See footnotes 43 and 153 on pages 56 and 135.

amount of electricity ε that would exert *a unit rotational moment* on the balance at the same distance when it is divided equally between the two balls of the torsion balance, in which one must take into account the non-uniform distribution of electricity on the surface of the ball. — In Section 7.10, one will find the determination of the *torsion* that would likewise exert *a unit rotational moment* on the balance from various observations. — With the help of the determinations that are contained in Sections 7.9 and 7.10, the amount of electricity e itself can be determined easily from the torsion that was found in Section 7.7, and then also the amount that remains in the Leyden jar, which will be done in Section 7.11, where the latter will be denoted by E', in order to distinguish it from the amount of electricity E that is employed by the discharge current whose magnetic effect is to be determined. — In the brief intervening time between the moment of the division to the moment of the discharge, the electricity that remains in the Leyden jar will change, namely, a small part of the charge in the jar will be lost to the air, and part of it will be lost to a change in the *residue* in the jar, and although that change during such a brief intermediate time of — say — only three seconds would be extremely negligible, from the superb quality of the jar that was selected for that experiment, it will still be included in the calculation in Section 7.12, from which, one will at least see how the change $E - E'$ would be determined for other jars and longer intermediate times.

Finally, with the help of the determination of τ that is contained Section 7.13 and mentioned on page 233,[188,189] the quantity $\frac{1}{2\tau} \cdot E$ will be calculated in Section 7.14, and with that, the problem that was posed above will be solved. The Section that follows it will include *applications*, for the most part, to which the determination of the *constant* c, which has been mentioned several times, belongs.

The two *Appendixes* include more precise descriptions of the *torsion balance* and the *tangent galvanometer*; for that of the *sine electrometer*, see Poggendorf's *Annalen* 88 (1853).[190]

It can be inferred from the satisfying agreement, without exception, between all published experiments (of which ones that were analyzed in Sections 7.6 and 7.7 were the most difficult) that the result can be considered to be accurate to within 1 to 2 percent. The calculation was performed with a precision of an even smaller fraction in order for the determination of the uncertainty in the results to depend upon merely the magnitude of the unavoidable observation error.

7.6 Determining the Conditions under which Electricity will be Divided between the Interior of a Leyden Jar and a Large Ball while the Exterior of the Jar is Connected to the Earth

The following Table gives the results of two series of observations that were performed with the *sine electrometer* of the decrease in charge in a Leyden jar by transferring it to a large uncharged ball that contacted the knob on the jar, while the exterior of the jar was connected to the Earth by a good conductor.

[188] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 621.

[189] [Note by AKTA:] [KW57, p. 621 of Weber's *Werke*], see also page 150 on Section 7.5.

[190] [Note by AKTA:] See footnote 186 on page 151.

The Leyden jar was previously connected to the sine electrometer with a conducting wire whose end was placed in a small indentation into the knob on the jar. Once the position of the sine electrometer had been observed, that end of the conducting wire was raised with a silk thread, and then the large ball contacted the knob of the jar, whereupon the exterior of the jar was connected to the Earth by a conductor. With double, triple and quadruple contacts, the individual contacts followed as quickly in succession as would permit the large ball to discharge completely in between them. If the sine electrometer, which suffered only a negligible loss to the air in between contacts, was then once more connected with the jar by a connecting wire that was kept insulated by a silk thread, then the needle of the electrometer, which was initially at rest, would be deflected only slightly in that way, since the jar had lost relatively little of its charge by contact with the ball, and since that loss was compensated approximately by the relatively smaller loss to the air that the jar suffered in comparison to the sine electrometer, which explained the shortness of the time during which the individual measurements could be performed in comparison to the end of each series of experiments.

Precise time measurements of the moment at which each individual contact was made could not be carried out, and the data that is contained in the following Table is based upon mere estimates, which can, however, be considered to be admissible to within 1-2 seconds, which is a precision that suffices completely for this. Both series were made on 2 April 1854 in the Physics Institute at Göttingen.

First Series			
No.	Time	Needle deflection on the sine electrometer	n
1.	$8^h 49'54''$	$32^o 36.2'$	
2.	$50'0''$	(quadruple contact)	1.032 4
3.	$51'25''$	$24^o 13.7'$	
4.	$53'46''$	$23^o 31.3'$	
5.	$53'52''$	(quadruple contact)	1.0299
6.	$54'42''$	$17^o 45.6'$	
7.	$58'56''$	$14^o 49.3'$	
8.	$59'2''$	(quadruple contact)	1.0167
9.	$59'55''$	$12^o 47.6'$	
10.	$9^h 2'7''$	$12^o 34.3'$	
11.	$2'13''$	(quadruple contact)	1.032 5
12.	$2'50''$	$9^o 41.7'$	
13.	$4'12''$	$9^o 41.7'$	
14.	$4'18''$	(quadruple contact)	1.0355
15.	$4'53''$	$7^o 21.3'$	
16.	$7'22''$	$7^o 30.2'$	
17.	$7'28''$	(quadruple contact)	1.0311
18.	$8'9''$	$5^o 51.2'$	
19.	$10'7''$	$4^o 48.3'$	
20.	$10'13''$	(quadruple contact)	1.0305
21.	$10'51''$	$4^o 32.9'$	

Second Series			
No.	Time	Needle deflection on the sine electrometer	n
1.	$9^h40'7''$	$46^o30.5'$	
2.	$41'57''$	$44^o9.0'$	
3.	$42'0''$	(single contact)	1.0330
4.	$42'23''$	$40^o23.9'$	
5.	$44'0''$	$39^o10.5'$	
6.	$44'3''$	(single contact)	1.0308
7.	$44'23''$	$36^o15.7'$	
8.	$46'24''$	$35^o11.7'$	
9.	$46'27''$	(single contact)	1.0379
10.	$46'51''$	$32^o24.6'$	
11.	$48'24''$	$32^o46.6'$	
12.	$48'27''$	(single contact)	1.0490
13.	$48'51''$	$29^o21.1'$	
14.	$51'41''$	$28^o31.0'$	
15.	$51'44''$	(single contact)	1.0390
16.	$52'9''$	$26^o14.2'$	
17.	$52'52''$	$26^o14.2'$	
18.	$52'55''$	(single contact)	1.0375
19.	$53'25''$	$24^o14.7'$	
20.	$58'30''$	$19^o41.9'$	
21.	$9^h58'33''$	(single contact)	1.0303
22.	$59'1''$	$18^o27.6'$	
23.	$10^h5'52''$	$17^o42.6'$	
24.	$5'56''$	(double contact)	1.032 8
25.	$6'28''$	$15^o30.1'$	
26.	$7'14''$	$15^o30.1'$	
27.	$7'19''$	(triple contact)	1.0338
28.	$7'45''$	$12^o38.7'$	
29.	$10'13''$	$12^o38.7'$	
30.	$10'19''$	(quadruple contact)	1.0315
31.	$11'27''$	$9^o50.0'$	
32.	$12'44''$	$9^o50.0'$	
33.	$12'50''$	(quadruple contact)	1.0292
34.	$13'27''$	$7^o47.8'$	

The last column in this Table, under n, gives the *ratio* of the charge in the jar before contact with the ball to the charge after contact, which is always made immediately before and after the moment of contact between the two. The *second* and *third* columns contain observations that are reckoned according to the following rule:

- $q_{''}^2$ and $q_{'}^2$ denote the sines of the observed deflections for the two previous times of observation,
- q'^2 and q''^2 denote the sines of the observed deflections for the two following times of observation,

- $-t_{''}$, $-t_{'}$, t', t'' are the associated observation times, measured from the moment of contact,
- m is the number of times the contact was repeated.

Hence:[191]

$$n = \sqrt[m]{\frac{t'' - t'}{t_{''} - t_{'}} \cdot \frac{t_{''} q_{'} - t_{'} q_{''}}{t'' q' - t' q''}} \ .$$

Indeed, some of the observations in these two series of observations are less definitive (which is almost unavoidable when three observers collaborate), and in that way one can find that is permissible to discard some values of n completely: for example, the one that

[191] [Note by KW:] The observations of the deflection of the needle in the third column and the time in the second column will immediately give the values of $q_{''}$, $q_{'}$, $q^{'}$, $q^{''}$ and the associated values of $-t_{''}$, $-t_{'}$, t', t'' at which the values of q_0 and q^0 should be calculated, which are true for the moments immediately before and after the contact. The cited rule will then be implied in the following way:

1) For the brief time duration of the experiment, it suffices to assume that the charge lost to the air over time and the charge at the moment of observation are proportional, from which, one will then get the following four values for the reduced observations at the moment of contact:

$$(1 - \alpha t_{''})\, q_{''} \ , \qquad (1 - \alpha t_{'})\, q_{'} \ , \qquad (1 + \alpha t')\, q^{'} \ , \qquad (1 + \alpha t'')\, q^{''} \ .$$

2) If one adds each of these values to the residue in the jar at the time in question then the first two, which represent the *total charge before contact*, must be equal, and similarly for the last two, which represent the *total charge after contact*. When one denotes the residue at time t by r_t, one will then get the equations:

$$(1 - \alpha t_{''})\, q_{''} + r_{-t_{''}} = (1 - \alpha t_{'})\, q_{'} + r_{-t_{'}} = q_0 + r_0 \ ,$$

$$(1 + \alpha t')\, q^{'} + r_{t'} = (1 + \alpha t'')\, q^{''} + r_{t''} = q^0 + r^0 \ .$$

However, the residue *before* and *after* contact (see Section 7.12) can be represented by:

$$r_t = \beta \left(1 - e^{-\gamma(\vartheta + t)^\delta}\right) \cdot (q_0 + r_0) \ , \qquad r_t = \beta \left(1 - e^{-\gamma(\vartheta' + t)^\delta}\right) \cdot \left(q^0 + r^0\right) \ .$$

The residue remains unchanged at the moment of contact, so $r_0 = r^0$. That easily implies that *for small values of* t *before* and *after* contact, r_t can be set to:

$$r_t = r_0 + at \ , \qquad r_t = r_0 + a't \ ,$$

where a and a' denote two coefficients that are determined from the observations. — By substituting those values in the equations above, in which one might likewise replace $\alpha q_{''}$ and $\alpha q_{'}$ with αq_0, and similarly replace $\alpha q^{'}$ and $\alpha q^{''}$ with αq^0, one will get:

$$q_0 = q_{'} - (a + \alpha q_0)\, t_{'} = q_{''} - (a + \alpha q_0)\, t_{''} \ ,$$

$$q^0 = q^{'} + \left(a' + \alpha q^0\right) t' = q^{''} + \left(a' + \alpha q^0\right) t'' \ ;$$

and as a result:

$$q_0 = \frac{t_{''} q_{'} - t_{'} q_{''}}{t_{''} - t_{'}} \ , \qquad q^0 = \frac{t'' q^{'} - t' q^{''}}{t'' - t'} \ ,$$

$$n = \sqrt[m]{\frac{q_0}{q^0}} = \sqrt[m]{\frac{t'' - t'}{t_{''} - t_{'}} \cdot \frac{t_{''} q_{'} - t_{'} q_{''}}{t'' q' - t' q''}} \ .$$

is cited in no. 8 in the first column and the ones in nos. 12, 15, 33 in the second column. However, it will follow that the removal of those values will have no appreciable effect on the determination of the mean value of n, since one finds that the mean values *with* and *without* removal are:

$$n = 1.032\,82 \ , \qquad n = 1.032\,97 \ ,$$

respectively.

A similar series of observations with the same jar and ball that was carried out earlier in Marburg yielded the following mean value for the ratio n:

$$n = 1.032\,63 \ .$$

Hence, the desired ratio will be henceforth assumed to be:

$$n = 1.032\,76 \ .$$

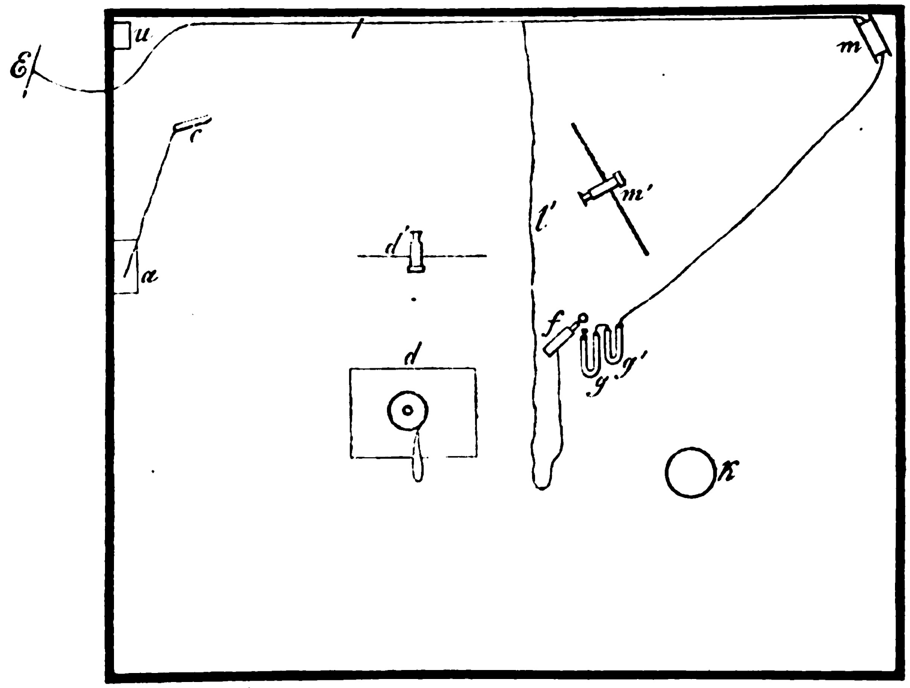
Finally, this ratio of the charge in the jar *before* and *after* contact with the large ball also yields the *ratio of the distribution of the electricity between the jar and the large ball at the moment of contact; namely it is equal to*:

$$1 : 0.032\,76 \ .$$

7.7 Corresponding Observations of the Deflection of the Tangent Galvanometer that is Produced by the Amount of Electricity E that Flows Through the Multiplier, and the Torsion in the Coulomb Torsion Balance Through which the Two Balls Charged with the Amount of Electricity e will be Maintained at the Same Distance as the Uncharged Ones

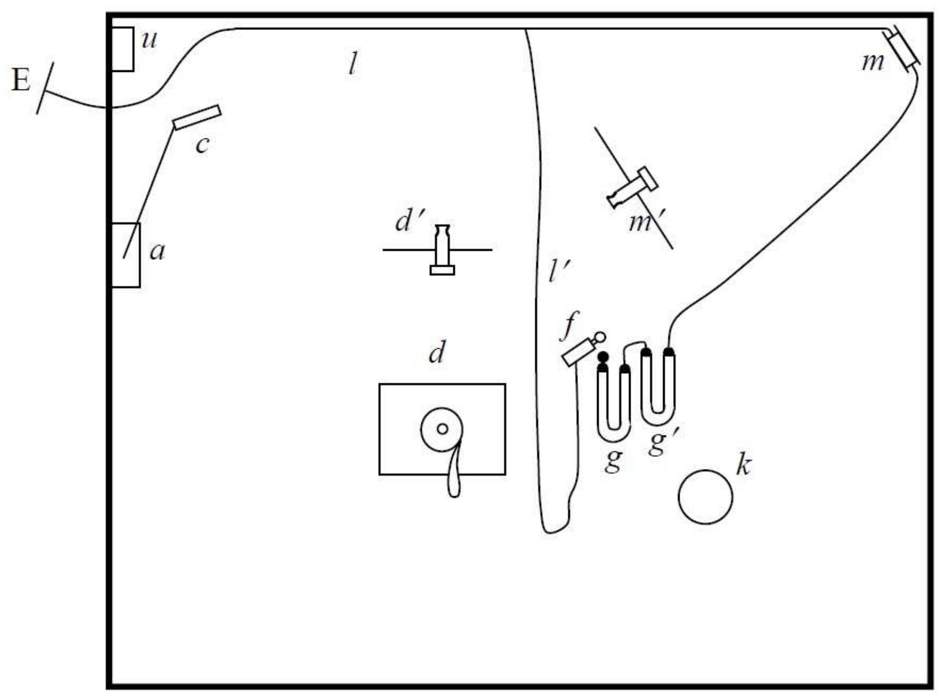
The arrangement of the instruments that were used in the experiment that was mentioned before in Section 7.5 is depicted in Figure 1, which will serve to make it more intuitive.[192]

Fig. 1.



The *tangent galvanometer* is denoted by m, whose multiplier wire is connected to the Earth at its one end by a conducting wire l that is soldered to a plate E that is buried in wet soil, while the other end of the wire leads through the air to the long U-shaped glass tubes

[192][Note by AKTA:] An improved version of Figure 1 has been prepared by D. H. Delphenich, namely:



g and g', which are filled with water. m' represents the scale and telescope for observing the needle of the tangent galvanometer, which is provided with a mirror.

d refers to the *Coulomb torsion balance*, which will be described in more detail at the end of this treatise in Appendix I. d' represents the scale and telescope for observing the state of the torsion balance. Namely, a long hanging shellac rod is fixed to the torsion wire under the arm that carries the moving ball, and it carries a mirror at its end, to which the telescope points. — The large ball hangs from the ceiling of the room by a silk thread at k. l' is a fork in the conducting wire l so one can connect the exterior of the jar f to the Earth. — u is a clock, and a is a hole in the ceiling of the room through which a wire from the conductor of an electrification machine that was found in the upper room was led to the small conductor c in order to charge the jar f.

Once the jar f was charged, and a clamping screw was fixed to the wire l', the jar was then contacted by the large ball k. The amount of electricity that remained in the jar after that contact will be denoted by E'. After three seconds, during which E' went to E by losing electricity to the air and the formation of a residue, the knob on the jar f, as is suggested in Figure 1, was contacted by a metal knob that stuck out of the U-shaped tube g, and the observer at the telescope m watched the elongation of the magnetic needle of the tangent galvanometer that was produced by the discharge current of the amount of electricity E that went through the multiplier.

Immediately after the jar f was discharged, the fixed ball on the Coulomb torsion balance, which had been kept on standby, was charged by the ball k and quickly placed into the torsion balance; however, the ball k itself was likewise discharged in that way.

Thereupon, the torsion was measured several times in brief intermediate times, which was necessary in order to keep the two balls in their positions, in which, the two radii that pointed from the rotational axis to center of the ball would define a right angle. The torsion that would exist at the moment when the large ball k was charged by the jar f (so the two balls in the torsion balance had also been charged and could be inserted) could then be *calculated* from the gradual decrease in that torsion according to Coulomb's law, which says that the charge decreases geometrically when time increases arithmetically.[193] The torsion that was first noticed for each number is *calculated* in that way in the following Table. The amount of electricity e that went from the large ball k to the fixed ball of the torsion balance at the moment of contact will be determined from it in Section 7.11.

The last column of the following Table, which is labeled with $A/\sqrt{T}$, contains the quotients that take the form of the deflection of the magnetic needle in the tangent galvanometer, expressed in scale divisions, divided by the square root of the torsion in the torsion balance, expressed in minutes. — The distance from the mirror to the scale of the tangent galvanometer was equal to:

$$6437\frac{1}{2} \text{ scale divisions .}$$

[193][Note by KW:] By a series of experiments in which the fixed ball was sometimes found to be outside the case of the torsion balance and sometimes inside of it between the individual determinations of torsion, it was confirmed that the loss of electricity to the air when it was inside the case was the same as the loss to the air when it was outside of it, which might have been expected from the size of the case. If that were not the case then the aforementioned application of Coulomb's law would not be directly applicable, since the fixed ball would be found outside of the case for some moments before it could be placed inside the torsion balance.

No.	Time	Deflection of the tangent galvanometer in scale divisions $= A$	Torsion of the torsion balance in minutes $= T$	$\frac{A}{\sqrt{T}}$
1.	$8^h 11' 8''$	73.5	$175.3'$	5.55
	$16' 13''$		$152.4'$	
	$21' 16''$		$136.1'$	
	$26' 35''$		$118.3'$	
	$32' 32''$		$99.9'$	
2.	$8^h 37' 8''$	80.0	$237.1'$	5.20
	$42' 4''$		$208.4'$	
	$45' 14''$		$189.1'$	
	$50' 10''$		$165.3'$	
	$54' 40''$		$148.1'$	
3.	$9^h 0' 37''$	96.5	$332.9'$	5.29
	$5' 14''$		$297.5'$	
	$9' 19''$		$270.6'$	
	$14' 11''$		$238.5'$	
	$18' 10''$		$218.3'$	
4.	$9^h 31' 14''$	91.1	$265.1'$	5.59
	$35' 17''$		$249.2'$	
	$41' 1''$		$226.2'$	
	$47' 43''$		$201.1'$	
	$55' 0''$		$178.0'$	
5.	$10^h 1' 46''$	97.8	$332.4'$	5.36
	$6' 24''$		$306.0'$	
	$10' 54''$		$280.4'$	
	$16' 31''$		$251.1'$	
	$22' 4''$		$228.6'$	

7.8 Calculating the Ratio of the Two Amounts of Charge $E' : e$

The radius of the large ball was:

$$a = 159.46 \text{ millimeters} ,$$

and the radius of the fixed ball in the Coulomb torsion balance was:

$$ba = 11.537 \text{ millimeters} .$$

If one now sets the ratio by which the electricity equal to $0.032\,76E'$ that is transferred from the jar to the first ball by contact with the latter equal to:

$$(0.032\,76E' - e) : e = A : b^2 B ,$$

as in Section 7.6, then, from Plana ("Mémoire sur la distribution de l'électricité à la surface de deux sphères conductrices," Turin, 1845, pages 64, 66):[194]

$$\frac{B}{h} = \frac{1}{1+b} + \frac{1}{(1+b)^2}\left\{k_2 + \frac{b}{1+b}k_3 + \frac{b^2}{(1+b)^2}k_4 + \frac{b^3}{(1+b)^3}k_5 \ ...\right\} \ ,$$

and when one sets $b/(1+b) = a$:

$$\frac{A}{h} = \frac{1}{2} + \frac{a^3}{1-a^2} + \frac{\pi a}{2}\cot \pi a + a^3 k_3 + a^5 k_5 + a^7 k_7 \ ... \ ,$$

where:

$$k_n = \frac{1}{2^n} + \frac{1}{3^n} + \frac{1}{4^n} + \frac{1}{5^n} + ...$$

That yields the cited value for the desired ratio:

$$(0.032\,76E' - e) : e = A : b^2 B = 1 : 0.007\,937\,7 \ ;$$

as a result:

$$E' : e = 3\,876 : 1 \ .$$

7.9 Calculating the Amount of Electricity ε with which the Two Balls in the Coulomb Torsion Balance Must be Charged in Order for Their Repulsion to Exert One Unit of Rotational Moment on the Torsion Balance

The radius of the fixed ball on the Coulomb torsion balance was equal to 11.537 millimeters, and the radius of the moving ball was equal to 11.597 millimeters, so one can then assume to no detriment that the mean radius of the two almost-equal balls in the following calculation is:

$$a = 11.567 \text{ millimeters} \ .$$

Furthermore, the distance from the rotational axis to the center of the fixed ball was equal to 93.53 millimeters, the distance from the rotational axis to the center of the moving ball was equal to 61.7 millimeters, and both centers defined a right angle with the axis of rotation. That yielded the distance between the centers as being equal to:

$$112.05 \text{ millimeters} \ ,$$

which was also confirmed by direct measurement of that distance.

Now, if each of the two balls contains one-half of the amount of electricity to be determined ε, then if one assumes that *this electricity is distributed uniformly on the surface of each ball* then, from known laws, that:

[194][Note by AKTA:] See footnote 153 on page 135.

1. A uniformly distributed amount of electricity on the surface of the ball will act upon all points in external space just as if it were concentrated at the center of the ball.
2. The force of repulsion that the amount of electricity that is concentrated at a point exerts upon another [amount of electricity] concentrated at a point, is equal to the quotient of the product of both amounts of electricity divided by the square of the distance between them,

one would obtain immediately the *force of repulsion between both balls*, namely:

$$\frac{1}{4} \cdot \frac{\varepsilon^2}{112.05^2} = \frac{\varepsilon^2}{502\,21} \ .$$

However, if that force of repulsion were to be found *precisely*, then the assumption above would be inadmissible, and one would have to determine the *non-uniformity of the distribution of electricity on the surface of every ball* precisely from the given magnitude and distance and include it in the calculation.

In Poisson's "Mémoire sur la distribution de l'éléctricité à la surface des corps conducteurs" (*Mèmoires de l'Institut. Année* 1811. *Première partie*, page 88),[195] one finds the following expression for the *density* z of the electricity on the surface of a small ball at a great distance from another ball when the *mean* density on the first ball is given to be equal to B and is equal to A on the latter:

$$z = B - \frac{3a^2A}{c^2} \cdot \mu_| + \frac{5a^2bA}{2c^3}\left(1 - 3\mu_|^2\right) \ ,$$

in which b and a are the radii of the two balls, c is the distance between their centers, and $\mu_|$ means the cosine of the angle φ that the radius of the first ball defines with the direction of c at the location in question. — If one wishes to apply that general rule to the foregoing case, then one must set:

$$A = B \ ,$$

$$a = b \ ,$$

and when one writes the value $\cos\varphi$ for $\mu_|$, it will follow that the *density* is:

$$z = A\left[1 - \frac{3a^2}{c^2}\cos\varphi + \frac{5}{2}\frac{a^3}{c^3}\left(1 - 3\cos^2\varphi\right)\right] \ .$$

Furthermore, that *density* implies the *outward-pointing electric pressure perpendicular to the surface of the ball* at the location in question from the known law that was proved by Poisson in the cited treatise, according to which, the pressure is proportional to the square of the density, or more precisely, it is equal to the square of the density z^2 multiplied by the number 2π:

$$2\pi \cdot z^2 \ .$$

If one then decomposes that *pressure* in the direction of the extended line c and a direction that is perpendicular to it, then one will get that the *component parallel to the extended line* c is equal to:

[195] [Note by AKTA:] See footnote 43 on page 56.

$$-2\pi z^2 \cdot \cos\varphi \ .$$

Finally, if one substitutes the value of z above in this [expression], then one will get the decomposed *pressure for two equal elements of the surface of the ball whose connecting line is parallel to the line* c, for which the value of φ between them is then extended to π, and *collected along the direction* of the extended line c from:

$$24\frac{\pi a^2}{c^2}A^2\left[1+\frac{5}{2}\frac{a^3}{c^3}\left(1-3\cos^2\varphi\right)\right]\cos^2\varphi \ ,$$

from which, one will find the *force of pressure* that is parallel to the extended line c, *first of all*, for the two zones of width $ad\varphi$, which both include the elements of the surface of the ball that belong to the values of φ, extended to π, upon multiplying by the area $2\pi a^2 \sin\varphi d\varphi$:

$$48\frac{\pi^2 a^4}{c^2}A^2\left[1+\frac{5}{2}\frac{a^3}{c^3}\left(1-3\cos^2\varphi\right)\right]\cos^2\varphi\sin\varphi d\varphi \ ,$$

and *secondly*, for the total surface of the ball, by integration:

$$\begin{aligned} 48\frac{\pi^2 a^4}{c^2}A^2\int_0^{\pi/2}&\left[1+\frac{5}{2}\frac{a^3}{c^3}\left(1-3\cos^2\varphi\right)\right]\cos^2\varphi\sin\varphi d\varphi \\ &= 16\frac{\pi^2 a^4}{c^2}\left(1-2\frac{a^3}{c^3}\right)A^2 \ , \end{aligned}$$

in which A is the *mean* density of the electricity on the surface of each of the two balls of radius a, and as a result:

$$4\pi a^2 \cdot A$$

will represent the *amount of electricity* that is distributed on the surface of each ball.

However, the *desired amount of electricity* that is distributed on both surfaces collectively (whose force of repulsion should exert a unit of rotational moment on the torsion balance) was denoted by ε above; as a result, one has:

$$\frac{1}{2}\varepsilon = 4\pi a^2 \cdot A \ ,$$

from which:

$$A = \frac{\varepsilon}{8\pi a^2} \ .$$

If one substitutes this value of A, one will get the *force of pressure* parallel to the direction of the extended line c; i.e., the *force of pressure on the two balls*:

$$\frac{1}{4}\left(1-2\frac{a^3}{c^3}\right)\frac{\varepsilon^2}{c^2} \ ,$$

or when one substitutes the aforementioned values for a and c in this [expression], namely:

$$a = 11.567 \ ,$$

$$c = 112.05 \ ,$$

one will get:[196]

$$\frac{\varepsilon^2}{50\,331} \ .$$

Finally, the product of the force of repulsion between the two balls in the direction from the rotational axis to the direction of that force — i.e., along the perpendicular dropped from the line c — gives the value of the *rotational moment* that this force of repulsion exerts upon the torsion balance, which should be equal to 1.

However, since the line c that connects the centers of both balls defines a right triangle at the rotational axis with the horizontals that are drawn from both centers to the rotational axis, the perpendicular that is dropped from the rotational axis to the hypotenuse of the rectangular triangle c will be equal to the product of the two catheti[197] divided by the hypotenuse, or since the two catheti are 93.53 and 61.7 millimeters long, and $c = 112.05$ millimeters, that expression will be equal to:

$$\frac{61.7 \times 93.53}{112.05} = 51.502\,5 \text{ millimeters} \ .$$

Now, it follows from this that the *rotational moment* that is exerted by the electric force of repulsion on the two balls of the torsion balance will be equal to:

$$51.502\,5 \cdot \frac{\varepsilon^2}{50\,331} = \frac{\varepsilon^2}{977} \ .$$

The requirement that the *rotational moment* that originates in the electric force of repulsion on the two balls should be equal to 1 will be satisfied in such a way that the *amount of electricity* that is contained in the two balls collectively will be:

$$\varepsilon = \sqrt{977} = 31.25 \ .$$

This determination of ε bases the unit for the amount of electricity as the amount that will make two equal amounts of electricity exert a unit force of repulsion when they are at a unit of distance and in a state of relative rest.

[196][Note by KW:] That implies that, due to its non-uniform distribution on the outer surface, the electricity that is contained in each ball cannot be thought of as concentrated at the *center* of the ball. — However, one has:

$$\frac{\varepsilon^2}{50\,331} = \frac{1}{4} \cdot \frac{\varepsilon^2}{112.1743^2} \ ,$$

which then implies that the force of repulsion between the two balls is the same as if the two halves of the total amount of electricity that is contained in them were concentrated at two points that are separated by 112.1734 millimeters, that is, since that distance is 0.1234 millimeters greater than the distance between the centers, at two points that lie at a distance of 0.0617 millimeters from the two centers.

[197][Note by DHD:] Viz., shorter sides.

7.10 Calculating the Torsion ϑ that the Wire from which the Coulomb Torsion Balance Hangs Must Possess in Order to Exert One Unit of Rotational Moment on the Torsion Balance by Its Force of Torsion

The *rotational moment* that is exerted upon the torsion balance by the torsion in the wire to which it [that is, the balance] hangs is known to be proportional to the *torsion* and the *torsion coefficient* of the wire — or more precisely — it is equal to the *product of the torsion angle, expressed in units of radii, with the directive force*[198] *that the wire exerts upon the torsion balance*. One therefore needs only to determine that *directive force* in order to infer from it the torsion angle ϑ for which the rotational moment that is exerted upon the torsion balance is equal to *one unit*.

From the known laws of the elasticity of solid bodies, the magnitude of the directive force that is exerted upon the wire is independent of the size and weight of the body that hangs from the wire, and other bodies, instead of the torsion balance, can therefore be hung from the wire and observed in order to determine the *directive force* of the wire.

First of all, one might hang a circular brass plate horizontally at its center from the wire, instead of the torsion balance. That brass plate has:

a *mass* of 191 112.4 milligrams,
a *radius* of 63.95 millimeters.

A small vertical cylinder with:

a *mass* of 2626.0 milligrams,
a *radius* of 3.25 millimeters,

will serve to connect the wire with the disc. The *period* t of the torsion oscillations of the plate was then observed and found to be:[199]

$$t = 47.139 \text{ seconds.}$$

However, from the foregoing data, the *moment of inertia of the oscillating plate* was:

$$K_1 = \frac{1}{2} \cdot 63.95^2 \cdot 191112.4 = 390\,790\,000 \ ,$$

and the *moment of inertia* of the small cylinder was:

$$K_2 = \frac{1}{2} \cdot 3.25^2 \cdot 2626 = 13\,868 \ ,$$

so when they are combined:

---

[198][Note by AKTA:] See footnote 66 on page 77.

Representing the rotational moment by τ, the torsion angle by ϑ and the directive force (or torsion coefficient) by D we have

$$\tau = -D\vartheta \ .$$

[199][Note by AKTA:] J. C. F. Gauss (1777-1855) and W. E. Weber utilized the French definition of the period of oscillation t which is half of the English definition of the period of oscillation T, that is, $t = T/2$, [Gil71a, pp. 154 and 180]. For instance, the period of oscillation for small oscillations of a simple pendulum of length ℓ is $T = 2\pi\sqrt{\ell/g}$, where g is the local free fall acceleration due to the gravity of the Earth, while $t = T/2 = \pi\sqrt{\ell/g}$.

$$K = K_1 + K_2 = 390\,603\,868 \ .$$

Now, from the known laws of such oscillations, one will get the value of the *directive force* D from that *moment of inertia* K and the observed *period of oscillation* t:[200]

$$D = \frac{\pi^2 K}{t^2} = 1\,735\,800 \ .$$

Secondly, a brass cylinder was hung horizontally by its center from the same wire. That cylinder had:

a *mass* of 58 897.1 milligrams,
a *length* of 269.7 millimeters,
a *radius* of 2.865 millimeters.

That same small vertical cylinder served to connect it with the wire, as it did in the foregoing experiment. The *period* t' of the torsional oscillation of that rod was then observed and found to be:

$t' = 44.9537$ seconds.

From the foregoing data, the *moment of inertia of the oscillating rod* was:

$$K_1' = \frac{1}{12}\left(269.7^2 + 3 \cdot 2.865^2\right) 58\,897.1 = 357\,130\,000 \ ,$$

and then the *total moment of inertia*, including the small vertical cylinder was:

$$K' = 357\,143\,868 \ .$$

Those observations then yielded the value of the *directive force* D as:

$$D = \frac{\pi^2 K'}{t'^2} = 1\,744\,200 \ .$$

As a result, the mean of the two series of observations was:

$$D = 1\,740\,000 \ .$$

Now, should the product of this value of D with the torsion angle, expressed in units of the radius — i.e., the *rotational moment* that the wire exerts upon the torsion balance — be equal to 1, then that would imply that the value of the angle of rotation or the desired *torsion in the wire* ϑ would be equal to the angle whose arc is equal to 1/1740000 of the radius, or:

$$\vartheta = 0.001\,975\,7 \text{ arcminutes} \ .$$

[200][Note by AKTA:] Weber is utilizing the equation of motion of the rigid body with moment of inertia K given by $\tau = -D\vartheta = K\ddot{\vartheta}$. Here $\tau = -D\vartheta$ is the torque or rotational moment acting on it due to the wire from which it hangs, when the lower portion of this wire undergoes a deflection ϑ relative to the upper portion, D being the torsion coefficient or directive force of this wire. The known solution of this equation yields a sinusoidal periodic motion with angular frequency $\omega = \sqrt{D/K}$. The period T of a complete oscillation is given by $T = 2\pi/\omega = 2\pi\sqrt{K/D}$. Gauss and Weber's period of oscillation discussed in footnote 199 on page 164 is then given by $t = T/2 = \pi\sqrt{K/D}$, such that $D = \pi^2 K/t^2$.

7.11 Calculating the Amounts of Electricity E' and e in the Observations that were Described in Section 7

In the experiments that were described in Section 7.7, the following values were found for the torsion angle in the Coulomb torsion balance when it was in equilibrium, where the various experiments are distinguished by numbers:

No.	Torsion angle in minutes
1.	175.3
2.	237.1
3.	332.9
4.	265.1
5.	332.4

However, the equilibrium of the torsion balance shows that the rotational moment that is exerted on the torsion balance by the wire is equal and opposite to the rotational moment of the force of repulsion between the two balls. — Nonetheless, the *first* rotational moment was found by dividing the observed torsion angle by the angle $\vartheta = 0.0019757$ arc minutes that was determined in the previous Section, which was the angle through which the wire would have to be rotated in order to exert *one unit of rotational moment* on the torsion balance. One then gets the *rotational moment* that the wire exerts on the torsion balance in the experiments that were described.

No.	Rotational moment of the wire
1.	88 728
2.	120 010
3.	168 500
4.	134 180
5.	168 240

From Section 7.9, the *last* of the rotational moments that originates in the electric repulsive forces between the two balls is:

$$\frac{e^2}{\varepsilon^2} = \frac{e^2}{977} ,$$

where e denotes the *amount of electricity* with which the two balls of the torsion balance are collectively charged, which one can then calculate in the five cited experiments from the *equality* of the two rotational moments, which is done in the following Table. In addition, the values of E' that are calculated from the proportion:

$$E' : e = 3\,876 : 1$$

that was found in Section 7.8 are entered the last column of that Table.

No.	e	E'
1.	9 310	36 086 000
2.	10 828	41 970 000
3.	12 830	49 730 000
4.	11 450	44 379 000
5.	12 821	49 593 000

7.12 Calculating the Correction that is Required by the Loss of Electricity and the Residue in the Leyden Jar During the Transfer of Electricity up to the Elapsed Time when the Jar is Discharged, which Equals $E' - E$

The amount of electricity E' that remains in the Leyden jar after the charging of the large ball will experience a small change during the time interval of *three seconds* up to its discharge, partly by loss to the air and partly by the formation of residue. The amount E that is still present in the jar can then be determined from E' in the following way:

In Poggendorf's Annalen 91 (1854), one will find a method given for determining the formation of the residue in a Leyden jar.[201] In accordance with it, if Q is an amount of electricity that is suddenly transferred to the jar, Q_t of which is lost to the air in t seconds, then a residue of r_t will have formed at time t whose equation is:

$$r_t = p\left(Q_t - Qe^{-\frac{b}{m+1}\cdot t^{m+1}}\right) . \qquad (I)$$

From the previous investigation, the constants of the jar that is used have the values:

$$p = 0.044\,94 \, , \qquad b = 0.183\,4 \, ,$$

while $m + 1$ possesses a magnitude equal to 0.4255, which is the same for all jars.

If those constants are determined for a jar, then the constant α that refers to the electricity lost to air can also be easily found. One suddenly transfers to the end an unknown charge Q from the jar to another jar and at the times:

$$t_1, t_2, ..., t_n \, ,$$

one observes the available charges:

$$L_{t_1}, L_{t_2}, ..., L_{t_n}$$

with the *sine electrometer*. Now, if ν_t denotes the amount of electricity that has leaked to the air up to time t then:

$$L_t = Q - r_t - \nu_t \, . \qquad (II)$$

However, for small values of t, one can set:

[201][Note by AKTA:] [Koh54a] and [Koh54b].

$$\nu_t = \alpha \cdot t\frac{Q+L_t}{2} \ ,$$

and if $Q - \nu_t$ is written for Q_t in equation (I), in addition, then one will get:

$$L_t = Q(1-\rho_t) - \alpha(1-p)t\frac{Q+L_t}{2} \ ,$$

in which $p\left(1 - e^{-\frac{b}{m+1}\cdot t^{m+1}}\right)$ has been replaced with ρ_t.

Now, that equation shall suffice for all observations. If one calculates ρ_t for the times of the first and last observations and substitutes those values in the equation, along with the observed values of L_t and t, then one will get two equations in the two unknown quantities Q and α.

Now, once a charge was suddenly imparted to the Leyden jar in the location where the previous experiments were carried out, the following results for the determination of α would be obtained from the observations:

t	L_t	ρ_t
23	0.667 6	0.036 19
65	0.657 6	0.041 42
128	0.648 3	0.043 44
226	0.638 9	0.044 35

One has $L_t = \sqrt{\sin\varphi}$ in this, and φ is the deflection that is observed in the *sine electrometer*. However, ρ_t is calculated from t and the constants of the jar. — Upon combining the first and last observations, one finds that:

$$Q = 0.695\,6 \ , \qquad \alpha = 0.000\,179\,35 \ .$$

Equation (III) then yields the following associated values for t and L_t with those values:

t	L_t
23	0.6676
65	0.6592
128	0.6506
226	0.6389

which deviate from the observed values so slightly that the values that were found for α can be employed precisely in order to find the correction to E'. In *three seconds* then, the *loss of electricity to the air* will amount to:

$$0.000538$$

times the total charge E'.

The *residue* that is created in the same time will be found in the following way:

Immediately before contact with the large ball, which results t seconds after the jar is charged, the latter will have an available charge of L_t and residue r_t that cannot be discharged. If one writes $Q - \nu_t$ in place of Q_t in equation (I), sets ν_t equal to its value of $\alpha \cdot t\frac{Q+L_t}{2}$, and sets Q equal to the value that is implied by equation (III), then one will get the residue at the time t, expressed in terms of the available charge that is present at that time:

$$r_t = \frac{\rho_t - \alpha t\left(p - \frac{1}{2}\rho_t\right)}{1 - \rho_t - \frac{1}{2}\alpha t(1-p)} \cdot L_t = \beta L_t \ . \qquad (IV)$$

After the ball has been charged, only an available charge of L_t/n will remain in the jar (Section 6), so an amount of electricity $\left(\frac{1}{n} + \beta\right) L_t$. Now, the form that the ratio of the residue will take after that partial discharge will depend upon whether the residue that forms βL_t is less than, equal to, or greater than the limiting value:

$$p\left(\frac{1}{n} + \beta\right) L_t$$

of the residue for the charge that is still present in the jar, which will, in turn, depend upon whether n is less than, equal to, or greater than $p/[\beta(1-p)]$, respectively.

In the present experiments, t was close to 60 seconds, in the mean. If one substitutes that value in equation (IV), then that will imply that:

$$\beta = 0.042\,86 \ , \qquad \frac{p}{\beta(1-p)} = 1.097\,8 \ .$$

Since it was found in Section 7.6 that $n = 1.032\,76$, so it is less than $p/[\beta(1-p)]$, it emerges that the residue will continue to increase. However, its growth will be slower than before the partial discharge, since the present limiting value of the residue that has already formed lies closer than it did before, and indeed the further formation will proceed as if the residue that is present βL_t were generated by the present charge $(1/n + \beta)L_t$. However, that would have required a time that follows from the equation:[202]

$$r_t = \beta L_t = \left(\frac{1}{n} + \beta\right) L_t \cdot p\left(1 - e^{-\frac{b}{m+1}t^{m+1}}\right) \ ,$$

from which, it will follow that:

$$\log t = \frac{1}{m+1} \log\left[-\frac{m+1}{b} \ln\left(1 - \frac{\beta}{\left(\frac{1}{n} + \beta\right) p}\right)\right] \ ,$$

which yields 85.9 seconds.

From the charge $E' = L_t/n$ that is present the moment after contact with the large ball, the resulting growth in the residue will then get lost in the *three seconds* up to the discharge of the jar, which is determined from:

$$\left[\left(\frac{1}{n} + \beta\right) p\left(1 - e^{-\frac{b}{m+1}88.9^{m+1}}\right) - \beta\right] L_t = 0.00010 L_t \ ,$$

or since $L_t = nE'$:

$$0.000\,103 \cdot E' \ .$$

That finally gives the desired correction:

$$E' - E = (0.000\,538 + 0.000\,103)E' = 0.000\,641 E' \ ,$$

[202][Note by KW:] That equation is formed according to the residue equation (I), in which one must now set $Q = (1/n + \beta)$ in place of Q_t.

and one will then get the corrected values E for the values of E' that were given in the previous Section, which will give the amount of electricity that is actually discharged to the multiplier, as follows:

No.	E
1.	36 060 000
2.	41 940 000
3.	49 700 000
4.	44 350 000
5.	49 660 000

7.13 Calculating the Time Duration that a Current with the Normal Strength that was Described in Section 4 Must Have in Order to Produce the Deflections of the Tangent Galvanometer that were Observed in Section 7

The *deflections of the tangent galvanometer* that were cited in Section 7.7 were observed in *scale divisions.* One will obtain those deflections in *arc values for a radius of* 1 by dividing them by the radius (or twice the distance from the mirror to the scale), expressed in scale divisions, which equals 12875.

No.	Deflection in scale divisions	Deflection in arc values for radius = 1 φ
1.	73.5	0.005 708 7
2.	80.0	0.006 213 6
3.	96.5	0.007 495 2
4.	91.1	0.007 075 7
5.	97.8	0.007 596 2

In the Second treatise on *Electrodynamic Measurements*, p. 363,[203,204] it was proved that a current of strength 1 that goes through a winding of a multiplier whose radius is a will exert a *force* F on a particle of the *North magnetic* fluid $+\mu$ or a particle of the *South magnetic* fluid $-\mu$ that is found at a distance of b from the plane of the multiplier winding, and whose projection onto that plane lies at a distance of x from the center, perpendicular to the plane of the winding of the multiplier:

$$F = \pm \frac{2\pi a^2 \mu}{(a^2+b^2+x^2)^{3/2}} \cdot \left\{ 1 + \frac{3}{4}\left(3a^2 - 2b^2 - 2x^2\right) \frac{x^2}{(a^2+b^2+x^2)^2} + ... \right\} ,$$

from which, it will follow that the same current will exert a *rotational moment* D on a *needle* that contains the particles $+\mu$ and $-\mu$ at a very small distance of 2ε apart that is parallel to the plane of the multiplier:

[203] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 454.

[204] [Note by AKTA:] [Web52c, p. 454 of Weber's *Werke*] with English translation in [Web21b].

$$D = \frac{4\pi a^2 \mu\varepsilon}{(a^2+b^2+x^2)^{3/2}} \cdot \left\{1 + \frac{3}{4}\left(3a^2 - 2b^2 - 2x^2\right)\frac{x^2}{(a^2+b^2+x^2)^2} + ...\right\} ,$$

where $2\mu\varepsilon$ denotes the *magnetic moment* of the needle or the *needle magnetism*.

Now, three different applications can be made of this equation: *First of all*, to the *normal conditions* that were assumed for the magnetic effects in the Section 7.1, *next*, to the *tangent galvanometer with a single multiplier loop*, and *finally*, to the *tangent galvanometer with multiple multiplier loops* that was used in the present experiments. The first two applications show only that, as was pointed out before in *loc. cit.* in relation to the current strengths, this equation is actually the basis for the current intensity unit that is derived from magnetic effects. The last application leads to the calculation of the *desired time interval* τ.

If one applies this equation *first of all* to the *normal conditions* that were assumed for the magnetic effects of a current in Section 7.1, then one will have $\pi a^2 = 1$, $b = 0$, $2\mu\varepsilon = 1$, $x = R$, and that a/R is a vanishingly-small fraction. *The equation above* will then yield the rotational moment D (without the sign, which depends upon the direction of the current):

$$D = \frac{1}{R^3} \qquad \text{or} \qquad R^3 D = 1 ,$$

which then agrees with the magnetic current effect that was established for a current intensity of 1 in Section 7.1. It follows from this that the equation above is the basis for the unit of current intensity that was derived from magnetic effects in Section 7.1.

Secondly, if one applies that equation to a *tangent galvanometer with a single multiplier loop* of radius R, where a small magnetic needle is in the center of loop, parallel to the plane of the loop, pointing to the magnetic meridian, then $a = R$, $b = 0$, $x = 0$. *The equation above* then yields the rotational moment that the current exerts on the needle when it is found *along the magnetic meridian*:

$$D = \frac{4\pi\mu\varepsilon}{R} .$$

For a *deflection* of the needle from the magnetic meridian that equals φ, that will go to:

$$D\cos\varphi = \frac{4\pi\mu\varepsilon}{R} \cdot \cos\varphi .$$

If T denotes the horizontal component of the Earth magnetism, then $-2\pi\varepsilon T \sin\varphi$ will be the rotational moment that *the Earth* exerts upon the needle. The sum of these two moments is equal to 0 when the needle persists at rest for the *deflection* φ; as a result:

$$\frac{2\pi}{R} = T\tan\varphi \qquad \text{or} \qquad \varphi = \arctan\frac{2\pi}{RT} .$$

However, this deflection is the same as the [deflection which a] *normal current* that was described in Section 7.4 should produce in a tangent galvanometer with a single loop.

Third, and finally, that same equation shall be applied to the *tangent galvanometer with multiple multiplier loops* that is used in the present experiment, and the rotational moment shall be determined that the aforementioned *normal current* that was described in Section 7.4 exerts upon the needle when it goes through all windings of the multiplier.

We next consider *one* winding of the multiplier that has radius a and whose plane is separated from the meridian plane of the needle by b. The rotational moment D' that this winding exerts upon the needle will be determined from the equation above:

$$D' = \frac{4\pi a^2 \mu\varepsilon}{(a^2+b^2+x^2)^{3/2}} \cdot \left\{1 + \frac{3}{4}\left(3a^2 - 2b^2 - 2x^2\right)\frac{x^2}{(a^2+b^2+x^2)^2} + ...\right\} ,$$

in which one can set $x = 0$, as in the previous application, if the length of the needle is a very small fraction of the diameter of the multiplier winding. Now, the length of the needle in our *tangent galvanometer* was, in fact, merely 60 millimeters, while the mean diameter of the multiplier windings amounted to 267 millimeters, which was, however, still not enough to be able to neglect x entirely. However, it sufficed to set x equal to an approximate value that suggested itself when one understood the $+\mu$ and $-\mu$ in the needle magnetism, [that is, in the magnetic moment of the needle,] which is equal to $2\mu\varepsilon$, to mean the combination of the north-magnetic and south-magnetic fluids that are distributed on the surface of the needle according to the *ideal distribution*, and accordingly determined 2ε, which then meant the distance from the center of mass of the north-magnetic fluid to that of the south-magnetic fluid, such that one would set $x = \varepsilon$. From the length and nature of the needle that was used, 2ε could not be very far from 40 millimeters, and one could then set:

$$x = \varepsilon = 20 \text{ millimeters}$$

with sufficient accuracy.

If one then lets a' and a'' denote the inner and outer radii of the multiplier ring and lets $2b'$ denote its width, then the cross-section of the entire ring will be equal to:

$$2(a'' - a')b' .$$

If one further denotes the part of the cross-section that the multiplier winding in question occupies (whose radius was equal to a, and whose plane was separated from the common center of the needle and the multiplier ring by b) by $da \cdot db$ then the product of those elements of the cross-section in the multiplier winding under consideration with the rotational moment that is exerted upon the galvanometer will be equal to:

$$\frac{4\pi a^2 \mu\varepsilon}{(a^2+b^2+\varepsilon^2)^{3/2}} \cdot dadb \left\{1 + \frac{3}{4}\left(3a^2 - 2b^2 - 2\varepsilon^2\right)\frac{\varepsilon^2}{(a^2+b^2+\varepsilon^2)^2} + ...\right\} ,$$

or since the terms that include the fourth and higher powers of the fraction ε/a can be neglected, due to the smallness of that fraction:

$$\frac{4\pi a^2 \mu\varepsilon}{(a^2+b^2)^{3/2}} \cdot dadb \left\{1 + \frac{3}{4}\frac{a^2 - 4b^2}{(a^2+b^2)^2} \cdot \varepsilon^2\right\} .$$

It then follows from this that the sum of the products of the cross-section of each winding with the rotational moment that is exerted upon it will be:

$$4\pi\mu\varepsilon \int_{a'}^{a''} a^2 da \int_{-b'}^{+b'} \frac{db}{(a^2+b^2)^{3/2}} \cdot \left\{1 + \frac{3}{4}\frac{a^2 - 4b^2}{(a^2+b^2)^2} \cdot \varepsilon^2\right\}$$

$$= 8\pi\mu\varepsilon b' \left\{\log \frac{a'' + \sqrt{a''^2 + b'^2}}{a' + \sqrt{a'^2 + b'^2}} + \frac{1}{4}\left(\frac{a''^3}{(a''^2+b'^2)^{3/2}} - \frac{a'^3}{(a'^2+b'^2)^{3/2}}\right) \cdot \frac{\varepsilon^2}{b'^2}\right\} .$$

Upon dividing this value by the cross-section of the entire ring, which is equal to $2(a''-a')b'$, one will get the rotational moment that is exerted upon the needle in the center of *one* multiplier winding, from which, after multiplying by the number of windings n, one will get the total *rotational moment* that the multiplier exerts upon the needle due to the *normal current* that flows through it, namely:

$$D = \frac{4\pi n\mu\varepsilon}{a''-a'}\left\{\log\frac{a''+\sqrt{a''^2+b'^2}}{a'+\sqrt{a'^2+b'^2}}+\frac{1}{4}\left(\frac{a''^3}{(a''^2+b'^2)^{3/2}}-\frac{a'^3}{(a'^2+b'^2)^{3/2}}\right)\cdot\frac{\varepsilon^2}{b'^2}\right\} .$$

That *rotational moment* D, when divided by the *moment of inertia of the needle* K, which is then equal to:

$$\frac{D}{K} ,$$

will give the angular acceleration of the needle in terms of the given *normal current*, and when that acceleration is multiplied by the *duration of the current* τ, which is very brief in comparison to the period oscillation, which equals t, will give the *angular velocity* that is given to the needle by the *normal current* during its brief duration, which equals:

$$\frac{D\tau}{K} .$$

Finally, the *deflection* — i.e., the *initial elongation width* φ — of the needle that is set into oscillation can be calculated from that angular velocity that is suddenly given to the needle at rest by known rules (see the Second treatise on *Electrodynamic Measurements*, p. 348),[205,206] namely, when the decrease in the arc of oscillation of the needle is given by the ratio of two successive oscillation arcs $e^\lambda : 1$:

$$\varphi = \frac{D\tau}{K}\cdot\frac{t}{\pi}\cdot\frac{e^{-\frac{\lambda}{\pi}\arctan\frac{\pi}{\lambda}}}{\sqrt{1+\frac{\lambda^2}{\pi^2}}} .$$

In order to not have to determine the value of the moment of inertia of the needle K and its magnetic moment $2\mu\varepsilon$ from special observations, one can eliminate both of them by consulting the known equation for the period of oscillation, but in which one must account for the force of torsion[207] of the wire. If $1 : \vartheta$ denotes the ratio of the geomagnetic directive force that acts upon the needle, which equals $2\mu\varepsilon T$, to the one that is exerted by the wire, then the equations for the period of oscillation t will be:

$$\frac{2\mu\varepsilon\cdot T}{K} = \frac{\pi^2}{t^2}\cdot\frac{1+\frac{\lambda^2}{\pi^2}}{1+\vartheta} ,$$

and a result, if one sets:

$$d = \frac{D}{2\mu\varepsilon} = \frac{2\pi n}{a''-a'}\left\{\log\frac{a''+\sqrt{a''^2+b'^2}}{a'+\sqrt{a'^2+b'^2}}\right.$$
$$\left.+\frac{1}{4}\left(\frac{a''^3}{(a''^2+b'^2)^{3/2}}-\frac{a'^3}{(a'^2+b'^2)^{3/2}}\right)\cdot\frac{\varepsilon^2}{b'^2}\right\}$$

[205][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 440.

[206][Note by AKTA:] [Web52c, p. 440 of Weber's *Werke*] with English translation in [Web21b].

[207][Note by AKTA:] In German: *Torsionskraft*.

and multiplies the foregoing equation by $\frac{D}{2\mu\varepsilon\cdot T} = \frac{d}{T}$ then:

$$\frac{D}{K} = \frac{d}{T} \cdot \frac{\pi^2}{t^2} \cdot \frac{1 + \frac{\lambda^2}{\pi^2}}{1 + \vartheta} .$$

If one substitutes that value in the equation for φ then one will get:

$$\varphi = \pi \frac{d}{T} \cdot \frac{\tau}{t} \cdot \frac{\sqrt{1 + \frac{\lambda^2}{\pi^2}}}{1 + \vartheta} \cdot e^{-\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} ,$$

and that will give the desired *duration of the normal current*:

$$\tau = t \cdot \frac{\varphi}{\pi} \cdot \frac{T}{d} \cdot \frac{1 + \vartheta}{\sqrt{1 + \frac{\lambda^2}{\pi^2}}} \cdot e^{\frac{\lambda}{\pi} \arctan \frac{\pi}{\lambda}} .$$

However, it was determined by measurement that for the multiplier of the tangent galvanometer that was used here:

$$2\pi a' = 709.0 \text{ millimeters},$$
$$2\pi a'' = 965.35 \text{ millimeters},$$
$$2b' = 72.04 \text{ millimeters},$$
$$n = 5635,$$

from which, with the aforementioned value of $\varepsilon = 20$ millimeters, one will get the value of d:

$$d = 262.1 .$$

If the value of ε also has an uncertainty of 1 millimeter, then that will imply the uncertainty in d, which only amounts to 0.4, out of 262, however (i.e., only 1/657), which is not worth considering.

In addition, the period of oscillation of the needle t, the horizontal component of the Earth's magnetism at the location of the tangent galvanometer T, the logarithmic decrement in the decrease of the arc of oscillation λ, and the ratio ϑ of the directive force of the wire to the one that is due to geomagnetism T were found in the usual way:

$$t = 9.244 \text{ seconds} ,$$

$$T = 1.798\,3 \text{ seconds} ,$$

$$\lambda = 0.070 \text{ seconds} ,$$

$$\vartheta = \frac{1}{691} .$$

If one substitutes these values in the equation for τ then one will get:

$$\tau = 0.020\,921 \cdot \varphi .$$

The values of φ that were obtained from the five experiments that were described in Section 7.7 were collected at the beginning of this Section. If one substitutes them in the equation for τ then one will get the following five results for the cited experiments:

No.	τ
1.	0.000 119 4
2.	0.000 130 0
3.	0.000 156 8
4.	0.000 148 0
5.	0.000 158 9

7.14 Calculating the Quantity $\frac{1}{2\tau} \cdot E$

Finally, it still remains for us to calculate the value of $\frac{1}{2\tau} \cdot E$ from the values of E and τ that were found. Namely, if we summarize the corresponding values of E and τ from the previous two Sections in the following Table:

No.	E	τ
1.	36 060 000	0.000 119 4
2.	41 940 000	0.000 130 0
3.	49 700 000	0.000 156 8
4.	44 350 000	0.000 148 0
5.	49 660 000	0.000 158 9

then that will yield the following five values of $\frac{1}{2\tau} \cdot E$ that result from the five measurements that were described in Section 7.7:

No.	$(1/2\tau) \cdot E$
1.	$151\,000 \cdot 10^6$
2.	$161\,300 \cdot 10^6$
3.	$158\,500 \cdot 10^6$
4.	$149\,800 \cdot 10^6$
5.	$156\,250 \cdot 10^6$

All of the measurement collectively then give the mean value:

$$\frac{1}{2\tau} \cdot E = 155\,370 \cdot 10^6 \ .$$

However, from Section 7.5:

$$\frac{1}{2\tau} \cdot E : 1$$

denotes the ratio of the *amount of positive electricity* that passes through the cross-section of the conductor in one second for a constant current that is composed of equally-large masses of positive and negative electricity that flow in opposite directions and whose intensity is equal to the *magnetic* measure of current intensity, to the amount that would exert a force at a distance of one millimeter that would impart a velocity of one millimeter per second to a mass of one milligram during one second, if an equal amount of electricity were concentrated into a point. That ratio was determined in the *problem* in Section 7.4 that remains to be solved, which shall now be done.

7.15 Reducing the Magnetic, Electrodynamic, and Electrolytic Units of the Current Intensity to Mechanical Units

However, the solution of the problem that was posed in Section 7.4 shall now be used to *reduce the magnetic, electrodynamic, and electrolytic units of the current intensity to mechanical units*.

From Section 7.2, for a constant current that is composed of equally-large masses of positive and negative electricity that flow in opposite directions whose intensity is equal to the *mechanical* unit of current intensity, the *amount of positive electricity* that passes through the cross-section of the conductor in one second shall be equal to one; i.e., it is equal to the amount of electricity concentrated into a point that would exert a force at another equal amount of electricity concentrated into another point at a distance of one millimeter that would impart a velocity of one millimeter per second to a mass of one milligram in one second.

However, from the foregoing Section, that *unit* amount of positive electricity has a ratio with the *amount of positive electricity* that passes through the cross-section in one second for a current whose intensity is given by the *magnetic* current unit of:

$$155\,370 \cdot 10^6 : 1\ .$$

Now, since the current intensities are proportional to the amounts of electricity that pass through the cross-section in equal time intervals, that will immediately imply the *reduction of the magnetic unit for current intensity to the mechanical unit*, since the *magnetic* current unit of the amount of electricity that passes through the cross-section in the same time interval will then be:

$$155\,370 \cdot 10^6$$

times greater than the amount in the *mechanical* unit of current. As a result, from the cited proportion, the *magnetic unit of the current intensity will itself also be* $155\,370 \cdot 10^6$ *times larger than the mechanical unit*.

Furthermore, since, from Section 7.1, page 223,[208,209] the *magnetic unit* of current intensity has a ratio of $\sqrt{2} : 1$ with the *electrodynamic* one, the *electrodynamic unit of current intensity will be* $109\,860 \cdot 10^6 (= 155\,370 \cdot 10^6 \cdot \sqrt{\frac{1}{2}})$ *times greater than the mechanical unit*.

Finally, since, from Section 7.1, page 224,[210,211] the *mechanical unit* of current intensity has a ratio of $1 : 106\frac{2}{3}$ with the *electrolytic* one, *the electrolytic unit of current intensity will be* $16\,573 \cdot 10^9 (= 106\frac{2}{3} \cdot 155\,370 \cdot 10^6)$ *greater than the mechanical unit*.

The problem in this treatise, as it was expressed in Section 7.2, of reducing those three units of current intensity to the mechanical unit, is then solved, and all that remains is to discuss the *applications* that can be made of the result that was found.

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[208] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 613.

[209] [Note by AKTA:] See page 144 of Section 7.1, or [KW57, p. 613 of Weber's *Werke*].

[210] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 614.

[211] [Note by AKTA:] See page 144 of Section 7.1, or [KW57, p. 614 of Weber's *Werke*]. See also footnote 146 on page 134.

II - Applications

7.16 Determining the Amount of Electricity that is Required to Liberate 1 Milligram of Hydrogen from 9 Milligrams of Water

The first application that we shall make of the results that were found is to the precise determination of the amount of electricity that is required to liberate 1 milligram of hydrogen from 9 milligrams of water, over which the determination that Buff found with the help of his tangent galvanometer and a long conducting wire and published in *Annalen der Chemie und Physik*, Vol. 86, p. 33 was referred to already in the footnote to Section 7.3, page 226.[212,213,214]

According to Buff, that amount of electricity was sufficient to charge a battery of 45480 Leyden jars, each of which were 480 millimeters high and 160 millimeters in diameter, up to a spark gap of 100 millimeters. That determination that Buff made lacked only more precise data on the amount of electricity that a Leyden jar contained when it had been charged as described.

Now, the results that were found in the present treatise imply that the amount of electricity that is required to liberate 1/9 milligram of hydrogen from 1 milligram of water is equal to the *amount of positive electricity* that passes through the cross-section of the conductor in one second for a constant current whose intensity has the *electrolytic* unit. However, the latter is, in proportion to the current intensities that correspond to the *electrolytic* and *magnetic* current units (see Section 7.1, page 224),[215,216] $106\frac{2}{3}$ times greater than the *amount of positive electricity* that passes through the cross-section in one second for a constant current whose intensity has the *magnetic* current unit, and from Section 7.14, that is:

$$155\,370 \cdot 10^6$$

times greater than the *unit* amount of electricity concentrated into a point that would exert a force at a distance of one millimeter, that would impart a velocity of one millimeter per second on a mass of one milligram during one second if an equal amount were concentrated into a point.

It follows form this that:

> $9 \cdot 106\frac{2}{3} \cdot 155\,370 \cdot 10^6 = 149\,157 \cdot 10^9$ *units, as it was just determined, are required to liberate 1 milligram of hydrogen from 9 milligrams of water.*

If such an amount of positive electricity were concentrated into a cloud and an equal amount of negative electricity were concentrated on the surface of the Earth at the location that is directly below it, then that would yield an attraction of the cloud to the Earth that would be equal to a weight of 45 000 hundredweights (= 2 268 000 kilograms) if they were at a distance of 1000 meters from each other.

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[212] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 616.

[213] [Note by AKTA:] See page 146 of Section 7.3, or [KW57, p. 616 of Weber's *Werke*].

[214] [Note by AKTA:] See footnote 178 on page 146.

[215] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 616.

[216] [Note by AKTA:] See page 144 of Section 7.1, or [KW57, p. 614 of Weber's *Werke*].

If one divides that number of units by the number of Leyden jars in the battery that Buff described (viz., 45 480), then one will get the precise data for the amount of electricity that is contained in the charge in one Leyden jar as described by Buff, namely:

$$3\,280 \cdot 10^6 \text{ units} .$$

However, from Buff's description, the charged surface of such a jar has an area of:

$$480 \cdot 160 \cdot \pi = 241\,300 \text{ square millimeters}$$

and as a result, each square millimeter will be charged with:

$$13\,600 \text{ units} ,$$

from which, one can determine the compression or condensation of electricity in the jar that is required for a spark gap of 100 millimeters.

7.17 Determining the Constant c

From the fundamental law of electrical action that was established in the first treatise on *Electrodynamic Measurements*,[217] which encompassed *electrostatics*, *electrodynamics*, and *induction*, the force that an amount of electricity e exerts upon an amount of electricity e' at a distance of r with a relative velocity of dr/dt and an acceleration of d^2r/dt^2 is expressed by:

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right] .$$

That force splits into two parts, the first of which, which is equal to ee'/r^2, can be called the *electrostatic* force, and the second of which, which is equal to $-(ee'/c^2r^2)(dr^2/dt^2 - 2rd^2r/dt^2)$, can be called the *electrodynamic* force. The ratio of those two forces is determined from the *constant* c. c means the value of relative velocity (assumed uniform) at which the *electrostatic* force would cancel the *electrodynamic* force. That *constant* c will now be determined in the following way:

In Section 7.14, the ratio $\frac{1}{2\tau}E : 1$ (that is, the *ratio of the magnetic unit of current intensity to the mechanical one*) was found to be:

$$155\,370 \cdot 10^6 : 1 .$$

In Section 26, page 261 of the Second treatise on *Electrodynamic Measurements*,[218,219] the ratio of the *magnetic* unit of current intensity to the *electrodynamic* one was given as:

$$\sqrt{2} : 1 ,$$

[217] [Note by AKTA:] [Web46] with partial French translation in [Web87] and a complete English translation in [Web07].

[218] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 360.

[219] [Note by AKTA:] [Web52c, p. 360 of Weber's *Werke*] with English translation in [Web21b].

and in Section 27, page 269,[220,221] the ratio of the *electrodynamic* unit for the current intensity to the *mechanical* one was given as:

$$c : 4 \; ,$$

from which, the *ratio of the magnetic unit of current intensity to the mechanical one* would follow:

$$c\sqrt{2} : 4 \; .$$

Setting this ratio equal to the one that was found in Section 7.14 of the treatise will then give:

$$c = 4 \cdot 155\,370 \cdot 10^6 \cdot \sqrt{\frac{1}{2}} = 439\,450 \cdot 10^6 \; .$$

From this determination of the *constant* c, one then sees that two electrical masses must move with a very large velocity with respect to each other if the *electrodynamic* force were to cancel the *electrostatic* one, namely, with a velocity of 439 million meters or 59 320 miles per second, which exceeds the speed of light significantly.

However, the speed of light is not the speed of motion of a body, but of a wave, while all of the speeds of actual motions of bodies that are known to us, even those of the celestial bodies, constitute only very small fractions of it. Now, if one observes that the ratio of the *electrodynamic* force to the *electrostatic* one corresponds to the square of that fraction, then that will imply that the electrodynamic force can always be considered to be vanishingly small in comparison to the electrostatic one. Indeed, we still have no knowledge of the speeds at which electric fluids move in metallic conductors.[222] However, in various situations, one can assume that the amount of neutral electricity that is contained in those conductors is exceptionally large. Nonetheless, the greater the latter gets, the less the speed of the actual motion will be, which is then implied by the *unit of current intensity* that is present. The speed of those motions probably defines only a very small fraction of the speed c then.

Furthermore, the large value of the *constant* c that was found implies the interesting consequence that such a dynamical part could also be attached to the *gravitational force on ponderable bodies* (which would exhibit a great analogy between the interactions of *ponderable* and *imponderable* bodies) without that dynamical part of the force having the slightest observable influence of the motions of the celestial bodies.

The fact that the effect of the *electrodynamic* force does not always vanish for electricity, but can emerge very apparently for galvanic currents, has its basis in merely the *complete cancellation of all electrostatic forces* that takes place during the *neutralization* of positive and negative electricity, against which those [electrostatic] forces would disappear. Wherever no such neutralization takes place, but free electricity is present, only the *electrostatic* force would come under consideration in the effect of free electricity. That explains why not all experiments that were intended to establish the fundamental laws of electrical action could be performed with merely *two* masses of free electricity, but some experiments had to be

[220] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 367.

[221] [Note by AKTA:] [Web52c, p. 367 of Weber's *Werke*] with English translation in [Web21b].

[222] [Note by AKTA:] Weber is referring here to the drift velocities of the electrified particles relative to the matter of the conductor.

performed with *two pairs* of electrical masses (viz., current elements) that were *neutralized electrostatically*.

For *ponderable* masses, for which the law of indifferent attraction is true, one can speak of *no neutralization of the masses*.

Remark. — At the beginning of this Section, the following equation for the determination of the *constant* c was presented:

$$c = \frac{E}{\tau} \cdot \sqrt{2} \, ,$$

in which $\frac{1}{2\tau} \cdot E : 1$ denoted the ratio that was found in Section 7.14 of the amount of *positive electricity* that passes through the cross-section of a conductor in one second for a constant current whose intensity is measured *magnetically* to the amount of electricity concentrated into a point that would exert at an equal amount of electricity concentrated into a point a force at a distance of one millimeter that would impart a velocity of one millimeter per second on a mass of one milligram in one second. — The Second treatise on *Electrodynamic Measurements*[223] was referred to in order to prove that equation. However, the validity of that equation can also be inferred directly from the *fundamental law of electrical action* and the *definition of the magnetic current measure*. To that end, one merely needs to consider the interaction of two equal current elements α and α of a current flowing along a straight line separated by a distance of r, about which, as it was already mentioned in the footnote on p. 224[224,225] that they repel each other with a force equal to:

$$\frac{\alpha^2}{r^2} i^2 \, ,$$

if i is expressed in terms of the *magnetic* current unit. As is known, that follows from *Ampère's fundamental law* and the relationship between *electromagnetism* and *electrodynamics* that it gives.

Assuming that, one proposes that the rectilinear conductor of our current should contain one unit of positive and negative electricity in each piece of it that is one millimeter long. (From Section 7.14), $\frac{1}{2\tau} \cdot E$ then denotes the number of millimeters that both electrical currents must traverse in the opposite directions in order to make:

$$i = 1 \, .$$

Those simple relationships give not only the amounts of electricity in the two current elements α and α, whose distance and the remaining relationship depends upon their force of repulsion (according to the fundamental law of electrical action), but also the *magnitude of that force of repulsion* itself; namely, since $i = 1$:

$$\frac{\alpha^2}{r^2} \, .$$

That merely depends upon the fact that this force of repulsion, which is known already, can be derived from the fundamental law of electrical action, so since c is contained in that

[223] [Note by AKTA:] [Web52c] with English translation in [Web21b].
[224] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 614.
[225] [Note by AKTA:] See page 144 on Section 7.1, or [KW57, p. 614 of Weber's *Werke*].

fundamental law, it will contain an expression for that force that depends upon c, and one needs only to set [the repulsive force] equal to the value that is known already in order to find c. However, the force of repulsion between the two current elements α and α can be derived very easily from the fundamental law of electrical action with the simple relationships that were described. We then decompose the total force that is given by the fundamental law into two parts, namely, into the *electrostatic* and *electrodynamic* forces. That will shed light upon the fact that the sum of the electrostatic forces between the two current elements is zero (due to the electrostatic neutralization that is present in both current elements). It will likewise illuminate the fact that no acceleration exists between the electrical masses in both current elements, so $d^2r/dt^2 = 0$. With that, the general expression for the electrical action:

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right]$$

will reduce to:

$$-\frac{1}{c^2}\frac{ee'}{r^2}\frac{dr^2}{dt^2}$$

in our case. Now, when that expression is applied:

1) to the two positive masses in the two current elements $e = +\alpha$ and $e' = +\alpha$, it will give a force of repulsion that is equal to zero, since the relative velocity of the masses $dr/dt = 0$ (because both of them move in the same direction with equal velocities);

2) the same thing will be true for two negative masses $e = -\alpha$ and $e' = -\alpha$;

3) however, when the same expression is applied to a positive mass $e = +\alpha$ and a negative one $e' = -\alpha$, it will give a force of repulsion that is equal to $+\frac{1}{c^2}\frac{ee'}{r^2} \cdot \frac{1}{\tau^2} \cdot E^2$, since the relative velocity of those masses is $dr/dt = E/\tau$ (because they both move in opposite directions with the velocity $\frac{1}{2\tau} \cdot E$);

4) the same thing will be true for a negative mass $e = -\alpha$ and a positive one $e' = +\alpha$.

It then follows from this that the sum of all forces of repulsion between the electrical masses that are contained in the two current elements is equal to:

$$2 \cdot \frac{1}{c^2}\frac{\alpha^2}{r^2} \cdot \frac{1}{\tau^2} \cdot E^2 \ ,$$

and if that sum is set equal to its value α^2/r^2 that is known already, then that will imply the following equation for the determination of c:

$$\frac{\alpha^2}{r^2} = 2 \cdot \frac{1}{c^2} \cdot \frac{\alpha^2}{r^2} \cdot \frac{1}{\tau^2} \cdot E^2 \ ,$$

or

$$c = \frac{E}{\tau} \cdot \sqrt{2} \ ,$$

which was to be proved.

7.18 The Electrical Laws, with the Numerical Determination of Their Constants

The electrical laws that were developed in the first and Second treatise on *Electrodynamic Measurements* are the following:

1) *The fundamental law of electrical action.* — According to it, the force the electrical mass e exerts upon the electrical mass e' at a distance of r with a relative velocity of dr/dt and an acceleration of d^2r/dt^2 is expressed by:

$$\frac{ee'}{r^2}\left[1-\frac{1}{c^2}\left(\frac{dr^2}{dt^2}-2r\frac{d^2r}{dt^2}\right)\right] .$$

2) *The fundamental law of electrodynamics.* — According it, the force that an unchanging and motionless current element of length α and current intensity i will exert upon an equal current element of length α' and current intensity i' at a distance of r when α makes an angle of ϑ with r, α' makes an angle of ϑ' with the extension of r, and α makes an angle of ε with α' is expressed by:

$$\frac{\alpha\alpha'}{r^2}ii'(3\cos\vartheta\cos\vartheta'-2\cos\varepsilon) .$$

3) *The law of voltaic induction for an unchanging current element that moves with respect to a conductor.* — According to it, the electromotive force that a current element of length α and current intensity i exerts upon an element of a conductor of length α' that moves with a velocity u at a distance of r when α makes an angle of ϑ with r, α' makes an angle of φ with r, u makes an angle of ϑ' with the extension of r, and α makes an angle of ε with u is expressed by:

$$\frac{2\sqrt{2}}{c}\cdot\frac{\alpha\alpha'}{r^2}\cdot ui\cos\varphi(3\cos\vartheta\cos\vartheta'-2\cos\varepsilon) .$$

4) *The law of voltaic induction for a variable current element that does not move with respect to a conductor.* — According to it, the electromotive force that a current element of length α whose current intensity grows uniformly by i in a time interval t exerts upon a conductor element of length α at a distance of r when α makes an angle of ϑ with r and α' makes an angle of ϑ' with the extension of r is expressed by:

$$-\frac{2\sqrt{2}}{c}\cdot\frac{\alpha\alpha'}{r}\cdot\frac{i}{t}\cos\vartheta\cos\vartheta' .$$

5) *The law of voltaic induction for a location where there is sliding.* — According to it, the electromotive force that a current of intensity i and sliding velocity v that goes through the sliding location exerts upon a conducting element of length α' at a distance of r when v makes an angle of ϑ with r, and α' makes an angle of ϑ' with the extension of r is expressed by:

$$-\frac{2\sqrt{2}}{c}\cdot\frac{\alpha'}{r}vi\cos\vartheta\cos\vartheta' .$$

A positive value in the expressions (1) and (2) means a force of repulsion, while a negative value means a force of attraction. The numerical values of our measurements give the magnitudes of the forces as ratios with the force that would impart a velocity of one millimeter per second on a mass of one milligram during one second. In the expression (2), as well as in all of the following ones, the current intensities i and i' are assumed to be measured in *magnetic* units, which can always be easily done with the *tangent galvanometer*. If one lets ε' denotes the *electrical capacity* of the conductor α' — i.e., the ratio of the amount of positive electricity that it contains (which is equal to that of the negative) to its length, — then for $\varepsilon' = 1$ the expressions (3), (4), (5) will give the difference between the two forces that act in the direction of α' on the amounts of positive and negative electricity that are contained in α', and in fact, they will give that *force difference* as a ratio with the force that would impart a velocity of one millimeter per second on a mass of one milligram during one second. — If ε' is not equal to 1, then the expressions (3), (4), (5) must be multiplied by ε' in order to get the given *force difference*.

A complete determination of all forces by means of the given laws requires that the *constant* c must be set equal to the numerical value that was found in the previous Section in all of the expressions above. One will then get:

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right]$$
$$= \frac{ee'}{r^2}\left[1 - \frac{1}{193\,120 \cdot 10^{18}}\left(\frac{dr^2}{dt^2} - 2r\frac{d^2r}{dt^2}\right)\right] , \qquad (1.)$$

$$\frac{\alpha\alpha'}{r^2} ii'(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon) , \qquad (2.)$$

$$\frac{2\sqrt{2}}{c} \cdot \frac{\alpha\alpha'}{r^2} \cdot ui\cos\varphi(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon)$$
$$= \frac{1}{155\,370 \cdot 10^6} \cdot \frac{\alpha\alpha'}{r^2} \cdot ui\cos\varphi(3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon) , \qquad (3.)$$

$$-\frac{2\sqrt{2}}{c} \cdot \frac{\alpha\alpha'}{r} \cdot \frac{i}{t}\cos\vartheta\cos\vartheta' = -\frac{1}{155\,370 \cdot 10^6} \cdot \frac{\alpha\alpha'}{r} \cdot \frac{i}{t}\cos\vartheta\cos\vartheta' , \qquad (4.)$$

$$-\frac{2\sqrt{2}}{c} \cdot \frac{\alpha'}{r} \cdot vi\cos\vartheta\cos\vartheta' = -\frac{1}{155\,370 \cdot 10^6} \cdot \frac{\alpha'}{r} \cdot vi\cos\vartheta\cos\vartheta' . \qquad (5.)$$

When all constants have been determined numerically, the law of electricity, in the last form, will satisfy all requirements *in practice*. However, for *theoretical* investigations, it can be necessary in many cases to substitute the values of i and i' that are derived from the *causes* of the current intensities (see Section 7.2) in the expressions above, instead of the current intensities i and i' that are measured in *magnetic* units. Namely, if $+\alpha\varepsilon$ and $-\alpha\varepsilon$ denote the amounts of positive and negative electricity, respectively, that are contained in the conductor α, and $+u$ and $-u$, respectively, are the velocities with which they move in the conductor, and if $+\alpha'\varepsilon'$, $-\alpha'\varepsilon'$, $+u'$, and $-u'$, respectively, denote the same things for the conductor α', then εu and $\varepsilon' u'$, respectively, will be the values of the current intensities, when determined in *mechanical* units, and from the relationships that were found in Section 7.15, those values must be divided by $155\,370 \cdot 10^6$ in order to obtain the values of the same current intensities, when expressed in *magnetic* units. As a result, one will have:

$$i = \frac{\varepsilon u}{155\,370 \cdot 10^6} , \qquad i' = \frac{\varepsilon' u'}{155\,370 \cdot 10^6} ,$$

in the expressions above, and those values can be substituted for i and i' in the expressions above, if that should be necessary.

7.19 Application to Electrolysis — Measurement of a Chemical Affinity Force

All electrical forces that are determined by means of the laws that were cited in the foregoing Section are forces that act directly upon only electrical masses. *However, all forces that act directly upon only electrical masses will also act indirectly upon the ponderable carriers of those electrical masses.* In that way, the application of electrical laws to the investigation of ponderable bodies opens up a broad field, since electricity will, in that way, become an instrument for us, with whose help we can make known forces act upon ponderable bodies by means of relationships for which no other known forces act.

When electrical masses are coupled with their ponderable carriers, the law above explains why the electrical masses cannot move without their carriers. However, even in metallic conductors, in which the electricity can move, while their ponderable carrier (the metal) remains at rest, so the electrical masses go from one metallic particle to another, one still finds a coupling between the electrical masses and the metallic particles that must be resolved before the electrical mass can go from one metallic particle to another. As long as that coupling exists, all forces that act upon only the electrical masses will, however, carry over directly to the metallic particles that they are coupled with, and only those forces that act upon the electrical masses, once they have been liberated from the metallic particles, will no longer carry over to those metallic particles, but will impart a certain velocity on those electrical masses until they arrive at the next metallic particle, but due to the coupling between those electrical masses and the next metallic particle, it will again be cancelled, which would have the same effect as if the electrical forces that produced that velocity were carried over to that next metallic particle. One calls all of those forces that emerge from the coupling of electrical masses with individual metallic particles *forces of resistance*, by which the metal opposes the motion of electricity in its interior, from which *Ohm's law* follows,[226] that the electricity in the metallic conductor can persist in a uniform motion only when it is driven forward continually by an equally-large force, and that current will momentarily vanish as soon as the driving force ceases. — It will then follow from this that, even in conductors, all forces that act upon the electricity in the conductor directly, will be transferred indirectly to the conductor itself due to the *resistance* of the conductor.

In *electrolysis*, one does not deal with a metallic conductor that remains at rest while the electrical fluid moves in it, but with a body (e.g., water) that is composed of various kinds of ponderable particles, of which, the one kind (viz., hydrogen particles) follows the motion of the *positive* electricity, while the other (viz., oxygen particles) follows the *negative* electricity. That then raises the question: What is the origin of the forces that produce the various motions of the two components of the water? The laws of electrolysis show that these motions must be an indirect effect of the electrical forces, if not also a direct one. Now, if the electrical forces act directly upon only the electrical masses that are bound to the hydrogen and oxygen particles, then the fact that the hydrogen particles follow the motion of the positive electricity and the oxygen particles follow the motion of the negative electricity shows that the one must be bound to positive electricity in water, and the other,

[226][Note by AKTA:] See footnote 128 on page 123.

with negative electricity, so it will remain in the water, regardless of whether it contains a quantity of neutral fluid, in addition to the free electricity. The strength of that coupling of the hydrogen particles with free positive electricity and the oxygen particles with the negative electricity might also go unmentioned, such as whether it is so strong that they cannot be separated at all, so the electricity will only move with its ponderable carrier under electrolysis, or if it behaves as it does in metallic conductors, such that the electricity will take on a motion that is independent of the motion of the ponderable carrier, in addition to the latter motion. However, in the latter case, the law that the decomposition of the different combined bodies that is due to that current will be proportional to the chemical equivalent will not be strictly valid, which has been shown by the most recent investigations of that case.

Now, if the electrical forces, which only seek to separate the electrical fluid directly, are transmitted to the components of the water by whatever bond that couples the fluid to the components, then one can achieve a closer determination of the *chemical separating forces*[227] that produce the separation of the ponderable components from a more precise knowledge of the electrical separating forces, and that is the reason for the special interest that electrolysis enjoys in comparison to the other methods of chemical separation. Namely, electricity can be used as an instrument by which we link each hydrogen and oxygen particle in the water by a thread and we can stretch both threads in opposite directions with known forces until the hydrogen and oxygen particles are torn from each other.

In order to employ that instrument, and in that way to actually determine the forces that are required to separate the chemically bound parts in terms of known measurements, we must give *the electrical law, along with the numerical determination of its constants*. Once that has been done, we would also like to attempt to apply that to the known results.

The forces that put the electrical fluid into current motion will be called *electromotive forces*. That special terminology (which will be used to distinguish *that type of force*, and not merely its effects) is merely based upon the fact that up to now those forces cannot be measured with known units, but can be determined only indirectly by the effects of the currents that they produce (e.g., thermal, chemical, and magnetic effects), by which they can indeed be compared to each other, but absolutely cannot be expressed in terms of known units, and therefore they also cannot be compared with other known forces. That argument breaks down when one determines those forces from the laws that were given in the foregoing Section, by which they will be expressed in terms of known units. One can also express the forces that one cannot calculate directly from the laws above in terms of known units by comparison them with the ones that can. — Finally, since one can determine the resistance in a closed circuit precisely, and for a constant current the electromotive force and resistance must always have the same ratio to each other, according to Ohm's law, one also learns how the electromotive forces are distributed over the various parts of the circuit. Thus, if a voltmeter is introduced into a circuit, then the electrical separating forces that act in the water can be ascertained precisely.

However, with water, one encounters the special circumstance that it defines a very bad conductor in its pure state and is very difficult to decompose. All electrolytic measurements then relate to water that has been mixed with sulfuric acid or other chemicals: One obtains different results in regard to decomposability for different mixtures. It is necessary to initially restrict oneself to a particular mixture then, and here we shall choose a mixture of water and sulfuric acid with a specific gravity of 1.25, following the investigations that Horsford

[227][Note by AKTA:] In German: *Chemischen Scheidungskräfte*, see also footnote 23 on page 47.

published in Poggendorf's *Annalen*, Vol. 70 (1847), p. 238, which is the easiest of all mixtures of water and sulfuric acid to decompose.[228]

For equal lengths and cross-sections, the *resistance* by which that mixture opposes the current that Horsford gave was found to be:

$$696\,700$$

times larger than the resistance of silver, or when one sets the ratio of the resistance of silver to that of copper equal to 1 : 0.7417, following Lenz (Poggendorf's *Annalen*, Vol. 34, p. 418, Vol. 45, p. 105):[229]

$$516\,750$$

times larger than the resistance of the copper that Lenz used. — From the measurements that were communicated in the *Abhandlungen der K. Gesellschaft der Wissenschaften in Göttingen*, Vol. 5 ("Über die Anwendung der magnetischen Induktion auf Messung der Inklination mit dem Magnetometer"),[230] the resistance of a copper wire of length one millimeter and a mass of one milligram (= 1/8.427 square millimeters of cross-section) was found to be equal to:[231,232]

$$2\,310\,000$$

in absolute units of the *magnetic* system; i.e., for a copper wire of length one millimeter and a cross-section of 1 square millimeter, it will be equal to:

$$274\,100.$$

That yields the resistance of the mixture above when it is one millimeter long and one square millimeter in cross-section as being:

$$141\,640 \cdot 10^6$$

in *magnetic* resistance units. However, that mixture contained about nine parts water to one part sulfuric acid by volume, and the pure water would then amount to only 9/10 of the total cross-section. If one assumes that the total current goes merely through the water (because if a part of the current were conducted by the sulfuric acid then that would define an auxiliary current, which would have to be excluded from any consideration of the decomposition of water) then the resistance would refer to just the water, and one would have to set it equal to:

$$127\,476 \cdot 10^6$$

for one millimeter of length and one square millimeter of cross-section.

Now, should this *resistance* to the current intensity in *magnetic* units be equal to $106\frac{2}{3}$ — namely, strong enough that, from Section 7.1, page 224,[233,234] one milligram of water would decompose in one second — then the electromotive force for each millimeter in *magnetic* units would have to amount to:

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[228][Note by AKTA:] Eben Norton Horsford (1818-1893), see [Hor47] and [Sto88].

[229][Note by AKTA:] [Len35] and [Len38].

[230][Note by AKTA:] [Web53e], [Web53a] and [Web53c].

[231][Note by KW:] In the cited place, [Wilhelm Weber's *Werke*, Vol. II, p. 319], one finds the resistances given for various types of copper, among which, one finds the one above, which corresponds to the copper that Jacobi used for his standard resistance (*Widerstands-Etalon*), which is the largest of them. That value was chosen because Lenz often referred to the same papers as Jacobi, so he probably appealed to the same types of copper as Jacobi in his experiments.

[232][Note by AKTA:] See [Web53e, p. 319 of Weber's *Werke*] and [Jac51].

[233][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 614.

[234][Note by AKTA:] See page 144 of Section 7.1, or [KW57, p. 614 of Weber's *Werke*].

$$106\frac{2}{3} \cdot 127\,476 \cdot 10^6 \ ,$$

which must be multiplied by $\frac{2\sqrt{2}}{c} = \frac{1}{155\,370 \cdot 10^6}$ in order to obtain its expression in *mechanical* units.

However, from the foregoing Section, that number means the *difference between the forces* that act in each direction of the current on *each unit* of free positive electricity (in the hydrogen particles) in a column of water that is one millimeter long and on *each unit* of the free negative electricity (in the sulfuric acid that is found in it), and that number must then be multiplied by n in order to obtain the *total force that acts*, if n is the number of units of free positive or free negative electricity that is contained in the hydrogen or oxygen particles, respectively, in a water column that is one millimeter long.

However, the hydrogen in one milligram of decomposed water gives up its free positive electricity to the electrode where it develops, which will then flow through the electrode (or, what amounts to the same thing, in effect, it will be neutralized by the supply of negative electricity in it) and will flow through the cross-section in one second. However, since the current intensity in *electrolytic* units is equal to 1, and from Section 7.15, with that current intensity, $106\frac{2}{3} \cdot 155\,370 \cdot 10^6$ units of positive electricity and just as much negative electricity will go through the cross-section in one second (when one-half of the free positive electricity that is on the electrode flows through the electrode, while the other half is neutralized by the negative electricity that the electrode supplies), which will yield:

$$\frac{1}{2}n = 106\frac{2}{3} \cdot 155\,370 \cdot 10^6 \ .$$

If one then multiplies that number by:

$$\frac{2\sqrt{2}}{c} \cdot n = 2 \cdot 106\frac{2}{3}$$

then the product

$$2 \cdot \left(106\frac{2}{3}\right)^2 \cdot 127\,476 \cdot 10^6$$

will give the *difference between the forces* that must act in the direction of the current on the hydrogen particles in one milligram of water that defines a column that is one millimeter long, which contain free positive electricity, and on the negative electricity that is contained in the oxygen particles (under the influence of the neighboring sulfuric acid) if the decomposition of the water is to result with a velocity of one millimeter per second, and indeed that *difference in forces* is determined from the number above as a ratio to the force that would impart a velocity of one millimeter per second on a mass of one milligram during one second.

The weight of one milligram is a force that will impart a velocity of 9 811 millimeters per second on a mass of one milligram in one second. Therefore, if one divides the given number by 9 811, then one will get that *force difference*, as expressed in milligram weights:

$$\frac{2}{9\,811} \cdot \left(106\frac{2}{3}\right)^2 \cdot 127\,476 \cdot 10^6 = 2 \cdot 147\,830 \cdot 10^6 \ .$$

One can express that result in the following way: *If all of the hydrogen particles in one milligram of water in a column one millimeter long were coupled by one thread and all of* *the oxygen particles were coupled with another thread, then both threads would each have to be tensed in opposite directions with a weight of:*

147 830 *kilograms*,

or about 2956 hundredweights, in order to produce a decomposition of the water with such a rate that one milligram of water would decompose in one second. The tension would remain the same for columns of different cross-sections but would increase in proportion to the length of the column.

Should the water decompose at a small rate under the same conditions — e.g., with a rate of one milligram per 2956 seconds — then the tension above would have to be proportionally smaller; viz., only one hundredweight. Above all, the tension could then be arbitrarily small, and decomposition would always result, but only at a lower rate as the tension become smaller. However, that is true only under the assumption that the *force of resistance* by which the water opposes its decomposition (the motion of the hydrogen and the oxygen in opposite directions), which is analogous to the *force of resistance* that opposes the motion of positive and negative electricity inside of a metallic conductor according to Ohm's law, is proportional to the rate of decomposition.[235] However, for metallic conductors, it is very likely that Ohm's law does not correspond to reality precisely, but that, strictly speaking, the force of resistance consists of two parts, one of which is proportional to the rate, while the other is constant, since it is only in that way that the better conductors (e.g., metals) can be included in the same law as the worse ones (e.g., insulators). The same thing is also probably true for the force of resistance by which the water opposes the motion of the hydrogen and oxygen in opposite directions in its interior. The *resistance* (viz., the force of resistance divided by the drift velocity)[236] will then be represented by the sum of a constant w and a part k/i that is inversely proportional to the drift velocity. Now, if one substitutes that sum for the *resistance* in Ohm's law then one will get the current intensity i, expressed in terms of the electromotive force E and the given sum, in the following way:

$$i = \frac{E}{w + k/i} ,$$

or

$$E = k + wi .$$

For metallic conductors, k is very small compared to the value of wi that comes from the measurements; for insulators, wi vanishes in comparison to k.

Now, no precise experiments involving water exist from which the value of the constant k could be measured. However, there do exist experiments in which it was shown that this constant does not vanish completely, although it is still very small. Namely, if one conducts magnetically induced currents through water, then one can infer from the measurable current effects that this induction would decompose more or less water according to whether it happened faster or slower, respectively, which would not be the case if one had $k = 0$. — For electrolytic measurements, wi is typically so large that k will not come under consideration in comparison to it.

[235][Note by KW:] From Ohm's law, the ratio of the force of resistance by which a conductor opposes the motion of the electricity inside of it to the velocity of that motion is a *constant* that is called the *resistance* of the conductor.

[236][Note by AKTA:] In German: *Stromgeschwindigkeit*. See footnote 52 on page 61.

One refers to the forces that define the resistance to the decomposition of the hydrogen and oxygen in water as *forces of chemical affinity*, which one is not, however, in a position to express in known units. In this Section, it will be shown in an example how the results of the foregoing investigation can actually be employed to implement such a determination. In that way, the path to a more detailed exploration of the *laws of forces of chemical affinity* will be blazed, but numerous measurements of those forces would be necessary for that, of which, only one measurement shall be given as an example.

7.20 Electricity Content in a Conductor

The intensity of the current that goes through a conductor is proportional to the velocity with which the positive and negative electricity flows through the *cross-section of the conductor* and therefore depends upon two factors:

1. The amount of electricity that is contained in each *element of length* of the conductor (which can be called the *capacity* of the conductor).

2. The velocity with which that amount of electricity (viz., positive and negative moving in opposite directions) advances in the conductor.

The intensity of the current that flows *through the cross-section of the conductor* — that is, the amount of positive and negative electricity — can be measured in known units, but neither the amount of electricity that is contained in an *element of length* in the conductor nor the *velocity* with which it advances in that conductor can be determined individually: That could happen only in those cases where the one kind of electricity does not move by itself, but the particles of the conductor in which it is contained move with it.

Now, whether that case comes about when the electricity jumps from one conductor to another (through a layer of air), whereby small particles of the one conductor break away and go over to the other conductor, has not, in fact, been ascertained experimentally, and it also cannot be ascertained completely and with certainty. However, it seems that under certain conditions, it can be established factually that small particles can break away from only the positively charged conductor and go over to the negative conductor. There is also no doubt that these small particles that break away are charged with free positive electricity and that the transfer of a well-defined amount of electricity from one conductor to another will be mediated by them. However, whether the transfer of only part of the positive electricity or all of it from one conductor to the other will be mediated in that way, and furthermore whether those small breakaway particles contain merely free positive electricity or also a well-defined amount of negative electricity, in addition, and finally, how the negative electricity on the other conductor behaves during the process, has not been subjected to a more detailed discussion up to now.

As far as the behavior of the electricity on the negatively-charged conductor is concerned, of which, no particles will break away and move to the positive conductor under the aforementioned conditions, it would seem to emerge from this that the negative charge on the conductor suffers some sort of deceleration under those conditions and therefore that before that charge has attained the strength that is required for the liberation of small particles, the particles that break away from the positively-charged conductor have already arrived at the negative one and hinder the growth in negative charge by transmitting their positive charge. Hence, no electricity at all would go from the negatively charged conductor to the positively charged one under those conditions.

As far as the other question is concerned, of whether the liberated particles contain merely positive electricity or whether they carry a well-defined quantity of neutral fluid with them, in addition, a definite opinion on that could only be based upon some fact about the liberated particles at the highest level of detail.

Namely, it is known that when a larger ball is separated from a smaller one after contact, the free electricity that is contained in both of them will split between them in a well-defined ratio, and indeed in such a way that the mean densities of the layers of electricity that are found on the surface of each ball will not be equal, but the mean density that is found on the surface of the smaller ball will be larger than the density that is found on the surface of the larger ball, and in fact that ratio will approach:

$$1.6449 : 1$$

as the two balls become the more unequal.

Now, a particle that breaks away can be considered to be only an extremely-small ball, and therefore when one denotes the density of amount of electricity that is present on the surface of the positively-charged conductor by ε, the density of liberated particles that are present on the surface will be set equal to $1.6449 \cdot \varepsilon$. Now, it is known that whereas ε vanishes in comparison to the radius of curvature of the surface of the positively-charged conductor, $1.6449 \cdot \varepsilon$ will also vanish in comparison to the radius of the smallest liberated particle, but in contrast, due to the extreme smallness of that particle, one must assume that its radius is smaller than $1.6449 \cdot \varepsilon$, or at least no larger than it. However, it would then follow that this layer of positive electricity would fill up the entire positive particle, and therefore no space would be left in that layer that might contain a well-defined amount of neutral fluid. The small liberated particle would then contain merely free positive electricity. Finally, in regard to the question of whether the free electricity goes from the positively-charged conductor to the negative conductor only by means of the liberated particles or if another quantity of positive electricity without ponderable carriers finds a path to the negatively-charged conductor by itself, as well, one can only assert that given the lack of any physical basis upon which it would depend, under exactly the same conditions, the one part of the electricity should move independently of its ponderable carrier, while the other part must move with its ponderable carrier. Since that would then actually establish that part of the transferred electricity was drawn along by its ponderable carrier, that must be assumed of all the transferred electricity until the contrary has been proved.

The case of a current for which the conducting particles, which contain only positive electricity, would advance would then actually exist. The amount of advancing electricity that goes from the one conductor to the other can now be determined precisely from the measurements that are obtained (by measuring the current intensity). As a result, all that remains is to measure precisely the amount of ponderable mass that simultaneously breaks away from the positive conductor and lands on the negative conductor. Although that ponderable mass might also be so small, nonetheless, it can still be clearly observed, and from that, one can assume that its weight can also be determined with the most accurate balance that we possess.

In any event, that implies that even for very large amounts of electricity that go from the positively-charged conductor to the negatively-charged one, the ponderable mass of the conducting particles that break away is very small, and as a result, the amount of electricity that is contained in each *element of length* in the conductor is exceptionally large. However, the larger that amount of electricity gets, the smaller that the *velocity* with which that amount of electricity advances in the conductor will be,[237] and that smaller velocity with which the electrical fluid moves in its conductor can then by no means be confused with the extremely large velocity with which the perturbation of the equilibrium in the electrical fluid propagates through the metallic conductor, to which the well-known experiment of Wheatstone referred.[238]

The facts that the amount of electricity that is contained in one element of length in a *metallic conductor* is very large and that the velocity with which the amount of electricity moves in the conductor is very small for all currents that are presented in reality, could have been expected beforehand by analogy with the results that were found in Section 7.15 for a *wet conductor* (e.g., water), because it was found there that for a current whose intensity is equal to 1 in *electrolytic* units, an amount of positive electricity of $106\frac{2}{3} \cdot 155\,370 \cdot 10^6$ units, together with 1/3 milligram of hydrogen, will move in one direction, while an equally-large amount of negative electricity that is bound to 8/9 milligram of oxygen will move in the opposite direction through the cross-section of the conductor in one second, from which, it would follow that $106\frac{2}{3} \cdot 155\,370 \cdot 10^6$ units of positive electricity and equally-much negative electricity must be contained in one milligram of water, but they (together with their ponderable carriers) advance only with the very small velocity of 1/2 millimeter in one second when the area of the cross-section of the wet conductor is only 1 square millimeter. If the cross-section were larger, then the velocity would be proportionally smaller.

7.21 Applying This to Units — Derivation of All Units from the Spatial Unit

The units that are useful in physics are divided into the *fundamental units* and the *derived units*. In general mechanics, where all forces are considered to be given individually, all units can be reduced to the known fundamental units of *space*, *time*, and *mass*. — In all of those branches of physics where the *law of gravitation* must be assumed to apply, all units can be reduced to merely the two fundamental units of *space* and *time*, since the units of *mass* can also be derived with the help of the law of gravitation. Namely, one can take the unit of mass to be the mass that would, if it were concentrated at a point, exert a force upon another mass at a unit distance that would impart a velocity to the latter that would equal one unit length per unit time according to the law of gravitation.

Now, it is interesting to note that this system of units is capable of being simplified even further, and that it is possible to derive all of the units that are used in physics *from the single basic unit of space* when one assumes two fundamental laws of nature to that end, namely, in addition to the *law of gravitation of ponderable masses*, one assumes *the fundamental law of electrical action*, since one can also *derive the unit of time from the unit of space* with the help of the latter. Namely, one can take that unit of time to be the time during which two electrical masses that move with uniform relative velocity must move towards or away from each other if they are to have no influence on each other according to that law.

If one chooses the *millimeter* to be the spatial unit, then the unit of time could be derived

[237][Note by AKTA:] Weber is referring here to the drift velocity, that is, the velocity of the electrified particle relative to the matter of the conductor.

[238][Note by AKTA:] See footnote 138 on page 129.

from it under the assumption of the fundamental law of electrical action, and it would be the:

$$439\,450 \textit{ millionth part of a second},$$

since when two electrical masses that move with uniform relative velocity approach or move apart from each other [the distance] of 1 millimeter in that small time interval, they will exert no effect on each other according to the fundamental law of electrical action.

Once the time unit has been derived from the spatial unit in that way, the unit of mass can also be derived from those two units under the assumption of the law of gravitation. Namely, from the law of gravitation, the Earth is a mass that, if it were concentrated into a point, would impart an acceleration equal to 9 811 upon another mass at a distance equal to the Earth radius if the millimeter were used as the spatial unit and the second were used as the unit of time. If one were to take the unit of time that was just derived instead of the second, which is 439 450 million times smaller, then the derived unit of acceleration would be 4 394 502 billion times larger, and the acceleration would be equal to:

$$\frac{9\,811}{439\,450^2 \cdot 10^{12}}$$

in that larger unit. Now, if one sets the radius of the Earth equal to $6\,370 \cdot 10^6$ (millimeter), then according to the law of gravitation, if the mass of the Earth were concentrated into a point, then it would impart an acceleration upon another mass at a unit of distance that would equal:

$$\frac{9\,811 \cdot 6\,370^2 \cdot 10^{12}}{439\,450^2 \cdot 10^{12}} \, ,$$

and as a result, a mass that amounts to $\frac{439\,450^2}{9\,811 \cdot 6\,370^2}$, or almost one-half the mass of the Earth, which is the mass that one will get as the *derived mass unit* from the law of gravitation, under the assumption that the millimeter is the spatial unit and with the help of the time unit that was derived from it before.

Finally, all of the remaining units that are used in physics can be derived from the millimeter as the spatial unit and the units of time and mass that were just derived from it in known ways.

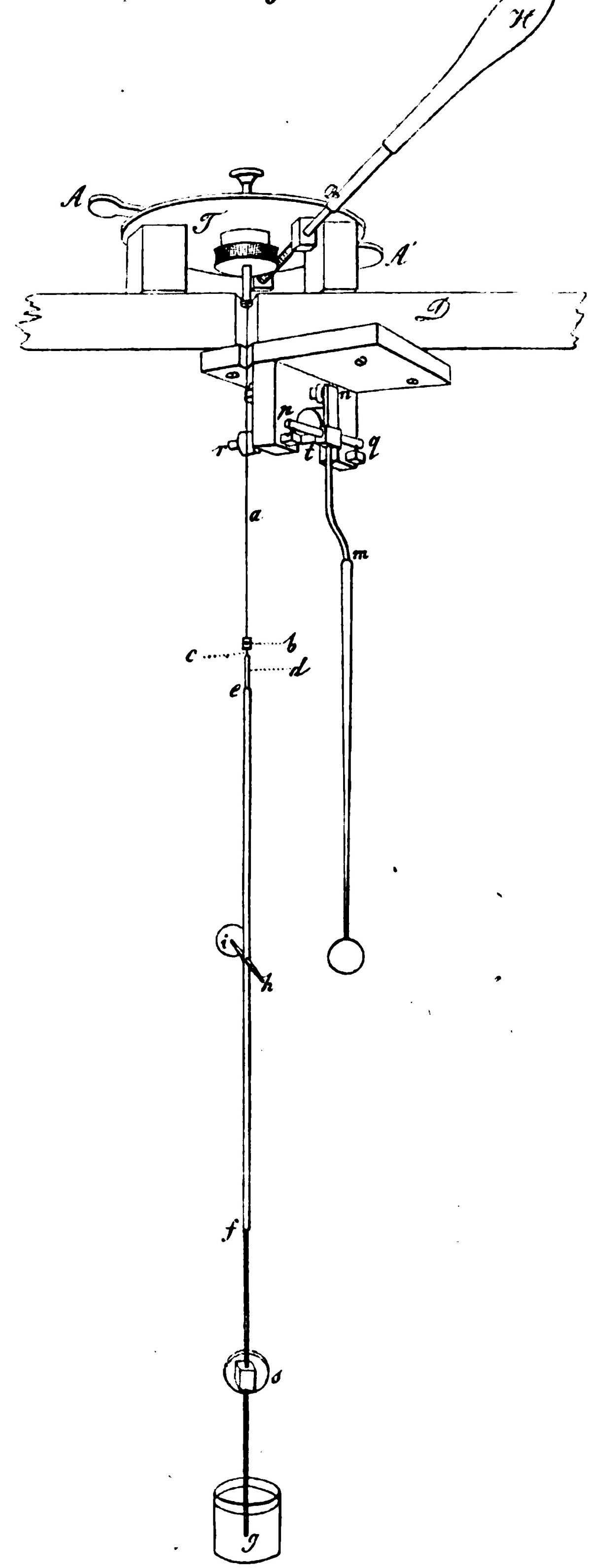
According to this system, in which all units can be derived from the single fundamental unit of distance, the force of attraction between two masses m and m' at a distance of r will be equal to mm'/r^2, and the force of repulsion of two amounts of electricity e and e' at a distance of r will be equal to $(ee'/r^2)(1 - dr^2/dt^2 + 2rd^2r/dt^2)$, without having to add constant factors to these expressions or individual terms.

Appendices

I. Description of the Torsion Balance

In order to avoid, as much as possible, an unequal reaction of the charged balls on the moving ball of the torsion balance due to the electrostatic induction of its electrified walls, the balance is usually associated with a very large scale. The case in which the balls were hung was a parallelepiped that was 1.16 meters long, 0.87 meter wide, and 1.44 meter high. The twelve edges of the parallelepiped were constructed from square posts (with a thickness of 80 mm) of hard wood. Once the framework was established on a large stone foundation, a heavy sheet of wood was laid upon it as a lid, but the side walls were draped with a tightly-stretched oilcloth in such a way that the edges of the posts would not protrude into the interior of the space. After that draping, which left merely the upper fourth of a wall open for one to hang the apparatus, the rigidity of the case was increased appreciably by bolted struts. For the measurement itself, once the fixed ball was introduced, the opening was closed with a slide. However, in addition, the entire case was covered with multiple layers of towels and blankets that rested upon the stone in order to keep the draft off of it. Nevertheless, it was necessary to make the observations at night in an unheated room, since the opening and closing of the doors in other parts of the building and the uneven warming of the floor by the Sun would give rise to air currents that would produce an occasional oscillation of the moving ball of up to one-half of a degree. However, at night, when the outside air was not too agitated, the ball did not oscillate by even one minute.

Fig. 2.



The torsion circle T was fixed over the center of the lid, whose cross-section is denoted by D in Figure 2, whose alidade AA' allowed one to read off the individual minutes from its vernier scale[239] and would lead to a finer adjustment of the torsion with a Hooke's joint[240] H or also by freely loosening it by hand. Furthermore, the definitions of the symbols in the Figure are:

- a the hard-drawn brass wire (no. 12), which is 398 mm long and fixed in the axis of the alidade;
- b a small brass cylinder with a side screw for clamping it fast to the lower end of a. Under it, is
- c a 5 mm protruding threaded spindle, in order to attach either the body whose period of oscillation is to be determined from the torsion coefficient, or the brass wire
- d, to which the 5 mm thick, 450 mm long, cylindrical rod ef of pure shellac was fused.[241]
- hi means the shellac lever for the moving ball, which was tapered on both sides with a length of about 60 mm up to 2.5 mm in thickness.
- fg is a wire that is immersed an inch deep in olive oil with a mirror s on it that is attached to wood. The oil has the effect of damping out not only the oscillation of the moving ball, but also the pendulum motions that arise from vibrations, in the shortest time, while on the other hand, it is no impediment that the lever follows the most imperceptible changes in torsion.

The two balls of the torsion balance consist of very thin Argentan sheet metal that were finely polished and gold-plated, and merely heat-glued to the shellac.

The long vertical shellac rod for the *fixed ball*, which was tapered below, was glued to a curved brass rod mn. A horizontal axis pq with two steel tips was solidly fixed to it, and at right angles to it, a brass rod rt with a running weight. The running weight pushed the upper end of the brass rod mn against an adjusting screw, such that precisely the same position of the torsion balance would result whenever the fixed ball was taken out or put in. If one pushed the brass rod mn forward in order to charge the moving ball until the rod tr joined up with an adjustment screw, then the charged fixed ball would be found to be near the moving one, so the former could attract and charge the latter without the latter needing to describe a large path.

Opposite to the mirror s, there was an opening in the wall of the torsion balance that was closed with flat glass. Outside at some distance, one found a horizontal scale whose mirror image could be observed in a telescope. The distance to the scale was chosen such that when the rotation of the level in the torsion balance amounted to one minute, the scale in the telescope would move by one scale division. At the same time, the scale was positioned such that when the centers of the two balls defined precisely a right angle with the axis of rotation,

[239][Note by AKTA:] In German: *Dessen Alhidade AA' die einzelne Minute durch ihre Nonien ablesen liess*. The measuring device "nonius" was named after its inventor, the Portuguese mathematician and cosmographer Pedro Nunes (Latin: Petrus Nonius) (1502-1578). The Vernier scale was derived from it, being due to the French mathematician and instrument maker Pierre Vernier (1580-1637).

[240][Note by DHD and AKTA:] In German: *Hook'schen Schlüssel*. It is also called a universal joint or universal coupling. It is named after Robert Hooke (1635-1703).

[241][Note by KW:] The length ef, and above all, the length Tg, are too negligible in comparison to the size of the upper part of the figure to be indicated. The balls were further away from the lid.

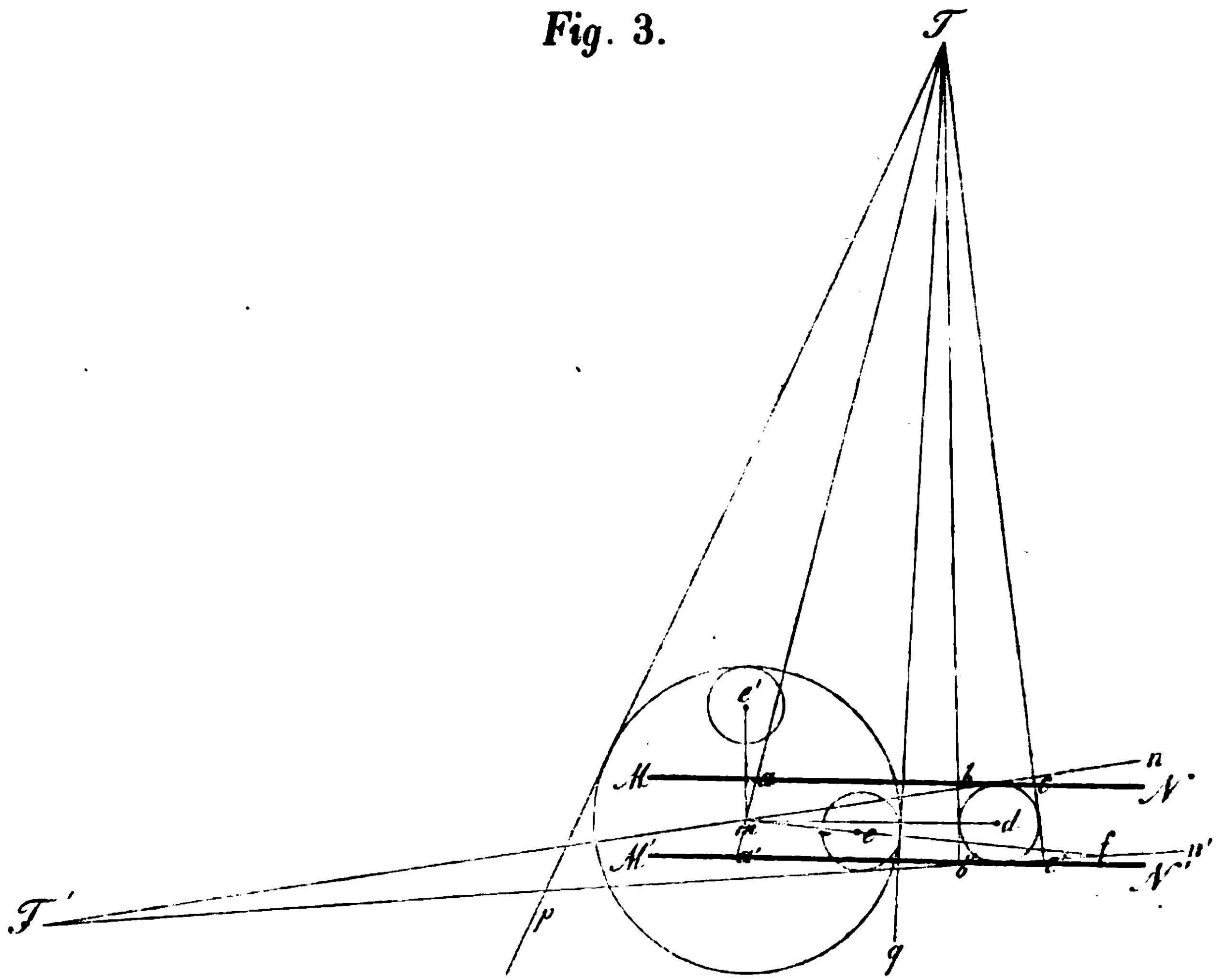
its zero-point, which was placed in the center and from which the scale was numbered on both sides outward, would appear in the crosshair of the telescope.

That was the position of the balls in which they should be observed, which could always be known with great accuracy in that way. Had the moving ball moved further from the fixed ball after they were electrified, then the observer who was found at the telescope could likewise read off how many degrees or minutes would be needed to correct the state of the moving ball by torsion. On the other hand, a disc was installed on the Hooke's joint that allowed one to see the rotation of that joint in minutes of the rotation of the alidade, and the torsion-adjusting second observer could bring about the correction on command[242] without needing to look at the vernier scale. Some practice with the timely assignment and performance of that command and the excellent effect of the oil soon brought one to the point that the moving ball, which was thought to be put into a state of violent motion by the charging, could be brought to rest completely in a relatively short time in such a way that the centers of the two balls would define an angle with the axis of rotation that was larger than a right angle by only a few minutes; i.e., such that the zero-point of the scale in the telescope would be at a distance of a few tick marks on the crosshair of the telescope. The loss of electricity would then bring the ball gradually closer to the fixed ball due to the torsion on it that was present, such that the time-point at which the zero-point of the slowly-drifting scale passed the crosshair of the telescope would be determined accurately. The torsion could be read off from that.

The state in which the centers of the two balls define precisely a right angle with the rotational axis of the torsion balance is found in the following way:

[242][Note by KW:] If one wished to bring the lever in an uncharged torsion balance from one position to another without producing long-lasting oscillations, then one would make one-half the correction suddenly when the lever was still at rest and the other half just as suddenly at the moment when attained its greatest elongation and began to reverse. It would then become more still the less the air resistance came into consideration in comparison to its moment of inertia. One will achieve the goal approximately for the charged torsion balance in that way.

Fig. 3.



Once a fine filament that was weighted down (whose projection m represents the axis of rotation in Figure 3), was fixed to the small cylinder on the torsion wire in place of the shellac rod, a theodolite T was placed at a distance of a few meters, and the distance Tm was measured precisely. From there, an ivory yardstick that was divided into millimeters was brought into the positions MN and $M'N'$ horizontally, such that it stood parallel to md each time and was tangent to the fixed ball at one-half its height. The vertical crosshair in the telescope of the theodolite allowed one to estimate the lengths ab, ac, $a'b'$, and $a'c'$ to one tenth of a millimeter due to its higher magnification. One then had:

$$md = \frac{1}{4}(ab + ac + a'b' + a'c') \ .$$

After that, a second theodolite was placed at a point T' such that the vertical line in its telescope covered the rotational axis m and was tangent to the fixed ball. Once $T'm$ was measured, the telescope was rotated into the position $T'n$ such that the line was tangent to the other side of the fixed ball, and it then remained unperturbed.

One then hung the shellac rod with the moving ball from the torsion wire again and measured the angle pTq with the theodolite T. The moving ball, which was protected from light reflection, stood out very sharply from the white background, and the theodolite pointed to the tangent to the circle inside of which it moved by slow rotation. The distance from the center of the moving ball to the axis of rotation was then:

$$me = Tm \sin \frac{1}{2} pTq - r' \ ,$$

in which r' is the previously measured radius of the moving ball.

The fixed ball was now taken out, and in order to avoid air currents, the case of the torsion balance was closed completely, except for two small openings in the already-known direction $T'n'$, and the moving ball was placed in such a way that it would be tangent to the direction $T'n'$ by means of the torsion wire.

It would then be necessary to rotate the moving ball through $90^o + dme$ in order to make its center come to the position e', in which it would describe a right angle with m and d. Now, the angle:

$$dme = mfT' + mT'f - nmd \ ,$$

while:[243]

$$mfT' = \arcsin \frac{T'm \sin mT'n' - r'}{me} \ ,$$

$$mT'f = 2 \cdot \arcsin \frac{r}{T'm + md \cos nmd} \ ,$$

$$nmd = \arcsin \frac{r}{md} \ .$$

Since everything in that has been given, dme could be easily calculated, and the rotation of the moving ball through $90^o + dme$ was accomplished by means of the torsion circle, so the zero-point of the observer scale was located correctly.

II. Description of the Tangent Galvanometer

The copper wire that was employed for the multiplier was wound quite tightly with silk, and then almost 2/3 of a mile of collodion was pulled along its entire length.[244] From the large roll on which it was then found, with the help of a very uniformly tensed pulley, it would be wound around the circular ring of the tangent galvanometer with 5635 windings. That metal ring, which defined a channel of rectangular cross-section, was previously given a thick coating of heated sealing wax everywhere that the wire was laid in it. After that, a 20 pound copper weight was placed into the ring as a damper. All of the remaining procedures are known.

The main idea was to confirm one's belief that all windings of the tangent galvanometer would actually be traversed by the discharge current, and that it would not perhaps jump over some of them by a spark that occurred deep within the windings, but perhaps not visibly. Now, a small multiplier of 1000 windings that had been used often at Marburg was

[243][Note by KW:] The multiplicity of these possibilities was due to the opacity of the hanging shellac rod.

[244][Note by KW:] Experiments concerned with whether the degree of insulation would actually increase in that way have not been performed, but one should, nonetheless, assume that is so. In any event, in that way, one will arrange that the silk not only adheres to the wire very firmly, but also that it does not become slightly rough on the surface. The process is simple: One leads the wire from the original roll to a small fixed roll with a horizontal axis, and from there, to a larger roll at a greater distance, around which it will be temporarily wound. The small fixed roll is immersed halfway in a container of collodion.

on hand, and it could be predicted from the dimensions of the two instruments that they would have roughly the same sensitivity to the discharge of a Leyden jar. Both multipliers were coupled in such a way that the same discharge from a larger Leyden jar, when retarded by a column of water, would have to flow through the windings in both of them. Now, since not only the predicted behavior of the sensitivity occurred, but upon raising the charge, the data from both galvanometers remained proportional to each other, as well as the data that corresponded to a sine electrometer, which allowed one to compare the charge on the Leyden jar in isolation when coupled to it, one could convince oneself that the large tangent galvanometer would serve its purpose. For all discharges that would be regulated by a specially-constructed pendulum, the knob on the jar remained coupled with the multiplier for the same time (and in fact, only 2/3 of a second) in order to allow only a very small (and in fact proportional) part of the residue to appear again. The results are as follows:

No.	*a.* Deflection φ of the sine electrometer	*b.* $\sqrt{\sin\varphi}$	*c.* Small multiplier. Elongation in scale divisions	*d.* Tangent-galvanometer. Elongation in scale divisions	d/c	d/b
1.	$9^o31'$	0.4078	41.75	170.40	4.1060	417.85
2.	$19^o59'$	0.5845	59.50	244.85	4.1151	418.91
3.	$34^o57'$	0.7569	76.95	316.10	4.1078	417.62
4.	$49^o54'$	0.8746	88.97	365.45	4.1076	417.85

Each of the numbers under c and d is the mean of 2 to 3 measurements that differed from each other by at most one scale division. The desired proportionality then emerged from this completely. Now, the distance from the mirror to the scale was 1633 for the small multiplier and 6437.6 scale divisions for the large one, and their sensitivities then had roughly the ratio that was required above, namely, 1 : 1.042 3.

Those measurements, the second of which could obviously be assumed to include an observation error in the tangent galvanometer, showed an extraordinary accuracy in the comparison of the available charge in a Leyden jar for all three instruments.

Chapter 8

[Kirchhoff, 1857a] On the Motion of Electricity in Wires

Gustav Kirchhoff[245,246]

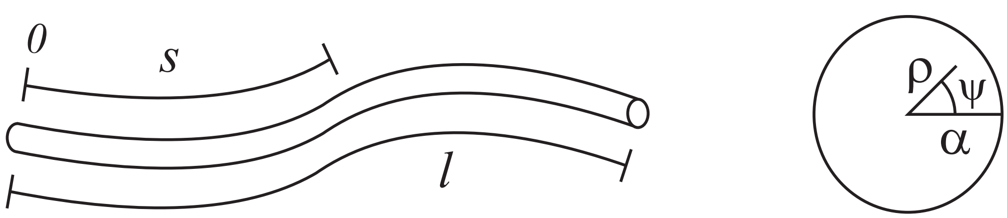
I have attempted to establish a general theory of the motion of electricity in an infinitely thin wire, by assuming certain facts which are observed in constant currents, and in currents whose intensity alters but slowly, to be universally valid. I will here develope this theory, and show its application to some cases of a simple nature.

I picture to myself a homogeneous wire possessing the same thickness throughout and of circular cross section. In the axis of this wire I take a fixed point and a variable one; the portion of the axis between both points I call s. Through the changeable point I permit a transverse section to pass, and call the polar coordinates of a point of this section, with reference to a system of ordinates whose origin is the centre, ρ and ψ.[247] I will calculate the electromotive force which tends to separate, in the direction of the length of the wire, the two electricities in the vicinity of the point determined by s, ρ and ψ. This force is partly derived from free electricity, partly from the induction which takes place in consequence of the alteration of the strength of the current in all parts of the wire. With regard to the first

[245] [Kir57b] with English translation in [Kir57a].

[246] Gustav Kirchhoff's Notes are represented by [Note by GK:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[247] [Note by AKTA:] The Figure of this footnote shows my representation of this configuration. Figure (a) shows a fixed point 0 at the left extremity of the wire and a variable point of length s along the curved axis of the wire of total length l. Figure (b) shows the circular cross section of the wire of radius α and the polar coordinates $(\rho,\ \psi)$ of a point inside the wire.



(a) (b)

portion, we may make use of the electrostatic law of Coulomb.[248] Let V be the potential of the free electricity with reference to the point under consideration; that is to say, the sum of all the single quantities of free electricity, each divided by its distance from the point. The quantities of electricity are here to be referred to a mechanical unit; the unit of electricity shall be that which, acting upon an equal quantity at the unit of distance, produces the unit of force. In general, all quantities which appear in this investigation — strength of current, resistance, etc. — shall be regarded as measured by a mechanical unit, in the manner often described by W. Weber in his "*Electrodynamic Determinations*".[249] We have there $-\partial V/\partial s$ as the force with which the free electricity strives to move the unit of positive electricity at the point under consideration in the direction in which s increases. An equal force tends to move the negative electricity in the opposite direction. Therefore we have $-2\partial V/\partial s$ as the electromotive force derived from the free electricity, and acting at the point in question.

In developing the value of V, I will assume that no other free electricity acts upon the wire than that which is in the wire itself. The quantity of free electricity which, at the time t, is contained in the element of the wire which corresponds to the element ds of the axis I will denote by eds;[250] let ds' be a second element of the axis, and $e'ds'$ the quantity of electricity contained in the corresponding element of the wire. I picture to myself a portion of the wire, whose centre lies in ds and the length of which is 2ε, where ε denotes a quantity which is to be regarded as infinitely small in comparison with the length of the whole wire, but as infinitely great in comparison with the radius of its cross section. When the element of the wire in which the quantity of electricity $e'ds'$ is contained lies outside the above portion, we can imagine, in the calculation of V, its electricity to be concentrated in the line ds', and the point to which V refers situated in the line ds. Hence the portion of V derived from the whole wire, with the exception of the portion alluded to, is

$$= \int \frac{e'ds'}{r} \, ,$$

where r denotes the distance of the elements ds and ds', the integration being extended over the whole of the central line, with the exception of the length 2ε.

With regard to the portion of V derived from the part separated, this can only be calculated when the distribution of the electricity within a cross section is known. I will assume that here, as in the case of a constant current, and of electricity in equilibrium, free electricity is to be found upon the surface only, and besides that its density is the same at all points of the periphery of a cross section. Denoting by α the radius of the cross section, we have, according to this, the density of the free electricity at any point of the surface of the portion of wire under consideration $= \frac{e}{2\pi\alpha}$; hence, as on account of its infinitely small length it may be regarded as straight, the quantity of V derived from it is

$$= \frac{e}{2\pi} \int_{-\varepsilon}^{+\varepsilon} \int_0^{2\pi} \frac{dx'd\psi'}{\sqrt{x'^2 + \alpha^2 + \rho^2 - 2\alpha\rho\cos(\psi' - \psi)}} \, .$$

In this expression x' has been written for $s' - s$, and ψ' denotes the angle between the

[248][Note by AKTA:] See footnote 43 on page 56.

[249][Note by AKTA:] In German: *elektrodynamischen Maassbestimmungen*. Kirchhoff is here referring to Wilhelm Weber's major Memoirs on Electrodynamic Measurements: [Web46] with partial French translation in [Web87] and complete English translation in [Web07]; [Web52c] with English translation in [Web21b]; [Web52b] with English translation in [Web21a]; and [KW57] with English translation in [KW21].

[250][Note by AKTA:] Therefore e means linear charge density.

radius drawn to an element of the surface of the wire and the line from which the angle ψ is reckoned. When the integration, according to x', is carried out, ε, in comparison with α and ρ, being regarded as infinitely great, we have the following expression:[251]

$$= \frac{e}{\pi} \int_0^{2\pi} d\psi' \left(\ln 2\varepsilon - \ln \sqrt{\alpha^2 + \rho^2 - 2\alpha\rho\cos(\psi' - \psi)} \right) \ ,$$

that is,

$$= 2e \left(\ln 2\varepsilon - \frac{1}{2\pi} \int_0^{2\pi} d\psi' \ln \sqrt{\alpha^2 + \rho^2 - 2\alpha\rho\cos(\psi' - \psi)} \right) \ .$$

Setting

$$\int_0^{2\pi} d\psi' \ln \sqrt{\alpha^2 + \rho^2 - 2\alpha\rho\cos(\psi' - \psi)} = U \ ,$$

the differential equation

$$\frac{\partial^2 U}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial U}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 U}{\partial \psi^2} = 0$$

must be satisfied, because the quantity under the sign of integration multiplied by $d\psi'$ satisfies this equation for all the values of ψ'; but it is easily seen, by setting $\psi' - \psi$ instead of ψ' as the variable according to which the integration is to be carried out, that U is independent of ψ; hence we must have

$$\frac{d^2 U}{d\rho^2} + \frac{1}{\rho}\frac{dU}{d\rho} = 0 \ ;$$

but from this it follows that

$$U = C_1 \ln \rho + C_2 \ ,$$

where C_1 and C_2 denote two unknown constants. These may be easily determined by assuming ρ as infinitely small in comparison with α; the carrying out of the integration in the expression for U gives then

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[251] [Note by AKTA:] In Kirchhoff's original paper, [Kir57b, p. 196 of the *Annalen der Physik und Chemie* or p. 134 of his *Gesammelte Abhandlungen*], this equation was written as:

$$= \frac{e}{\pi} \int_0^{2\pi} d\psi' \left(\lg 2\varepsilon - \lg \sqrt{\alpha^2 + \rho^2 - 2\alpha\rho\cos(\psi' - \psi)} \right) \ .$$

In the English translation of this paper, [Kir57a, p. 395 of the *Philosophical Magazine*], this equation was written as:

$$= \frac{e}{\pi} \int_0^{2\pi} d\psi' \left(\log 2\varepsilon - \log \sqrt{\alpha^2 + \rho^2 - 2\alpha\rho\cos(\psi' - \psi)} \right) \ .$$

This last expression gives the impression that Kirchhoff's logarithm of a magnitude m, which he wrote as $\lg m$, should be understood as the common logarithm base 10, $\log_{10} m = \log m$. However, this is not the case. What Kirchhoff wrote as $\lg m$ should be understood as the natural logarithm which has Euler's number $\mathrm{e} \approx 2.718...$ as its base, namely, $\lg m = \log_{\mathrm{e}} m = \ln m$.

In the translation presented in this book I am utilizing the modern symbol, ln, instead of Kirchhoff's original symbol, lg. In this way his original mathematical reasoning can be better understood, avoiding misunderstandings. In Section 9.1 of Chapter 9, page 221, I present a modern solution of this integral.

$$U = 2\pi \ln \alpha \ ,$$

from which it follows that C_1 is $= 0$, and U has this constant value for all the values of ρ. Consequently the portion of V derived from the piece 2ε of the wire is

$$= 2e \ln \frac{2\varepsilon}{\alpha} \ ,$$

and hence

$$V = 2e \ln \frac{2\varepsilon}{\alpha} + \int \frac{e' ds'}{r} \ , \tag{1}$$

where the integration is to be extended to the whole wire, with the exception of the portion 2ε.

We have now to form the expression for the electromotive force induced in the point under consideration, by the alteration of the intensity of the current in all portions of the wire.

When in the element of a conductor, the length of which is l', the intensity of the current denoted by i' changes, an electromotive force will be induced by this change in a second element of the conductor, which, with reference to the unit of the quantity of electricity, according to Weber,[252,253] is equal to[254]

$$= -\frac{8}{c^2} \frac{\partial i'}{\partial t} \frac{l'}{r} \cos\theta \cdot \cos\theta' \ ,$$

where θ and θ' denote the angles formed by the two elements with the line drawn from the first to the second, r the length of this line, and c the constant velocity with which two particles of electricity must move towards each other, so that they may exercise no force upon each other.

For all parts of the wire, excepting the piece of the length 2ε already alluded to, the electric current may be regarded as concentrated in the central line: the portion of the induced electromotive force now sought, which is derived from the wire, with the exception of the piece already mentioned, is therefore

$$-\frac{8}{c^2} \int \frac{\partial i'}{\partial t} \frac{ds'}{r} \cos\theta \cdot \cos\theta' \ ,$$

where i' is the intensity of the current which passes through the cross section of the wire at the place ds', θ and θ' the angles which the elements ds and ds' form with the line which is drawn from the latter to the former, r the length of this line, and where the integration is to

[252][Note by GK:] *Elektrodynamische Maassbestimmungen*, 1846, p. 354; and 1856, p. 268.

[253][Note by AKTA:] [Web46, p. 354 of Weber's original 1846 paper and pp. 187-188 of Weber's *Werke*], [Web07, pp. 121-122]; [KW57, p. 268 of the *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften, mathematisch-physische Klasse* or pp. 655-656 of Weber's *Werke*] and [KW21, pp. 55-56]. See also item (4) of page 182 on Section 7.18.

[254][Note by AKTA:] In German: "so wird dadurch in einem zweiten Leiterelemente eine elektromotorische Kraft inducirt, die bezogen auf die Einheit der Elektricitätsmenge, nach Weber ist = ...". This expression was translated as, [Kir57a, p. 396]: "an electromotive force will be induced by this change in a second element of the conductor, which, with reference to the unit of electricty of Weber, is = ...". I modified a little this translation.

be extended throughout the whole wire, with the exception of the portion already referred to.

In this portion the current must not be regarded as concentrated in the central line, but in lieu of this it may be regarded as straight and parallel to ds. Through the first point of ds' let a transverse plane be placed cutting the wire, and let ρ' and ψ' be the polar coordinates of a point of the plane, with reference to a system of coordinates whose origin is the centre, and whose axis is parallel to the line from which the angle ψ is reckoned: if then the density of the current in the points determined by ρ' and ψ' be J', we obtain for the portion of the induced electromotive force due to the portion of wire 2ε, the expression

$$-\frac{8}{c^2}\int_0^\alpha\int_0^{2\pi}\int_{-\varepsilon}^{+\varepsilon}\frac{\partial J'}{\partial t}\cdot\frac{\rho' d\rho' d\psi' x'^2 dx'}{\left(x'^2+\rho^2+\rho'^2-2\rho\rho'\cos(\psi'-\psi)\right)^{3/2}} .$$

As J' may be regarded as independent of x', the integration according to x' may be easily accomplished: making use of the fact that ε is infinitely great in comparison with all values of ρ and ρ', we obtain[255]

$$-\frac{16}{c^2}\int_0^\alpha\int_0^{2\pi}\frac{\partial J'}{\partial t}\rho' d\rho' d\psi'\left[\ln 2\varepsilon-1-\ln\sqrt{\rho^2+\rho'^2-2\rho\rho'\cos(\psi'-\psi)}\right] .$$

But as

$$\int_0^\alpha\int_0^{2\pi}J'\rho' d\rho' d\psi=i ,$$

this expression is

$$=-\frac{16}{c^2}\left[(\ln 2\varepsilon-1)\frac{\partial i}{\partial t}-\int_0^\alpha\int_0^{2\pi}\frac{\partial J'}{\partial t}\rho' d\rho' d\psi'\ln\sqrt{\rho^2+\rho'^2-2\rho\rho'\cos(\psi'-\psi)}\right] .$$

Hence the entire induced electromotive force is

$$=-\frac{8}{c^2}\frac{\partial W}{\partial t} ,$$

where

$$W=\int i'\frac{ds'}{r}\cos\theta\cos\theta'+2i\,(\ln 2\varepsilon-1)$$
$$-2\int_0^\alpha\int_0^{2\pi}J'\rho' d\rho' d\psi'\ln\sqrt{\rho^2+\rho'^2-2\rho\rho'\cos(\psi'-\psi)} .$$

In the case of a stationary electric current,[256] the density of the current is equal to the product of the electromotive force, referred to the unit of quantity of electricity, and the

[255] [Note by AKTA:] Due to a misprint, the original equation was written as:

$$-\frac{16}{c^2}\int_0^\alpha\int_0^{2\pi}\frac{\partial J'}{\partial t}\rho' d\rho' d\psi'\left[\ln 2\varepsilon-1-\ln\sqrt{\rho^2+\rho'^2-2\rho\rho\cos(\psi'-\psi)}\right] .$$

[256] [Note by AKTA:] That is, in the case of a constant or steady electric current which does not change in time.

conductivity; I will assume that the same also holds good when the current is not stationary. This assumption will be fulfilled when the forces acting upon the electricity, and which constitute the resistance, are so powerful that the time during which a particle of electricity remains in motion after the cessation of the accelerating forces, and in virtue of its inertia, may be regarded as infinitely small, even in comparison with the small space of time which comes into consideration in the case of a non-stationary electric current. According to this assumption, if k be the conductivity of the wire,[257] J the density of the current at the point determined by the values of s, ρ and ψ at the time t, we have the equation

$$J = -2k\left(\frac{\partial V}{\partial s} + \frac{4}{c^2}\frac{\partial W}{\partial t}\right) .$$

From this expression for the density J, I deduce an expression for the strength i of the current, by multiplying the above with $\rho d\rho d\psi$, and integrating the expression with reference to ρ from 0 to α, and in reference to ψ from 0 to 2π; as V is independent of ρ and ψ, when I make

$$w = \frac{1}{\pi\alpha^2}\int_0^\alpha\int_0^{2\pi} W\rho d\rho d\psi ,$$

I obtain

$$i = -2\pi k\alpha^2\left(\frac{\partial V}{\partial s} + \frac{4}{c^2}\frac{\partial w}{\partial t}\right) . \tag{2}$$

We have here[258]

$$w = \int i'\frac{ds'}{r}\cos\theta\cos\theta' + 2i\left(\ln 2\varepsilon - 1\right)$$

$$-\frac{2}{\pi\alpha^2}\int_0^\alpha\int_0^{2\pi}\int_0^\alpha\int_0^{2\pi} J'\rho' d\rho' d\psi' \rho d\rho d\psi \ln\sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\psi' - \psi)} .$$

The integral

$$\int_0^{2\pi} d\psi \ln\sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\psi' - \psi)}$$

is of the same form as that already considered and denoted by U: from the conclusions there stated, it follows that the integral is $= 2\pi\ln\rho'$, when $\rho' > \rho$, and $= 2\pi\ln\rho$ when $\rho' < \rho$. Multiplied by $\rho d\rho$, and integrated from 0 to α, it therefore gives this expression:

$$\pi\alpha^2\left(\ln\alpha - \frac{\alpha^2 - \rho'^2}{2\alpha^2}\right) .$$

As we may set

$$\int_0^\alpha\int_0^{2\pi} J'\rho' d\rho' d\psi' = i ,$$

[257] [Note by AKTA:] In German: *Leitungsfähigkeit*. This expression was translated as "conductive capacity" in [Kir57a]. I preferred the translation "conductivity".

[258] [Note by AKTA:] Due to a misprint, the first expression in the parenthesis was written as $(\ln 2r - 1)$.

the third member in the expression for w will be

$$= -2i\ln\alpha + \int_0^\alpha \int_0^{2\pi} \frac{\alpha^2 - {\rho'}^2}{\alpha^2} J'\rho' d\rho' d\psi' \ ;$$

and hence we obtain

$$w = \int i' \frac{ds'}{r} \cos\theta\cos\theta' + 2i\left(\ln\frac{2\varepsilon}{\alpha} - 1\right) + \int_0^\alpha \int_0^{2\pi} \frac{\alpha^2 - {\rho'}^2}{\alpha^2} J'\rho' d\rho' d\psi' \ .$$

The remaining double integral cannot be reduced to a simple form, as J' is an unknown function of ρ'; its value, however, can be neglected in comparison with the member $2i\left(\ln\frac{2\varepsilon}{\alpha} - 1\right)$, and for this we may set $2i\ln\frac{2\varepsilon}{\alpha}$, if the thickness of the wire be only small enough in comparison with the dimensions of the figure formed by its axis; for then ε can be so chosen that $\ln\frac{2\varepsilon}{\alpha}$ shall be a number infinitely great, and ε notwithstanding infinitely small in comparison with the dimensions of the figure alluded to. In accordance with this supposition we have

$$w = 2i\ln\frac{2\varepsilon}{\alpha} + \int i'\frac{ds'}{r}\cos\theta\cos\theta' \ , \tag{3}$$

where the integration is to be extended over the whole wire, with the exception of the length 2ε.

To the equations (1), (2) and (3), between the four quantities i, e, V, w, a fourth may be added.

Let two transverse sections be supposed to pass through the commencing and terminal points of ds; through the first point passes in the time dt into the element of the wire bounded by both, the quantity idt of positive electricity; through the second point passes in the same time out of the element of the wire the quantity of positive electricity $\left(i + \frac{\partial i}{\partial s}ds\right)dt$; the element loses, therefore, in the time dt the quantity $\frac{\partial i}{\partial s}dsdt$ of positive electricity; the negative electricity flows in equal quantity and in the opposite direction through both cross sections; the element of the wire gains, therefore, in the time dt as much of negative electricity as it loses of positive; its free electricity, that is, the difference between its negative and positive, diminishes therefore in the element of time by $2\frac{\partial i}{\partial s}dsdt$: this free electricity is, however, eds, and hence we have

$$2\frac{\partial i}{\partial s} = -\frac{\partial e}{\partial t} \ . \tag{4}$$ [259]

I will now develope further the theory contained in the four equations distinguished by numbers, under the supposition that the form of the central line of the wire is such, that the distance between two of its points, between which a finite portion of the wire lies, is never infinitely small. By this supposition the case is excluded, that induction spirals are contained in the circuit. In this way we greatly simplify the equations (1) and (3).

[259] [Note by GK:] The deduction of this equation is based on the supposition, that in the case of a non-stationary current equal quantities of the opposite electricities pass through every cross section of the conductor in equal times. If this supposition be, however, rejected, the equations would nevertheless hold good; it would then be merely necessary to define the intensity of the current as the arithmetic mean of the two quantities of electricity, which in the unit of time move in opposite directions through the cross section of the conductor.

Let A denote the position of the element ds, and B and C two points upon the wire, at both sides of A and at a finite distance from it; then the integral

$$\int \frac{e' ds'}{r} ,$$

extended over the whole wire, with the exception of the piece BAC, is a finite quantity, hence infinitely small in comparison with $2e \ln \frac{2\varepsilon}{\alpha}$; hence in the equation (1) this integral must only be extended over the portion BAC, with the exception of the portion 2ε. Denoting, therefore, by σ the arc between A and a variable point of the wire, the integral mentioned may be set

$$= \int_{\varepsilon}^{AB} \frac{e' d\sigma}{r} + \int_{\varepsilon}^{AC} \frac{e' d\sigma}{r} .$$

The quantity e'/r is a function of σ, which approximates to the value e/σ when σ approaches 0; the integrals[260]

$$\int_{\varepsilon}^{AB} \left(\frac{e'}{r} - \frac{e}{\sigma} \right) d\sigma \qquad \text{and} \qquad \int_{\varepsilon}^{AC} \left(\frac{e'}{r} - \frac{e}{\sigma} \right) d\sigma$$

have therefore finite values, for the function to be integrated will never be infinitely large; hence instead of the integral in equation (1) we may set

$$\int_{\varepsilon}^{AB} \frac{e d\sigma}{\sigma} + \int_{\varepsilon}^{AC} \frac{e d\sigma}{\sigma} ;$$

that is,

$$e \ln \frac{AB}{\varepsilon} + e \ln \frac{AC}{\varepsilon} .$$

The choice of the lengths AB and AC is arbitrary, only they must be finite in comparison to the length of the wire; for both we may set the half of this length: denoting the whole length by l, the equation (1) will be

$$V = 2e \ln \frac{2\varepsilon}{\alpha} + 2e \ln \frac{l}{2\varepsilon} ,$$

that is,

$$V = 2e \ln \frac{l}{\alpha} .$$

Through considerations of the same kind it will be seen that the equation (3) receives a similar form; thus we have

$$w = 2i \ln \frac{l}{\alpha} .$$

These values of V and w are to be substituted in equation (2); when this is done, setting, for the sake of brevity,

[260] [Note by AKTA:] Due to a misprint, the last integral appeared in the original paper as:

$$\int_{\varepsilon}^{AC} \left(\frac{e_|}{r} - \frac{e}{\sigma} \right) d\sigma .$$

$$\ln \frac{l}{\alpha} = \gamma \ ,$$

and denoting the resistance of the entire wire, that is, the quantity

$$\frac{l}{k\pi\alpha^2} \ ,$$

by r, we obtain

$$i = -4\gamma \frac{l}{r} \left(\frac{\partial e}{\partial s} + \frac{4}{c^2} \frac{\partial i}{\partial t} \right) \ .$$

From this equation, in connexion with equation (4), viz.

$$2\frac{\partial i}{\partial s} = -\frac{\partial e}{\partial t} \ ,$$

we have to determine i and e as functions of s and t.[261]

A particular solution of the differential equation is found by setting

$$e = X \sin ns \ ,$$

$$i = Y \cos ns \ ,$$

where n denotes an arbitrary constant, and X and Y are unknown functions of t. By this the equations become

$$Y = -4\gamma \frac{l}{r} \left(nX + \frac{4}{c^2} \frac{dY}{dt} \right) \ ,$$

$$2nY = \frac{dX}{dt} \ .$$

From this we obtain, by eliminating Y,

$$\frac{d^2X}{dt^2} + \frac{c^2 r}{16\gamma l} \frac{dX}{dt} + \frac{c^2 n^2}{2} X = 0 \ .$$

The general integral of this differential equation is

$$X = C_1 \mathrm{e}^{-\lambda_1 t} + C_2 \mathrm{e}^{-\lambda_2 t} \ ,$$

where C_1 and C_2 are two arbitrary constants, e the basis of the hyperbolic logarithms,[262] and λ_1 and λ_2 the roots of the quadratic equation

$$\lambda^2 - \frac{c^2 r}{16\gamma l} \lambda + \frac{c^2 n^2}{2} = 0 \ .$$

According to this the values of λ_1 and λ_2 are

[261][Note by AKTA:] In Section 9.2 of Chapter 9, page 222, I discuss the telegraph equation obtained by Kirchhoff.

[262][Note by AKTA:] That is, e = 2.718... is the basis of the natural logarithm.

$$\frac{c^2 r}{32\gamma l}\left[1 \pm \sqrt{1 - \left(\frac{32\gamma}{cr\sqrt{2}} nl\right)^2}\right] .$$

In order to form an idea as to whether these roots are real or imaginary, a particular case shall be considered. Let the wire be of the standard wire of Jacobi,[263] the resistance of which has been measured by Weber.[264] This is a copper wire of 7.620 metres in length[265] and 0.333 millimetres radius. The value of γ is, according to this, very nearly $= 10$. Weber[266,267] found its resistance according to the electro-magnetic unit as[268]

$$= 598 \cdot 10^7 ,$$

regarding the millimetre and second as units of length and time. To find the resistance according to the mechanical unit, that is, the value of r, we must multiply the above value by $8/c^2$. Now, as according to the same units we have[269,270]

$$c = 4.39 \cdot 10^{11} ,$$

we obtain

$$r = 2.482 \cdot 10^{-13} ,$$

and from this we obtain

$$\frac{32\gamma}{rc\sqrt{2}} = 2\,070 .$$

The quantity n, which is still left undetermined, shall subsequently be so chosen that nl may be a multiple of π. The negative member under the vinculum in the expressions for λ_1 and λ_2 will then be so large in comparison with 1 that it may be regarded as infinitely great. This circumstance carries with it a considerable simplification of the question. In the following we shall only investigate the case in which the same circumstance takes place, viz. where

$$\frac{32\gamma}{rc\sqrt{2}}$$

[263][Note by AKTA:] Moritz Hermann von Jacobi (1801-1874). See [Jac51].

[264][Note by AKTA:] [Web51] with English translation in [Web61a].

[265][Note by AKTA:] In the original German paper we have the length as $7^m, 620$. In the English translation this length was incorrectly expressed as 7.620 inches.

[266][Note by GK:] *Elektrodynamische Massbestimmungen*, 1850, p. 252.

[267][Note by AKTA:] Maybe Kirchhoff was referring to Weber's paper of 1851, [Web51, p. 262 of the *Annalen der Physik* or p. 292 of Weber's *Werke*] with English translation in [Web61a, p. 262]; or to Weber's paper of 1852, [Web52c, p. 252 of the *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften, mathematisch-physische Klasse* or p. 351 of Weber's *Werke*] with English translation in [Web21b].

[268][Note by AKTA:] In German: "Den Widerstand desselben nach elektromagnetischem Maasse hat Weber $= 598 \cdot 10^7$ gefunden ...". This expression was translated as: "Its resistance, according to the electro-magnetic unit of Weber, was found to be $= 598 \cdot 10^7$, ...", [Kir57a, p. 402]. I modified a little this translation.

[269][Note by GK:] Ibid. (Weber and Kohlrausch) 1856, p. 264.

[270][Note by AKTA:] Kirchhoff was referring to Kohlrausch and Weber's work of 1857: [KW57, p. 264 of the *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften, mathematisch-physische Klasse* or p. 652 of Weber's *Werke*] and [KW21, p. 52]. See also page 179 on Section 7.17.

may be regarded as infinitely great in comparison to 1: this assumption will be the more nearly fulfilled the smaller the resistance of the wire, while the ratio of its length to its radius remains constant; this resistance, however, may be considerably greater than that of Jacobi's wire, without prejudicing the validity of the results which we shall obtain.

According to the above assumptions, the values of λ_1 and λ_2 will be

$$h \pm \frac{cn}{\sqrt{2}}\sqrt{-1}\ ,$$

where, for the sake of brevity, we have placed

$$\frac{c^2 r}{32\gamma l} = h\ .$$

Introducing new constants in the place of C_1 and C_2, the expression for X may be brought to the form

$$X = \mathrm{e}^{-ht}\left(A\cos\frac{cnt}{\sqrt{2}} + B\sin\frac{cnt}{\sqrt{2}}\right)\ .$$

Hence we obtain

$$Y = -\frac{\mathrm{e}^{-ht}}{2}\left\{\left(\frac{h}{n}A - \frac{c}{\sqrt{2}}B\right)\cos\frac{cnt}{\sqrt{2}} + \left(\frac{c}{\sqrt{2}}A + \frac{h}{n}B\right)\sin\frac{cnt}{\sqrt{2}}\right\}\ .$$

I will assume that for $t = 0$, i is $= 0$, hence also $Y = 0$; this condition gives

$$B = \frac{A}{\frac{nc}{h\sqrt{2}}}\ ;$$

The quantity n, as above remarked, shall be set equal to a multiple of π/l; hence the denominator of the expression for B will be a multiple of

$$\pi\cdot\frac{c}{hl\sqrt{2}}\ ;$$

but the quantity here multiplied by π is

$$= \frac{32\gamma}{rc\sqrt{2}}\ ;$$

that is, the precise quantity which has been assumed to be infinitely great. Hence B will be infinitely small in comparison with A, and we may set

$$X = A\cdot\mathrm{e}^{-ht}\cdot\cos\frac{cnt}{\sqrt{2}}\ ,$$

$$Y = -\frac{c}{2\sqrt{2}}A\mathrm{e}^{-ht}\sin\frac{cnt}{\sqrt{2}}\ .$$

Multiplying these expressions respectively by $\sin ns$ and $\cos ns$, and setting the products equal to e and i, we obtain a particular solution of the differential equations for e and i. This solution may be generalized by adding in it to s an arbitrary constant; we thus obtain

$$e = \mathrm{e}^{-ht}\cos\frac{cnt}{\sqrt{2}}\left(A\sin ns + A'\cos ns\right)\ ,$$

$$i = -\frac{c}{2\sqrt{2}} e^{-ht} \sin \frac{cnt}{\sqrt{2}} \left(A \cos ns - A' \sin ns \right) \ .$$

A particular solution of another form, which also satisfies the condition that for $t = 0$, i vanishes, is

$$e = a + bs \ ,$$

$$i = -\frac{c^2}{8h} b \left(1 - e^{-2ht}\right) \ ,$$

where a and b denote two arbitrary constants. That the two differential equations are satisfied by this is easily seen, by observing that by introducing the quantity h, one of them assumes the form[271]

$$2hi = -\left(\frac{c^2}{4} \frac{\partial e}{\partial s} + \frac{\partial i}{\partial t} \right) \ .$$

A solution is obtained which can be made to agree with the further conditions of the problem, where e and i are made equal to the sums of particular solutions of the forms stated.

We shall now examine more particularly the case in which the wire is one returning into itself. In this case e and i must have equal values for $s = 0$ and for $s = l$; and this must moreover take place whatever the origin of s may be; this requires that e and i are functions of s, which are periodic with regard to l; for this it is necessary that

$$b = 0 \qquad \text{and} \qquad n = m\frac{2\pi}{l} \ ,$$

where m denotes an integer. We have thus for e and i the following expressions:

$$e = e^{-ht} \sum_{m=1}^{\infty} A_m \cos m \frac{2\pi}{l} \frac{c}{\sqrt{2}} t \cdot \sin m \frac{2\pi}{l} s$$

$$+ \ a + e^{-ht} \sum_{m=1}^{\infty} A'_m \cos m \frac{2\pi}{l} \frac{c}{\sqrt{2}} t \cdot \cos m \frac{2\pi}{l} s \ ,$$

$$i = -\frac{c}{2\sqrt{2}} e^{-ht} \sum_{m=1}^{\infty} A_m \sin m \frac{2\pi}{l} \frac{c}{\sqrt{2}} t \cdot \cos m \frac{2\pi}{l} s$$

$$+ \ \frac{c}{2\sqrt{2}} e^{-ht} \sum_{m=1}^{\infty} A'_m \sin m \frac{2\pi}{l} \frac{c}{\sqrt{2}} t \cdot \sin m \frac{2\pi}{l} s \ .$$

[271] [Note by AKTA:] In the *Philosophical Magazine* the next equation appeared as, [Kir57a, p. 404]:

$$2hi = -\left(\frac{c^2}{4} \frac{\partial e}{\partial s} + \frac{\partial i}{\partial l} \right) \ .$$

The constants a, A, A' may be determined by the proposition of Fourier,[272] when for $t = 0$, e is given as a function of s. The solution may, however, be reduced to another form, which shows its characteristics more plainly.

For $t = 0$, let

$$e = f(s) \ .$$

Let the expressions under the sign of summation in e be modified according to the equations

$$\cos x \sin y = \frac{1}{2}\sin(y+x) + \frac{1}{2}\sin(y-x) \ ,$$

$$\cos x \cos y = \frac{1}{2}\cos(y+x) + \frac{1}{2}\cos(y-x) \ ,$$

$$\sin x \sin y = -\frac{1}{2}\cos(y+x) + \frac{1}{2}\cos(y-x) \ .$$

When it is considered that the function f is necessarily periodic with regard to l, we see that the expressions for e and i may be written as follows:

$$e = a + \frac{1}{2}\mathrm{e}^{-ht}\left[f\left(s + \frac{c}{\sqrt{2}}t\right) + f\left(s - \frac{c}{\sqrt{2}}t\right) - 2a\right] \ ,$$

$$i = -\frac{c}{4\sqrt{2}}\mathrm{e}^{-ht}\left[f\left(s + \frac{c}{\sqrt{2}}t\right) - f\left(s - \frac{c}{\sqrt{2}}t\right)\right] \ .$$

The quantity a is here determined by the equation

$$a = \frac{1}{l}\int_0^l f(s)ds \ ;$$

that is, la is the quantity of free electricity which the whole wire contains.

The expression for e shows a very remarkable analogy between the propagation of electricity in the wire, and the propagation of a wave in a tended wire or an elastic rod vibrating longitudinally. When $a = 0$, that is, when the total quantity of electricity $= 0$, the electricity resolves itself, if I may use the expression, into two waves of equal strength, which run in opposite directions through the wire with the velocity $c/\sqrt{2}$. Here the density of the electricity diminishes everywhere proportionally with e^{-ht}. This diminution, however, in comparison with the velocity of the waves, is very slow. The time required by both waves for a revolution is $l\sqrt{2}/c$, and hence the ratio of the electric densities at a point before and after the revolution is that of[273]

$$1 : \mathrm{e}^{-hl\sqrt{2}/c} \ .$$

This ratio differs from 1 by an infinitely small quantity, as the exponent of e, according to the assumption already made, is infinitely small. In comparison to velocities which come

[272][Note by AKTA:] Jean-Baptiste Joseph Fourier (1768-1830), [Fou22] with English translation in [Fou52].

[273][Note by AKTA:] Due to a misprint, the next equation appeared in the English translation as

$$1 : \mathrm{e} - \frac{hl\sqrt{2}}{c} \ .$$

within the range of our conceptions, the diminution of the density of electricity will certainly be always very speedy. If the wire were the standard wire of Jacobi, then $1/h$ would be very nearly the $\frac{1}{2\,000}$th of a second; and hence in this small time the electric density would diminish in the ratio of e : 1, that is, of 2.7 : 1.

When a is not $= 0$, or when the mean density of the electricity is not $= 0$, the expression for e shows that the excess of density over the average changes exactly as if the mean density were equal 0.

The velocity of propagation of an electric wave is here found to be $= c/\sqrt{2}$, hence it is independent of the cross section, of the conductivity of the wire, also, finally, of the density of the electricity: its value is 41 950 German miles in a second, hence very nearly equal to the velocity of light *in vacuo*.

When the wire is not one which returns into itself, the quantity b cannot be $= 0$, and the quantities n may have other values than in the case just considered. As regards the ends of the wire, certain equations are to be fulfilled according to the conditions to which the ends are subjected. If one end be insulated, at this end i must always be $= 0$; if the end be placed in complete connexion with the earth, the potential V, and also e for all values of t must here vanish. There is no difficulty in forming the expressions for e and i for the cases that both ends are insulated, both connected with the earth, or one of them insulated and the other connected with the earth. In all cases a reflexion of the wave occurs at the end at which it arrives. If the end is connected with the earth, a reversion of the wave accompanies its reflexion, that is, negative electricity proceeds from the end after it has been struck by positive; at an insulated end the reflexion takes place without reversion. Hence when the end is connected with the earth, it corresponds in some measure to the fixed end of a rod vibrating longitudinally; the insulated end, on the contrary, corresponds to the free end of the rod.

We shall enter more fully here into the consideration of another case. We shall examine how the electricity moves in the connecting wire of a galvanic battery before the current has become stationary. I will assume that the resistance of the battery is infinitely small in comparison with that of the wire connecting its poles, and that one of its poles stands in perfect connexion with the earth. With this pole let the commencement of the wire be connected, and with the other pole the end of the wire at the time $t = 0$. We may then assume that at the commencement of the wire, or for $s = 0$, the potential is always $= 0$; and at the end of the wire, or for $s = l$, it has a constant value depending upon the electromotive force of the circuit. When K denotes the electromotive force, this value must be $K/2$. The conditions to be satisfied by the expressions for e and i are therefore the following:

$$\begin{gathered}\text{for } s = 0 \text{ we must have } e = 0\ , \\ \text{for } s = l \text{ we must have } e = \tfrac{1}{4\gamma}K\ , \\ \text{for } t = 0 \text{ we must have } e = 0\ .\end{gathered}$$

On account of the first condition, we must have the quantities $A' = 0$ and also $a = 0$. As for $s = l$, e is to be independent of t, the quantities n must satisfy the condition

$$\sin nl = 0\ ;$$

that is, we must have

$$n = m\frac{\pi}{l}\ ,$$

where m denotes a whole number. Further, in order that for $s = l$, e shall have the required value, we must make

$$b = \frac{1}{4\gamma l} K \ .$$

Setting, for the sake of shortness,

$$\frac{\pi}{l} \frac{c}{\sqrt{2}} t = \tau$$

and

$$\frac{\pi}{l} s = \varphi \ ,$$

we obtain for e the equation

$$e = \frac{K}{4\gamma l} s + \mathrm{e}^{-ht} \sum_{m=1}^{\infty} A_m \cos m\tau \sin m\varphi \ .$$

The constants A may be determined by the last condition: according to this, for all values of φ between 0 and π we must have

$$\frac{K}{4\gamma\pi} \varphi = - \sum_{m=1}^{\infty} A_m \sin m\varphi \ .$$

But, by Fourier's proposition, between the same limits we have the following equation:

$$\varphi = -2 \sum_{m=1}^{\infty} (-1)^m \frac{1}{m} \sin m\varphi \ .$$

Hence we have to set

$$A_m = (-1)^m \frac{K}{4\gamma\pi} \frac{1}{m} \ ;$$

and we thus obtain

$$e = \frac{K}{4\gamma} \left\{ \frac{s}{l} + \frac{2}{\pi} \mathrm{e}^{-ht} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \cos m\tau \sin m\varphi \right\} \ .$$

If the corresponding expression be formed for i, remembering the equation by which it has been defined, we obtain

$$i = -\frac{K}{r} \left(1 - \mathrm{e}^{-2ht}\right) - \frac{cK}{4\sqrt{2}\gamma\pi} \mathrm{e}^{-ht} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \sin m\tau \cos m\varphi \ .$$

The meaning of these expressions shall now be developed; in the first place that of the expression for i. It is our chief object here to find the value of the summation which appears in the expression. We are to regard φ as a constant, and as a function of τ; this function is periodic as regards 2π; it has further opposite values for τ and $2\pi - \tau$; it is sufficient, therefore, to find the values through which it passes when τ lies between 0 and π. We have

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{m} \sin m\tau \cos m\varphi = \frac{1}{2} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \sin m(\tau+\varphi) + \frac{1}{2} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \sin m(\tau-\varphi) \ .$$

But when x lies between $-\pi$ and $+\pi$ we have the sum

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{m} \sin mx = -\frac{x}{2} \ ;$$

and because it is periodic as regards 2π, it is in general

$$= -\frac{1}{2}(x - 2p\pi) \ ,$$

where p denotes that whole number, for which $x - 2p\pi$ lies between $-\pi$ and $+\pi$. With the limits which have been assumed for τ, $\tau - \varphi$ lies always between $-\pi$ and $+\pi$, because for all points of the wire the value of φ is between 0 and π. Hence we have

$$\sum \frac{(-1)^m}{m} \cdot \sin m(\tau-\varphi) = \frac{\tau-\varphi}{2} \ .$$

With regard to the value of $\tau+\varphi$, this can be either greater or less than π. We have[274]

$$\sum \frac{(-1)^m}{m} \sin m(\tau+\varphi) = -\frac{\tau+\varphi}{2} \ , \qquad \text{when} \qquad \varphi < \pi - \tau \ ,$$

$$= -\frac{\tau+\varphi}{2} + \pi, \qquad \text{when} \qquad \varphi > \pi - \tau \ .$$

From this it follows that

$$\sum \frac{(-1)^m}{m} \cos m\tau \cos m\varphi = -\frac{\tau}{2} \ , \qquad \text{when} \qquad \varphi < \pi - \tau \ ,$$

$$= -\frac{\tau}{2} + \frac{\pi}{2} \ , \qquad \text{when} \qquad \varphi > \pi - \tau \ .$$

It is here supposed that τ lies between 0 and π; if it lies between π and 2π, we have the same sum,

$$= \pi - \frac{\tau}{2} \ , \qquad \text{when} \qquad \varphi < \tau - \pi \ ,$$

and

$$= \frac{\pi}{2} - \frac{\tau}{2} \ , \qquad \text{when} \qquad \varphi > \tau - \pi \ .$$

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[274][Note by AKTA:] Due to a misprint, the following equation appeared in the original text as:

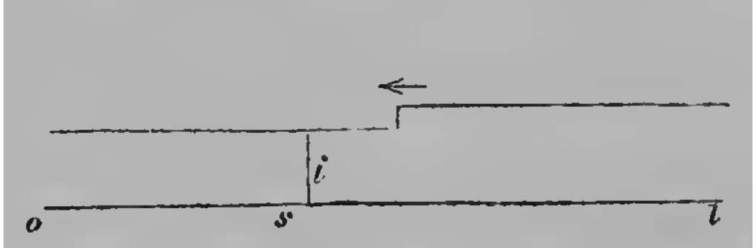
$$\sum \frac{(-1)}{m} \sin m(\tau+\psi) = -\frac{\tau+\varphi}{2} \ , \qquad \text{when} \qquad \varphi < \pi - \tau \ ,$$

$$= -\frac{\tau+\varphi}{2} + \pi, \qquad \text{when} \qquad \varphi > \pi - \tau \ .$$

To find the sum for greater values of τ, it is to be remembered that it is periodic as regards 2π.

From this it appears, that at every moment a point exists in the wire in which the intensity of the current suffers a sudden change or break. This point, at the time $t = 0$, lies at the end of the wire, but moves from this with the velocity $c/\sqrt{2}$ towards the commencement, after reaching which it returns with the same velocity towards the end; turns here again, and thus travels perpetually to and fro over the length of the wire. In each of the two portions into which the wire is at each moment divided by this point, the same intensity exists everywhere at that moment; so that if s and i be regarded as rectangular coordinates of a point, a line is described of the form of Figure 1 .

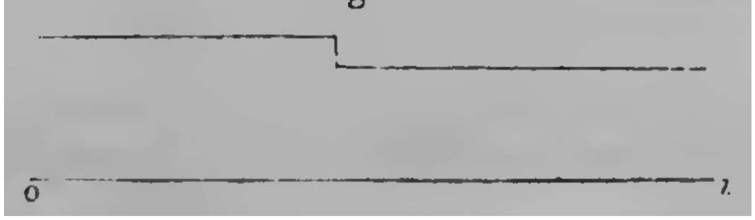
Fig. 1.



The intensity before the point at which the break occurs, considered without regard to its sign, is always the smaller, that behind the point the greater, the words *before* and *behind* being used with reference to the direction in which the point moves. The Figure 1 is therefore true only for a moment in which the point moves from the end towards the commencement of the wire.

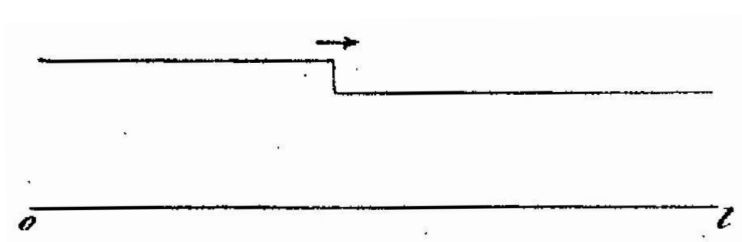
The Figure 2 refers to a moment in which the opposite takes place.[275]

Fig. 2.



The magnitude of the break is

[275][Note by AKTA:] The arrow pointing to the right was not included in Figure 2 of the English translation appearing in the Philosophical Magazine. The original Figure of the *Annalen der Physik* including the arrow appears in this footnote.



$$= \frac{cK}{8\sqrt{2}\gamma}\mathrm{e}^{-ht} ;$$

or if we denote by J the value to which i approximates as the time is increased, that is, the value of K/r,

$$= J \cdot \frac{cr}{8\sqrt{2}\gamma}\mathrm{e}^{-ht} .$$

This quantity has its greatest value when $t = 0$; but this, in accordance with an assumption already made, is also infinitely small in comparison to J. The expression for the magnitude of the break may be more shortly written, when the time is introduced required by the point at which it takes place, or the time required by an electric wave to move through the length of the wire. Denoting this time by T, that is, setting

$$T = \frac{l\sqrt{2}}{c} ,$$

the expression is easily found to be

$$= J \cdot 2hT\mathrm{e}^{-ht} .$$

As the time increases, the magnitude of the break diminishes, but so slowly that during the time T only an infinitely small diminution takes place.

To obtain a complete view of the process, it is now only necessary to examine the alterations of the strength of the current at the commencement of the wire. Let this, that is to say, the value of i for $s = 0$, be i_0; then making use of the symbols J and T, we find

$$i_0 = J\left(1 - \mathrm{e}^{-2ht}\right) + \frac{J4hT}{\pi}\mathrm{e}^{-ht}\sum_{m=1}^{\infty}\frac{(-1)^m}{m}\sin m\tau .$$

Setting for the sum its value, and remembering that

$$\frac{\tau}{\pi} = \frac{t}{T} ,$$

we obtain

$$i_0 = J\left(1 - \mathrm{e}^{-2ht}\right) + J2h\mathrm{e}^{-ht}(2pT - t) ,$$

where p denotes the whole number for which

$$\frac{t - 2pT}{T}$$

is a proper fraction, positive or negative. p may also be defined as the greatest integer which is contained in the fraction

$$\frac{t + T}{2T} .$$

For values of t, for which the number p is not very great, the expression for i_0 is capable of a considerable simplification. For such the quantity ht is infinitely small; and by neglecting members of higher orders, the equation for i_0 may be thus written:

$$i_0 = J \cdot 2ht + J2h(2pT - t) \ ,$$

that is,

$$i_0 = pJ4hT \ .$$

This expression shows that the intensity at the commencement of the wire is 0 up to the time when $t = T$; here and at the times $t = 3T$, $t = 5T$, etc., it alters itself by jumps; and moreover the jump is twice as great as at other points of the wire. During the intervening times the intensity is constant.

In a similar manner the expression for e may be discussed. We have[276]

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{m} \cos m\tau \sin m\varphi = \frac{1}{2} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \sin m(\tau + \varphi) - \frac{1}{2} \sum_{m=1}^{\infty} \frac{(-1)}{m} \sin m(\tau - \varphi) \ ;$$

or as soon as τ lies between 0 and π,

$$= -\frac{\varphi}{2} \ , \qquad \text{when} \qquad \varphi < \pi - \tau \ ,$$

$$= -\frac{\varphi}{2} + \frac{\pi}{2} \ , \qquad \text{when} \qquad \varphi > \pi - \tau \ ;$$

if τ lies between π and 2π, we have the same sum,

$$= -\frac{\varphi}{2} \ , \qquad \text{when} \qquad \varphi < \tau - \pi \ ,$$

$$= -\frac{\varphi}{2} + \frac{\pi}{2} \ , \qquad \text{when} \qquad \varphi > \tau - \pi \ .$$

The second fact follows from the first, when it is considered that the sum has the same value for τ and for $2\pi - \tau$. For greater values of τ, the value of the sum is found when we remember that it is periodic with reference to 2π.

From this it follows that at each moment at some one point of the wire, e also suffers a break. This point always coincides with that in which the break for i takes place. e is always greater on the side of this point on which the end of the wire lies, and smaller on the side of the commencement. The magnitude of the break is

$$= \frac{K}{4\gamma} \mathrm{e}^{-ht} \ ;$$

or, denoting by E the constant value of e at the end of the wire,

$$= E\mathrm{e}^{-ht} \ .$$

At that side of the break on which the commencement of the wire lies, we have

[276][Note by AKTA:] In the *Philosophical Transactions* this equation appeared as

$$\sum_{m=1}^{\infty} \frac{(-1)^m}{m} \cos m\tau \sin m\phi = \frac{1}{2} \sum_{m=1}^{\infty} \frac{(-1)^m}{m} \sin m(\tau + \phi) \ .$$

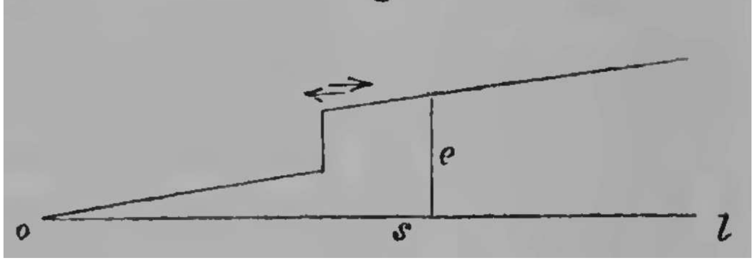
$$e = E \cdot \frac{s}{l}\left(1 - \mathrm{e}^{-ht}\right) \;;$$

and on the side towards the end,

$$e = E\left\{\frac{s}{l}\left(1 - \mathrm{e}^{-ht}\right) + \mathrm{e}^{-ht}\right\} \;.$$

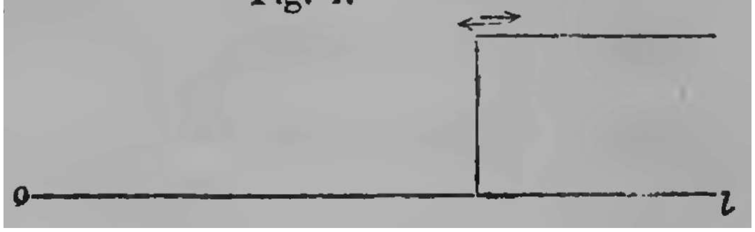
If e and s be made the rectangular coordinates of a point, then for a certain value of t we obtain a line of the form shown in Figure 3.

Fig. 3.



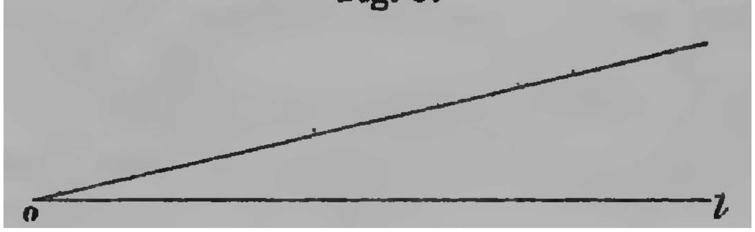
When t does not exceed a moderate multiple of T, the line has the form shown in Figure 4.

Fig. 4.



The more t increases, the more nearly does the Figure approximate to the straight line, Figure 5.

Fig. 5.



Chapter 9

Editor's Comments on Kirchhoff's 1857 Paper on the Motion of Electricity in Wires

A. K. T. Assis[277]

9.1 Solution of an Integral Appearing in Kirchhoff's Paper

In this paper Kirchhoff arrived at the following Equation (see page 203):

$$\frac{e}{2\pi}\int_{-\varepsilon}^{+\varepsilon}\int_{0}^{2\pi}\frac{dx'd\psi'}{\sqrt{x'^2+\alpha^2+\rho^2-2\alpha\rho\cos(\psi'-\psi)}} . \tag{9.1}$$

He said (see page 203) that the integration in x' of this Equation has the following solution when $\varepsilon \gg \alpha > 0$ and $\varepsilon \gg \rho \geq 0$:

$$\frac{e}{\pi}\int_{0}^{2\pi}d\psi'\left(\lg 2\varepsilon - \lg\sqrt{\alpha^2+\rho^2-2\alpha\rho\cos(\psi'-\psi)}\right) . \tag{9.2}$$

Replacing Kirchhoff's "lg" with the natural logarithm of base e = 2.718..., "ln", yields:

$$\frac{e}{\pi}\int_{0}^{2\pi}d\psi'\left(\ln 2\varepsilon - \ln\sqrt{\alpha^2+\rho^2-2\alpha\rho\cos(\psi'-\psi)}\right) . \tag{9.3}$$

Let me show in detail how to arrive at Kirchhoff's result. I will call $\alpha^2+\rho^2-2\alpha\rho\cos(\psi'-\psi) = a^2 \geq 0$. The following integral needs to be solved:

$$I = \int_{-\varepsilon}^{\varepsilon}\frac{dx'}{\sqrt{x'^2+a^2}} . \tag{9.4}$$

To this end I utilize that

$$\int\frac{dx'}{\sqrt{x'^2+a^2}} = \ln|x'+\sqrt{x'^2+a^2}| + C , \tag{9.5}$$

[277] Homepage: www.ifi.unicamp.br/~assis

where C is an arbitrary constant. I then obtain:

$$I = \ln|\varepsilon + \sqrt{\varepsilon^2 + a^2}| - \ln|-\varepsilon + \sqrt{\varepsilon^2 + a^2}| = \ln\frac{\sqrt{\varepsilon^2 + a^2} + \varepsilon}{\sqrt{\varepsilon^2 + a^2} - \varepsilon} \ . \tag{9.6}$$

Multiplying the numerator and denominator by $\sqrt{\varepsilon^2 + a^2} + \varepsilon$ yields:

$$I = \ln\frac{(\sqrt{\varepsilon^2 + a^2} + \varepsilon)^2}{a^2} = \ln\frac{(\sqrt{\varepsilon^2 + a^2} + \varepsilon)^2}{(\sqrt{a^2})^2} = 2\ln\frac{\sqrt{\varepsilon^2 + a^2} + \varepsilon}{\sqrt{a^2}} \ . \tag{9.7}$$

Assuming that $\varepsilon^2 \gg a^2$ yields

$$I = 2\ln\frac{2\varepsilon}{\sqrt{a^2}} = 2\left[\ln(2\varepsilon) - \ln\sqrt{a^2}\right] = 2\left[\ln(2\varepsilon) - \ln\sqrt{\alpha^2 + \rho^2 - 2\alpha\rho\cos(\psi' - \psi)}\right] \ . \tag{9.8}$$

This is the result presented by Kirchhoff. This deduction shows clearly that the symbol "lg" which he presented in his original paper[278] should be understood as the natural logarithm represented in modern textbooks as "ln".

9.2 The Telegraph Equation Obtained by Kirchhoff

In his work Kirchhoff arrived at his Equation (4) on page 207:

$$2\frac{\partial i}{\partial s} = -\frac{\partial e}{\partial t} \ , \tag{9.9}$$

He also arrived (see page 209) at another equation which he might have numbered as Equation (5), namely:

$$i = -4\gamma\frac{l}{r}\left(\frac{\partial e}{\partial s} + \frac{4}{c^2}\frac{\partial i}{\partial t}\right) \ . \tag{9.10}$$

The partial derivative of Equation (9.9) with respect to t and the partial derivative of equation (9.10) with respect to s yield, respectively:

$$2\frac{\partial^2 i}{\partial t\partial s} = -\frac{\partial^2 e}{\partial t^2} \ , \tag{9.11}$$

and

$$\frac{\partial i}{\partial s} = -4\gamma\frac{l}{r}\left(\frac{\partial^2 e}{\partial s^2} + \frac{4}{c^2}\frac{\partial^2 i}{\partial s\partial t}\right) \ . \tag{9.12}$$

I now assume well behaved functions such that

$$\frac{\partial^2 i}{\partial t\partial s} = \frac{\partial^2 i}{\partial s\partial t} \ . \tag{9.13}$$

Finally, by combining Equations (9.11), (9.12) and (9.13), the complete telegraph equation is obtained, namely:

[278] [Kir57b, p. 196 of the *Annalen der Physik und Chemie* or p. 134 of his *Gesammelte Abhandlungen*].

$$\frac{\partial^2 e}{\partial s^2} - \frac{2}{c^2}\frac{\partial^2 e}{\partial t^2} = \frac{r}{8\gamma l}\frac{\partial e}{\partial t} \ . \tag{9.14}$$

By following the same procedure but now beginning with the partial derivative of Equation (9.9) with respect to s and the partial derivative of Equation (9.10) with respect to t yields:

$$\frac{\partial^2 i}{\partial s^2} - \frac{2}{c^2}\frac{\partial^2 i}{\partial t^2} = \frac{r}{8\gamma l}\frac{\partial i}{\partial t} \ . \tag{9.15}$$

Kirchhoff also obtained that $V = 2e \ln \frac{l}{\alpha}$. This means that the potential of the free electricity, V, will also satisfy the same equation, namely:

$$\frac{\partial^2 V}{\partial s^2} - \frac{2}{c^2}\frac{\partial^2 V}{\partial t^2} = \frac{r}{8\gamma l}\frac{\partial i}{\partial t} \ . \tag{9.16}$$

Equations (9.14), (9.15) and (9.16) are the complete telegraph equations for the linear charge density e, for the current intensity i and for the potential of the free electricity V. These equations were obtained by taking into account the resistance of the wire, the electromotive force due to the free electricity spread over the whole surface of the wire, and the induction which takes place in consequence of the alteration of the strength of the current in all parts of the wire. In modern terminology, Kirchhoff deduced the complete telegraph equation by taking into account not only the resistance and capacitance of the wire, but especially its self-inductance. All of this was accomplished beginning with Weber's electrodynamics.

For a wire of negligible resistance, that is, when the right side of Equations (9.14), (9.15) and (9.16) go to zero, they reduce to the the wave equation with the signal propagating along the wire with velocity $c/\sqrt{2}$:

$$\frac{\partial^2 e}{\partial s^2} - \frac{1}{(c/\sqrt{2})^2}\frac{\partial^2 e}{\partial t^2} = \frac{\partial^2 i}{\partial s^2} - \frac{1}{(c/\sqrt{2})^2}\frac{\partial^2 i}{\partial t^2} = \frac{\partial^2 V}{\partial s^2} - \frac{1}{(c/\sqrt{2})^2}\frac{\partial^2 V}{\partial t^2} = 0 \ . \tag{9.17}$$

Weber's constant c was first measured by Weber and Kohlrausch,[279] namely, $c = 4.39 \times 10^{11}\ mm/s$. This yields:

$$\frac{c}{\sqrt{2}} = \frac{4.39 \times 10^{11}\ mm/s}{\sqrt{2}} = 3.1 \times 10^8\ \frac{m}{s} \ . \tag{9.18}$$

This number has the same value of light velocity in vacuum. For this reason Kirchhoff concluded on page 214 that:

> The velocity of propagation of an electric wave is here found to be $= c/\sqrt{2}$, hence it is independent of the cross section, of the conductivity of the wire, also, finally, of the density of the electricity: its value is 41 950 German miles in a second, hence very nearly equal to the velocity of light *in vacuo*.

[279] [KW57, p. 264 of the *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften, mathematisch-physische Klasse* or p. 652 of Weber's *Werke*] and [KW21, p. 52]. See also page 179 on Section 7.17.

Representing light velocity in vacuum by $v_L = c/\sqrt{2}$, Equation (9.17) can then be written as:

$$\frac{\partial^2 e}{\partial s^2} - \frac{1}{v_L^2}\frac{\partial^2 e}{\partial t^2} = \frac{\partial^2 i}{\partial s^2} - \frac{1}{v_L^2}\frac{\partial^2 i}{\partial t^2} = \frac{\partial^2 V}{\partial s^2} - \frac{1}{v_L^2}\frac{\partial^2 V}{\partial t^2} = 0 \ . \tag{9.19}$$

This was a remarkable result indicating a deep connection between electrodynamics and optics, as first obtained by Weber's electrodynamics.

In the sequence of this first paper of 1857 Kirchhoff analysed the solutions of Equations (9.14), (9.15) and (9.16) not only in this particular case of negligible resistance, but also by taking into account the resistance of the wire. He considered open and closed wires, see page 209 and the following.

A deduction of the complete telegraph equation from Weber's electrodynamics in different geometries (cylindrical wire, coaxial cable etc.) utilizing the modern International System of Units MKSA has been discussed elsewhere.[280]

280 [Ass99], [Ass00], [HA00], [HA01], [Ass03a], [AH05], [AH07], [AH09], [AH13], [Ass14a] and [Ass19a].

Chapter 10

[Poggendorff, 1857] Comment on the Paper by Prof. Kirchhoff

Johann Christian Poggendorff[281,282,283,284,285]

Allow me to add the remark concerning the paper on page 193 of this issue,[286] that when I spoke to Professor W. Weber, during his recent stay in Berlin, about Professor Kirchhoff's investigations, Professor Weber showed me a complete treatise on the same subject elaborated by him, which, however, he did not yet intend to submit for printing, because he first wanted to have the results of an experimental investigation that he had undertaken jointly with R. Kohlrausch on this subject.[287] Professor Kirchhoff's visit to Berlin a few days later gave him the opportunity to comment on the coincidence of their results — a coincidence which can be called a pleasant one, as both works, starting from essentially the same basis, have led to identical results. This identity certainly deserves special attention in the case of a subject so little researched as the laws of current formation have been so far.

[281] [Pog57] with English translation in [Pog21].

[282] Translated and edited by A. K. T. Assis, `www.ifi.unicamp.br/~assis`. I thank Laurence Hecht for relevant suggestions.

[283] The Notes by H. Weber, the Editor of the fourth volume of Wilhelm Weber's *Werke*, are represented by [Note by HW:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[284] [Note by HW:] The above remark by J. C. Poggendorff, to which W. Weber refers in the paper on page 130, has been included here because of its historical interest.

[285] [Note by AKTA:] Poggendorff's paper is related to Kirchhoff's paper of 1857 and to Weber's work of 1864, see Chapters 8 and 18. Weber will mention Poggendorff's paper on page 289, Section 18.6 of his Fifth major Memoir on Electrodynamic Measurements, [Web64, p. 130 of Weber's *Werke*].

[286] [Note by AKTA:] Poggendorff is referring to Kirchhoff's paper of 1857 published in page 193 of Volume 100 of the *Annalen der Physik und Chemie*, [Kir57b], see Chapter 8.

[287] [Note by AKTA:] See footnote 132 on page 125.

Chapter 11

Editor's Comments on Poggendorff's 1857 Paper

A. K. T. Assis[288]

Wilhelm Weber and Carl Friedrich Gauss (1777-1855) invented in 1833 the world's first operational electromagnetic telegraph.[289] It was a 3 km long twin lead connecting Göttingen University, where Weber was Professor of Physics, with the Astronomical Observatory (*Sternwarte*), directed by Gauss. This telegraph worked based on Faraday's law of induction discovered two years earlier.[290] A representation of their telegraph appears in Figure 11.1.

Twenty four years later, in 1857, Weber and Kirchhoff were the first to derive theoretically the complete telegraph equation. As pointed out by Poggendorff in the present paper, they worked independently from one another and arrived simultaneously at the same result. Utilizing the modern concepts and usual terminology of circuit theory, we can say that they were the first to take into account not only the capacitance and resistance of the wire, but also its self-inductance. Both of them worked with Weber's electrodynamics.

Johann Christian Poggendorff (1796-1877) edited the *Annalen der Physik und Chemie* from 1824 to 1876, where many of Weber and Kirchhoff's papers were published. The modern *Annalen der Physik* is the successor to this Journal. Kirchhoff's 1857 paper was published in this Journal. Poggendoff's comment on this paper was published in the same Volume of Kirchhoff's work.

Rudolf Kohlrausch (1809-1858) collaborated with Weber on the first measurement of Weber's fundamental constant c, which should not be confused with the modern constant c appearing in the textbooks. Weber's constant c is written in the modern International System of Units MKSA as $\sqrt{2}/\sqrt{\mu_o \varepsilon_o}$, while the modern constant c is written as $1/\sqrt{\mu_o \varepsilon_o}$. Their experiment was performed in 1854-1855 and they published three works on this subject in 1855, 1856 and 1857, see Chapters 5, 6 and 7. They obtained the value of Weber's constant c as given by 4.39450×10^8 m/s, see page 179 on Section 7.17. Therefore

[288] Homepage: www.ifi.unicamp.br/~assis

[289] [LB67, Section 66: Gauss and Weber's telegraph, pp. 41-42], [Ano89], [Fey33a], [Fey33b], [Wie60, Chapter 5, pp. 17-20], [Wie67, pp. 85-90], [Tim05], [Wol05], [MRGL10] and https://www.uni-goettingen.de/de/historische-sammlung/47114.html.

[290] [Far32a] with German translation in [Far32b] and [Far89], and with Portuguese translation in [Far11].

Figure 11.1: The telegraph of Gauss and Weber.

$$\frac{c}{\sqrt{2}} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = \frac{4.39450 \times 10^8 \ m/s}{\sqrt{2}} = 3.1 \times 10^8 \ m/s = v_L \ . \tag{11.1}$$

That is, $c/\sqrt{2}$ has essentially the same value as light velocity in vacuum, v_L.

Weber and Kirchhoff deduced the telegraph equation utilizing Weber's 1846 force law between electrified particles.[291] They showed, in particular, that when the conductor had negligible resistance, the velocity of propagation of an electric wave is very nearly equal to the velocity of light in vacuum. This result indicated a direct connection between electromagnetism and optics, as Kirchhoff pointed out in the paper which is being discussed here (see page 214). This result of Kirchhoff and Weber was obtained several years before Maxwell (1831-1879).

Kohlrausch, who was collaborating with Weber on some experiments related with the propagation of electromagnetic waves, died in 1858. Weber's work has been delayed in publication and appeared only in 1864. He compared his results with those of Kirchhoff and mentioned Poggendorff's paper on Section 6 of his paper.[292]

[291] [Web46] with English translation in [Web07].

[292] [Web64, Section 6, pp. 130-132 of Weber's *Werke*] with English translation in [Web21d, Section 6]. See page 289 of Section 18.6 of this book.

Chapter 12

[Kirchhoff, 1857b] On the Motion of Electricity in Conductors

Gustav Kirchhoff[293,294]

In an earlier paper[295,296] I developed a theory of the motion of electricity in linear conductors. I will now show how the former considerations can be generalized to conductors of any form.

The Cartesian coordinates x, y, z locate a point in the conductor. The current which at time t flows through this point we resolve along the three coordinate axes to give the current density components u, v, w. These current densities have to be equal to the products of the components of the electromotive force[297] and electrical conductivity at point $(x,\ y,\ z)$ and are assumed to involve one unit of electrical charge. The electromotive force is partly due to the presence of free electricity, and partly due to induction which arises in all parts of the conductor because of changes in the current. If Ω represents the potential function of the free electricity relative to the point $(x,\ y,\ z)$, then the component of the first part of the electromotive force are

$$-2\frac{\partial\Omega}{\partial x}\ , \qquad -2\frac{\partial\Omega}{\partial y}\ , \qquad -2\frac{\partial\Omega}{\partial z}\ .$$

In order to derive the components of the second part, I denote the coordinates of a second point of the conductor by x', y', z', while u', v', w' are the values of u, v, w for this point. Let r be the distance between the points $(x,\ y,\ z)$ and $(x',\ y',\ z')$ and write:

$$U = \int\int\int \frac{dx'dy'dz'}{r^3}\,(x-x')\,[u'(x-x')+v'(y-y')+w'(z-z')]\ ,$$

$$V = \int\int\int \frac{dx'dy'dz'}{r^3}\,(y-y')\,[u'(x-x')+v'(y-y')+w'(z-z')]\ ,$$

[293][Kir57c] with English translation by the late Peter Graneau (1921-2014) in [GA94]. See also [Ass14b].
[294]Gustav Kirchhoff's Notes are represented by [Note by GK:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].
[295][Note by GK:] This *Annalen* Vol. 100, p. 193.
[296][Note by AKTA:] [Kir57b] with English translation in [Kir57a]. See Chapter 8.
[297][Note by AKTA:] In German: *elektromotorischen Kraft*. In English: "electromotive force," abbreviated *emf*.

$$W = \int\int\int \frac{dx'dy'dz'}{r^3}(z - z')\left[u'(x - x') + v'(y - y') + w'(z - z')\right] ,$$

where the integrations extend over all of the volume of the conductor. According to Weber's law of induction,[298] the components of the second part of the electromotive force under consideration are:

$$-\frac{8}{c^2}\frac{\partial U}{\partial t} , \qquad -\frac{8}{c^2}\frac{\partial V}{\partial t} , \qquad -\frac{8}{c^2}\frac{\partial W}{\partial t} ,$$

where c is the constant velocity with which two electric charges have to move toward each other so that they will not exert a force on each other. If k is the conductivity of the conductor, we have

$$u = -2k\left(\frac{\partial\Omega}{\partial x} + \frac{4}{c^2}\frac{\partial U}{\partial t}\right) , \qquad (1)$$

$$v = -2k\left(\frac{\partial\Omega}{\partial y} + \frac{4}{c^2}\frac{\partial V}{\partial t}\right) , \qquad (2)$$

$$w = -2k\left(\frac{\partial\Omega}{\partial z} + \frac{4}{c^2}\frac{\partial W}{\partial t}\right) . \qquad (3)$$

It must not be assumed that the free electricity is confined to the surface of the conductor, as in equilibrium cases or at constant current. In fact, it will be shown that, in general, the opposite is true. I denote by ε the [volume] density of free electricity at point $(x,\ y,\ z)$, by ε' the density at $(x',\ y',\ z')$, by e the [surface] density in a surface element dS, and by e' the same for a second surface element dS'. Then we have:

$$\Omega = \int \frac{dx'dy'dz'}{r}\varepsilon' + \int \frac{dS'}{r}e' , \qquad (4)$$

where the first integration is over the volume, and the second over the surface of the conductor.

To these equations we can add two more which deal with the time changes of the density of free electricity. For every point inside the conductor we have therefore:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{2}\frac{\partial\varepsilon}{\partial t} ; \qquad (5)$$

and if we denote the normal to element dS directed inward by N, then further for every point of the surface:

$$u\cos(N, x) + v\cos(N, y) + w\cos(N, z) = -\frac{1}{2}\frac{\partial e}{\partial t} . \qquad (6)$$

From these equations we can derive a remarkable relationship between ε and Ω. Substituting the values of u, v, w from (1), (2), and (3) into (5), and using:

[298][Note by AKTA:] [Web46, p. 354 of Weber's original 1846 paper and pp. 185-189 of Weber's *Werke*], [Web07, pp. 120-122]; [KW57, p. 268 of the *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften, mathematisch-physische Klasse* or pp. 655-657 of Weber's *Werke*] and [KW21, pp. 55-58].

$$\frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial y^2} + \frac{\partial^2\Omega}{\partial z^2} = -4\pi\varepsilon \ ,$$

one finds

$$\frac{\partial\varepsilon}{\partial t} = -16k\left[\pi\varepsilon - \frac{1}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}\right)\right] \ .$$

As the equation for U may be written:

$$U = -\int dx'dy'dz'\frac{\partial\frac{1}{r}}{\partial x}\left[u'(x-x') + v'(y-y') + w'(z-z')\right] \ ,$$

it follows that:

$$\frac{\partial U}{\partial x} = -\int dx'dy'dz'\frac{\partial\frac{1}{r}}{\partial x}u' - \int dx'dy'dz'\frac{\partial^2\frac{1}{r}}{\partial x^2}\left[u'(x-x') + v'(y-y') + w'(z-z')\right] \ .$$

Forming the value of $\partial V/\partial y$ and $\partial W/\partial z$ in a similar manner, one obtains:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = -\int dx'dy'dz'\left(u'\frac{\partial\frac{1}{r}}{\partial x} + v'\frac{\partial\frac{1}{r}}{\partial y} + w'\frac{\partial\frac{1}{r}}{\partial z}\right) \ ;$$

because of:

$$\frac{\partial^2\frac{1}{r}}{\partial x^2} + \frac{\partial^2\frac{1}{r}}{\partial y^2} + \frac{\partial^2\frac{1}{r}}{\partial z^2} = 0$$

for all points $(x', \ y', \ z')$ which do not coincide with point $(x, \ y, \ z)$; and extend through the infinitely small volume surrounding point $(x, \ y, \ z)$, the integrals of the second parts of $\partial U/\partial x$, $\partial V/\partial y$, $\partial W/\partial z$ are infinitely small. It is easy to convince ourselves of the validity of this last assumption by the method which Gauss used to prove that the contribution to the potential at a point by masses infinitely near to the point is negligible compared to the contribution from continuously distributed matter throughout space.[299,300] If in the integral on the right side of the equation the differential coefficients with respect to x, y, z, are replaced with the negative coefficients with respect to x', y', z' and the result is divided in three partial differentials with respect to x', y' and z', one obtains:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}$$

$$= -\int\frac{dS'}{r}\left[u'\cos(N', x) + v'\cos(N', y) + w'\cos(N', z)\right] - \int\frac{dx'dy'dz'}{r}\left(\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'}\right) \ ;$$

where N' is the inward directed normal of the surface element dS'. In view of equations (6), (5) and (4), this equation may be written:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = \frac{1}{2}\frac{\partial\Omega}{\partial t} \ .$$

[299] [Note by GK:] Resultate aus den Beobachtungen des magnetischen Vereins; 1839 p. 7.

[300] [Note by AKTA:] [Gau40, p. 7 of the *Resultate*] with English translation in [Gau43, pp. 158-159].

From this it follows that:

$$\frac{\partial \varepsilon}{\partial t} = -8k \left(2\pi\varepsilon - \frac{1}{c^2} \frac{\partial^2 \Omega}{\partial t^2} \right) . \qquad (7)$$

This equation shows clearly that $\varepsilon = 0$ is a special case, and in general we find free electricity inside of conductors. It is probable that the so called mechanical actions of the discharge current of a Leyden jar, as for example in the pulverization of a fine wire, the internal free electricity plays an important role.

I would like to apply the theory developed here to the case considered in the initially mentioned paper, *i.e.* the case in which the conductor is an infinitely thin wire with no electrical bodies in its vicinity. I will show that the theory furnishes the same results which I obtained previously, and in addition it supplies answers to questions which so far remained unanswered.

To begin with I will simplify the general equation by the assumption that the conductor is cylindrical of circular cross-section, and that the current, as well the distribution of free electricity, is symmetrical about the axis. I take the axis as the x-direction, and for y and z I introduce the new coordinates ρ and φ, so that:

$$y = \rho \cos\varphi \ , \qquad z = \rho \sin\varphi \ ,$$

and correspondingly:

$$y' = \rho' \cos\varphi' \ , \qquad z' = \rho' \sin\varphi' \ .$$

Furthermore, I denote the current density, perpendicular to the current along the axis — positive for the progressive direction of the axis — at point $(x,\ y,\ z)$ by σ, and at point $(x',\ y',\ z')$ by σ'. We then have:

$$v = \sigma \cos\varphi \ , \qquad w = \sigma \sin\varphi \ ,$$

$$v' = \sigma' \cos\varphi' \ , \qquad w' = \sigma' \sin\varphi' \ .$$

Hence:

$$u = -2k \left(\frac{\partial \Omega}{\partial x} + \frac{4}{c^2} \frac{\partial U}{\partial t} \right) , \qquad (8)$$

where[301]

$$U = \int \frac{dx'\rho' d\rho' d\varphi'}{r^3} (x - x') \left[u'(x - x') + \sigma' \left(\rho \cos(\varphi - \varphi') - \rho' \right) \right] . \qquad (9)$$

[301] [Note by AKTA:] Due to a misprint, the original text presented the following equation:

$$U = \int \frac{dx'\rho' d\rho' d\varphi'}{r^3} (x - x') \left[u'(x - x') + \sigma' \left(\rho \cos\varphi - \varphi' - \rho' \right) \right] . \qquad (9)$$

If we ignore the action of the free electricity on the end-faces of the cylinder, then, with α being the radius of the cylinder, equation (4) may be written:

$$\Omega = \int \frac{dx'\rho' d\rho' d\varphi'}{r}\varepsilon' + \alpha \int \frac{dx' d\varphi'}{r} e' \ . \qquad (10)$$

Equation (5) becomes:

$$\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial \rho\sigma}{\partial \rho} = -\frac{1}{2}\frac{\partial \varepsilon}{\partial t} \ ; \qquad (11)$$

and equation (6), which refers to the surface, becomes:

$$\sigma = \frac{1}{2}\frac{\partial e}{\partial t} \ . \qquad (12)$$

The expressions for Ω and U are greatly simplified if it is assumed that the cross-section of the cylinder is infinitely small, while the wire is of finite length. I call this length l, and the origin of the coordinates is taken to be the middle of the cylinder. The limits of the integrations in the x'-direction are then $-l/2$ and $+l/2$. For brevity I will take:

$$x' - x = \xi \ ;$$

for dx' the integrand may then be written $d\xi$. The integration along ξ then has the limits $-l/2 - x$ and $l/2 - x$, of which the first one is always negative and the second one is always positive. The quantity r of the integrals is determined by the equation:

$$r^2 = \xi^2 + \beta^2 \ ,$$

where

$$\beta^2 = \rho^2 + \rho'^2 - 2\rho\rho' \cos(\varphi - \varphi') \ .$$

For the transformation of the second part of Ω in the integral:[302]

$$\int_{-\frac{l}{2}-x}^{\frac{l}{2}-x} \frac{d\xi e'}{\sqrt{\beta^2 + \xi^2}} \ ,$$

I will develop e' according to Taylor's theorem[303] in powers of ξ, that is:

$$e' = e + \frac{\partial e}{\partial x}\xi + \frac{\partial^2 e}{\partial x^2}\frac{\xi^2}{1 \cdot 2} + ... \ ;$$

the individual terms into which the integral has been split then take the form:

$$\frac{1}{1 \cdot 2 \cdots n}\frac{\partial^n e}{\partial x^n}\int \frac{\xi^n d\xi}{\sqrt{\beta^2 + \xi^2}} \ .$$

But we have:

[302][Note by AKTA:] Due to a misprint, the original text presented the following equation:

$$\int_{-\frac{l}{x}-2}^{\frac{l}{2}-x} \frac{d\xi e'}{\sqrt{\beta^2 + \xi^2}} \ .$$

[303][Note by AKTA:] This theorem is named after the mathematician Brook Taylor (1685-1731).

$$\int \frac{\xi^n d\xi}{\sqrt{\beta^2+\xi^2}} = \frac{1}{n}\xi^{n-1}\sqrt{\beta^2+\xi^2} - \frac{n-1}{n}\beta^2 \int \frac{\xi^{n-2}d\xi}{\sqrt{\beta^2+\xi^2}} \ ,$$

and[304]

$$\int \frac{d\xi}{\sqrt{\beta^2+\xi^2}} = \ln\left(\xi + \sqrt{\beta^2+\xi^2}\right) \ ,$$

$$\int \frac{\xi d\xi}{\sqrt{\beta^2+\xi^2}} = \sqrt{\beta^2+\xi^2} \ .$$

When β is infinitely small, which occurs when α is infinitely small, the first — and only the first — term becomes infinitely large. One may therefore neglect all following terms compared to the first one, and write:

$$\int \frac{e' d\xi}{\sqrt{\beta^2+\xi^2}} = 2e \ln \frac{\sqrt{l^2-4x^2}}{\beta} \ ,$$

or also, by neglecting finite terms compared to the infinite term:

$$= 2e \ln \frac{l}{\beta} \ .$$

Furthermore:

$$\int_0^{2\pi} \ln \beta d\varphi' = 2\pi \ln \rho' \ , \qquad \text{when} \qquad \rho' > \rho \ .$$

In the second part of Ω we have $\rho' = \alpha$. The second part therefore is:

$$\alpha \int \frac{dx' d\varphi'}{r} e' = 4\pi\alpha e \ln \frac{l}{\alpha} \ .$$

Similar considerations may be applied to the first part of Ω. Denoting the value of ε at the point $(x,\ \rho',\ \varphi')$ by ε'_o, then these considerations lead to:

$$\int \frac{\varepsilon' dx'}{r} = 2\varepsilon'_o \ln \frac{l}{\beta} \ .$$

Furthermore:

$$\int \ln \beta d\varphi' = 2\pi \ln \rho' \ , \qquad \text{when} \qquad \rho' > \rho$$

$$= 2\pi \ln \rho \ , \qquad \text{when} \qquad \rho > \rho' \ .$$

[304][Note by AKTA:] The next equation appeared in the original text as:

$$\int \frac{d\xi}{\sqrt{\beta^2+\xi^2}} = \lg\left(\xi + \sqrt{\beta^2+\xi^2}\right) \ .$$

What Kirchhoff represented as the logarithm of a magnitude m, $\lg m$, will be replaced everywhere in this translation by the natural logarithm which has Euler's number e $\approx$ 2.718... as its base, namely, $\lg m = \log_{\mathrm{e}} m = \ln m$. See also Section 9.1.

For both of these expression we may write $2\pi \ln \alpha$ when ignoring finite quantities compared to infinite quantities. Therefore:

$$\int \frac{dx'\rho' d\rho' d\varphi'}{r} \varepsilon' = 4\pi \ln \frac{l}{\alpha} \int_0^\alpha \rho' d\rho' \varepsilon_o' \ .$$

Let:

$$2\pi\alpha e + 2\pi \int_0^\alpha \rho' d\rho' \varepsilon_o' = E \ ,$$

that is, if Edx is the amount of free electricity contained in the element dx of the wire,[305,306] then we find:

$$\Omega = 2E \ln \frac{l}{\alpha} \ . \qquad (13)$$

The expression of U in equation (9) can be treated in the same way. In this expression I am thinking of u' and σ' to be developed in powers of ξ, and the values of u and σ at the point $(x,\ \rho',\ \varphi')$ to be denoted by u_o' and σ_o'. In the parts into which the expression can be split we find integrals of the form:

$$\int \frac{\xi^n d\xi}{(\beta^2 + \xi^2)^{3/2}} \ .$$[307]

We have:

$$\int \frac{\xi^n d\xi}{(\beta^2 + \xi^2)^{3/2}} = \frac{1}{n-2} \frac{\xi^{n-1}}{\sqrt{\beta^2 + \xi^2}} - \frac{n-1}{n-2} \beta^2 \int \frac{\xi^{n-2} d\xi}{(\beta^2 + \xi^2)^{3/2}} \ ,$$[308]

$$\int \frac{\xi d\xi}{(\beta^2 + \xi^2)^{3/2}} = -\frac{1}{\sqrt{\beta^2 + \xi^2}} \ ,$$

$$\int \frac{\xi^2 d\xi}{(\beta^2 + \xi^2)^{3/2}} = -\frac{\xi}{\sqrt{\beta^2 + \xi^2}} + \ln\left(\xi + \sqrt{\beta^2 + \xi^2}\right) \ .$$[309]

Of the specified integrals taken from a negative to a positive finite limit, only for $n = 2$ do we obtain an infinity, provided β is infinitely small. All other integrals can be neglected

[305][Note by GK:] E is the same quantity which in the former paper was denoted by e.

[306][Note by AKTA:] E here is the linear charge density.

[307][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$\int \frac{\xi^n d\xi}{(\beta_2 + \xi)^{3/2}} \ .$$

[308][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$\int \frac{\xi^n d\xi}{(\xi^2 + \xi^2)^{3/2}} = \frac{1}{n-2} \frac{\xi^{n-1}}{\sqrt{\beta^2 + \xi^2}} - \frac{n-1}{n-2} \beta^2 \int \frac{\xi^{n-2} d\xi}{(\beta^2 + \xi^2)^{3/2}} \ ,$$

[309][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$\int \frac{\xi^2 d\xi}{(\beta^2 + \xi^2)^{3/2}} = -\frac{\xi}{\sqrt{\beta^2 + \xi^2}} + \ln\left(\xi + \sqrt{\beta^2 + \xi^2)}\right. \ .$$

compared with this, and the finite part of the infinite term can also be neglected. A factor of it is:

$$u_o' - \frac{\partial \sigma_o'}{\partial x}\left(\rho\cos(\varphi-\varphi')-\rho'\right)\ ,$$ [310]

but, because of the smallness of ρ and ρ', we can replace this by u_o'. Using the same method utilized before for the calculation of Ω, we obtain:

$$U = 4\pi \ln\frac{l}{\alpha}\int \rho' d\rho' u_o'\ .$$

If we denote by i the quantity of electricity which in unit time passes through the cross-section of the wire, *i.e.* the current intensity, the equation can be simplified to:

$$U = 2i\ln\frac{l}{\alpha}\ .$$

Substituting this value of U and the value of Ω from (13) into the equation (8), we obtain:

$$u = -4\ln\frac{l}{\alpha}k\left(\frac{\partial E}{\partial x}+\frac{4}{c^2}\frac{\partial i}{\partial t}\right)\ .$$

The right-hand side of this equation is independent of ρ, and since u is independent of ρ we have:

$$i = \pi\alpha^2 u\ ;$$

hence:

$$i = -4\pi\alpha^2 k\ln\frac{l}{\alpha}\left(\frac{\partial E}{\partial x}+\frac{4}{c^2}\frac{\partial i}{\partial t}\right)\ . \qquad (14)$$

A second equation between the quantities E and i can be derived from equations (11) and (12). If one multiplies the first one with $\rho d\rho d\varphi$, then integrates it over the cross-section of the wire, and subtracts from the result the second equation, after having multiplied it by $2\pi\alpha$, one obtains:

$$\frac{\partial i}{\partial x} = -\frac{1}{2}\frac{\partial E}{\partial t}\ . \qquad (15)$$

The derivation of equations (14) and (15) presupposes that the wire is straight. But since these equations show that the electrical state at a point inside the wire is independent of the electrical state at all other points at a finite distance from the former, the equations will also be valid for bent wires. The radius of curvature, however, has to be everywhere finite, so that the distance between two points, with a finite piece of wire between them, cannot be infinitely close to each other. Equations (14) and (15) are the very same equations which I derived for the same case in the earlier paper. The more general theory developed here, therefore, leads to the same results obtained before, but it leads to further consequences. If,

[310][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$u_o' - \frac{\partial \sigma_o'}{\partial x}\left(\rho\cos\varphi-\varphi'-\rho'\right)\ ,$$

for example, (14) and (15) are used to determine E and (13) to determine Ω, it is possible to calculate ε from (7), *i.e.* the density of free electricity inside the wire, so long as ε is given for zero time. If the initial value of ε is independent of ρ, then ε remains independent of it, that is the density of electricity is the same at all points of the cross-section, for according to (13) Ω is independent of ρ, and ρ does not appear in equation (7). After calculating ε one can find e. If the initial value of ε is independent of ρ, as has been assumed, we make use of the equation:

$$E = 2\pi\alpha e + \pi\alpha^2\varepsilon \ .$$

With the same assumption it is easy to calculate σ from ε because:

$$\sigma = \frac{1}{2}\frac{\rho}{\alpha}\frac{\partial e}{\partial t} \ .$$

That this equation is valid for $\rho = \alpha$ we learn from equation (12), and that σ is proportional to ρ from equation (11). If one multiplies it by $\rho d\rho$ and integrates, remembering that u and ε are independent of ρ, one finds:

$$\sigma = -\frac{\rho}{2}\left(\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial \varepsilon}{\partial t}\right) + \frac{\text{constant}}{\rho} \ .$$

The constant of integration has to be zero, because, for $\rho = 0$, σ must not be infinite. In fact the opposite is true; it has to disappear, because along the axis of the wire the current has to be in the direction of the axis.

In the previous paper I discussed the solution of equations (14) and (15) for the special case which is approached the smaller the resistance of the wire is made. I proved that in this case the electricity in the wire progresses like a wave in a taut string[311] with the velocity of light in empty space. It is of interest to consider the opposite case which is approached the greater the resistance of the wire is made. I will do this here on the assumption that the two ends of the wire are connected with each other.

As in the previous paper, I let the resistance of the wire to be r, and write:

$$\ln\frac{l}{\alpha} = \gamma \ ;$$

then the solution of the differential equations (14) and (15), whatever the value of r, is as follows:

$$E = \sum \left(C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t}\right)\sin nx + \left(C_1' e^{-\lambda_1 t} + C_2' e^{-\lambda_2 t}\right)\cos nx \ ,$$

$$i = \sum -\frac{1}{2n}\left(\lambda_1 C_1 e^{-\lambda_1 t} + \lambda_2 C_2 e^{-\lambda_2 t}\right)\cos nx + \frac{1}{2n}\left(\lambda_1 C_1' e^{-\lambda_1 t} + \lambda_2 C_2' e^{-\lambda_2 t}\right)\sin nx \ ,$$

where n is a multiple of $2\pi/l$, and λ_1 and λ_2 have the values:

[311] [Note by AKTA:] In German: *In einer gespannten Saite*. This expression can also be translated as "in a stretched string".

$$\frac{c^2 r}{32\gamma l}\left[1 \pm \sqrt{1-\left(\frac{32\gamma}{cr\sqrt{2}}nl\right)^2}\,\right] \ ,$$

and C_1, C_2, C_1', and C_2' are arbitrary constants. The summation is over all values of n. The C-constants are easily determined if E and i are given for $t=0$. If the functions of x, which must transform to E and i for $t=0$, have the form:

$$\sum\left(E_n \sin nx + E_n' \cos nx\right) \ ,$$

and

$$\sum\left(-i_n \cos nx + i_n' \sin nx\right) \ ,$$

one obtains the equations:

$$E_n = C_1 + C_2 \ ,$$

$$i_n = \frac{1}{2n}\left(\lambda_1 C_1 + \lambda_2 C_2\right) \ ;$$ [312]

and

$$E_n' = C_1' + C_2' \ ,$$

$$i_n' = \frac{1}{2n}\left(\lambda_1 C_1' + \lambda_2 C_2'\right) \ ;$$ [313]

their solutions are:

$$C_1 = \frac{\lambda_2 E_n - 2ni_n}{\lambda_2 - \lambda_1} \ ,$$

$$C_2 = \frac{-\lambda_1 E_n + 2ni_n}{\lambda_2 - \lambda_1} \ ,$$ [314]

$$C_1' = \frac{\lambda_2 E_n' - 2ni_n'}{\lambda_2 - \lambda_1} \ ,$$

[312][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$i_n = \frac{1}{2n}\left(\lambda_1 C_1 + \lambda_2 C_2 \ ;\right.$$

[313][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$i_n' = \frac{1}{2n}\left(\lambda_1 C_1' + \lambda_2' C_2'\right) \ ;$$

[314][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$C_2' = \frac{-\lambda_1 E_n + 2ni_n}{\lambda_2 - \lambda_1} \ ,$$

$$C_2' = \frac{-\lambda_1 E_n' + 2ni_n'}{\lambda_2 - \lambda_1} \ .$$

In the earlier paper we examined the case in which:

$$\frac{32\gamma}{cr\sqrt{2}} \ ,$$

can be treated as infinitely large. It will now be assumed that this quantity is infinitely small. The two roots λ_1 and λ_2 are then real. If λ_2 is the greater root, so by ignoring terms of lower order:

$$\lambda_2 = \frac{c^2 r}{16\gamma l} \ , \qquad \lambda_1 = \frac{8\gamma l}{r} n^2 \ .$$ [315]

From this it follows:

$$\frac{\lambda_1}{\lambda_2} = \left(\frac{16\gamma}{cr\sqrt{2}} nl\right)^2 \ ;$$

this expression is infinitely small, because nl is a multiple of 2π, which is finite. The expressions of the C-coefficients may then be written:

$$C_1 = E_n - \frac{2n}{\lambda_2} i_n \ , \qquad C_1' = E_n' - \frac{2n}{\lambda_2} i_n' \ ,$$

$$C_2 = -\frac{\lambda_1}{\lambda_2} E_n + \frac{2n}{\lambda_2} i_n \ , \qquad C_2' = -\frac{\lambda_1}{\lambda_2} E_n' + \frac{2n}{\lambda_2} i_n' \ .$$

The coefficient of $\sin nx$ in the expression of E is therefore:

$$E_n \left(e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2} e^{-\lambda_2 t}\right) - \frac{2n}{\lambda_2} i_n \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \ ,$$ [316]

or

$$E_n e^{-\lambda_1 t} - \frac{2n}{\lambda_2} i_n \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \ ,$$ [317]

and the coefficient of $-\cos nx$ in the expression of i:

[315][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$\lambda_2 = \frac{c^2 r}{16\gamma l} \ , \qquad \lambda_2 = \frac{8\gamma l}{r} n^2 \ .$$

[316][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$E_n \left(e^{-\lambda_1 t} - \frac{\lambda_3}{\lambda_1} e^{-\lambda_2 t}\right) - \frac{2n}{\lambda_2} i_n \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \ ,$$

[317][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$E_n e^{-\lambda_1 t} - \frac{2n}{\lambda_2} i_n \left(e^{-\lambda_1 t} - e^{-\lambda_1 t}\right) \ ,$$

$$E_n \frac{\lambda_1}{2n}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) - i_n \left(\frac{\lambda_1}{\lambda_2} e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) .$$[318]

By setting E'_n and i'_n for E_n and i_n, one obtains the coefficients of $\cos nx$ in E, and of $\sin nx$ in i. Excluding the case when the initial value of i is infinitely large, compared to the value which i assumes for constant initial values of E, the expression can be simplified when the initial value of $i = 0$. It can be seen that when $i = 0$ for $t = 0$, that is when $i_n = 0$, the value of i is of the order of $E\lambda_1/2n$. Under the same circumstances i_n is of the order of $E_n\lambda_1/2n$. The coefficients of $\sin nx$ in E and of $-\cos nx$ in i may be written

$$E_n e^{-\lambda_1 t} ,$$

and

$$E_n \frac{\lambda_1}{2n} e^{-\lambda_1 t} + \left(i_n - E_n \frac{\lambda_1}{2n}\right) e^{-\lambda_2 t} .$$

If one excludes from these considerations the values of t which are so small that $\lambda_1 t$ becomes infinitely small, then $\lambda_2 t$ becomes infinitely large. Hence, the second term in the second expression can be neglected compared with the first one. As the same considerations with respect to the coefficients of $\cos nx$ and $\sin nx$ are valid in the expressions of E and i, then, substituting for λ_1 the previously obtained value, we have:

$$E = \sum (E_n \sin nx + E'_n \cos nx) e^{-\frac{8\gamma l}{r} n^2 t} , \qquad (16)$$

$$i = \frac{4\gamma l}{r} \sum n \left(-E_n \cos nx + E'_n \sin nx\right) e^{-\frac{8\gamma l}{r} n^2 t} . \qquad (17)$$

These expressions are independent of c. When one considers c infinitely large, the solutions of the differential equations (14) and (15) become:

$$i = -\frac{4\gamma l}{r} \frac{\partial E}{\partial x} ,$$

$$\frac{\partial i}{\partial x} = -\frac{1}{2} \frac{\partial E}{\partial t} .$$

Eliminating i, one obtains:

$$\frac{\partial E}{\partial t} = \frac{8\gamma l}{r} \frac{\partial^2 E}{\partial x^2} ,$$

which is an equation of the same form as the one which determines the conduction of heat in the conductor. Therefore, in the case considered here, the electricity propagates through the metal like heat does.

With the assumptions made with regard to the resistance r in equations (16) and (17), it is easily proved *a posteriori* that (16) and (17) are real solutions of (14) and (15). It is

[318][Note by AKTA:] Due to a misprint, this equation appeared in the original text as:

$$E_n \frac{\lambda_1}{2n}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) i_n \left(\frac{\lambda_1}{\lambda_2} e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) .$$

possible to convince oneself without difficulty that $(4/c^2)(\partial i/\partial t)$ is infinitely small compared with $\partial E/\partial x$ when i and E are taken from (17) and (16).

The case in which the ends of the wire are separated from each other, and are subject to two potential values, can be treated in a similar manner as the case where the wire forms a closed loop. In the open circuit, and provided the resistance of the wire is large enough, one finds the same analogy between the conduction of electricity and heat.

With Jacobi's resistance standard,[319] a copper wire of 7.62 m length, 0.333 mm diameter, as shown in the previous paper, is:

$$\frac{32\gamma}{rc\sqrt{2}} = 2\,070 \ .$$

For a wire of the same material, the same cross-section, and a length of 1 000 km this quantity is 0.034. By way of an approximation, it can be treated as infinitely large in the first case, and as infinitely small in the second case. In the first case the electricity propagates like a wave in a taut string, and in the second case it travels like heat.

Thomson[320,321] has examined the motion of electricity in an underwater telegraph wire. He assumed — without checking the reliability of this assumption — that induction makes no significant contribution to the phenomena. For this case he showed that electricity propagates like heat. The present considerations have proved that this conclusion is also justified in the case of a simple wire, provided it is long enough. It will be all the more correct in the underwater telegraph wire, in which the motion of the electricity is considerably slowed down on account of conduction in the seawater.

[319][Note by AKTA:] See footnote 263 on page 210.

[320][Note by GK:] *Phil. Mag. Ser. IV, Vol. II, p.* 157.

[321][Note by AKTA:] See [Tho56a] and [Tho56b] with Portuguese translation in [TBA18].

Chapter 13

[Weber, 1858] Report on Some Experiments Made at the Physics Institute in Göttingen

Wilhelm Weber[322,323,324,325]

Presented to the Königl. Societät on April 10, 1858.

In the Physics Institute of this University, in addition to the regular course of physics lectures, practical physics exercises[326] are held, in which the members of the mathematics-physics seminar take part. Those who choose physics as their main subject and have acquired greater practice will find the opportunity to do special work for themselves, such as for example in last year was done by Dr. Arndtsen and Dr. Christie from Christiania,[327] who will make their carried out work known in more detail in Poggendorff's *Annalen*.[328] Their results will be reported here briefly.

[322][Web58] with English translation in [Web21e].

[323]Translated by Peyman Ghaffari, IMAAC-next, Tech Park of Fuerteventura, Puerto del Rosario, Las Palmas 35600, Spain; Center for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal; pgsaid@fc.ul.pt. Edited by A. K. T. Assis.

[324]The Notes by H. Weber, the editor of the fourth volume of Weber's *Werke*, are represented by [Note by HW:]; the Notes by Peyman Ghaffari are represented by [Note by PG:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[325][Note by HW:] Nachrichten von der G. A. Universität und der Königl. Gesellschaft der Wissenschaften zu Göttingen. April 16. 1858. No. 6. p. 67-76.

[326][Note by PG:] In German: *praktische physikalische Uebungen*. By "exercise" it is meant "practical physics exercises" during studying physics at University. This system still exist in the German University system.

[327][Note by AKTA:] Hartvig Caspar Christie (1826-1873), a Norwegian mineralogist and physicist, who studied in Göttingen under Weber from 1857 to 1859. He measured, for instance, diamagnetism in bismuth. Adam Arndtsen (1829-1919), a Norwegian professor and physicist, who also studied in Göttingen under Weber in 1857. Cristiania is the former name of Oslo, the capital of Norway.

[328][Note by AKTA:] See, for instance, [Chr58]. Johann Christian Poggendorff (1796-1877) edited the *Annalen der Physik und Chemie* from 1824 to 1876, where many of Weber's papers were published. The modern *Annalen der Physik* is the successor to this Journal.

In view of the particular interest that *diamagnetism* still arouses as one of the latest discoveries promising information about the inner nature and coherence of bodies,[329] and in the view of the still existing lack of *quantitative* determinations, the *diamagnetometer*[330] manufactured by Mr. Leyser in Leipzig,[331] according to specifications by Professor Weber, with which Mr. John Tyndall made many interesting experiments in London and reported them in the Philosophical Transactions for 1856 (Further Researches on the Polarity of the Diamagnetic Force),[332] and of which a second copy is in the local Physics Institute, was used as a measuring device for some *quantitative* determinations on diamagnetism.

In his experiments, Mr. Tyndall had found an almost equal deflection of the astatic magnetic needle[333] by the diamagnetic body when the diamagnetism was excited by a current from *two*, *three* or *four* cells,[334] whereby the *proportional growth* of the diamagnetic force with the force which excites diamagnetism has been questioned. This doubt does not exist anymore due to more complete *measurements* of Dr. Christie performed with the same instruments, in that Dr. Christie associated a determination of *sensitivity*[335] of the astatic needle with each observation of *deflection*, from which the sensitivity resulted *variable* with the current intensity. From this it followed that the observed deflections first had to be reduced to the *same sensitivity*, before they could serve as a measure of the excited diamagnetism. After this reduction and after precise measurements of the *current intensities* (which, as is well known, must not be set proportional to the number of cells) with the help of the *tangent galvanometer*,[336] the law of the proportionality of the diamagnetic force with the galvanic force that excites it has been carefully tested and confirmed. The current intensity, determined according to known absolute measures, was increased from 16 to 44 units.

The same instrument now offered at the same time the opportunity, to measure more accurately the constant relationship between the diamagnetic force and the galvanic force that excites it. This ratio is called the *diamagnetic constant*,[337] and was determined by Professor Weber with an instrument which was constructed according to the mentioned principles, but which had not received such a fine mechanical execution, for the first time in the [paper] "Electrodynamic Measurements" (*Abhandlungen der mathematisch-physischen Klasse der Königl. Sächs. Gesellschaft der Wissenschaften*, Vol. I, Leipzig 1852).[338,339]

[329][Note by PG:] The German word "Körper" is translated as "body".

[330][Note by AKTA:] In German: *Diamagnetometer*.

[331][Note by AKTA:] See footnote 102 on page 101.

[332][Note by AKTA:] John Tyndall (1820-1893). See, for instance, [Tyn54], [Tyn55b], [Tyn55a], [Tyn55c] and [Tyn56].

[333][Note by AKTA:] The adjective "astatic" is used in physics with the meaning of something having no tendency to take a definite position or direction. An astatic needle can be a combination of two parallel magnetized needles having equal magnetic moments, but with their poles turned opposite ways, that is, in antiparallel position. The arrangement protects the system from the influence of terrestrial magnetism. It was invented by Ampère, [Amp21] and [LA98]. An earlier system composed of a single magnetized needle had also been created by Ampère, [Amp20b, p. 198] with Portuguese translation in [CA09, p. 133], [Amp20a, p. 239] and [Amp, p. 2], see also [AC15, p. 57].

[334][Note by PG and AKTA:] In German: *Becher*. Weber is referring here to a voltaic cell, element, battery or pile producing an electromotive force.

[335][Note by PG:] In German: "Empfindlichkeit".

[336][Note by PG and AKTA:] See footnote 12 on page 22.

[337][Note by PG:] In German: *Diamagnetische Konstante*.

[338][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 473.

[339][Note by PG and AKTA:] In German: *Elektrodynamischen Maassbestimmungen*. This work is the Third of Weber's 8 major Memoirs with the general title of Electrodynamic Measurements, [Web52b] with English

A repetition of this measurement with a finer measuring instrument therefore seemed of particular interest and was also carried out by Dr. Christie.[340]

This measurement was given a special sharpness[341] by the fact that it was not based on a comparison of the diamagnetic bismuth with magnetic iron, which apart from other circumstances, is not capable of much sharpness because of the different distribution of magnetism in iron and diamagnetism in the bismuth, but was based on a comparison of the diamagnetic bismuth with a *solenoid* (a spiral-shaped wire), through which passes a weak current precisely measured by the tangent galvanometer. This solenoid had a cylindrical shape of the same diameter and height as the used bismuth-cylinder. From the number of its spiral windings and the strength of the passing current, the torque[342] could be determined, with which it acted from the same place as the diamagnetic bismuth-cylinder on the astatic needle; the current could also be easily regulated in such a way that this effect was close to that of the bismuth-cylinder.

From these measurements it has been shown that one unit of the exciting force (according to absolute measure, after which the horizontal earth magnetic force in Göttingen is at present = 1.81) in 1 milligram bismuth produces a moment, which according to the Gaussian absolute measure[343] is = 0.000 001 488 5, while the same moment for iron was found by Professor Weber = 5.6074.[344] The diamagnetism of the bismuth is thus 3.8 million times smaller than the magnetism of iron. This result is somewhat smaller than that found by Weber, which, apart from the greater precision of the measurement which the instrument used here permits, is explained by the difference in bismuth, which in both cases could not have been obtained of absolute purity. — To give a clear idea of the size of these diamagnetic forces, it should be noted that 1 milligram of steel from a strong magnetic needle has on average about one moment = 400 according to the same units, which is 269 million times greater than the aforementioned moment of bismuth.

Finally, in third place, Dr. Christie used the same instrument to investigate the polarity of diamagnetic bodies, which Weber and Tyndall had put beyond doubt, by studying the *distribution of diamagnetism* more closely. It has been shown that, according to Gauss's principle of the *ideal distribution*[345] in a cylindrical bismuth rod, in which everywhere the same exciting force acts parallel to its axis, almost all diamagnetism can be thought of as being distributed over the two circular end faces, a result which is entirely in accordance with what seems to be expected with the theory.

In all these experiments carried out with the mentioned *diamagnetometer*, only one circumstance remained in the dark, namely, from where the *sensitivity* of this instrument results being so *variable*, which, according to its theory, if all the prescribed conditions were exactly fulfilled in the construction and regulation,[346] there would be no reason for it. It is now

translation in [Web21a], see Chapter 2.

[340][Note by AKTA:] [Chr58].

[341][Note by PG:] In German "Schärfe" meaning accuracy and sharpness.

[342][Note by PG:] In German: "Moment". That is, "torque" or "moment of force".

[343][Note by AKTA:] In German: *Gauss'schen absoluten Maasse*. That is, Gaussian absolute measure or unit. In 1832-1833 Gauss introduced the absolute system of units for magnetism and obtained the intensity of the horizontal component of terrestrial magnetic force in Göttingen in absolute measure as given by $T = 1.78$. See footnote 47 on page 59.

[344][Note by AKTA:] This value can be found in [Web52b, Section 27] with English translation in [Web21a, Section 27]; and [Web52f] with English translation in [Web53b] and [Web66b]. See, in particular, page 81 and 120 on Section 2.27 and on Subsection 3.2.6, respectively.

[345][Note by AKTA:] See footnote 7 on page 11.

[346][Note by PG and AKTA:] In German: "Regulirung". This word can also be translated as adjustment.

evident that by this construction and regulation of the instrument, a compensation of very large forces, which excited diamagnetism, should be achieved in such a way that they have no influence at all on the extremely sensitive astatic magnetic needle; while the forces then to be measured with the instrument, namely the diamagnetic forces themselves, are very small, accordingly it can be expected that the required compensation cannot be produced practically with the required accuracy. More precisely, the production of the required compensation breaks down into two different tasks, namely *first* in relation to the equilibrium position of the astatic needle, and *second* in relation to its sensitivity. In relation to the *first*, after both problems had been approximately solved, a *finer correction* had been made, without which the astatic needle with telescope, mirror and scale could not have been observed. As regards the *sensitivity*, however, a finer correction was dismissed in order not to complicate the instrument too much, due to the fact that lack of the same does not make any significant contribution to the measurements, if only the variation in sensitivity are precisely determined and taken into account. However, since these variations in sensitivity were of an unexpected size and importance, it was necessary, in order to fully control all the essential elements in these fine measurements, to examine and investigate the causes of these variations more precisely. This actual fine examination has been carried out by Dr. Arndtsen with the best results, and it has emerged from this, how these variations can be mastered and, if found necessary, can be eliminated altogether. With the accuracy with which these variations can be taken into account, there is usually no reason to avoid them, rather, since it is entirely up to one to decide whether the sensitivity of the instrument is to be increased or decreased by the variation, one can often benefit from this for the measurements themselves. Since here the description of the instrument has to be dispensed, so the theory of these experiments cannot be discussed in more detail.

On the other hand, another investigation from Dr. Arndtsen made with the same instruments should be mentioned. It is clear that the same instrument which is used to examine the polarity of diamagnetic bismuth can also be used to examine the magnetic polarity of those bodies which were previously thought to be non-magnetic or weakly magnetic, in order to obtain *quantitative* determinations which are still completely lacking for those bodies. In particular, it seemed important to investigate whether in these bodies, just as Joule,[347] Müller[348] and Weber have found in iron, a deviation of the magnetism from the proportionality of the magnetizing force can be demonstrated with increasing magnetizing force, as this circumstance is of great importance for the study of the inner causes of magnetism and its variations. These tests were carried out by Dr Arndtsen with *iron-sulfate*,[349] *iron-chloride solution*, *cyan-iron-potassium* and *nickel*. It should be noted that the magnetizing force that acted on these bodies, and which was not generated by electromagnets, but by mere galvanic currents, could be brought only up to the strength = 600 by available means according to absolute measure (after which the horizontal earth-magnetic force was presently in Göttingen = 1.81), whereas in the carried-out experiments with iron by Weber it had been driven up to over 3000. Add to this, that this measured magnetizing force can be described as an *external* force, and that in *pure iron* also a significant *inner* force resulting from the magnetic interaction applies on the *individual* particles, which disappears almost completely in the above-mentioned bodies. According to this, it would therefore to be expected that in the above-mentioned bodies the questioned deviation from proportionality usually under the

347[Note by AKTA:] [Jou40].

348[Note by AKTA:] [Mül51b] and [Mül51a].

349[Note by PG:] In German the word "Eisenvitriol" was used, i.e. iron-sulfate or $FeSO_4$.

same condition would be perceptible later than in the case of pure iron, namely only when the *external* force alone acted just as strongly as the *external* and *internal* force combined in the case of iron.

Under these circumstances it is not surprising that in several of the bodies examined there was no deviation from the proportionality noticeable; it is more interesting however, that with *nickel* it has emerged that this deviation from proportionality occurs much earlier than with iron, so that the *nickel-magnetism* has almost reached its highest limiting value as a result of magnetizing forces, at which the iron-magnetism hardly deviates noticeable from the initial proportionality.

In addition to these experiments, Dr. Arndtsen also carried out a comprehensive investigation into the *resistance*[350] of metals with special consideration of their *temperature*, of which the following Table gives a brief overview of the results. Under the title *Resistance*, the resistance of a cylinder with 1 millimetre in height and 1 millimetre in diameter at 0° temperature is given in absolute measure; under the title *Correction due to the temperature*, the factor is given by which the resistance at 0° temperature must be multiplied in order to obtain the resistance at t degrees on the scale divided by 100 parts.

Metal	Resistance	Correction due to the temperature
Silver	241 190	$1 + 0.003\,414\,20 \cdot t$
Copper	244 370	$1 + 0.003\,940\,25 \cdot t$
Aluminum No. 1	476 218	$1 + 0.003\,407\,90 \cdot t$
Aluminum No. 2	427 616	$1 + 0.003\,638\,60 \cdot t$
Brass	949 086	$1 + 0.001\,661\,9 \cdot t + 0.000\,002\,734 \cdot t^2$
Argentan	1 289 815	$1 + 0.000\,387\,36 \cdot t + 0.000\,000\,557\,8 \cdot t^2$
Iron	1 626 643	$1 + 0.004\,130\,4 \cdot t + 0.000\,005\,271\,3 \cdot t^2$
Lead	2 631 490	$1 + 0.003\,767\,68 \cdot t$

Also Dr. Christie made a few more magnetic observations and experiments, which gave interesting results. The local mechanic, Inspector Meyerstein,[351] constructed the instruments for two complete magnetic observatories on the order of the Brazilian government, for scientific expeditions, tested by Dr. Christie. The set-up of the associated portable magnetometer for measuring *declination* and *intensity* is described in detail elsewhere. Only the *induction-magnetometers* for measuring the *inclination*, which, according to Professor Weber, were made on a smaller scale in order to be used on the journey, should deserve a mention, as the ones tested by Dr. Christie have shown that the same advantages are achieved for measuring the *inclination* not only in fixed observatories, but also on a journey, as the other magnetometer for *declination* and *intensity*. — The observations and experiments made by these induction magnetometers have now also given cause for Dr. Christie to determine anew the variability of needle magnetism, which is important for measuring the intensity, with the needle in *normal* and *transverse* positions. This determination was made

[350][Note by PG:] In German: "Leitungswiderstand".

[351][Note by AKTA:] Moritz Meyerstein (1808-1882). See [Hen04], [Hen05], [Hen07] and [Hen20].

by Professor Weber with the help of *magnetic induction* for only smaller needles, in his treatise in the 6th volume of the treatises of our society (Göttingen 1855);[352,353] but it seemed important that the same determination should be used again for the two larger needles with which all measurements of the intensity in the local magnetic observatory since 1834 have been carried out. These attempts have led to the results, which are given in the last column under the title *"Change"*. There the factor is given by which the magnetic directive force,[354] expressed in absolute units, must be multiplied in order to obtain the change in the magnetic moment of the needle produced by it. Since now, during the transition from the *transversal* position to the *normal*, the horizontal earth-magnetic force (presently = 1.81) begins to act on the needle, it follows that one obtains the increase in needle magnetism associated with that transition by multiplying the factor specified in the last column by 1.81.

Needle number	Weight in gram	Magnetic moment	Change
1	1 770	$714 \cdot 10^6$	450 000
2	1 750	$674 \cdot 10^6$	462 000

[352] [Note by HW:] Wilhelm Weber's *Werke*, Vol. II, p. 333.
[353] [Note by AKTA:] [Web56], see also [Web54].
[354] [Note by AKTA:] See footnote 66 on page 77.

Chapter 14

[Weber, 1861] On the Intended Introduction of a Galvanic Resistance Etalon or Standard

Wilhelm Weber[355,356,357]

I allow myself, to make a brief announcement to the [Göttingen] Royal Society regarding the intended introduction of a galvanic resistance etalon or standard. The proposal is based on reasons of practical need, which could be expected from the ever expanding technical applications of galvanism. All galvanic *piles*[358] used for chemical analysis, galvanoplastic and other technical purposes, even if they are called constant, are continually subject to minor and often major changes which one must be aware of in order to be able to control them. But even if these columns were completely unchangeable, their *effect* would soon be larger or smaller, according to the variety of applications. To master these *effects* does not only require a knowledge of the pile itself, but also of all the objects through which the current of the pile is supposed to pass, namely knowledge of their *resistance*. That is why resistance measurements have become indispensable and indeed the need for them has emerged most urgently for telegraphic use.

However, a *resistance unit*[359] is required for resistance measurements. Without such a unit, the objects through which the current is to be passed can only be described, while, once a unit for resistance has been established, a *number* suffices to express everything that is essential, and indeed much more precisely than is possible through all descriptions, since very large differences in the resistance can still exist even though the descriptions of the bodies may completely agree with one another.

Basically, to avoid descriptions, such unit was applied at an early stage by comparing the various bodies through which currents were to be conducted with copper wires of known

355[Web61b] with English translation in [Web20b].

356Translated by H. Härtel, haertel@astrophysik.uni-kiel.de and `http://www.astrophysik.uni-kiel.de/~hhaertel/index_e.htm`. Edited by A. K. T. Assis.

357The Notes by H. Weber, the editor of the fourth volume of Weber's *Werke*, are represented by [Note by HW:]; the Notes by Wilhelm Weber are represented by [Note by WW:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

358[Note by AKTA:] In German: *Alle galvanischen Säulen*.

359[Note by AKTA:] In German: *Ein Widerstandsmaass*.

length and cross-section. It is obvious that this implies, if only implicitly, that the resistance of a copper wire of standard length and standard cross section is taken as basis for a *resistance unit*. However the first who brought up the need for a specific resistance unit was Jacobi in Petersburg in 1846.[360]

Jacobi himself said about this topic:[361]

> "As important as the determination of the current, is that physicists are using a common unit when expressing their resistance measurement. So far, however, no absolute determination can take place because it seems that there are differences in the resistances even of the chemically purest metals, which cannot be explained by a difference in external dimensions alone. In case the physicist would have related their ohmmeters and multipliers to a copper wire 1 meter long and 1 millimeter thick, we would still not be convinced whether their copper wire and ours would have the same resistance coefficient. All these difficulties could be solved if we let any chosen copper or other wire send around the physicists and ask them to refer their resistance measuring instruments to this wire as etalon and to publish their measurements only according to this unit."

Of such a resistance etalon chosen by Jacobi at will (a copper wire 25 feet long and weighing $22\frac{3375}{10000}$ gram), a lot of copies have actually been made and used for resistance measurements. But whether it is that the necessary care was not taken in making these copies, regardless of the finest means of comparison by using Wheatstone bridges,[362] or whether these resistance etalons have undergone a change over time, later very important differences showed up.

For this reason, Siemens in Berlin in 1860,[363] with special consideration of the increasingly urgent needs of technical physics, and due to various concerns raised about the Jacobian resistance standard, proposed something that met all requirements, namely that it could be represented by everyone with ease and with the necessary accuracy as a new unit of resistance. This new standard is based on the resistance of *mercury* as that metal which can be obtained everywhere or manufactured in sufficient, almost perfect purity. As long as it is liquid, it has no different molecular properties modifying its conductivity, and is also less dependent on temperature changes in its resistance than other metals. Finally it offers particular convenience for the application through the size of its specific resistance.

With the establishment of this new resistance unit, Siemens has also pointed to the importance of *resistance scales* as a necessary and indispensable mediator between the standard unit and the objects to be measured. He has constructed such scales in large numbers and perfection in such a way that all resistances between (in whole numbers) 1 and 10 000 standard units can easily be formed.

Finally, in England, too, it is currently intended to set up a certain standard to the measure of resistance, and it is hoped that this goal will best be achieved if the British Association and the Royal Society adopt suitable measures to provide every experimenter

[360][Note by AKTA:] See footnote 263 on page 210.

[361][Note by AKTA:] A French version of Jabobi's text can be found in [Jac51].

[362][Note by AKTA:] In German: *Wheatstone'schen Waage*. The so-called Wheatstone bridge was invented by S. H. Christie (1784-1865) in 1833 and popularized by C. Wheatstone (1802-1875) in 1843, [Chr33], [Whe43, p. 325] with French translation in [Whe44b] and German translation in [Whe44a]. See also [Eke01].

[363][Note by AKTA:] E. W. v. Siemens (1816-1892), [Sie60] with English translation in [Sie61]. See also [GT19].

in the whole world with such a standard, especially all who are occupied with investigations and tests pertaining to the electrical telegraph. Such a standard should not only be valid for a certain temperature, but should also provide an exact indication of its variation for a certain temperature change, as well as, finally, to determine its galvanic significance, *with a precise indication of the force which is required to excite a certain current in it*. — I owe this communication to our correspondent, Professor W. Thomson in Glasgow,[364] one of the most thorough researchers in the field of electricity.

Some time ago I was occupied with a more precise determination of such a standard under the title of *absolute resistance measurements*. I determined the galvanic meaning of the Jacobian resistance etalon, for instance, by stating that an electromotive force of 5980 million is required in order to excite a current of the unit of intensity established by Gauss.[365,366,367] A similar determination of another copper circuit was presented by me to the [Göttingen] Royal Society in 1853.[368,369,370] The purpose of these previous determinations, however, was more focussed on the method and the significance of the results which could be obtained with it than about a quantitative execution. The latter were only achieved as a test with the methods and instruments actually available for other investigations.

If, however, these absolute resistance measurements would find further applications, namely to give all quantitative results of important galvanic observations and research a *lasting* expression, then a similar case would arise as with the measurement of the seconds pendulum length and other fundamental determinations: the need would arise for an absolute resistance measurement to be carried out according to the strictest regulations, with the most perfect instruments and with all the knowledge about most accurate observations. This is a task which can only be solved by very skilful hands, with the most undisturbed leisure and with more solid arrangements than are now available for physical research. But if the British Association and the Royal Society really take the task into their own hands, everything that is necessary will certainly be procured somehow and from somewhere.

This finest execution of an absolute resistance measurement must be preceded by various investigations, some of which I present here.

A distinction is made between *galvanometers* and *galvanoscopes*. Those to which the tangent galvanometer[371] belongs are only used for stronger currents, the intensity of which is obtained in terms of precisely determined units; these, on the other hand, serve to observe the slightest traces of currents of which nothing else can be perceived. The highest sensitivity of the latter, however, is only achieved by the closest encirclement of the needle with its multiplier, whereby the more precise knowledge of the scale is lost, which in the case of the tangent galvanometer resulted automatically from its construction. In order to use such a galvanoscope anyway for real measurements, some kind of observation is necessary as a measure of the sensitivity of the instrument, besides the observation of the deflection produced by the current. As a rule, one seeks to establish this standard once and for all by making corresponding observations on the galvanometer and galvanoscope beforehand.

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[364] [Note by AKTA:] William Thomson (1824-1907).

[365] [Note by WW:] Abhandlung der Königl. Sächs. Gesellschaft der Wissenschaften, I, p. 252.

[366] [note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 351.

[367] [Note by AKTA:] [Web52c, p. 351 of Weber's *Werke*] with English translation in [Web21b]. Weber is referring to the absolute system of units introduced by C. F. Gauss (1777-1855).

[368] [Note by WW:] Abhandlungen der Königl. Gesellschaft der Wissenschaften zu Göttingen, Vol. 5.

[369] [Note by HW:] Wilhelm Weber's *Werke*, Vol. II, p. 277.

[370] [Note by AKTA:] [Web53e]. See also [Web53a] and [Web53c].

[371] [Note by AKTA:] See footnote 12 on page 22.

Apart from the fact that such corresponding observations do not give an exact result because of the very different sensitivities of the two instruments, that standard for very sensitive galvanoscopes is by no means constant and can therefore not be determined in advance. On the other hand, another observation can be combined with the observation of the deflection, namely that of the damping of the oscillation, which directly gives that standard we are looking for.

On this combination of these two observations rests the possibility of using the most sensitive galvanoscope for the most precise measurements, which is the necessary condition for the execution of absolute resistance measurements. The theory of such galvanoscopes, which are suitable for precise measurements, requires a special development, since they have to have a construction that is completely different from ordinary galvanoscopes. The development of this theory offers particular interest because the use of galvanoscopes also opens the way for many other more detailed investigations, where they were previously unusable.

The construction of the galvanoscope must make it possible to observe the deflection and attenuation simultaneously with the greatest possible accuracy, while with conventional galvanoscopes only the magnification of the deflection was decisive for the construction. However, what increases the deflection does not always increase the damping and vice versa. In addition, there is a maximum deflection, which must not be exceeded, and there is as well a certain level of damping which allows the most accurate determination, namely the level of damping at which two consecutive oscillation maxima of the galvanometer needle behave as 2.7182... : 1.

This results in several interesting tasks in the theory of magnetoscopes suitable for precise measurements, which I will not discuss in detail here. I only note that the construction of such galvanoscopes, which deviates from the usual ones, is mainly due to the need for strong magnets as galvanoscope needles. For the sake of damping, and for the purpose of measuring absolute resistance, the need for a longer period of oscillation and a galvanoscope needle, whose zero point can be changed as little as possible should be added. The last two requirements lead to the use of two equally strong magnets connected to an astatic system and suspended on a metal wire,[372] the strength of which can regulate the period of oscillation. In the application of the astatic system this magnetoscope resembles the usual one; only with the difference that what happens here with very small needles has to be done there with larger and much stronger needles.

On the basis of these *galvanoscopes*, prepared for absolute resistance measurements, trial tests were finally made to experimentally determine the extreme limit of accuracy that can be achieved in absolute resistance measurements, the result of which can be expressed as follows.

The absolute resistance measurement presupposes the knowledge of the intensity of the terrestrial magnetism (measured in absolute units) at the place and at the time of the absolute resistance measurement, which must be given all the more exactly, as the influence of an error of this intensity is doubled in the resistance measurement. There is no need to enter into a discussion of the accuracy that can be achieved for the intensity of terrestrial magnetism, because this was done by Gauss in the "Intensitas",[373,374] according to which

[372] [Note by AKTA:] See footnote 333 on page 244.

[373] [Note by HW:] Gauss' *Werke*, Vol. V, p. 79.

[374] [Note by AKTA:] For information on Gauss' work on the intensity of the Earth's magnetic force reduced to absolute measure, see footnote 47 on page 59.

these measurements belong to the fundamental determinations in this area of physics. If one now, while taking absolute resistance measurements, distinguishes between the probable error resulting from the measurement of the earth's magnetism and the probable error resulting from the rest of the measurement, it can always be achieved that the latter is, if not smaller, by no means larger than the former. But it is easy to see that doing more would bring no significant benefit since all observations in this area depend on the earth's magnetism which is influencing all needles and currents.

I conclude with a remark which was stimulated by the result of the absolute determination of the Siemens resistance scale.

According to galvanic principles, the ratio of an electromotive force to an intensity of the current determines a *velocity*, just as by the ratio of the length of a path to a time. The ratio of the force which is required to excite a certain current in a given circuit to the intensity of this current is thus also determined by a *velocity*. According to Ohm's laws,[375] the ratio of that force to this current intensity for a *given circuit* is constant, and this gives immediately the force which is required to excite a current of intensity = 1 in the given circuit.

An absolute resistance measurement is synonymous with determining this *velocity*. This idea could now be supported by physically representing it, which can be done with the help of the induction inclinatorium,[376] described by me in the "Resultaten und den Beobachtungen des magnetischen Vereins im Jahre 1837" (Results and Observations of the Magnetic Association in 1837).[377,378] The direct measurement of such a velocity, displayed in real, is associated with great difficulties. Hence, preference is given to an indirect measurement method such as that to which the above discussions relate. This makes the actual physical representation of that velocity superfluous, which is of no importance to the matter. The result found still remains a velocity.

The above-mentioned result of the absolute resistance measurement of the Jacobian resistance etalon was therefore a certain *velocity*, whereby according to Gauss, millimeters were the basis for length and seconds for time. The basis of this measure can be expressed by adding the designation millimeter/(second) to the number 5 980 000 000 given there, which is equivalent to 5 980 000 meters/(second).

Likewise, the result of the absolute resistance measurement for the unit of the Siemens' resistance scale has now also been obtained expressed as a certain *velocity*, namely the number of meters, which is close to 10 million, i.e. equal to the length of the *earth's quadrant* (to be divided with one second).

Even if it does not matter how large or small a resistance standard is chosen, it makes sense that, according to the proposal of the British Association and Royal Society, this standard should always be accompanied, in order to determine its galvanic significance, by the exact specification of the force, which is required to excite a certain current in it. In addition, it appears to be very useful to set up the otherwise completely arbitrary choice

[375][Note by AKTA:] See footnote 128 on page 123.

[376][Note by AKTA:] In German: *Induktions-Inklinatorium*.

The dip circle, dip needle, inclinometer or inclinatorium is an instrument used to measure the angle between the horizon and terrestrial magnetism (the dip angle). It consists essentially of a magnetic needle pivoted at the center of a vertical graduated circle.

Weber's *Induktions-Inklinatorium* is a new instrument which he presented in 1837, [Web38b]. It offered a novel way to circumvent the two main problems with dip circles: the effect of gravity, and the need to reverse the polatiry of the needle, [WSH03].

[377][Note by HW:] Wilhem Weber's *Werke*, Vol. II, p. 75.

[378][Note by AKTA:] [Web38b].

in such a way that this information which should be added can briefly and succinctly be expressed by the mere designation earth-quadrant/(second). — The fact that a resistance that is, according to Siemens, very close to this standard can then be produced easily and safely under all conditions by using a mercury pile with a cross section of 1 square millimeter and a length of 1 meter would often be of practical use under conditions, where the true standard is not available.

Chapter 15

Translator's Introduction to Weber's 1863 Paper

Peter Marquardt[379]

Wilhelm Eduard Weber, **his Time, his Research, and his German**

Dear Readers,

In Weber's time (he lived from 1804 till 1891), the conditions for a scientific publication were considerably different from today's: 1st scientists and science journals were scarce then; 2nd the scientists could afford to write in a lengthy and, quite contrary to concise writing, circumstantial style. The 19th century German language in particular lent itself for "mile long" sentences, compilations of interwoven secondary clauses; the German term is *Schachtelsatz*. "Involved period" sounds a bit too innocent, however; German is among the world champions, maybe second only to Latin, when it comes to constructing a jungle of ideas in one go. Such style may have been considered then as "elegant". Scientists adapted their writing to their thinking and they were cautious about the details of their publications. Their extensive way of formulating may also be considered as trademark of honesty. We find other German writers in that tradition, take Einstein or Woldemar Voigt, whose messages are presented in an unusually rich bouquet of words. Reading Weber's original, we meet an exceptionally detailed writer with the knack of complicated formulations, often offering a tedious text to his readers who have to struggle through the syntax. Weber was a meticulous and cautious researcher, sticking to all (often unnecessary) details, thereby repeating quite a few of them that burden the text. Nobody writes like that any more. In my high school days, we were strictly advised not to overload our sentences. The German proverb "In der Kürze liegt die Würze" (brevity is the soul of wit) is an official present day motto — it certainly was not Weber's.

The 19th century nomenclature, too, differs from what we are used to today. Some examples: *Elektricität* is best understood as charge (Weber uses *Ladung*, too, but mostly *Elektricität*); *Kette* (chain) is translated by circuit; current intensity (*Intensität*) and current density (*Dichte*) occur interchangeably; we take density; the "encounter of waves" (*Begegnung von Luftwellen in Orgelpfeifen*) is our interference; a "steady, or persistent, current"

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(*beharrlicher Strom*) is our direct current, DC; Weber's title *Maassbestimmungen* is ambiguous — it hints both at *measurements* and at his aim to establish "*absolute units of measure*" by measurements. We chose "measurements", always keeping in mind also the units of measure so important for Weber.

To give you a taste of Weber, here is a single(!) sentence from his present introduction to the treatise "Electrodynamic Measurements" followed by a "modernized" translation containing the essential information. The version that tries to stay close to Weber makes part of the whole translation.

> *Die Frage über die Fortpflanzungsgeschwindigkeit elektrischer Bewegungen in Leitungsdrähten lässt sich danach überhaupt nicht so einfach beantworten und noch weniger durch eine Messung, wie sie Wheatstone auszuführen versucht hat, entscheiden, wie daraus ein leuchtet, dass sehr verschiedene Geschwindigkeiten bei diesen Fortpflanzungen zu unterscheiden sind, und dass zumal bei längeren Leitungs drähten, wie der Wheatstone'sche oder die zu Telegraphen gebrauchten, die Fortpflanzungsgeschwindigkeit der grösseren Wellen, welche bei kürzeren Drähten der des Lichts nahe kommt oder sie noch übersteigt, sogar bis auf Null herabsinken kann, und dass darüber hinaus, wo der Ausdruck der Fortpflanzungsgeschwindigkeit imaginär wird, von Fortpflanzung der Bewegung durch Wellen gar nicht mehr im gewöhnlichen Sinne die Rede sein kann, sondern blos von einer asymptotischen Annäherung der Bewegung an ein bestimmtes Gleichgewicht, die als reine Dämpfung oder Absorption betrachtet werden kann, und die bei der Wichtigkeit, die sie für längere Leitungsdrähte, namentlich für Telegraphendrähte, hat, noch nähere Untersuchung verdient.*

> *The question concerning the velocity at which the motion of charges propagates in conductors is, thus, not at all an easy one to answer, let alone one to be decided by a measurement like Wheatstone's effort. Clearly very different propagation velocities have to be distinguished. Above all, in quite long wires as used by Wheatstone or in telegraphy, the velocity of long wavelengths may even drop to zero, while in short wires it may approach or even surpass that of light. Furthermore, when the expression for the velocity becomes imaginary, ordinary wave propagation is out of the question, leaving only the approach to an equilibrium, to be considered as pure damping or absorption. For long wires, especially as in telegraphy, its importance deserves closer investigation.*

Why then are we offering a tentative translation that tries to stay strangely literal at the risk of looking like old fashioned "German-flavored English"? Staying closer to the original may help to understand Weber better in the context of his time and of the spirit of 19th century science. You may have to read sentences twice, likewise to rework the somewhat complicated and circumstantial mathematical part that seems to fit the linguistic part. Anyway, a tedious study may prove rewarding when it lets the reader pause to reconsider the information contained in Weber's honest and cautious way of presenting science. Enjoy!

Chapter 16

[Weber, 1863] On the Treatise "Electrodynamic Measurements, relating specially to Electric Oscillations"

Wilhelm Weber[380,381,382]

The task to examine more closely the *forces* mutually exerted by electric particles or which are exerted on them by other bodies, which has been the main subject of the preceding treatises on *Electrodynamic Measurements*,[383] is closely followed by a second task, namely, to examine carefully the *motions* performed by the electric particles driven by all these forces or to establish the *laws of the motion of electricity* derived from the laws of those forces; for the knowledge of these forces above all is to serve to gain a more exact knowledge of these motions than is possible by direct observation.

This second far-reaching task of electrodynamics has found but little attention yet, and it is justified to ask for the reason why it happened that it has been hardly tried to develop further the foundation given by the knowledge of the forces. Obviously, the reason is that the foundation itself has not yet been considered as completely finished and secured. That is to say, it could be called into doubt whether all forces acting on the electric masses were indeed already known, or whether besides the known electric forces acting *at all distances*, any yet unknown *electric molecular forces*, limited to immeasurably small regions of influence, co-act which should be investigated before one tried to develop the laws of motion of electric masses depending on them. Even the reliability of the *resistance law of ponderable conductors* could be called to doubt, at least when the same should be applied to the development of laws for

[380][Web63] with English translation in [Web21f]. Related to [Web64] with English translation in [Web21d], see Chapter 18.

[381]Translated by P. Marquardt, marquardtp@gmail.com. Edited by A. K. T. Assis. We thank Robert W. Gray for relevant suggestions.

[382]The Notes by H. Weber, the editor of the fourth volume of Weber's *Werke*, are represented by [Note by HW:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[383][Note by AKTA:] [Web46] with partial French translation in [Web87] and a complete English translation in [Web07]; [Web52c] with English translation in [Web21b]; [Web52b] with English translation in [Web21a]; [KW57] with English translation in [KW21].

non-uniform and *rapidly increasing* motions; because this law, first formulated by Ohm,[384] can be considered as safely established only for *steady* currents.

The only effort to solve this task in a somewhat more general way has been reported by Kirchhoff in Poggendorff's Annals 1857, Vol. 100 and 102,[385] but, as declared by Kirchhoff himself, it is restricted to very thin conducting wires and to the assumption of a more general validity of Ohm's law than has been proven, namely, its validity also for non-uniform and rapidly increasing currents. Furthermore, the development of the laws, as far as it has been conducted up to now, does not allow a more detailed test by experience.

In particular, the following two objections against the existing development are in order, namely, *first* that, should the requirements of the fineness of the conducting wire be merely approximately fulfilled, the wire should be much finer than all wires available or producible by existing means; *second*, that, apart from this, the assumption of a more general validity of Ohm's law would not be compatible with such a fineness of the conducting wire; because the finer the wire, the more pronounced become the deviations from Ohm's law for non-uniform and rapidly changing currents.

The establishment of the laws of motion of electricity have therefore been tried in *closed* conductors independent of those more or less unrealizable and dubious assumptions and as far as necessary to develop the latter at least for the simplest case when the closed conductor is a *circle* in order to test the theory by means of experience.

The result has been that, after each perturbation of the equilibrium of the electricity in a closed conductor, indeed *propagations* of electric motions take place with determinable *velocities* which could be called *electric waves*; but those *electric waves* are fundamentally different from *air* or *aether waves*, through which sound and light are propagated, which for example is evident from their velocity being dependent on the *length of the path* (the length of the closed conducting wire) they have to pass through which completely contradicts the laws of propagation of other waves. Likewise, the *wavelength* in each wave train to which a certain *velocity of propagation* belongs, is in a certain ratio to the length of the path: namely, it always represents an *aliquot part* of the total length of the closed conducting wire as is usually assumed for those standing air oscillations that are produced by their interference in organ pipes. But the laws of decomposition of those types of wave trains that are valid in air are not applicable to electricity, because here wave trains with different wavelengths have different *propagation velocities*.

Consequently, the question about the *propagation velocity* of electric motion in conducting wires is not at all an easy one to answer and even less to decide by a measurement, like Wheatstone tried to perform,[386] as is clear from the various velocities that have to be distinguished for these propagation velocities, and in particular the propagation velocity of the longer waves which may approach that of the light or may even surpass it in shorter wires, may even decrease to zero especially in longer wires like those utilized by Wheatstone or those used for telegraphs; and moreover from the propagation of motion [of electricity] by means of waves in the usual sense which is out of the question when the expression for the propagation velocity becomes imaginary, but is just an asymptotic approach of the motion to a certain equilibrium which can be considered as pure damping or absorption and which deserves closer investigation in view of its importance for longer conducting wires, namely,

[384][Note by AKTA:] See footnote 128 on page 123.

[385][Note by AKTA:] [Kir57b] and [Kir57c], with English translations in [Kir57a] and [GA94], respectively. See Chapters 8 and 12.

[386][Note by AKTA:] See footnote 138 on page 129.

telegraph wires.

The case when the expression for the propagation velocity for the bigger wave trains becomes imaginary (where for this part of the motion, as already noticed by Thomson and Kirchhoff,[387] similar laws like that for thermal conduction may hold) deserves special attention when another part of the motion always remains which produces smaller wave trains for which the expression for the propagation velocity stays real. Hence there are indeed wave trains with certain propagation velocities in such a wire after each disturbance of equilibrium, however, they do not constitute a pure wave motion but are mixed with motions that are subject to other laws, namely, those analogous to heat conduction.

If one considers all relations that arise from such a mixture of motions which change according to completely different laws, then it becomes self-evident that the *non-simultaneity of sparks* at very distant ruptures of a long conducting wire observed by Wheatstone by no means allows a conclusion of a definite propagation velocity, that Wheatstone's method of observation, be it as practical as is, is not suited at all for the present purpose, and that one may succeed with difficulty to find other methods to determine the laws of all changes of motion of the electricity in a conductor after a disturbed equilibrium by *pure experimentation*. The purpose of the observation rather seems to be restricted to *test* the laws obtained from otherwise acquired knowledge of electricity, for which purpose it is thus necessary to place this derivation before the laws, all the more as the laws derived and to be tested must themselves serve as *guide* in order to find the methods of observation that are best suited for the test.

Such a test, if it is to be exact, will always demand fine-tuned measurements. If one considers that the finest measurements in physics concern either *equilibrium phenomena*, or *steady motions*, or *periodically occurring motions* (oscillations), it is manifest, apart from constant currents, to establish a test method also for the observation of *periodically regularly occurring motions*, or *oscillations*, of electricity in conductors, taking for granted that there are means for the fine execution of such observations.

Periodically occurring motions of electricity in a conductor, however, cannot arise all by themselves, but always by repeated excitation, and the quick rotation of a small magnet around an axis perpendicular to its magnetic axis offers itself for their *production* as the simplest and, for finer observations and measurements, most practicable method, as well as for their *observation* the effects they bring about when the electrodynamometer is switched on. In order to obtain a practicable guide for such observations, however, the laws of such *electric oscillations* have to be developed first.

From this development it follows that, with the magnet in continuous rotation, the electricity in all parts of the closed conductor will be set in regular continuing oscillation which is oppositely equal for positive and negative electricity. The period of an oscillation is equal to the period of half a rotation of the magnet. However it also follows that the *oscillation amplitudes* and the *oscillation phases* of the electricity at different positions of the closed conductor have to coincide perfectly, not only when the electromotive forces simultaneously exerted by the rotating magnet are equal everywhere, but that they should exhibit almost unnoticeable differences even when these forces are quite randomly distributed in the circuit. Thus, in general, the oscillations may be considered completely equal and simultaneous which extremely simplifies the observations of electrical oscillations in closed circuits which, according to the theory, should prove almost correct also in very long circuits.

[387][Note by AKTA:] [Tho56b] and [Tho56a] with Portuguese translation in [TBA18]; [Kir57b] and [Kir57c], with English translations in [Kir57a] and [GA94], respectively. See Chapters 8 and 12.

This remarkable result has been tested *first* by performing observations of oscillations under conditions that are favorable for the comparison of the *amplitude* at various positions of a long circuit which showed that the deflection of the switched-on dynamometer that is proportional to the square of the oscillation amplitude deviated by less than 1/3 of a scale unit out of 846 scale units as an average from six observations for two positions almost five miles apart; with respect to unavoidable observational uncertainties, this means that there was no difference of the oscillation amplitude.

Second, the oscillations have been observed under conditions favorable for the determination of the *phase differences* at different positions of a long circuit which resulted in the *difference* between two observed deflections of the dynamometer, which should be closely proportional to the phase difference at two positions of the circuit almost five miles apart, was less than 3/5 scale units out of 844; with respect to unavoidable observational uncertainties, this means that practically no phase difference at all could be detected. — In these observations, the oscillation period corresponded to 1/520 second, or to 260 turns per second of the little magnet.

Furthermore, from this theory, confirmed by practical tests, it follows that there is no such *velocity* that would be as important and meaningful for this kind of propagation as that claimed for the propagation velocities of sound and light in air and in the light aether, the exact measurement of which is among the most important tasks in physics because they have to be considered as true fundamental measurements for the exact knowledge of these media.

Should there be no such *velocity* serving as fundamental determination also for the motions propagating through the electric medium, then this leads to the question whether the theory would not offer another issue suited for a fundamental determination, leaving equilibrium out of consideration, which would have a similar meaning for the knowledge about the medium and replace that velocity in the present context.

According to the theory, such a topic should reveal certain *deviations* from Ohm's law which, with increasing *refinement of the conducting wires*, set in with very unsteady and rapidly changing currents. According to the theory the validity of Ohm's law, firmly established by experiment for steady currents and indeed also for variable currents, should hold only as far as a certain *coefficient*, $c^2/r\mathfrak{E}$, *depending on the nature of the electric fluid and of the conducting wire*, may be considered as vanishingly small. Whenever this coefficient, as is the case when the conducting wire is made finer, increases above a value that cannot be neglected compared with unity, then certain *deviations* of the manifestations of electric oscillations from the determinations derived from Ohm's law should become the more pronounced the faster the electricity oscillates. If these *deviations* could be observed and measured, they would lead to the knowledge of that *coefficient* which, depending on the nature of the electric fluid and of the conducting wire, is of utmost importance for the science of electricity.

The *physical meaning* of this coefficient is that of a ratio of the square of the known velocity c (which determines in the fundamental law the ratio of the static and the dynamic part of the electric force) divided by the force that would be exerted by the total amount of positive electricity contained in one length unit of the conducting wire, assumed as concentrated in a point, on *1 milligram of the electric fluid* at the unit distance. Thus, this force would be determined if the *deviations* from Ohm's law, caused by the acceleration of the oscillation and refinement of the conducting wire, could be exactly observed and measured.

On the other hand, this force may be expressed as the *product* $r\mathfrak{E}$ of the amount of positive electricity, $\mathfrak{E}$, contained in a unit length of the conductor, expressed in electrostatic units, times the amount of electrostatic units, r, contained in *1 milligram of the electric fluid*, which, if $\mathfrak{E}$ and hence r were known, any electrostatically determined amount of electricity and likewise the masses of ponderable bodies could be expressed *in milligrams*.

If, as shown in a former treatise (see Transactions of the Royal Saxonian Society of Science, Vol. V, Sections 15 and 20),[388,389] this amount of electricity, $\mathfrak{E}$, were determinable at least for certain conductors, namely, electrolytes like water, then the observation of oscillations would offer the *possibility* to determine the measure of electric masses like that of ponderable masses, even if the execution requires various preparatory work. This knowledge of the mass could never be obtained by means of *electrostatic* observations.

However, the execution of such a determination of the mass constitutes a new task that had to be reserved for a special treatise, even if the method had been completely established. This is a similar case like the magnetic measurements, the practicability of which in terms of absolute measures was demonstrated theoretically by Poisson,[390] which, however, would have been fruitless without Gauss' investigations which led to the control of all details.[391]

The same holds also for other applications allowed by the theory of electric motion in conductors, for instance, of an exact determination of all processes in *telegraphs* or Rühmkorff's *machines*,[392] because it is clear that the theory, first developed just for *circular* conductors, even if it were impeccable, would not be sufficient with respect to telegraphs or Rühmkorff's machines with completely different shapes of conductors, and that various investigations will be needed in order to control all details under such circumstances, necessary to successfully perform such determinations.

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[388] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, pp. 648 and 664.

[389] [Note by AKTA:] [KW57, Sections 15 and 20, pp. 648 and 664 of Weber's *Werke*] and [KW21, Sections 15 and 20, pp. 48 and 65]. See Sections 7.15 and 7.20 of this book.

[390] [Note by AKTA:] See footnote 43 on page 56.

[391] [Note by AKTA:] For information on Gauss' work on the intensity of the Earth's magnetic force reduced to absolute measure, see footnote 47 on page 59.

[392] [Note by AKTA:] Rühmkorff's machines or induction coils were named after Heinrich Daniel Rühmkorff (1803-1877), a German instrument maker.

Chapter 17

Translator's Introduction to Weber's Fifth Memoir on Electrodynamic Measurements

Peter Marquardt[393]

Dear Readers,

Who venture into studying the translation of Weber's 1864 treatise "Electrodynamic Measurements, Fifth Memoir, relating specially to Electric Oscillations" which makes part of his "*Electrodynamische Maassbestimmungen*":

If you have already studied Weber's Introduction to this Treatise,[394] you may be familiar with some special aspects, hopefully encouraging you to go on.

The present work by Wilhelm Eduard Weber on oscillations in conductors is an exhaustive and detailed description, in theory and practice, of a series of pioneering experiments by one of the prominent researchers of his time. The copy of the German original used here, accessible on internet, is from the library of the Deutsche Museum, München. The frontispiece bears a hand-written personal dedication to Gustav Wiedemann, known from the Wiedemann-Franz rule on the thermal conductivity of metals ("Herrn Professor G. Wiedemann vom Verf." - to prof. G. Wiedemann by the author).

The Treatise on oscillations consists of two main parts: Laws of motion (an Introduction followed by Sections (Weber calls them "*Artikel*") 1 through 24) and Observations of Oscillations (Sections 25 through 36 followed by the list of contents).

Like its Introduction, the translated Treatise suggests some comments on Weber's science, language, and presentation. With respective changes, the following remarks may also apply to other translated parts of his extensive work.

393 E-mail: marquardtp@gmail.com
394 Foreword in [Web63] with English translation in [Web21f], See Chapter 16.

17.1 About Weber's Science

Weber's science rightfully receives revived attention. In the present Treatise Weber addresses what he calls a "second task" following the previous work ("first task") on *various forces* investigated by Coulomb, Ampère, Faraday, Neumann for various experimental situations - static charges, current elements, induction, currents or circuits in motion, respectively. Its author considers this second task of electrodynamics, turning to the *motions of electric masses* due to the *forces* treated in the first task, as even more far-reaching, hitherto neglected, and hence all the more important.

Weber, a child of the 19th century, formulates very cautiously, one of the reasons for the kind of language he uses (see below). For instance, he does not take the validity of Ohm's law for granted when he turns to what we call alternating currents ("oscillations"). Clearly these investigations are of great importance for telegraphy, Weber's pioneering subject in collaboration with Gauss, and for the later triumphant career of alternating currents in general. The validity of Ohm's law for rapidly varying currents is questioned. Do we sense here an early taste of the skin effect in spite of the then still quite low frequencies?

His repeated and very meticulous experiments (assisted by Rudolf Kohlrausch) require a few remarks.

Of particular interest are his rotating magnet to produce well defined oscillations (Section 20) and his solution how to handle wires 5 miles long in a laboratory. Aiming at telegraphy, these experiments on long conductors called for the strategy to spool them as twin wires (bifilar winding) with opposite current directions to avoid inductive losses (Section 26).

Referring to Kirchhoff , Weber rightfully mentions the great importance of the agreement between the propagation velocity of electric waves and of light in free space.

The occurrence of a factor $\sqrt{2}$ when Weber addresses the propagation velocity of wave trains (Section 16) may appear strange; it has been commented in recent years by several scientists. We may speculate and seek its connection with Weber's novel velocity dependent modification of the static Coulomb potential where the relative velocity $dr/dt = \dot{r}$ between two charges enters by means of the dynamic factor $(1 - \dot{r}^2/2c^2)$.

17.2 About Weber's Language

Weber's language is 19th century German, in his case with extensively long sentences, with technical terms not in use any more, with circumstantial formulations and intertwined syntax. The German grammar is suited to fill a person with awe when it comes to long chains of nouns linked together and to construct extended periods (just take a look at the very end of Section 36; yes — that is one single sentence!)

Yet this here is the attempt to convey the text in Weber's spirit to the readers of our time, trying to stay close to the original at the cost of sounding strange. For instance, these many conjunctions (*but, however, thus, (w)hence, finally, now*, and more of the like) are indeed present in the original.

You may notice some compound nouns with a "German taste", like *rotation direction*, *oscillation phase*, *oscillation amplitude*. They are to avoid some of the many "of - of - type" constructions for better readability.

The following glossary exemplifies Weber's old-time German by means of some technical terms in 19th century formulation and spelling → here translated as:

Ablenkung → deflection (Weber uses *Elongation*, too).

Beharrlicher Strom → steady current (our DC, constant in density and direction; today *beharrlich* = tenacious or stubborn).

Beruhigung → damping (Weber also uses Absorption or Dämpfung; calling his device, intended to dampen, a "*Beruhigungsmittel*" — meaning tranquillizer today).

(Freie) Electricität → "(free) electricity" (corresponds to the scientific state of the art of the mid 19th century when Weber was among those scientists to already postulate a smallest indivisible charge long before the electron was identified as the mobile carrier; to be understood as *(free) charge*. Sometimes Weber also uses *Ladung*).

Kette → chain, translated as circuit.

Linearer Leiter → "linear", or thin in the sense of 1-D, or line shaped, or straight conductor.

Maass → unit (this fits better than *measure* because Weber set great value on his "absolute units" as is also expressed in his use of *Maasseinheit* and *Maassbestimmungen*).

Säule → voltaic pile.

Schwingungsdauer → (oscillation) period.

Schwingungszahl → (oscillation) number.

Stärke der Ladung → amount of charge.

Strom or Strömung → current (Weber uses both).

Stromdichtigkeit or *Stromintensität* → current density (Weber does not clearly distinguish current density and intensity).

Tangentenbussole → tangent galvanometer (bussole means a compass with a graduated circle and a line of sighting).

Verhältniss → quotient in the sense of proportion or ratio.

Commutator, (electro)dynamometer, multiplier, magnetoelectric, electrodiamagnetic and others of the like have similar meanings in German and English; they are somewhat obsolete names, maybe not listed in a standard dictionary.

17.3 About Weber's Presentation

Some additional illustrative figures of the experimental setup to assist or, better still, to reduce the text, would have been helpful. Weber contents himself to just 5 figures (see Section 25, The Commutators).

His nomenclature is not easy to follow, and so are some of his formulas.

His "partial differential equations" (e. g. Sections 17 and 20) are not formulated as "partial" according to our usage.

Weber's treatment of a "line element", ds, is different from our infinitesimal differentials: his $ds = l$ may be considered very small compared to the circumference of a conducting ring, yet large enough compared to its thickness, so as to consider the finite length l as straight, "*geradlinig*" (Section 10).

You will definitely not fail to notice that the essence of this long treaty could be offered in a concise form. Brevity ("the wit of wisdom"), so it seems, was not in the spirit of Weber's time. If interested to get just a taste of that spirit, feel free to resist the temptation to follow *all* the details.

Anyway, if you venture into the text and get acquainted with its peculiarities, you will find yourself rewarded entering a past world of great science. Enjoy!

PS Apologies for all mistakes and linguistic lapses which have survived.

Chapter 18

[Weber, 1864, EM5] Electrodynamic Measurements, Fifth Memoir, relating specially to Electric Oscillations

Wilhelm Weber[395,396,397]

Introduction

The *first* task of these treatises on *Electrodynamic Measurements*[398] has been to exactly and completely determinate the various forces exerted by electric masses. A fundamental law[399] has been set up from which have been derived and determined *first* the forces of electrostatic interactions and their laws discovered by Coulomb,[400] *second* the mutual electrodynamic forces between current elements and their laws discovered by Ampère,[401] *third* the forces of induction discovered by Faraday (Volta-induction)[402] — including a current co-moving

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[395] [Web64] with English translation in [Web21d]. Foreword in [Web63] with English translation in [Web21f], see Chapter 16.

[396] Translated by P. Marquardt, marquardtp@gmail.com. Edited by A. K. T. Assis. We thank Thomas Herb for sharing with us his partial translation of the first 2 Sections of this work.

[397] Wilhelm Weber's Notes are represented by [Note by WW:]; the Notes by H. Weber, the editor of the fourth volume of Weber's *Werke*, are represented by [Note by HW:]; the Notes by A. K. T. Assis are represented by [Note by AKTA:]; while the Notes by the translator P. Marquardt are represented by [Note by PM:].

[398] [Note by AKTA:] [Web46] with partial French translation in [Web87] and a complete English translation in [Web07]; [Web52c] with English translation in [Web21b]; [Web52b] with English translation in [Web21a]; and [KW57] with English translation in [KW21].

[399] [Note by AKTA:] Weber is referring to his force law presented in the first treatise on *Electrodynamic Measurements*, [Web46] with partial French translation in [Web87] and a complete English translation in [Web07].

[400] [Note by AKTA:] See footnote 43 on page 56.

[401] [Note by AKTA:] See footnote 44 on page 57.

[402] [Note by AKTA:] Michael Faraday (1791-1867). The expression utilized by Weber, *Volta-Induktion*, had been first suggested by Faraday himself in paragraph 26 of his first paper on electromagnetic induction presented in 1831, see [Far32a, § 26] and [Far52, § 26, p. 267]. Portuguese translation in [Far11, p. 159]:

> For the purpose of avoiding periphrasis, I propose to call this action of the current from the voltaic battery, *volta-electric induction*.

with its carrier and a current changing in its stationary carrier, and also what Neumann first discovered and observed,[403] [namely, induction with] the passage of a current through a sliding contact — and their laws.

Apart from these various forces due to *purely* electric interactions, also forces exerted by *magnetism* on electricity have been considered, namely *electromagnetic* and *magnetoelectric induction* forces due to magnetism moving relative to electric masses — including magnetism co-moving with its carrier and merely within its carrier. — The laws could also be derived for these forces beginning with the established fundamental electric law, namely when, following Ampère, molecular currents were substituted for molecular magnetism. The same was valid for *electrodiamagnetic* forces.

Finally, also the laws for the forces exerted by *ponderable bodies* on the electric masses moving within them and exerted on the latter, have been considered; and which are called the galvanic resistance forces of the ponderable bodies. On the basis of Ohm's law,[404] established for steady currents, a more general fundamental law for these forces has been tentatively set up.

The investigation of these *forces* is closely tied to a *second* task of electrodynamics, namely the exploration of the *motions* of the electric masses driven by all these forces and the exploration of their laws in terms of these forces acting on the electric masses. Hence an exact and complete knowledge of all forces and their exploration is mandatory in order to determine these motions, the exploration of these forces may be considered as the means and the exploration of these motions as the aim to be arrived at in this way.

This second very general task of electrodynamics has found but little attention and we may rightfully ask why so little has been done to extend the foundation based on the knowledge of the forces? Obviously one can hesitate to consider this foundation as safely established and finished. The knowledge of all forces acting on the electric masses could be called in question, namely, whether some yet unknown co-acting *electric molecular forces* limited to immeasurably small scales, must be investigated besides the known purely electric forces acting at all distances, before one tried to develop the laws of motion depending on them. There was also some doubt about the reliability of the resistance law in *ponderable conductors* as applied for the development of the laws for *high frequency* electric motions, because Ohm's law was established for *steady* currents only and the generalization has been only been tentative. — To conclude, add to this that the knowledge of the *forces* is not the only necessary requisite to fulfill the second task, but moreover a more specific knowledge of the *masses* subject to motion besides other not yet sufficiently known details is required.

Nevertheless, Kirchhoff made a very thorough effort to fulfill the second task, in fact in such a comprehensive way as the conditions allowed, and published the results in Poggendorff's *Annalen* 1857, Vols. 100 and 102.[405] Irrespective of the above objections, this first attempt has rightfully received wide attention because the decision whether and, if so, how far the objections are justified can hardly be found by any way other than by experiment. — Kirchhoff indeed tried to set up *a general theory on the motion of electricity in an infinitely long thin wire*, however, indicating himself that he considered as generally valid some known

This phenomenon of Volta-induction is nowadays called Faraday's law of induction.

[403][Note by AKTA:] Franz Ernst Neumann (1798-1895). See [Neu48a] and [Neu49]. See also [Web49] with English translation in [Web21c].

[404][Note by AKTA:] See footnote 128 on page 123.

[405][Note by AKTA:] [Kir57b] and [Kir57c], with English translations in [Kir57a] and [GA94], respectively. See Chapters 8 and 12.

facts which take place for constant currents, or for those whose intensity varies only slowly. His procedure will be considered in more detail in the following Section.

I - Laws of Motion

18.1 Kirchhoff on the Propagation of Electricity in Conductors

Let x, y, z denote the rectangular co-ordinates of a point of the conductor and u, v, w the $(x,\ y,\ z)$ components of the *current densities* which is present at time t in this point of the conductor. — We understand *current density* here as the product of the velocity of the moving carriers times the amount of positive electricity per unit volume of the conductor. Assuming Ohm's resistance law as generally valid, this is tantamount to the product of the *electromotive force* acting on the point $(x,\ y,\ z)$ under consideration times the *specific conductivity* of the metal conductor. Hence, if A denotes the electromotive force at point $(x,\ y,\ z)$ — that is the difference of the forces acting on the unit measure of positive and negative electricity at the point $(x,\ y,\ z)$ —, and α, β, γ as the angles between this force and the three co-ordinate axes, and k as the specific conductivity of the metal we have

$$u = A\cos\alpha \cdot k, \qquad v = A\cos\beta \cdot k, \qquad w = A\cos\gamma \cdot k \ ,$$

where the *mechanical measures*[406] always used by Kirchhoff are to be assumed for forces and conductivity.[407]

[406] [Note by AKTA:] In German: *mechanischen Maasse*. This expression can be translated as "mechanical measures", "mechanical units" or "mechanical units of measure".

[407] [Note by WW:] Let ξ, η, ζ denote the displacement of an electric particle at point $(x,\ y,\ z)$ after time t, hence $d\xi/dt$, $d\eta/dt$, $d\zeta/dt$ the velocity components of the flowing electricity, then, when $\mathfrak{E}$ represents the amount of positive electricity in the unit volume of the conductor, we have according to the first relation:

$$u = \mathfrak{E}\frac{d\xi}{dt}, \qquad v = \mathfrak{E}\frac{d\eta}{dt}, \qquad w = \mathfrak{E}\frac{d\zeta}{dt} \ .$$

But, according to Ohm's law for steady currents, the current intensity i, if steady, in a linear conductor is proportional to, or, in mechanical measure, equal to the sum of all electromotive forces, that is $\int Adl$ along the total length l of the conductor divided by the total resistance, that is $\int dl/ks$, where s is the cross section and k the specific conductivity of the metal, hence we have $i = \int Adl/[\int dl/ks]$. But in this form of Ohm's law the current intensity i through the cross section s of the conducting wire is understood as the product of the velocity of the flowing electricity, that is $d\sigma/dt$, and the amount $\mathfrak{E}$ of positive electricity in the volume unit of the conductor, hence putting

$$\frac{d\xi}{dt} = \frac{d\sigma}{dt}\cdot\cos\alpha, \qquad \frac{d\eta}{dt} = \frac{d\sigma}{dt}\cdot\cos\beta, \qquad \frac{d\zeta}{dt} = \frac{d\sigma}{dt}\cdot\cos\gamma \ ,$$

[one gets]

$$\frac{\int Adl}{\int \frac{dl}{ks}} = \mathfrak{E}s\frac{d\sigma}{dt} \ .$$

Assuming that Ohm's law holds in general for each length element in the circuit, one gets

$$\frac{Adl}{\frac{dl}{ks}} = Aks = \mathfrak{E}s\frac{d\sigma}{dt} \ ,$$

or $Ak = \mathfrak{E}[d\sigma/dt]$, and from this [relation], decomposing for the coordinate axes

$$A\cos\alpha\cdot k = \mathfrak{E}\frac{d\xi}{dt} = u, \qquad A\cos\beta\cdot k = \mathfrak{E}\frac{d\eta}{dt} = v, \qquad A\cos\gamma\cdot k = \mathfrak{E}\frac{d\zeta}{dt} = w \ .$$

The electromotive force, A, however, originates partly from the *free electricity* distributed in the whole circuit and partly from the *induction* which acts in all parts of the conducting circuit due to the change of intensity of the current. To start with, we exclude all *external* electromotive forces, for example magnetoelectric induction forces. Excluding resistive forces that could be taken into account, all other known forces acting on electric masses do not contribute to the *electromotive force* (if the resistive forces, that could be taken into account, are excluded), like for example the electrodynamic forces discovered by Ampère, resulting from interacting current elements, from which it is known that the difference between the forces acting on positive electricity and those acting on negative electricity is always zero, from which hence no *electromotive* force results.

The components of the *first* part of the electromotive force, which comes from the *free electricity* distributed in the conducting circuit, are represented by *doubling* the negative values of the partial differentials of Ω, taken as the value of the *potential function of the free electricity* at point $(x,\ y,\ z)$, with respect to the three coordinate axes, that is these components are represented by[408]

$$-2\frac{d\Omega}{dx}, \qquad -2\frac{d\Omega}{dy}, \qquad -2\frac{d\Omega}{dz}\ ,$$

as is readily realized taking into account that the electromotive force, that is the difference of the forces acting on the unit positive and negative electricities, is *double* the force acting only on one unit of *positive* electricity.

In order to determine the components of the *second* part of the electromotive force which comes from the *induction* caused by changes of current intensities in all parts of the conducting circuit, we denote the coordinates of a second point in the circuit by x', y', z', further the values of u, v, w at this point [are represented] by u', v', w', and the distance between $(x,\ y,\ z)$ and $(x',\ y',\ z')$ by r.

From the fundamental law of electric action we get the electromotive force exerted by the electricity in volume element $dx'dy'dz'$, moving *along the direction of the x axis* with velocity $d\xi'/dt$, remembering that, according to the previous footnote, $u' = \mathfrak{E}[d\xi'/dt]$, [acting] at point $(x,\ y,\ z)$ *along the x-axis*, expressed in *mechanical* measure [as given by:]

$$= -\frac{8}{c^2}\cdot\frac{dx'dy'dz'}{r^3}\cdot(x-x')^2\cdot\frac{du'}{dt}\ .$$

Hence, the electromagnetic force due to a current element of length α with its current intensity uniformly increasing by [a factor of] i during the time t that acts on a point at distance r equals (see *Electrodynamic Measurements*, Vol. 5 of these *Abhandlungen*, p. 268, number 4)[409,410]

$$= -\frac{2\sqrt{2}}{c}\cdot\frac{\alpha}{r}\cdot\frac{i}{t}\cdot\cos\vartheta\cos\vartheta'\ ,$$

[408][Note by AKTA:] Nowadays these partial derivatives would be written as:

$$-2\frac{\partial\Omega}{\partial x}, \qquad -2\frac{\partial\Omega}{\partial y}, \qquad -2\frac{\partial\Omega}{\partial z}\ .$$

In this English translation we are maintaining Weber's original notation.

[409][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 655.

[410][Note by AKTA:] [KW57, Section 18, number 4, p. 655 of Weber's *Werke*] with English translation in [KW21, Section 18, number 4, p. 55]. See also item (4) of page 182 on Section 7.18.

along the direction which makes an angle ϑ' with the extension of r, if α makes an angle ϑ with r. Here, however, the current intensity i, usually determined by means of a galvanometer, is to be expressed in terms of absolute *magnetic* measure; which can be replaced by the value expressed in *mechanical* measure if multiplied by $2\sqrt{2}/c$. In *mechanical* measure the above current intensity, on the other hand, is $= u'dy'dz'$. Putting $i = [2\sqrt{2}/c] \cdot u'dy'dz'$, hence

$$\frac{i}{t} = \frac{di}{dt} = \frac{2\sqrt{2}}{c} \cdot \frac{du'}{dt} \cdot dy'dz' \ ,$$

and notice that in the above case $\cos\vartheta = \cos\vartheta' = (x - x')/r$ and $\alpha = dx'$, then we find the electromotive force we were looking for

$$= -\frac{2\sqrt{2}}{c} \cdot \frac{dx'}{r} \cdot \frac{2\sqrt{2}}{c} \cdot \frac{du'}{dt} \cdot dy'dz' \cdot \frac{(x - x')^2}{r^2} \ ,$$

which equals the value given above.

Considering the motion of electricity in element $dx'dy'dz'$ *along the y or z axis* instead of along the x axis, the value of $\cos\vartheta$ is given by $(y-y')/r$ or by $(z-z')/r$ instead of $(x-x')/r$, and dv'/dt or dw'/dt instead of du'/dt, from which the total electromotive force exerted by the element $dx'dy'dz'$ at point $(x,\ y,\ z)$ *along the x axis* results as equal to

$$= -\frac{8}{c^2} \cdot \frac{dx'dy'dz'}{r^3}(x - x')\left(\frac{du'}{dt}(x - x') + \frac{dv'}{dt}(y - y') + \frac{dw'}{dt}(z - z')\right) \ .$$

Likewise, putting $(y - y')/r$ or $(z - z')/r$ instead of $(x - x')/r$ for $\cos\vartheta'$, one finds the total electromotive force exerted *along the y or z axis* as equal to

$$= -\frac{8}{c^2} \cdot \frac{dx'dy'dz'}{r^3} \cdot (y - y')\left(\frac{du'}{dt}(x - x') + \frac{dv'}{dt}(y - y') + \frac{dw'}{dt}(z - z')\right) \ ,$$

or

$$= -\frac{8}{c^2} \cdot \frac{dx'dy'dz'}{r^3} \cdot (z - z')\left(\frac{du'}{dt}(x - x') + \frac{dv'}{dt}(y - y') + \frac{dw'}{dt}(z - z')\right) \ .$$

Putting for brevity

$$U = \int\int\int \frac{dx'dy'dz'}{r^3}(x - x')\Big(u'(x - x') + v'(y - y') + w'(z - z')\Big) \ , \qquad (1)$$

$$V = \int\int\int \frac{dx'dy'dz'}{r^3}(y - y')\Big(u'(x - x') + v'(y - y') + w'(z - z')\Big) \ , \qquad (2)$$

$$W = \int\int\int \frac{dx'dy'dz'}{r^3}(z - z')\Big(u'(x - x') + v'(y - y') + w'(z - z')\Big) \ , \qquad (3)$$

one gets the components of the *second* part of the electromotive force which comes from the *induction* as a result from changes of current intensities in all parts of the conducting circuit equal to

$$= -\frac{8}{c^2} \cdot \frac{dU}{dt}, \qquad = -\frac{8}{c^2} \cdot \frac{dV}{dt}, \qquad = -\frac{8}{c^2} \cdot \frac{dW}{dt} \ .$$

However, the above components of the *total* electromotive force have been expressed as

$$A\cos\alpha, \qquad A\cos\beta, \qquad A\cos\gamma \ ,$$

thus one gets

$$A\cos\alpha = -2\left(\frac{d\Omega}{dx} + \frac{4}{c^2} \cdot \frac{dU}{dt}\right) \ ,$$

$$A\cos\beta = -2\left(\frac{d\Omega}{dy} + \frac{4}{c^2} \cdot \frac{dV}{dt}\right) \ ,$$

$$A\cos\gamma = -2\left(\frac{d\Omega}{dz} + \frac{4}{c^2} \cdot \frac{dW}{dt}\right) \ .$$

Finally, putting these values into the above equations for the current densities u, v, w at point $(x,\ y,\ z)$, one gets the following equations

$$u = -2k\left(\frac{d\Omega}{dx} + \frac{4}{c^2} \cdot \frac{dU}{dt}\right) \ , \qquad (4)$$

$$v = -2k\left(\frac{d\Omega}{dy} + \frac{4}{c^2} \cdot \frac{dV}{dt}\right) \ , \qquad (5)$$

$$w = -2k\left(\frac{d\Omega}{dz} + \frac{4}{c^2} \cdot \frac{dW}{dt}\right) \ , \qquad (6)$$

As a special prerequisite to determine the value Ω of the *potential function* at point $(x,\ y,\ z)$ of the total *free* electricity distributed in the whole circuit, the density of *free* electricity in the inner part of the conductor where current motions take place must not be equated to zero, as for a conductor with electricity at rest. Denoting by ε' the non-zero [volume] density of *free* electricity at point $(x',\ y',\ z')$, if *inside* the conductor, and by e', when located on the surface element dS', thus denoting the [surface] density of *free* electricity in the *surface element* dS' by e', we then get the following value of Ω, namely

$$\Omega = \int\int\int \frac{dx'dy'dz'}{r} \cdot \varepsilon' + \int\int \frac{dS'}{r} \cdot e' \ . \qquad (7)$$

In addition, the distribution of free electricity *inside* the whole conducting circuit as well as *at its surface* which is determined by the values of ε' and e', may indeed change with time, but these changes depend on the motion of electricity in the circuit, whence there must be two equations to represent the *partial differential coefficients* of ε' and e' with respect to time in their dependence on the *motion* of electricity.

The difference between the positive electricity leaving the element $dx'dy'dz'$ along the x, y, z axis during the time element dt and the electricity entering it is

$$dx'dy'dz' \cdot \frac{du'}{dx'}dt, \qquad dx'dy'dz' \cdot \frac{dv'}{dy'}dt, \qquad dx'dy'dz' \cdot \frac{dw'}{dz'}dt \ .$$

The *sum* of these differences yields the *decrease* of the free electricity $dx'dy'dz' \cdot \varepsilon'$ contained in $dx'dy'dz'$ during the time element dt which is produced by motion of the *positive* electricity. There is, however, another equal decrease resulting from the *opposite motion of negative electricity* during the same time element dt; consequently this sum equals *half of the total decrease* of the free electricity contained in the element $dx'dy'dz'$ during the time element dt, that is *half* of $= -dx'dy'dz' \cdot [d\varepsilon'/dt] \cdot dt$, hence we have

$$\frac{du'}{dx'} + \frac{dv'}{dy'} + \frac{dw'}{dz'} = -\frac{1}{2}\frac{d\varepsilon'}{dt} \ . \tag{8}$$

Finally, denoting the angles between the inward normal on the surface element dS' and the x, y, z axes by $(N',\ x')$, $(N',\ y')$, $(N',\ z')$, the amount of *positive* electricity flowing back inwards from the surface element dS' during the time element dt equals

$$= \Big(u' \cos(N',\ x') + v' \cos(N',\ y') + w' \cos(N',\ z')\Big) dS' \cdot dt \ ,$$

and, because an equal amount of *negative* electricity flows from the interior towards the surface element dS' during the same time, this amount equals half of the *total decrease of free electricity* $e'dS'$ at the surface element dS' during the time element dt, that is, $= -\frac{1}{2}[de'/dt] \cdot dS'dt$, hence we have

$$u' \cos(N',\ x') + v' \cos(N',\ y') + w' \cos(N', z') = -\frac{1}{2}\frac{de'}{dt} \ . \tag{9}$$

As general as this derivation of the equations of motion of electricity in any conductor by Kirchhoff may be in other respects, it is based on three limiting assumptions, namely:

1. the assumption that the value of the electromotive force in a point, as was done above, may simply be determined by *doubling the force exerted on the positive electricity*, that is, assuming equal amounts of positive and negative electricity in all parts of the conductor, or, more precisely, that this would strictly mean that the densities ε' and e' of free electricity inside and on the surface of the conductor would always and everywhere be equal to zero, which is not the case, that at least the present *free* electricity may be considered as vanishingly small compared with the amount of a *neutral mixture* of both electricities at the same position;

2. the assumption that always equal amounts of positive and negative electricity pass through each cross section simultaneously in opposite directions, which is only justified, when in addition we can assume everywhere an arbitrary motion of the neutral fluid, on the grounds that such an added motion of a neutral fluid, if it were really present, would have no influence at all on the *observations*;

3. the assumption of a more general validity of Ohm's law which, as is to be shown later, may be reduced to the assumption that the *mass* of the electric fluid would vanish everywhere compared with the *mass* of its ponderable carrier, which, however, is usually assumed in general.

18.2 Derivation of the Expression for the Electromotive Force Exerted by the Free Electricity and by the Electric Motions in a Small Piece of the Conducting Wire, Considered as a Cylinder, on Any Point of the Middle Circular Cross Section of This Piece

The more precise determination of the electromotive force acting in any point of the cross section of the conducting wire suggests to divide the latter into two parts, namely the part that comes from the element of the conducting wire which contains the point under consideration, and the part that comes from all other elements that lie in greater measurable distances from the point under consideration.[411]

Let the element of the conducting wire which contains the point under consideration be a cylinder whose radius is very small compared with its length. Kirchhoff assumes the distribution of free electricity as well as that of the electric motions in that cylinder as *symmetric with respect to the cylinder axis*. With respect to the coordinates, let the x axis coincide with the cylinder axis and put

$$y = \rho\cos\varphi, \qquad y' = \rho'\cos\varphi' \ ,$$

$$z = \rho\sin\varphi, \qquad z' = \rho'\sin\varphi' \ .$$

Further, distinguishing the current densities parallel to the cylinder axis and perpendicular to the cylinder axis, the latter is everywhere *radial* under the assumption of the symmetry of the motions, that is, in any point its direction coincides with the cylinder radius through that point. Hence, denoting σ this *radial current density* at point $(x,\ y,\ z)$ and σ' at point $(x',\ y',\ z')$, it follows that

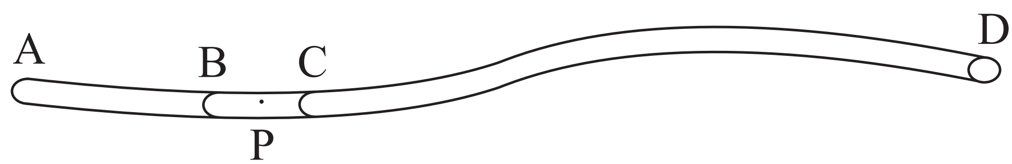
$$v = \sigma\cos\varphi, \qquad v' = \sigma'\cos\varphi' \ ,$$

$$w = \sigma\sin\varphi, \qquad w' = \sigma'\sin\varphi' \ ,$$

where the values of σ and σ' are independent of φ and φ'.

Substituting these values in the expressions of Ω and U in the previous Section and taking α as the radius of the cylinder, we get

[411][Note by AKTA:] Weber will calculate the electromotive force at a point P inside the wire. He then divides the whole circuit AD below into two parts.



The first part, BC, contains all points closed to the point P and will be considered as cylindrical. The point P is located in the cross section of the wire located in the middle of BC. The second part, composed of pieces AB and CD, contains the points which are at great distances from P.

$$\Omega = \int\int\int \frac{dx' \cdot \rho' d\rho' d\varphi'}{r} \cdot \varepsilon' + \alpha \int\int \frac{dx' d\varphi'}{r} \cdot e' \ , \tag{1}$$

$$U = \int\int\int \frac{dx' \cdot \rho' d\rho' d\varphi'}{r^3} (x - x') \Big(u'(x - x') + \sigma'(\rho \cos(\varphi - \varphi') - \rho') \Big) \ . \tag{2}$$

Further, this substitution yields

$$\frac{dv'}{dy'} = \frac{d \cdot \sigma' \cos\varphi'}{dy'} \ ,$$

where σ' depends only on the variable ρ' for a given value of x'. Hence putting $\sigma' = f(\rho') = f\left(\sqrt{y'^2 + z'^2}\right)$, we get

$$\frac{dv'}{dy'} = \frac{d}{dy'} \cdot \left(\frac{y' f\left(\sqrt{y'^2 + z'^2}\right)}{\sqrt{y'^2 + z'^2}} \right) = \frac{y'^2}{\rho'^2} \cdot \frac{d\sigma'}{d\rho'} + \frac{\rho'^2 - y'^2}{\rho'^3} \cdot \sigma' \ .$$

Likewise, we get

$$\frac{dw'}{dz'} = \frac{z'^2}{\rho'^2} \cdot \frac{d\sigma'}{d\rho'} + \frac{\rho'^2 - z'^2}{\rho'^3} \cdot \sigma' \ .$$

hence

$$\frac{dv'}{dy'} + \frac{dw'}{dz'} = \frac{d\sigma'}{d\rho'} + \frac{\sigma'}{\rho'} = \frac{1}{\rho'} \cdot \frac{d \cdot \rho' \sigma'}{d\rho'} \ .$$

Adding du'/dx' and substituting the respective value for the sum $du'/dx' + dv'/dy' + dw'/dz'$ in Equation (8) of the preceding Section, yields the equation

$$\frac{du'}{dx} + \frac{1}{\rho'} \cdot \frac{d \cdot \rho' \sigma'}{d\rho'} = -\frac{1}{2} \frac{d\varepsilon'}{dt} \ . \tag{3}$$

Finally one finds the following values for the angles between the normal of the surface element dS' pointing inwards and the directions of the three coordinate axes:

$$(N',\ x') = \frac{\pi}{2}, \qquad (N',\ y') = \varphi' + \pi, \qquad (N',\ z') = \varphi' + \frac{\pi}{2} \ ;$$

hence we have[412]

$$u' \cos(N',\ x') = 0 \ ,$$

[412][Note by AKTA:] The second and third equation below were written in the original text as, respectively:

$$v' \cos(N',\ y') = -\sigma' \cos\varphi'^2 \ ,$$

$$w' = \cos(N',\ z') = -\sigma' \sin\varphi'^2 \ .$$

In order to avoid confusion, in this translation we are replacing the notations $\sin\varphi'^2$ and $\cos\varphi'^2$ used in Weber's time by their modern counterparts, namely, $\sin^2\varphi'$ and $\cos^2\varphi'$, respectively.

$$v' \cos(N',\ y') = -\sigma' \cos^2 \varphi' \ ,$$

$$w' = \cos(N',\ z') = -\sigma' \sin^2 \varphi' \ .$$

Substituting these values in Equation (9) of the preceding Section then yields

$$\sigma' = \frac{1}{2}\frac{de'}{dt} \ . \qquad (4)$$

Putting for brevity

$$x' - x = \lambda, \qquad \text{hence} \qquad dx' = d\lambda \ ,$$

$$\rho^2 + \rho'^2 - 2\rho\rho' \cos(\varphi - \varphi') = \beta^2, \qquad \text{hence} \qquad r^2 = \beta^2 + \lambda^2 \ ,$$

then, according to Equations (1) and (2), we have

$$\Omega = \int\int \rho' d\rho' d\varphi' \int_{-l/2}^{l/2} \frac{\varepsilon' d\lambda}{\sqrt{\beta^2 + \lambda^2}} + \alpha \int d\varphi' \int_{-l/2}^{l/2} \frac{e' d\lambda}{\sqrt{\beta^2 + \lambda^2}} \ ,$$

$$U = \int\int \rho' d\rho' d\varphi' \int_{-l/2}^{l/2} \frac{u' \lambda^2 d\lambda}{(\beta^2 + \lambda^2)^{3/2}}$$
$$+ \int\int \rho'^2 \left(1 - \frac{\rho}{\rho'} \cos(\varphi - \varphi')\right) d\rho' d\varphi' \int_{-l/2}^{l/2} \frac{\sigma' \lambda d\lambda}{(\beta^2 + \lambda^2)^{3/2}} \ .$$

where l denotes the length of the cylinder and the point $(x,\ y,\ z)$ lies on the cross section that cuts this length in half.[413]

Calculating the series expansion of ε' and e' in powers of λ in the first equation, namely

$$e' = e + \frac{de}{dx} \cdot \lambda + \frac{1}{1 \cdot 2} \cdot \frac{d^2 e}{dx^2} \cdot \lambda^2 + \ldots \ ,$$

$$\varepsilon' = \varepsilon'_0 + \frac{d\varepsilon'_0}{dx} \cdot \lambda + \frac{1}{1 \cdot 2} \cdot \frac{d^2 \varepsilon'_0}{dx^2} \cdot \lambda^2 + \ldots \ .$$

where ε'_0, apart from time, depends only on the variable ρ', then for very small values of β^2/l^2 which follow with necessity from small values of α/l because β^2 can never be greater than $4\alpha^2$, we may put[414]

$$\int_{-l/2}^{l/2} \frac{d\lambda}{\sqrt{\beta^2 + \lambda^2}} = 2 \log \frac{l}{\beta} \ ,$$

$$\int_{-l/2}^{l/2} \frac{\lambda d\lambda}{\sqrt{\beta^2 + \lambda^2}} = 0 \ ,$$

[413][Note by AKTA:] In footnote 407 on page 270 of Section 18.1, Weber had called l the total length of the curved and thin conductor. He is now representing by the same letter the length of the small cylinder BC represented in footnote 411 on page 275.

[414][Note by AKTA:] What Weber represents by the symbol "log" in the next equations should be understood as the natural logarithm represented nowadays as "ln".

$$\int_{-l/2}^{l/2} \frac{\lambda^2 d\lambda}{\sqrt{\beta^2+\lambda^2}} = \frac{l^2}{4} \ ,$$

whence for small values of l we get

$$\Omega = \int\int \rho' d\rho' d\varphi' \left(2\varepsilon_0' \log\frac{l}{\beta} + \frac{1}{8}\frac{d^2\varepsilon_0'}{dx^2}l^2\right) + \alpha \int d\varphi' \left(2e\log\frac{l}{\beta} + \frac{1}{8}\frac{d^2 e}{dx^2}l^2\right) \ .$$

The integration is to be carried out from $\varphi' = 0$ to $\varphi' = 2\pi$ and from $\rho' = 0$ to $\rho' = \alpha$, whence we get

$$\begin{aligned}\Omega = 2\pi \int_0^\alpha \rho' d\rho' \left(2\varepsilon_0' \log l + \frac{1}{8}\frac{d^2\varepsilon_0'}{dx^2}l^2\right) + 2\pi\alpha \left(2e\log l + \frac{1}{8}\frac{d^2 e}{dx^2}l^2\right) \\ - 2\int_0^\alpha \rho' d\rho' \cdot \varepsilon_0' \int_0^{2\pi} d\varphi' \cdot \log\beta - 2\alpha e \int_0^{2\pi} d\varphi' \cdot \log\beta \ .\end{aligned}$$

Considering that

$$\int_0^{2\pi} d\varphi' \cdot \log\beta = \frac{1}{2}\int_0^{2\pi} d\varphi' \cdot \log\left(\rho^2 + \rho'^2 - 2\rho\rho'\cos(\varphi-\varphi')\right)$$

either equals $2\pi\log\rho'$ when $\rho' > \rho$, or equals $2\pi\log\rho$ when $\rho > \rho'$, then we get for the part referring to the *surface* for which $\rho' = \alpha$,

$$-2\alpha e \int_0^{2\pi} d\varphi' \cdot \log\beta = -4\pi\alpha e \log\alpha \ .$$

The part referring to the *interior* [of the conductor] is decomposed into two parts, namely

$$-2\int_0^\alpha \rho' d\rho' \cdot \varepsilon_0' \int_0^{2\pi} d\varphi' \cdot \log\beta = -4\pi\log\rho \int_0^\rho \rho' d\rho' \cdot \varepsilon_0' - 4\pi \int_\rho^\alpha \rho' d\rho' \cdot \varepsilon_0' \log\rho' \ ,$$

which hence reduces to

$$-4\pi\log\alpha \cdot \int_0^\alpha \rho' d\rho' \cdot \varepsilon_0' \ ,$$

in the limit where $\rho = \alpha$, and [reduces] to

$$-4\pi \int_0^\alpha \rho' d\rho' \cdot \varepsilon_0' \log\rho' \ ,$$

in the other limit where $\rho = 0$, which expressions differ the less from each other the smaller [the value of] α,[415] so that, with sufficient precision for very small values of α one may put

[415][Note by WW:] Under the assumption of the symmetric distribution of free electricity in the wire, ε_0' approaches a *constant* with decreasing values of ρ'. Thus, if for small values of α it is allowed to put ε_0' equal to a *constant* for all values of $\rho' < \alpha$, the value found for the first limit turns into

$$-4\pi\varepsilon_0' \log\alpha \int_0^\alpha \rho' d\rho' = -2\pi\alpha^2\varepsilon_0' \log\alpha \ ,$$

$$2\int_0^\alpha \rho' d\rho' \cdot \varepsilon_0' \int_0^{2\pi} d\varphi' \cdot \log\beta = -4\pi \log\alpha \cdot \int_0^\alpha \rho' d\rho' \cdot \varepsilon_0'$$

for the part referring to the interior; hence

$$\begin{aligned}\Omega = 2\pi \int_0^\alpha \rho' d\rho' \left(2\varepsilon_0' \log l + \frac{1}{8}\frac{d^2\varepsilon_0'}{dx^2} l^2\right) + 2\pi\alpha \left(2e \log l + \frac{1}{8}\frac{d^2 e}{dx^2} l^2\right) \\ - 4\pi \log\alpha \cdot \left(\alpha e + \int_0^\alpha \rho' d\rho' \cdot \varepsilon_0'\right) \ ,\end{aligned}$$

or, more concisely

$$\Omega = 4\pi \log\frac{l}{\alpha} \cdot \left(\alpha e + \int_0^\alpha \rho' d\rho' \cdot \varepsilon_0'\right) + \frac{1}{4}\pi l^2 \cdot \left(\alpha \frac{d^2 e}{dx^2} + \int_0^\alpha \rho' d\rho' \cdot \frac{d^2 \varepsilon_0'}{dx^2}\right) \ .$$

Finally, putting

$$2\pi\alpha e + 2\pi \int_0^\alpha \rho' d\rho' \cdot \varepsilon_0' = E \ ,$$

that means, denoting the amount of *free* electricity contained in the conductor element dx, partly at its surface, partly in the interior, by Edx,[416] then differentiating twice one gets

$$2\pi\alpha \cdot \frac{d^2 e}{dx^2} + 2\pi \int_0^\alpha \rho' d\rho' \cdot \frac{d^2\varepsilon_0'}{dx^2} = \frac{d^2 E}{dx^2} \ ,$$

hence

$$\Omega = 2E \log\frac{l}{\alpha} + \frac{1}{8}\frac{d^2 E}{dx^2} \cdot l^2 \ . \qquad (5)$$

Likewise, the values of u' and σ' may be developed in the above equation for U by a series expansion in powers of λ, namely

$$u' = u_0' + \frac{du_0'}{dx} \cdot \lambda + \frac{1}{1 \cdot 2} \cdot \frac{d^2 u_0'}{dx^2} \cdot \lambda^2 + \ldots \ ,$$

$$\sigma' = \sigma_0' + \frac{d\sigma_0'}{dx} \cdot \lambda + \frac{1}{1 \cdot 2} \cdot \frac{d^2 \sigma_0'}{dx^2} \cdot \lambda^2 + \ldots \ ,$$

where, apart from the time, u_0' and σ_0' depend only on the variable ρ' for a given value of x'.

Now, for very small values of β^2/l^2 corresponding to very small values of α^2/l^2, one can put

$$\int_{-l/2}^{l/2} \frac{\lambda d\lambda}{(\beta^2 + \lambda^2)^{3/2}} = 0 \ ,$$

and the one for the latter limit [turns] into

$$-4\pi\varepsilon_0' \int_0^\alpha \rho' d\rho' \log\rho' = -2\pi\alpha^2\varepsilon_0' \left(\log\alpha - \frac{1}{2}\right) \ ,$$

which expressions differ the less, the smaller α.

[416][Note by AKTA:] Therefore the magnitude E means the linear charge density of the wire, that is, the amount of free charge per unit length.

$$\int_{-l/2}^{l/2} \frac{\lambda^2 d\lambda}{(\beta^2+\lambda^2)^{3/2}} = 2\left(\log\frac{l}{\beta} - 1\right) = 2\log\frac{l}{e\beta} \ ,$$

$$\int_{-l/2}^{l/2} \frac{\lambda^3 d\lambda}{(\beta^2+\lambda^2)^{3/2}} = 0 \ ,$$

$$\int_{-l/2}^{l/2} \frac{\lambda^4 d\lambda}{(\beta^2+\lambda^2)^{3/2}} = \frac{1}{4}l^2 \ ,$$

where e is the base of the natural logarithms. Hence one gets the following equation for U:

$$U = \int\int \rho' d\rho' d\varphi' \cdot \left(2u_0' \cdot \log\frac{l}{e\beta} + \frac{1}{8}\frac{d^2u_0'}{dx^2} \cdot l^2\right)$$
$$+ \int\int \rho'^2 \left(1 - \frac{\rho}{\rho'}\cos(\varphi-\varphi')\right) d\rho' d\varphi' \left(2\frac{d\sigma_0'}{dx}\log\frac{l}{e\beta} + \frac{1}{24}\frac{d^3\sigma_0'}{dx^3}l^2\right) \ .$$

The latter part of this value of U may be considered very small when α is very small, because the integration for ρ' has to be carried out from $\rho' = 0$ to $\rho' = \alpha$, hence

$$U = \int\int \rho' d\rho' d\varphi' \cdot \left(2u_0' \log\frac{l}{e\beta} + \frac{1}{8}\frac{d^2u_0'}{dx^2}l^2\right) \ ,$$

where the integration has to be carried out from $\varphi' = 0$ to $\varphi' = 2\pi$ and from $\rho' = 0$ to $\rho' = \alpha$, thus

$$U = 2\pi \int_0^\alpha \rho' d\rho' \cdot \left(2u_0' \log\frac{l}{e} + \frac{1}{8}\frac{d^2u_0'}{dx^2} \cdot l^2\right) - 2\int_0^\alpha \rho' d\rho' \cdot u_0' \int_0^{2\pi} d\varphi' \log\beta \ .$$

As

$$\int_0^{2\pi} d\varphi' \cdot \log\beta = \frac{1}{2}\int_0^{2\pi} d\varphi' \cdot \log\left(\rho^2 + \rho'^2 - 2\rho\rho'\cos(\varphi-\varphi')\right)$$

equals either $2\pi\log\rho'$ if $\rho' > \rho$, or equals $2\pi\log\rho$ if $\rho > \rho'$, one gets

$$U = 2\pi \int_0^\alpha \rho' d\rho' \cdot \left(2u_0' \log\frac{l}{e} + \frac{1}{8}\frac{d^2u_0'}{dx^2}l^2\right)$$
$$- 4\pi\log\rho \int_0^\rho \rho' d\rho' \cdot u_0' - 4\pi \int_\rho^\alpha \rho' d\rho' \cdot u_0' \log\rho' \ ,$$

for which one can also write

$$U = 4\pi\log\frac{l}{e\alpha} \int_0^\alpha \rho' d\rho' \cdot u_0' + \frac{1}{4}\pi l^2 \int_0^\alpha \rho' d\rho' \cdot \frac{d^2u_0'}{dx^2}$$
$$+ 4\pi\log\frac{\alpha}{\rho} \cdot \int_0^\rho \rho' d\rho' \cdot u_0' + 4\pi \int_\rho^\alpha \rho' d\rho' \cdot u_0' \log\frac{\alpha}{\rho'} \ .$$

But now, when α is very small, the latter two parts of this value of U may be considered as vanishing, then one may put

$$U = 4\pi \log \frac{l}{e\alpha} \cdot \int_0^\alpha \rho' d\rho' \cdot u_0' + \frac{1}{4}\pi l^2 \cdot \int_0^\alpha \rho' d\rho' \cdot \frac{d^2 u_0'}{dx^2} \ .$$

Finally putting

$$2\pi \int_0^\alpha \rho' d\rho' \cdot u_0' = i \ ,$$

that means, denoting the amount of positive electricity flowing through the cross section of the conducting wire during the time element dt by idt, where i expresses the current intensity in mechanical measure, then differentiating twice yields

$$2\pi \int_0^\alpha \rho' d\rho' \cdot \frac{d^2 u_0'}{dx^2} = \frac{d^2 i}{dx^2} \ ,$$

hence

$$U = 2i \log \frac{l}{e\alpha} + \frac{1}{8} \frac{d^2 i}{dx^2} \cdot l^2 \ .$$

Hereafter the electromotive force exerted by the *free electricity* in a small piece of the conducting wire, considered as a cylinder, on any point of the middle part of this piece, is determined more precisely, namely, from the value of Ω, [through]

$$-2\frac{d\Omega}{dx} = -4\frac{dE}{dx} \cdot \log \frac{l}{\alpha} - \frac{1}{4} \frac{d^3 E}{dx^3} \cdot l^2 \ ,$$

and likewise the electromotive force exerted by *induction* of the *electric motions* in the same piece on the same point [is determined], namely, from the value of U, [through]

$$-\frac{8}{c^2} \frac{dU}{dt} = -\frac{16}{c^2} \cdot \frac{di}{dt} \cdot \log \frac{l}{e\alpha} - \frac{1}{c^2} \cdot \frac{d^3 i}{dx^2 dt} \cdot l^2 \ .$$

Finally, assuming a very large number for the value of $\log[l/\alpha]$ as Kirchhoff did, one may put

$$-2\frac{d\Omega}{dx} = -4\frac{dE}{dx} \cdot \log \frac{l}{\alpha} \ ,$$

$$-\frac{8}{c^2} \frac{dU}{dt} = -\frac{16}{c^2} \cdot \frac{di}{dt} \cdot \log \frac{l}{e\alpha} \ ,$$

or, when 1 vanishes completely compared with $\log[l/\alpha]$,

$$-\frac{8}{c^2} \frac{dU}{dt} = -\frac{16}{c^2} \cdot \frac{di}{dt} \cdot \log \frac{l}{\alpha} \ .$$

18.3 Simplification of the General Equations

Following a more exact determination of the electromotive forces acting on a point $(x, \ y, \ z)$ of the conducting wire, that come partly from free electricity, partly from the electric motions in a small part of the conducting wire to be considered as a cylinder, Kirchhoff has tried to simplify the general equations presented in Section 18.1 under the following conditions, namely

1. that the radius of the conducting wire, α, be so small compared to the length l of its element, considered as cylindrical, that $\log[l/\alpha]$ represents a very large number as was assumed already in the preceding Section for the simplification of the expression of the electromotive forces;

2. that the electromotive forces acting on a point $(x,\ y,\ z)$ in such a thin conducting wire originating from the free electricity and from the electric motions, except from the single small piece considered as a cylinder whose central cross section contains the point $(x,\ y,\ z)$, be vanishingly small compared to the electromotive forces acting on the same point originating from the electric motions in this small piece. — In addition, we have the assumption already used also for the development of the general equations in Section 18.1;

3. that Ohm's law be separately valid for all current elements, even if the current intensities in these elements are very different and vary rapidly.

If now, according to the *first* assumption, $\log[l/\alpha]$ is a very large number and if, according to the *second* assumption, only the electromotive forces determined more precisely in the preceding Section are taken into consideration, compared to which the others due to the more distant parts of the conducting wire are vanishingly small, one finds after the conclusion of the preceding Section the *complete expression of the electromotive force* along the axis of the conducting wire, [namely]

$$-2\left(\frac{d\Omega}{dx}+\frac{4}{c^2}\frac{dU}{dt}\right)=-4\log\frac{l}{\alpha}\left(\frac{dE}{dx}+\frac{4}{c^2}\frac{di}{dt}\right) .$$

If this [formula] is now the expression of the total electromotive force, then it yields according to Section 18.1, multiplied by the specific conductivity k, in accordance with the *third* assumption, the current density u along the direction of the conducting wire in the point $(x,\ y,\ z)$, namely

$$u=-4k\log\frac{l}{\alpha}\cdot\left(\frac{dE}{dx}+\frac{4}{c^2}\frac{di}{dt}\right) .$$

Considering, finally, that the current density at the point $(x,\ y,\ z)$, hereafter independent of ρ and, consequently, equal for all points of the cross section of the wire, multiplied by the wire cross section $\pi\alpha^2$, yields therefore the current intensity i, then, multiplying the previous equation by $\pi\alpha^2$ one gets the following equation derived from the seven first general equations of Section 18.1:

$$i=-4\pi\alpha^2k\log\frac{l}{\alpha}\cdot\left(\frac{dE}{dx}+\frac{4}{c^2}\frac{di}{dt}\right) .$$

Hence there only remain the last two of the general equations derived in Section 18.1, which have been reduced in Section 18.2 to

$$\frac{du}{dx}+\frac{1}{\rho}\cdot\frac{d\cdot\rho\sigma}{d\rho}=-\frac{1}{2}\frac{d\varepsilon}{dt} ,$$

$$\sigma=\frac{1}{2}\frac{de}{dt} .$$

Multiplying the first equation by $\rho d\rho d\varphi$ and integrating over the total cross section of the conducting wire, and finally subtracting the second equation multiplied by $2\pi\alpha$, one gets

$$\pi\alpha^2 \cdot \frac{du}{dx} = -\pi\alpha \cdot \frac{de}{dt} - \pi \int_0^\alpha \rho d\rho \cdot \frac{d\varepsilon}{dt} .$$

But now, according to Section 18.2, for $\rho' = \rho$ we have

$$2\pi\alpha e + 2\pi \int_0^\alpha \rho d\rho \cdot \varepsilon = E ,$$

whence

$$2\pi\alpha \cdot \frac{de}{dt} + 2\pi \int_0^\alpha \rho d\rho \cdot \frac{d\varepsilon}{dt} = \frac{dE}{dt} ,$$

and so, because we had $\pi\alpha^2 u = i$, from which follows $\pi\alpha^2 \cdot (du/dx) = di/dx$, the two last equations of Section 18.1 yield the following [equation]:

$$\frac{di}{dx} = -\frac{1}{2}\frac{dE}{dt} .$$

Based on this reduction from nine general equations to two, namely

$$i = -4\pi\alpha^2 k \log\frac{l}{\alpha} \cdot \left(\frac{dE}{dx} + \frac{4}{c^2}\frac{di}{dt}\right) ,$$

$$\frac{di}{dx} = -\frac{1}{2}\frac{dE}{dt} ,$$

we can, eliminating i, finally derive the law that allows to determine the distribution of free electricity, E, in the circuit for any moment, namely[417]

$$\frac{d^2E}{dt^2} - \frac{c^2}{2}\frac{d^2E}{dx^2} + \frac{c^2}{16\pi\alpha^2 k \log\frac{l}{\alpha}} \cdot \frac{dE}{dt} = 0 ,$$

or we can, eliminating E, derive the law that allows to determine the current intensity, i, for any point of the circuit and for any moment, namely

[417] [Note by AKTA:] The next equation can be expressed in more familiar terms utilizing the modern symbol ∂ for partial derivatives, the modern symbol "ln" for the natural logarithm instead of Weber's "log", and multiplying all terms by $-2/c^2$. After rearranging the terms we get the following equation for the linear charge density $E(x,\ t)$ along the thin wire:

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2/2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{8\pi\alpha^2 k \ln\frac{l}{\alpha}}\frac{\partial E}{\partial t} .$$

Weber and Kohlrausch had measured Weber's constant c in 1855-6. They obtained $c = 4.39450 \times 10^8$ m/s, [KW57] with English translation in [KW21], see also page 179 on Section 7.17. Therefore $c/\sqrt{2} = 3.1 \times 10^8$ m/s, essentially the same value of light velocity in vacuum, as pointed out by Kirchhoff in 1857, [Kir57b] with English translation in [Kir57a], see page 214 on Chapter 8.

Therefore the next equation presented by Weber is the modern telegraph equation describing the propagation of an electric disturbance along a resistive wire. When the resistance is negligible, we obtain the wave equation for a signal propagating at light velocity $v_L = c/\sqrt{2}$, namely

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2/2}\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} - \frac{1}{v_L^2}\frac{\partial^2 E}{\partial t^2} = 0 .$$

$$\frac{d^2 i}{dt^2} - \frac{c^2}{2}\frac{d^2 i}{dx^2} + \frac{c^2}{16\pi\alpha^2 k \log\frac{l}{\alpha}} \cdot \frac{di}{dt} = 0 \ .$$

As is easy to see, the distribution of free electricity as well as the current intensities in all parts, however, would have followed all by itself from the *motions* of all electric particles in the conducting wire, were the law of the latter known. Vice versa the latter law is easily derived from the known law of the distribution and the current intensities where it suffices to formulate it for the motions of all *positive* electric particles in the conducting wire, because the opposite motions of all *negative* electric particles follow all by themselves.

Let s denote any point of the conducting wire[418] and $\mathfrak{E}ds$ the total amount of positive electricity which is contained in the length element, ds, of the conducting wire, and further σ the displacement of one particle of this positive electricity after time t from its initial equilibrium, thus $d\sigma/dt$ the velocity of this particle in the conducting wire and $d\sigma/ds$ the dilution of the positive electricity at the point s of the conducting wire at the end of time t, which always corresponds to an equally great dilution of negative electricity; then the current intensity i at the point s of the conducting wire at the end of time t equals the product $\mathfrak{E}d\sigma/dt$, and the [linear] density E of *free electricity*, that is the surplus of positive electricity over negative in the element ds at the end of time t, equals double the negative product $\mathfrak{E}d\sigma/ds$, thus

$$i = \mathfrak{E} \cdot \frac{d\sigma}{dt} \ ,$$

$$E = -2\mathfrak{E} \cdot \frac{d\sigma}{ds} \ .$$

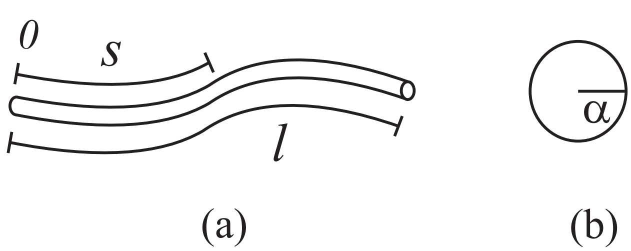
However, substituting these values in the preceding equations we obtain the two equations

$$\frac{d^3\sigma}{dt^3} - \frac{c^2}{2}\frac{d^3\sigma}{ds^2 dt} + \frac{c^2}{16\pi\alpha^2 k \log\frac{l}{\alpha}} \cdot \frac{d^2\sigma}{dt^2} = 0 \ ,$$

$$\frac{d^3\sigma}{ds dt^2} - \frac{c^2}{2}\frac{d^3\sigma}{ds^3} + \frac{c^2}{16\pi\alpha^2 k \log\frac{l}{\alpha}} \cdot \frac{d^2\sigma}{ds dt} = 0 \ ,$$

whence, taking into consideration that σ was equal to zero in the whole conducting wire during the initial equilibrium of electricity, the law of the motion of all *positive* electric particles in the conducting wire follows, namely

[418][Note by AKTA:] In Section 18.1 Weber had considered $s = \pi\alpha^2$ as the area of the curved and thin wire with a circular cross section of radius α and total length l. Now he will consider s as the position of a point along the curved axis of this thin wire measured from a given origin 0, as represented in the Figure of this footnote.



$$\frac{d^2\sigma}{dt^2} - \frac{c^2}{2}\frac{d^2\sigma}{ds^2} + \frac{c^2}{16\pi\alpha^2 k \log\frac{l}{\alpha}} \cdot \frac{d\sigma}{dt} = 0 \ .$$

18.4 Test of the Prerequisites Made in the Previous Section

At the beginning of the previous Section the prerequisites for the simplification of the equations have been compiled, from which already one was the basis for the development presented in Section 18.1. Concerning now the assumption added for the first simplification, namely the assumption of a very thin conducting wire, that seems to be so natural that it barely needs any further test, but goes without saying if it is a question of simplification; upon closer inspection, however, it is easily seen that the fineness of the conducting wire is and must be demanded here to such a degree that is never met in reality, so that any practical application of its consequences becomes questionable. In addition yet we have the special concern whether this prerequisite may not come into conflict with the prerequisite for the development of Section 18.1 concerning Ohm's law, because the latter must apparently be limited to less fine conducting wires.

If namely there is no objection against considering the thickness of the conducting wire compared to its total length for linear conductors as vanishingly small, the consideration of this thickness as vanishing compared to a single element, still considered as straight, of the conducting wire is more far reaching; and still more far reaching is the assumption of the logarithm of the ratio of the length of such a small element to that thickness as a large number, against which the number one is considered as insignificant, as was done in that prerequisite. For, taking just 20 as such a large number, would demand a wire whose smallest piece, still considered as straight, would have to be longer than thick by a factor of more than 200 millions, which does not exist.

More important, however, is the other objection whether the assumption of such a thin conduction wire, if it existed, would come into conflict with the prerequisite concerning Ohm's law. In any case, it must be called to doubt whether the latter prerequisite is *generally and strictly* valid or whether it *holds approximately for less fine wires*, and, as is easily seen, this doubt can only be remedied by a *development of the laws of motion* independent of this very prerequisite. We shall try to present such a development, at least in so far as seems to be necessary for the test of the indicated doubt, preliminarily sticking to the first prerequisite, namely a wire so thin, that the logarithm of the ratio of the length of the elements, still considered as straight, to their thickness be so large as to neglect the number one by comparison. This development rests on the following consideration.

Were all forces really known which act on the electric particles in the conducting wire and were all these forces expressed exactly in known mechanical measures, then it would be self evident that a *development of the laws of motion* of these electric particles in the conducting wire *is possible quite independent of the prerequisite of Ohm's law*; for the resultant of all forces acting on any particle divided by the acceleration of that particle must, as with all bodies, yield always the same quotient, which in mechanics is usually called the *mass of the particle*.

18.5 On the Derivation of the Equation of Motion Independent of Assuming Ohm's Law

Hence we first try to enumerate all forces acting on an electric particle in the conducting wire and to express them by mechanical measure, namely

1. those electric forces, already determined by Kirchhoff, acting at small distances from which, under the assumption of the fineness of the wire, yielded the electromotive force

$$= -4 \log \frac{l}{\alpha} \cdot \left(\frac{dE}{ds} + \frac{4}{c^2} \frac{di}{dt} \right)$$

for a point s of the conducting wire expressed by mechanical measure. This electromotive force is the difference between the two forces that act on the positive and on the negative electric unit of measure (as is defined in electrostatics) if they are present in that point. As these two forces are equal, apart from their opposite directions, it follows that half of this electromotive force, namely

$$= -2 \log \frac{l}{\alpha} \cdot \left(\frac{dE}{ds} + \frac{4}{c^2} \frac{di}{dt} \right) ,$$

is the force acting on any positive electric unit of measure at the point s. But the number of positive electric units of measure contained in the length element, ds, of the conducting wire has been denoted earlier in Section 18.3 by $\mathfrak{E} ds$, where it has been noted that $\mathfrak{E} d\sigma/dt = i$ and $-2\mathfrak{E} d\sigma/ds = E$. Multiplying the above force by the number $\mathfrak{E} ds$ and substituting the above values we get the force acting on the positive electricity in the element ds expressed in mechanical units, namely

$$= 4\mathfrak{E}^2 \log \frac{l}{\alpha} \cdot \left(\frac{d^2\sigma}{ds^2} - \frac{2}{c^2} \frac{d^2\sigma}{dt^2} \right) \cdot ds .$$

In addition to these previously determined forces we have to add

2. the forces exerted by the ponderable conductor particles on the positive electricity in the element ds which we try to determine as follows.

According to Ohm's law, established for *steady* currents as shown in the footnote of Section 18.1,[419] the electromotive force in one point of the conductor $= u/k$, which is independent of the ponderable particles of the conducting wire, or $= i/[\pi\alpha^2 k]$, because $\pi\alpha^2 u = i$ according to Section 18.3. But the *steadiness* of the current, that is the constant velocity of the electric particles in the conducting wire, proves that, apart from this electromotive force independent of ponderable particles, a second electromotive force of equal value and of opposite direction must exist which must obviously originates from the action of ponderable conductor particles on the electricity in the conductor, which hence is given by

$$= -\frac{i}{\pi\alpha^2 k} .$$

[419][Note by AKTA:] See footnote 407 on page 270.

Half of this electromotive force, namely

$$= -\frac{i}{2\pi\alpha^2 k} ,$$

then is, as is clear from the previous statement, the force which is exerted by the ponderable conductor particles on each positive electric unit of measure in the point s under consideration. Hence multiplying this force by the number of positive units of measure $\mathfrak{E}ds$ contained in the element ds and substituting also here as before $\mathfrak{E}d\sigma/dt$ by i, we find the force exerted on the positive electricity contained in the element ds expressed by mechanical measure, namely

$$= -\frac{1}{2\pi\alpha^2 k} \cdot \mathfrak{E}^2 \cdot \frac{d\sigma}{dt} \cdot ds .$$

Considering finally that the cases of *non-steady* currents differ from those of *steady* currents only regarding situations coming from different interactions *between the electric particles*, wherein the forces due to the interaction between *ponderable conductor particles on the electric* particles have no direct dependence, it seems justified to assume that the presented law for the determination of the latter forces, when it is valid for all cases of *steady* currents, holds in general, also in the cases of *non-steady* currents.

In order to take into account all forces which act on the electric particle under consideration, we finally summarize

3. all forces acting from a distance, wherever they may originate, and understand among them in particular also all forces originating from the interaction of the electricity [located on distant points of the conductor and acting] on the electricity in the point under consideration, apart from those [originating from electric particles located] in the element ds itself which contains the point under consideration, and which Kirchhoff has assumed as vanishingly small. We denote by S the electromotive force originating from this at point s in mechanical measure, half of which multiplied by $\mathfrak{E}ds$ yields the force exerted on the positive electricity in element ds, expressed in mechanical measure,

$$= \frac{1}{2}\mathfrak{E}Sds .$$

As all these forces, expressed in mechanical measure, that means by parts of that force which conveys the unit of velocity (one millimeter during one second) during the time unit (during the time of a second) to the ponderable unit of mass (the mass of one milligram), [then] it follows, according to the law of motion valid for all bodies, that the quotient of the sum of all these unidirectional forces and the acceleration, that is of the velocity conveyed by the sum of forces due to the positive electricity in the element ds acting on them during the unit time, namely

$$\frac{d^2\sigma}{dt^2} ,$$

yields the definition of the *mass* of the positive electricity contained in the element ds, expressed in the unit mass (milligram) defined for all bodies.

It is remarkable that one is led hereby to a new kind of *absolute determination of an amount of electricity*, about which the following remark, for comparison of this new method of *absolute determination* with the already known methods may find room here for the application to the present consideration.

Arranging the different methods of *absolute determinations* of an amount of electricity according to the exactness they allow in practice, the *absolute determinations by the galvanometric method* have to be placed topmost by which the amount of electricity, present as part of the *neutral fluid*, is obtained expressed as part of the amount of electricity which passes during the unit of time through the cross section of the conductor with the *unit of current intensity determined galvanometrically*. — Then follow the *absolute determinations by means of electrostatic measurement*, by which an existing amount of *free* electricity is obtained expressed as part of that amount of electricity which exerts the unit of force on the same amount [of free electricity] from a unit distance according to the electrostatic law. This determination is applied only to *small* amounts of electricity occurring as *free* in comparison with the large amounts of electricity in the neutral fluid determined galvanometrically. — Especially important is the knowledge of the ratio of the *units of measure* determined by the two methods, obtained by measuring twice one and the same amount of electricity, galvanometrically as well as by the electrostatic method, namely the ratio $155\ 370 \cdot 10^6 : 1$ (see the previous *Abhandlung*, Vol. V, p. 261)[420,421] — To these two *absolute methods* one may now add as *third* one that by which an existing amount of electricity is to be expressed by its *mass* in parts of the unit of mass (milligram) determined for all bodies; here, however, we have to remark that until now no existing amount of electricity has been expressed by this method because no way has yet been discovered which would just approximately lead to such a knowledge. Consequently, there is a complete lack of knowledge concerning the *ratio* of the *units of measure* due to the latter method and that due to the previous method because no measurement of one and the same amount of electricity could be carried out according to these different methods. Were this *ratio* $= r : 1$ known, then the *mass* of this amount of electricity in *milligrams* expressed as $= [1/r] \cdot \mathfrak{E} ds$ could be obtained from the number $\mathfrak{E} ds$ of electrostatic units of measure of positive electricity contained in the conductor element ds.

Introducing this expression for the *mass* and equating it with the above *quotient*, one obtains the following equation:

$$\frac{1}{\frac{d^2\sigma}{dt^2}} \cdot \left(4\mathfrak{E}^2 \log\frac{l}{\alpha} \cdot \left[\frac{d^2\sigma}{ds^2} - \frac{2}{c^2}\frac{d^2\sigma}{dt^2}\right] ds - \frac{1}{2\pi\alpha^2 k} \cdot \mathfrak{E}^2 \cdot \frac{d\sigma}{dt} ds + \frac{1}{2}\mathfrak{E} S ds\right) = \frac{1}{r}\mathfrak{E} ds \ ,$$

or, arranging and putting

$$\frac{c^2}{8\log\frac{l}{\alpha} \cdot r\mathfrak{E}} = \lambda \ ,$$

one obtains

[420][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 649.

[421][Note by AKTA:] [KW57, Section 15, p. 649 of Weber's *Werke*] with English translation in [KW21, Section 15, p. 48]. See, in particular, page 176 of Section 7.15.

$$\frac{d^2\sigma}{dt^2} - \frac{c^2}{2(1+\lambda)} \cdot \frac{d^2\sigma}{ds^2} + \frac{c^2}{16\pi\alpha^2 k \log\frac{l}{\alpha} \cdot (1+\lambda)} \cdot \frac{d\sigma}{dt} = \frac{c^2}{16\mathfrak{E} \log\frac{l}{\alpha} \cdot (1+\lambda)} \cdot S \ .$$

18.6 Comparison of the Results

This more general equation is seen to contain Kirchhoff's above equation, namely under the two assumptions that $S = 0$ and $\lambda = 0$; then we have

$$\frac{d^2\sigma}{dt^2} - \frac{c^2}{2} \cdot \frac{d^2\sigma}{ds^2} + \frac{c^2}{16\pi\alpha^2 k \log\frac{l}{\alpha}} \cdot \frac{d\sigma}{dt} = 0 \ ,$$

in total agreement with the equation developed at the end of Section 18.3.

Here it may be remarked that Poggendorff's Note to Kirchhoff's treatise in the 1857 *Annalen*, Vol. 100, p. 351, refers to this more general equation, just derived, and to its agreement with Kirchhoff's equation.[422,423]

The assumption that $S = 0$ does not just contain Kirchhoff's previously made assumption, that no external electromotive force shall act on the electricity in the conducting wire, but especially also the second assumption from the three made at the beginning of the third Section,[424] namely that all electromotive forces originating from the free electricity and from the electric motions in the whole conducting wire, apart from the small piece considered as cylindrical with the point under consideration in its center, are vanishingly small compared to those electromotive forces acting on the same point originating from the free electricity and from the electric motions in the cylindrical small piece itself.

The assumption that $\lambda = 0$, on the other hand, agrees with Kirchhoff's assumption of the general validity of Ohm's law. It may, however, seem that $\lambda = c^2/[8r \cdot \mathfrak{E}\log(l/\alpha)]$ vanishes for $\log(l/\alpha) = \infty$, and that the assumption $\lambda = 0$ would approximately be fulfilled by Kirchhoff's assumption that α vanishes compared to l; but this is not the case, [we have] rather $\lambda = \infty$ when α vanishes, as is easily seen because the number, $= \mathfrak{E}$, of positive units of measure contained in the unit length of the conducting wire is proportional to the square of the radius α, and, denoting the constant number of positive electric units of measure contained in the *unit volume of the conducting wire* by $\mathfrak{E}_0$, is given by

$$\mathfrak{E} = \pi\alpha^2 \cdot \mathfrak{E}_0 \ ,$$

whence it follows that the product

$$\mathfrak{E} \log\frac{l}{\alpha} = \pi\mathfrak{E}_0 \cdot \alpha^2 \log\frac{l}{\alpha}$$

vanishes together with α and thus

$$\lambda = \frac{c^2}{8r \cdot \mathfrak{E}\log\frac{l}{\alpha}}$$

becomes infinite.

[422] [Note by HW:] This Note can be found at the end of this paper under number VI.

[423] [Note by AKTA:] See [Pog57], reprinted in [Web94b, Paper number VI, p. 242 of Weber's *Werke*]. English translation in [Pog21]. See Chapters 10 and 11.

[424] [Note by AKTA:] See Section 18.3.

Hence it follows that in *thicker* conducting wires, with larger values of α, Ohm's law indeed could approximately hold in general as assumed by Kirchhoff, namely for a very small value of the constant quotient $c^2/[r\mathfrak{E}_0]$; that, on the other hand, Ohm's law would lose this more general validity for *thinner* conducting wires, particularly if this refinement is to be pushed so far as to make $\log(l/\alpha)$ a very large number, whence the explicit objection about the incompatibility of the two assumptions presented under (1) and (3) at the beginning of Section 18.3 seems to be well founded.

On the other hand, if by observation cases of thinner conducting wires could be demonstrated where Ohm's law does not receive this more general validity, but measurable deviations became obvious from which λ could be determined, it would yield the knowledge of the constant quotient

$$\frac{c^2}{r\mathfrak{E}_0} = 8\pi\alpha^2 \log\frac{l}{\alpha} \cdot \lambda \ ,$$

and the knowledge of the ratio $r : 1$, that is the number of electrostatic units of measure per milligram, would merely depend on the exploration of the number of electrostatic units of measure, $\mathfrak{E}_0$, which are contained in 1 cubic millimeter of the conductor.

18.7 Development of the Expression for the Electromotive Force which is Exerted by the Free Electricity and by the Electric Motions in the Whole Conductor on One Point of a Closed Thin Conductor, apart from that Element which Contains the Point under Consideration

If the forces which could not be determined, acting on a point s of the conducting wire from a distance including those which act from more distant parts of the conducting wire itself and those acting from outside were put equal $= 0$, then according to the developments of the preceding Section one obtains the following partial differential equation for the displacement σ of the positive particle in the point s:[425]

$$\frac{d^2\sigma}{dt^2} - a\frac{d^2\sigma}{ds^2} + b\frac{d\sigma}{dt} = 0 \ ,$$

where the only difference was that the meaning of the constant coefficients a and b in this equation after Section 18.3 was a bit different from that used in Section 18.6, a difference which possibly does not need to be considered, namely if the experience should show that the quotient in the previous Section denoted by $c^2/[r\mathfrak{E}_0]$ had a vanishingly small value for all kinds of conductors.

This agreement, however, does not at all make the above equation suited to really determine the motions of electricity in a conducting wire; even if there were cases with no external electromotive forces acting on the electricity in the conducting wire, there would be no case where also no electromotive forces would be acting [originating] from the more distant parts of the conducting wire itself, if any disturbance of the equilibrium of the electricity has

[425][Note by AKTA:] We are maintaing Weber's notation for partial derivatives, see footnote 408 on page 271.

happened. Thus, in order to arrive at an equation which would really serve to determine the motion of electricity in a conducting wire, the development of the electromotive forces from Section 18.2, exerted by the free electricity and by the electric motions of the single element ds containing the point s is not sufficient, but also those electromotive forces remain to be determined which are exerted by the free electricity and by the electric motions in all remaining parts of the conducting wire on the point s. Therefore, Kirchhoff's conclusions from the above equation remain to be tested with respect to the influence of these latter forces.

As the dimensions of the elements in question, ds and ds', vanish compared to their distance, it suffices indeed for the development of these forces to consider only the total values of the [linear] densities of free electricity and the current intensities E, E', i, i', respectively, for the total cross section which are merely functions of s and t or [functions of] s' and t'. But it is self evident that, unlike in Section 18.2, these functions cannot be expanded in series according to Taylor's theorem,[426] because the same can be arbitrarily given in the first moment $t = 0$; instead one must try to represent them in terms of sine and cosine series.

Hence putting for a closed conducting wire of length $2\pi a$

$$E' = \sum \left(a_n \sin \frac{ns'}{a} + b_n \cos \frac{ns'}{a} \right) ,$$

$$i' = \sum \left(c_n \sin \frac{ns'}{a} + \partial_n \cos \frac{ns'}{a} \right) ,$$

where n takes all successive integer numbers, and denoting the distance between the points s and s' by r and the angles which ds and ds' form with r by ϑ and ϑ', we get according to Section 18.1

$$\Omega = \int \frac{E' ds'}{r} = \int \frac{ds'}{r} \sum \left(a_n \sin \frac{ns'}{a} + b_n \cos \frac{ns'}{a} \right) ,$$

$$U = \int \frac{ds'}{r} \cos\vartheta \cos\vartheta' \cdot i' = \int \frac{ds'}{r} \cos\vartheta \cos\vartheta' \cdot \sum \left(c_n \sin \frac{ns'}{a} + \partial_n \cos \frac{ns'}{a} \right) .$$

In addition, we still have the equation found in Section 18.3,

$$\frac{di'}{ds'} = -\frac{1}{2} \frac{dE'}{dt} ,$$

or expressed in terms of sine and cosine series,

$$\frac{1}{a} \sum n \left(c_n \cos \frac{ns'}{a} - \partial_n \sin \frac{ns'}{a} \right) = -\frac{1}{2} \sum \left(\frac{da_n}{dt} \cdot \sin \frac{ns'}{a} + \frac{db_n}{dt} \cos \frac{ns'}{a} \right) .$$

Hence it follows, because this equation is to be valid for all values of s'

$$c_n = -\frac{a}{2n} \cdot \frac{db_n}{dt} ,$$

[426] Note by AKTA:] See footnote 303 on page 233.

$$\partial_n = +\frac{a}{2n} \cdot \frac{da_n}{dt} \ .$$

Aiming now to determine the electromotive force acting on the point s of the closed conducting wire from the obtained expressions for Ω and U, one may put $s' - s = \sigma$, and substitute $s + \sigma$ for s' and $d\sigma$ for ds' in the expressions for Ω and U. This then yields

$$\Omega = \sum \int \frac{d\sigma}{r} \left(a_n \sin\left(\frac{n\sigma}{a} + \frac{ns}{a}\right) + b_n \cos\left(\frac{n\sigma}{a} + \frac{ns}{a}\right)\right) \ ,$$

$$U = \sum \int \frac{d\sigma}{r} \cos\vartheta \cos\vartheta' \cdot \left(c_n \sin\left(\frac{n\sigma}{a} + \frac{ns}{a}\right) + \partial_n \cos\left(\frac{n\sigma}{a} + \frac{ns}{a}\right)\right) \ .$$

Expanding the sum in terms of sine and cosine, one gets

$$\begin{aligned} \Omega = &\sum \left(a_n \cos\frac{ns}{a} - b_n \sin\frac{ns}{a}\right) \cdot \int \frac{\sin\frac{n\sigma}{a} \cdot d\sigma}{r} \\ + &\sum \left(a_n \sin\frac{ns}{a} + b_n \cos\frac{ns}{a}\right) \cdot \int \frac{\cos\frac{n\sigma}{a} \cdot d\sigma}{r} \ . \end{aligned}$$

$$\begin{aligned} U = &\sum \left(c_n \cos\frac{ns}{a} - \partial_n \sin\frac{ns}{a}\right) \cdot \int \frac{\cos\vartheta \cos\vartheta' \sin\frac{n\sigma}{a} d\sigma}{r} \\ + &\sum \left(c_n \sin\frac{ns}{a} + \partial_n \cos\frac{ns}{a}\right) \cdot \int \frac{\cos\vartheta \cos\vartheta' \cos\frac{n\sigma}{a} d\sigma}{r} \ . \end{aligned}$$

Here, r, $\cos\vartheta$ and $\cos\vartheta'$ are functions of σ which result from the equation of the curve of the conducting wire. It follows that for each consecutive number n the four integrals to be taken between the limits from $\sigma = \frac{l}{2}$ to $\sigma = 2\pi a - \frac{l}{2}$ (where l is the length of the same piece of the conducting wire as in Section 18.2)

$$\int \frac{\sin\frac{n\sigma}{a} d\sigma}{r}, \quad \int \frac{\cos\frac{n\sigma}{a} d\sigma}{r}, \quad \int \frac{\cos\vartheta \cos\vartheta' \sin\frac{n\sigma}{a} d\sigma}{r}, \quad \int \frac{\cos\vartheta \cos\vartheta' \cos\frac{n\sigma}{a} d\sigma}{r}$$

are given and determined by the *shape of the conductor*, the values of which shall be denoted by

$$N, \qquad N', \qquad M, \qquad M' \ .$$

Then one has

$$\Omega = \sum \left((a_n N' - b_n N) \sin\frac{ns}{a} + (a_n N + b_n N') \cos\frac{ns}{a}\right) \ ,$$

$$U = \sum \left((c_n M' - \partial_n M) \sin\frac{ns}{a} + (c_n M + \partial_n M') \cos\frac{ns}{a}\right) \ ,$$

from which now the electromotive forces can be determined, namely

$$-2\frac{d\Omega}{ds} = -\frac{2}{a} \sum n \left((a_n N' - b_n N) \cos\frac{ns}{a} - (a_n N + b_n N') \sin\frac{ns}{a}\right) \ ,$$

$$-\frac{8}{c^2}\cdot\frac{dU}{dt} = -\frac{8}{c^2}\sum\left[\left(\frac{dc_n}{dt}\cdot M' - \frac{d\partial_n}{dt}\cdot M\right)\sin\frac{ns}{a}\right.$$
$$\left. + \left(\frac{dc_n}{dt}\cdot M + \frac{d\partial_n}{dt}\cdot M'\right)\cos\frac{ns}{a}\right] ,$$

or, substituting the above values of c_n and ∂_n in the latter equation

$$-\frac{8}{c^2}\cdot\frac{dU}{dt} = +\frac{4a}{c^2}\sum\frac{1}{n}\left[\left(\frac{d^2b_n}{dt^2}\cdot M' + \frac{d^2a_n}{dt^2}\cdot M\right)\sin\frac{ns}{a}\right.$$
$$\left. + \left(\frac{d^2b_n}{dt^2}\cdot M - \frac{d^2a_n}{dt^2}\cdot M'\right)\cos\frac{ns}{a}\right] .$$

18.8 Equation of Motion of the Electricity in a Closed Conductor

In order to formulate the equation of motion of the electricity in a closed conductor according to the method presented in Sections 18.4 to 18.5, at first all forces have to be taken into account which act on the positive electricity in an element ds of the conducting wire and [it is necessary] to express the value of these forces by mechanical measure.

1. At the end of Section 18.2 the electromotive forces acting *in the vicinity* of the point s of the conducting wire have been found:

$$-2\frac{d\Omega}{ds} = -4\frac{dE}{ds}\cdot\log\frac{l}{\alpha} - \frac{1}{4}\frac{d^3E}{ds^3}\cdot l^2 ,$$

$$-\frac{8}{c^2}\frac{dU}{dt} = -\frac{16}{c^2}\frac{di}{dt}\cdot\log\frac{l}{e\alpha} - \frac{1}{c^2}\frac{d^3i}{ds^2dt}\cdot l^2 .$$

But here we can substitute according to the previous Section

$$E = \sum\left(a_n\sin\frac{ns}{a} + b_n\cos\frac{ns}{a}\right) ,$$

$$i = -\frac{a}{2}\sum\frac{1}{n}\left(\frac{db_n}{dt}\cdot\sin\frac{ns}{a} - \frac{da_n}{dt}\cdot\cos\frac{ns}{a}\right) ,$$

hence

$$-2\frac{d\Omega}{ds} = -\frac{4}{a}\sum\left(n\log\frac{l}{\alpha} - \frac{1}{16}\frac{n^3l^2}{a^2}\right)\left(a_n\cos\frac{ns}{a} - b_n\sin\frac{ns}{a}\right) ,$$

$$-\frac{8}{c^2}\frac{dU}{dt} = \frac{8}{c^2}\sum\left(\frac{a}{n}\log\frac{l}{e\alpha} - \frac{1}{16}\frac{nl^2}{a}\right)\left(\frac{d^2b_n}{dt^2}\cdot\sin\frac{ns}{a} - \frac{d^2a_n}{dt^2}\cdot\cos\frac{ns}{a}\right) .$$

2. At the end of the previous Section the electromotive forces acting *from a distance* on the point s of the conducting wire have been found

$$-2\frac{d\Omega}{ds} = -\frac{2}{a}\sum n\left[(a_nN' - b_nN)\cos\frac{ns}{a} - (a_nN + b_nN')\sin\frac{ns}{a}\right] ,$$

$$\begin{aligned}-\frac{8}{c^2}\frac{dU}{dt} = +\frac{4a}{c^2}\sum\frac{1}{n}\Bigg[&\left(\frac{d^2b_n}{dt^2}\cdot M' + \frac{d^2a_n}{dt^2}\cdot M\right)\\ &+ \left(\frac{d^2b_n}{dt^2}\cdot M - \frac{d^2a_n}{dt^2}\cdot M'\right)\cos\frac{ns}{a}\Bigg] .\end{aligned}$$

Thus, putting

$$N' + 2\log\frac{l}{\alpha} - \frac{1}{8}\frac{n^2l^2}{a^2} = N'' ,$$

$$M' + 2\log\frac{l}{e\alpha} - \frac{1}{8}\frac{n^2l^2}{a^2} = M'' ,$$

the electromotive forces acting from the vicinity and from a distance taken together are

$$-2\frac{d\Omega}{ds} = -\frac{2}{a}\sum n\left[(a_nN'' - b_nN)\cos\frac{ns}{a} - (a_nN + b_nN'')\sin\frac{ns}{a}\right] ,$$

$$\begin{aligned}-\frac{8}{c^2}\frac{dU}{dt} = +\frac{4a}{c^2}\sum\frac{1}{n}\Bigg[&\left(\frac{d^2b_n}{dt^2}\cdot M'' + \frac{d^2a_n}{dt^2}\cdot M\right)\sin\frac{ns}{a}\\ &+ \left(\frac{d^2b_n}{dt^2}\cdot M - \frac{d^2a_n}{dt^2}\cdot M''\right)\cos\frac{ns}{a}\Bigg] .\end{aligned}$$

Now these electromotive forces are the differences of those forces which act on the positive and the negative electric unit of measure at the point s. As, however, the force acting on the positive unit of measure equals that acting on the negative unit of measure, apart from the opposite direction, it follows that half of these electromotive forces are those acting on each positive unit of measure in the point s. But the number of the positive units of measure contained in the length element, ds, of the conducting wire has been denoted by $\mathfrak{E}ds$ in Section 18.3; multiplying half of the above electromotive forces by $\mathfrak{E}ds$, one finds the forces which act on the positive electricity in the element ds, expressed in mechanical measure,

$$\begin{aligned}= -\frac{\mathfrak{E}ds}{a}\sum n&\left[(a_nN'' - b_nN)\cos\frac{ns}{a} - (a_nN + b_nN'')\sin\frac{ns}{a}\right]\\ &+\frac{2a\mathfrak{E}ds}{c^2}\sum\frac{1}{n}\Bigg[\left(\frac{d^2b_n}{dt^2}\cdot M'' + \frac{d^2a_n}{dt^2}\cdot M\right)\sin\frac{ns}{a}\\ &\qquad + \left(\frac{d^2b_n}{dt^2}\cdot M - \frac{d^2a_n}{dt^2}\cdot M''\right)\cos\frac{ns}{a}\Bigg] .\end{aligned}$$

3. The resistive force originating from the *ponderable conductor particles* acting on the positive electricity in the element ds was found in Section 18.5 and is given by, expressed in mechanical measure

$$= -\frac{1}{2\pi\alpha^2 k} \cdot \mathfrak{E}^2 \cdot \frac{d\sigma}{dt} ds \ ,$$

wherein

$$\frac{\mathfrak{E} d\sigma}{dt} = i = -\frac{a}{2} \sum \frac{1}{n} \left(\frac{db_n}{dt} \cdot \sin\frac{ns}{a} - \frac{da_n}{dt} \cdot \cos\frac{ns}{a} \right) \ ,$$

which yields this force as

$$= +\frac{a\mathfrak{E} ds}{4\pi\alpha^2 k} \cdot \sum \frac{1}{n} \left(\frac{db_n}{dt} \cdot \sin\frac{ns}{a} - \frac{da_n}{dt} \cdot \cos\frac{ns}{a} \right) \ .$$

In addition, finally, we have

4. the force acting *from outside* on the positive electricity in the element ds which, according to Section 18.5 (3),[427] yields

$$= +\frac{1}{2} \mathfrak{E} S ds \ ,$$

where S denotes here only the external electromotive force acting on the point s. Expanding now S in sine and cosine series

$$S = \sum \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a} \right) \ ,$$

then this force is represented by

$$= +\frac{1}{2} \mathfrak{E} ds \cdot \sum \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a} \right) \ .$$

As now all these forces are expressed in *mechanical measure*, that is in parts of that force which conveys the unit of velocity[428] to the unit of ponderable mass (milligrams) during the unit of time (second), it follows that, according to the well known law of motion, valid for all bodies, the quotient of the sum of all these forces and of the acceleration, $= d^2\sigma/dt^2$, conveyed to the positive electricity in the element ds on which they act, expresses the definition of the *mass* of this amount of electricity, where the measure of mass (milligram) has been denoted by $[1/r]\mathfrak{E} ds$ milligram in Section 18.5. Multiplying the equation thus obtained by $[1/\mathfrak{E} ds] \cdot [d^2\sigma/dt^2]$ and putting

$$\mathfrak{E} \frac{d^2\sigma}{dt^2} = \frac{di}{dt} = -\frac{a}{2} \sum \frac{1}{n} \left(\frac{d^2 b_n}{dt^2} \sin\frac{ns}{a} - \frac{d^2 a_n}{dt^2} \cdot \cos\frac{ns}{a} \right) \ ,$$

one gets the desired equation of motion of the electricity in a closed conducting wire as follows:

[427][Note by AKTA:] That is, item 3 in Section 18.5.

[428][Note by AKTA:] That is, a constant force increasing in 1 mm/s the velocity of the mass under consideration.

$$
\begin{aligned}
&-\frac{1}{a}\sum n\left[(a_n N'' - b_n N)\cos\frac{ns}{a} - (a_n N + b_n N'')\sin\frac{ns}{a}\right] \\
&\qquad + \frac{2a}{c^2}\sum\frac{1}{n}\left[\left(\frac{d^2 b_n}{dt^2}\cdot M'' + \frac{d^2 a_n}{dt^2}\cdot M\right)\sin\frac{ns}{a}\right. \\
&\qquad\qquad \left. + \left(\frac{d^2 b_n}{dt^2}\cdot M - \frac{d^2 a_n}{dt^2}\cdot M''\right)\cos\frac{ns}{a}\right] \\
&+ \frac{a}{4\pi\alpha^2 k}\sum\frac{1}{n}\left(\frac{db_n}{dt}\sin\frac{ns}{a} - \frac{da_n}{dt}\cos\frac{ns}{a}\right) + \frac{1}{2}\sum\left(f_n\sin\frac{ns}{a} + g_n\cos\frac{ns}{a}\right) \\
&\qquad = -\frac{a}{2r\mathfrak{E}}\cdot\sum\frac{1}{n}\left(\frac{d^2 b_n}{dt^2}\sin\frac{ns}{a} - \frac{d^2 a_n}{dt^2}\cos\frac{ns}{a}\right) .
\end{aligned}
$$

As N, N'', M, M'' depend only on the equation of the shape of the conductor, they can be presented as function of s. In the single case where this shape is a *circle*, each of these quantities has the *same value* for all points s and then the above equation can be split into the two simpler equations, namely, putting $c^2/[4M''r\mathfrak{E}] = \lambda$,

$$
\begin{aligned}
&\frac{d^2 a_n}{dt^2} + \frac{c^2}{8\pi\alpha^2 k M''(1+\lambda)}\cdot\frac{da_n}{dt} + \frac{n^2c^2N''}{2a^2M''(1+\lambda)}\cdot a_n - \frac{nc^2}{4aM''(1+\lambda)}\cdot g_n \\
&\qquad = \frac{M}{M''(1+\lambda)}\cdot\frac{d^2 b_n}{dt^2} + \frac{n^2c^2N}{2a^2M''(1+\lambda)}\cdot b_n ,
\end{aligned}
$$

$$
\begin{aligned}
&\frac{d^2 b_n}{dt^2} + \frac{c^2}{8\pi\alpha^2 k M''(1+\lambda)}\cdot\frac{db_n}{dt} + \frac{n^2c^2N''}{2a^2M''(1+\lambda)}\cdot b_n + \frac{nc^2}{4aM''(1+\lambda)}\cdot f_n \\
&\qquad = -\frac{M}{M''(1+\lambda)}\cdot\frac{d^2 a_n}{dt^2} - \frac{n^2c^2N}{2a^2M''(1+\lambda)}\cdot a_n .
\end{aligned}
$$

Hereby the treatment of the case of a *conductor of circular shape* is considerably simplified and deserves to be considered in particular. In all other cases N, N'', M, M'' as functions of s would have to be expanded further in series of sine and cosine whereby the equations would considerably lose their simplicity.

18.9 Equation for the Mean Values of the Electromotive Forces and Current Intensities in Closed Conductors with Arbitrary Shape

Considerations and applications of closed circuits often occur which do not demand the knowledge of the electromotive forces and current intensities in individual points of the circuit, but where the knowledge of their *mean values* for the total length of the conducting wire suffices. Hence before entering the special development of the laws of motion of the electricity in a *circular* conductor, the laws just found shall be applied in order to derive from them the equation for the *mean values* of the electromotive forces and current intensities in closed conductors *of arbitrary shape*.

This equation results when the terms of the equation found in the previous Section are multiplied by ds and are integrated from $s = 0$ to $s = 2\pi a$. It is considerably simplified because *first*, according to a known theorem, the value of the integral of the electromotive forces originating from the free electricity in the conducting wire is always equal to zero,[429] and because *second* the integral value of the external electromotive forces can usually be considered as given. Hence one obtains *first*

$$\int_0^{2\pi a} \frac{ds}{a} \sum n \left((a_n N'' - b_n N) \cos \frac{ns}{a} - (a_n N + b_n N'') \sin \frac{ns}{a} \right) = 0 \ ,$$

second, denoting by S the integral value of the external electromotive forces,

$$\int_0^{2\pi a} ds \sum \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a} \right) = S \ .$$

As now further, putting

$$i_n = -\frac{a}{2n} \left(\frac{db_n}{dt} \cdot \sin \frac{ns}{a} - \frac{da_n}{dt} \cdot \cos \frac{ns}{a} \right) \ ,$$

we had $i = \sum i_n$; hence one gets

$$\begin{aligned} \int ds \cdot \frac{2a}{c^2} \sum \frac{1}{n} \Bigg[\left(\frac{d^2 b_n}{dt^2} M'' + \frac{d^2 a_n}{dt^2} M \right) \sin \frac{ns}{a} & \\ + \left(\frac{d^2 b_n}{dt^2} M - \frac{d^2 a_n}{dt^2} M'' \right) \cos \frac{ns}{a} \Bigg] & \\ = -\frac{4}{c^2} \int ds \sum M'' \frac{di_n}{dt} - \frac{4a}{c^2} \sum \frac{1}{n} \int \frac{d^2 i_n}{ds dt} M ds \ . & \end{aligned}$$

Now one has

$$\int \frac{d^2 i_n}{dt^2} M ds = M \frac{di_n}{dt} - \int \frac{di_n}{dt} \cdot \frac{dM}{ds} ds \ ;$$

hence

$$\int_0^{2\pi a} \frac{d^2 i_n}{ds dt} M ds = -\int_0^{2\pi a} \frac{di_n}{dt} \cdot \frac{dM}{ds} ds \ ,$$

thus also

[429][Note by AKTA:] That is, in modern terms,

$$\oint_C \vec{E} \cdot d\vec{\ell} = 0 \ ,$$

where $\vec{E}$ is the electric field due to free charges, C is a closed circuit of arbitrary shape and $d\vec{\ell}$ is an infinitesimal element of length.

$$\int_0^{2\pi a} ds \cdot \frac{2a}{c^2} \sum \frac{1}{n} \left[\left(\frac{d^2 b_n}{dt^2} M'' + \frac{d^2 a_n}{dt^2} M \right) \sin \frac{ns}{a} \right.$$
$$\left. + \left(\frac{d^2 b_n}{dt^2} M - \frac{d^2 a_n}{dt^2} M'' \right) \cos \frac{ns}{a} \right]$$
$$= -\frac{4}{c^2} \int_0^{2\pi a} ds \sum M'' \frac{di_n}{dt} + \frac{4a}{c^2} \sum \frac{1}{n} \int_0^{2\pi a} \frac{di_n}{dt} \cdot \frac{dM}{ds} ds \ .$$

Finally adding that

$$\int \frac{ads}{4\pi\alpha^2 k} \sum \frac{1}{n} \left(\frac{db_n}{dt} \sin \frac{ns}{a} - \frac{da_n}{dt} \cos \frac{ns}{a} \right) = -\frac{1}{2\pi\alpha^2 k} \cdot \int ids \ ,$$

$$\int \frac{ads}{2r\mathfrak{E}} \sum \frac{1}{n} \left(\frac{d^2 b_n}{dt^2} \sin \frac{ns}{a} - \frac{d^2 a_n}{dt^2} \cos \frac{ns}{a} \right) = -\frac{1}{r\mathfrak{E}} \cdot \int \frac{di}{dt} ds \ ,$$

one gets the following equation for the mean values of the electromotive forces and current intensities, $\frac{1}{2\pi a} \cdot S$ and $\frac{1}{2\pi a} \cdot \int_0^{2\pi a} ids$, respectively:

$$S = \frac{1}{\pi\alpha^2 k} \cdot \int_0^{2\pi a} ids + \frac{8}{c^2} \int_0^{2\pi a} ds \sum M'' \frac{di_n}{dt}$$
$$- \frac{8a}{c^2} \sum \frac{1}{n} \int_0^{2\pi a} \frac{di_n}{dt} \cdot \frac{dM}{ds} ds + \frac{2}{r\mathfrak{E}} \int_0^{2\pi a} \frac{di}{dt} ds \ .$$

Now these mean values obviously come into primary consideration when there is no difference at all for the electric motions in the different elements of the conducting wire, or so small that it can be totally neglected. Thus in all these cases i and di/dt are quantities independent of s, and one may put $i = i_0$ and $di/dt = di_0/dt$, hence $di_n/dt = 0$ for $n > 0$, from which

$$S = \frac{2\pi a}{\pi\alpha^2 k} i_0 + \left(\frac{8}{c^2} \int_0^{2\pi a} M_0'' ds + \frac{4\pi a}{r\mathfrak{E}} \right) \frac{di_0}{dt} \ ,$$

where $2\pi a/[\pi\alpha^2 k] = w$ is the resistance of the whole circuit. Putting

$$\frac{8}{c^2} \int_0^{2\pi a} M_0'' ds + \frac{4\pi a}{r\mathfrak{E}} = p \ ,$$

and writing i for i_0, one gets

$$S = wi + p\frac{di}{dt} \ ,$$

wherein S, i and di/dt are only functions of t. Integrating one gets

$$i = \frac{1}{p} e^{-wt/p} \cdot \int e^{wt/p} \cdot Sdt \ .$$

18.10 Laws of Motion of the Electricity in a Circular Conducting Wire

When the shape of a closed conductor is given, [then] the values of N, N', M, M', that is the values of the definite integrals

$$\int_{l/2}^{2\pi a-l/2}\frac{\sin\frac{n\sigma}{a}d\sigma}{r}\ , \qquad \int_{l/2}^{2\pi a-l/2}\frac{\cos\frac{n\sigma}{a}d\sigma}{r}\ ,$$

$$\int_{l/2}^{2\pi a-l/2}\frac{\cos\vartheta\cos\vartheta'\sin\frac{n\sigma}{a}d\sigma}{r}\ , \qquad \int_{l/2}^{2\pi a-l/2}\frac{\cos\vartheta\cos\vartheta'\cos\frac{n\sigma}{a}d\sigma}{r}\ ,$$

can be determined for any point s of the conductor. As an example take a conductor with the shape of a circle with radius $= a$.[430]

With this circular shape the distance r between two point s and s' equals the chord of the arc $(s'-s)/a = \sigma/a$; hence we have

$$r = 2a\sin\frac{\sigma}{2a}\ .$$

Furthermore, the angle ϑ between the element ds and r equals the angle ϑ' between the element ds' and r, and both equal the angle between the tangent to the circle at the point s and the chord of the arc σ/a, that is

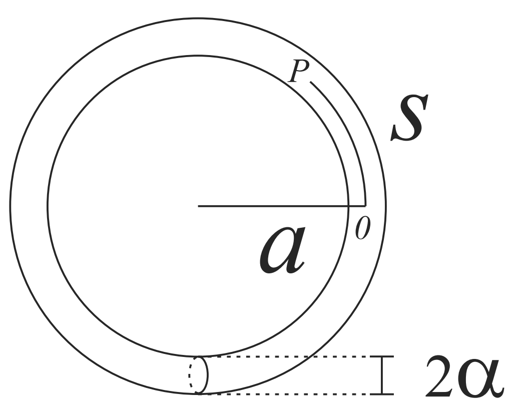
$$\vartheta = \vartheta' = \frac{\sigma}{2a}\ .$$

Hence we have

$$N = \frac{1}{2a}\int_{l/2}^{2\pi a-l/2}\frac{\sin\frac{n\sigma}{a}d\sigma}{\sin\frac{\sigma}{2a}}\ ,$$

$$N' = \frac{1}{2a}\int_{l/2}^{2\pi a-l/2}\frac{\cos\frac{n\sigma}{a}d\sigma}{\sin\frac{\sigma}{2a}}\ ,$$

[430][Note by AKTA:] Weber will consider a conductor in the shape of a ring with larger radius a, smaller radius α, with s being the arc length of a specific point P measured from a fixed origin 0, as shown in the Figure of this footnote:



$$M=\frac{1}{2a}\int_{l/2}^{2\pi a-l/2}\frac{\left(\cos\frac{\sigma}{2a}\right)^2\cdot\sin\frac{n\sigma}{a}d\sigma}{\sin\frac{\sigma}{2a}}=N-\frac{1}{2a}\int_{l/2}^{2\pi a-l/2}\sin\frac{n\sigma}{a}\sin\frac{\sigma}{2a}d\sigma\ ,$$

$$M'=\frac{1}{2a}\int_{l/2}^{2\pi a-l/2}\frac{\left(\cos\frac{\sigma}{2a}\right)^2\cdot\cos\frac{n\sigma}{a}d\sigma}{\sin\frac{\sigma}{2a}}=N'-\frac{1}{2a}\int_{l/2}^{2\pi a-l/2}\cos\frac{n\sigma}{a}\sin\frac{\sigma}{2a}d\sigma\ .$$

Putting now $\sigma/2a=z$, hence

$$N=\int_{l/(4a)}^{\pi-l/(4a)}\frac{\sin 2nz\cdot dz}{\sin z}\ ,$$

$$N'=\int_{l/(4a)}^{\pi-l/(4a)}\frac{\cos 2nz\cdot dz}{\sin z}\ ,$$

$$M=N-\int_{l/(4a)}^{\pi-l/(4a)}\sin 2nz\cdot\sin z dz\ ,$$

$$M'=N'-\int_{l/(4a)}^{\pi-l/(4a)}\cos 2nz\cdot\sin z dz\ ,$$

and considering that

$$\int\frac{\sin 2nz\cdot dz}{\sin z}=+2\int\cos(2n-1)z\cdot dz+2\int\cos(2n-3)z\cdot dz$$
$$+\ldots+2\int\cos z dz\ ,$$

$$\int\frac{\cos 2nz\cdot dz}{\sin z}=-2\int\sin(2n-1)z\cdot dz-2\int\sin(2n-3)z\cdot dz$$
$$-\ldots-2\int\sin z dz+\int\frac{dz}{\sin z}\ ,$$

then one finds, taking all integrals between $z=l/(4a)$ and $z=\pi-[l/(4a)]$,

$$N=0\ ,$$

$$N'=-4\left(\cos\frac{l}{4a}+\frac{1}{3}\cos\frac{3l}{4a}+\ldots+\frac{1}{2n-1}\cos\frac{(2n-1)l}{4a}\right)-2\log\tan\frac{l}{8a}\ .$$

Furthermore, as

$$\int\sin 2nz\cdot\sin z dz=\frac{1}{2(2n-1)}\sin(2n-1)z-\frac{1}{2(2n+1)}\sin(2n+1)z\ ,$$

$$\int\cos 2nz\cdot\sin z dz=\frac{1}{2(2n-1)}\cos(2n-1)z-\frac{1}{2(2n+1)}\cos(2n+1)z\ ,$$

taking also these integrals between the limits from $z=l/(4a)$ to $z=\pi-[l/(4a)]$, one finds

$$M = 0\ ,$$

$$M' = N' + \frac{1}{2n+1}\cos(2n+1)\frac{l}{4a} - \frac{1}{2n-1}\cos(2n-1)\frac{l}{4a}\ .$$

Hence follows finally, according to Section 18.8,

$$N'' = -4\left(\cos\frac{l}{4a} + \frac{1}{3}\cos\frac{3l}{4a} + ... + \frac{1}{2n-1}\cos(2n-1)\frac{l}{4a}\right) - 2\log\tan\frac{l}{8a} + 2\log\frac{l}{\alpha} - \frac{1}{8}\frac{n^2l^2}{a^2}\ ,$$

$$M'' = -4\left(\cos\frac{l}{4a} + \frac{1}{3}\cos\frac{3l}{4a} + ... + \frac{1}{2n-1}\cos(2n-1)\frac{l}{4a}\right) - 2\log\tan\frac{l}{8a} + 2\log\frac{l}{e\alpha} - \frac{1}{8}\frac{n^2l^2}{a^2} + \frac{1}{2n+1}\cos(2n+1)\frac{l}{4a} - \frac{1}{2n-1}\cos(2n-1)\frac{l}{4a}\ .$$

But here l denotes the length of the conductor element ds, considered as linear, with the point under consideration in its center. Between certain limits, this length is arbitrary, its choice is only limited by the quantities α/l and l/a being considered as vanishingly small values which must be the case if the conductor is to be considered as *linear*. The difference in the values of l, which are possible within these limits, does not have a noticeable influence on the values of N'' and M''. We may therefore put

$$l = \sqrt{a\alpha}\ ,$$

because for every conductor to be considered as *linear*, this value must lie between the specified limits. It also becomes clear that then $\tan(l/8a)$ may be replaced by $l/8a$. Putting for brevity

$$\frac{n^2\alpha}{8a} = 2\log\nu\ ,$$

[and]

$$\frac{n^2\alpha}{8a} + \frac{1}{2n-1}\cos\frac{2n-1}{4}\sqrt{\frac{\alpha}{a}} - \frac{1}{2n+1}\cos\frac{2n+1}{4}\sqrt{\frac{\alpha}{a}} = 2\log\mu\ ,$$

yields

$$N'' = -4\left(\cos\frac{1}{4}\sqrt{\frac{\alpha}{a}} + \frac{1}{3}\cos\frac{3}{4}\sqrt{\frac{\alpha}{a}} + ... + \frac{1}{2n-1}\cos\frac{2n-1}{4}\sqrt{\frac{\alpha}{a}}\right) + 2\log\frac{8a}{\nu\alpha}\ ,$$

$$M'' = -4\left(\cos\frac{1}{4}\sqrt{\frac{\alpha}{a}} + \frac{1}{3}\cos\frac{3}{4}\sqrt{\frac{\alpha}{a}} + ... + \frac{1}{2n-1}\cos\frac{2n-1}{4}\sqrt{\frac{\alpha}{a}}\right) + 2\log\frac{8a}{\mu e\alpha}\ .$$

Now substituting the values of N, N'', M, M'' found for a circular conductor into the equations found at the end of the Section 18.8 one gets the following two equations for the motions of the electricity in a circular conductor

$$\frac{d^2a_n}{dt^2}+\frac{c^2}{8\pi\alpha^2kM''(1+\lambda)}\cdot\frac{da_n}{dt}+\frac{n^2c^2N''}{2a^2M''(1+\lambda)}\cdot a_n-\frac{nc^2}{4aM''(1+\lambda)}\cdot g_n=0\ ,$$

$$\frac{d^2b_n}{dt^2}+\frac{c^2}{8\pi\alpha^2kM''(1+\lambda)}\cdot\frac{db_n}{dt}+\frac{n^2c^2N''}{2a^2M''(1+\lambda)}\cdot b_n+\frac{nc^2}{4aM''(1+\lambda)}\cdot f_n=0\ ,$$

where N'' and M'' have the above values.

18.11 Equilibrium of Electricity in a Circular Conductor

For the case of equilibrium of the electricity one has in all parts of the conductor

$$i=0 \quad \text{and} \quad \frac{di}{dt}=0\ .$$

Putting the value for i from Section 18.8 (3)[431] one gets

$$-\frac{a}{2}\sum\frac{1}{n}\left(\frac{db_n}{dt}\sin\frac{ns}{a}-\frac{da_n}{dt}\cos\frac{ns}{a}\right)=0\ ,$$

$$-\frac{a}{2}\sum\frac{1}{n}\left(\frac{d^2b_n}{dt^2}\sin\frac{ns}{a}-\frac{d^2a_n}{dt^2}\cos\frac{ns}{a}\right)=0\ ,$$

whence follows

$$\frac{da_n}{dt}=0,\qquad\frac{db_n}{dt}=0,\qquad\frac{d^2a_n}{dt^2}=0,\qquad\frac{d^2b_n}{dt^2}=0\ ,$$

where it has to be added that also for $n=0$ one need to have

$$\frac{1}{n}\frac{da_n}{dt}=\frac{1}{n}\frac{d^2a_n}{dt^2}=0\ .$$

The equations of motion, established at the end of the preceding Section, then turn into the following *equations of equilibrium*, namely, when $n>0$,

$$\frac{nN''}{a}\cdot a_n-\frac{1}{2}g_n=0\ ,$$

$$\frac{nN''}{a}\cdot b_n+\frac{1}{2}f_n=0\ ,$$

where $g_0=0$ still has to be added. Hence follows as equilibrium condition for the electricity that the sum of all external electromotive forces acting on the circular conductor, must equal

[431] [Note by AKTA:] That is, from item 3 of Section 18.8.

$$S = \int ds \sum \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a} \right) = 0 \ ,$$

in complete agreement with the known Ohm's law according to which the current intensity is proportional to the sum of all these forces and hence can only be zero together with this sum.

18.12 Steady Currents of Electricity in a Circular Conductor

The motion of electricity in a conductor is called a *steady* current if it stays constant in any point of the conductor. In the case of such a steady current one has for all points of the closed conductor

$$i = \text{constant},$$

hence

$$\frac{di}{dt} = -\frac{a}{2} \sum \frac{1}{n} \left(\frac{d^2 b_n}{dt^2} \sin \frac{ns}{a} - \frac{d^2 a_n}{dt^2} \cos \frac{ns}{a} \right) = 0 \ ,$$

whence

$$\frac{d^2 a_n}{dt^2} = 0 \ , \qquad \frac{d^2 b_n}{dt^2} = 0 \ ,$$

where it still has to be added that one must have $[1/n][d^2 a_n/dt^2] = 0$, also for $n = 0$.

The equations of motion given at the end of Section 18.10 then turn into the following equations of motion for *steady* currents, namely when $n > 0$

$$\frac{1}{4\pi\alpha^2 k} \cdot \frac{da_n}{dt} + \frac{n^2 N''}{a^2} \cdot a_n - \frac{n}{2a} g_n = 0 \ ,$$

$$\frac{1}{4\pi\alpha^2 k} \cdot \frac{db_n}{dt} + \frac{n^2 N''}{a^2} \cdot b_n + \frac{n}{2a} f_n = 0 \ ,$$

where also $[1/n][da_n/dt] =$ constant for $n = 0$, hence $a_0 =$ constant has to be added. It follows that for *steady* current the sum of all external electromotive forces acting on the circular conductor must be given by

$$\begin{aligned} S &= \int ds \sum \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a} \right) \\ &= \frac{2}{a} \int ds \sum n N'' \left(a_n \cos \frac{ns}{a} - b_n \sin \frac{ns}{a} \right) \\ &- \frac{a}{2\pi\alpha^2 k} \int ds \sum \frac{1}{n} \left(\frac{db_n}{dt} \sin \frac{ns}{a} - \frac{da_n}{dt} \cos \frac{ns}{a} \right) \ ; \end{aligned}$$

hence, because

$$-\frac{a}{2}\sum\frac{1}{n}\left(\frac{db_n}{dt}\sin\frac{ns}{a}-\frac{da_n}{dt}\cos\frac{ns}{a}\right)=i\ ,$$

and because

$$\int ds\sum nN''\left(a_n\cos\frac{ns}{a}-b_n\sin\frac{ns}{a}\right)=0\ ,$$

as is easily seen considering that a_0 has a constant value and hence $na_n=0$ for $n=0$,

$$S=\frac{1}{\pi\alpha^2 k}\cdot\int ids\ .$$

But now $[1/(2\pi a)]\cdot\int ids=J$ is the *mean value of the current intensity in the whole conductor*, and $2\pi a/[\pi\alpha^2 k]=w$ is the *resistance of the whole conductor*; thus $S=Jw$, that is, the sum of the external electromotive forces in the whole conductor must be equal the product of the resistance and the average current intensity of the whole conductor, quite in agreement with Ohm's law that yields the electromotive force of the circuit as the product of the resistance and the current, which is identical with the above results when it is assumed that there are no differences of the current intensities in the various points of the conductor. This need not be the case according to the above theory; but should there be current intensities in various points differing from the steady current in any single point, then the electromotive forces acting from outside must change *in proportion to time*, a case that does not occur in reality and therefore has been left out of the consideration of Ohm's law that is founded on experience. It is, namely, clear that if

$$i=-\frac{a}{2}\sum\frac{1}{n}\left(\frac{db_n}{dt}\sin\frac{ns}{a}-\frac{da_n}{dt}\cos\frac{ns}{a}\right)$$

is to have different values in different parts of the conductor, either da_n/dt or db_n/dt must have a non-zero value $=A$ for a non-zero value of n, whence follows either $a_n=At+B$ or $b_n=At+B$. Substituting in one case $At+B$ for a_n in the first of the above equations for the condition for steady currents, one gets

$$\frac{1}{4\pi\alpha^2 k}\cdot A+\frac{n^2N''}{a^2}(At+B)-\frac{n}{2a}g_n=0\ ,$$

whence follows that g_n changes *in proportion to time*. Substituting in the other case $At+B$ for b_n in the second conditional equation, it follows in a similar way that f_n changes *in proportion to time*. Hence in both cases also the electromotive force

$$S_n=f_n\sin\frac{ns}{a}+g_n\cos\frac{ns}{a}$$

would change *in proportion to time*.

18.13 Laws of Motion of the Electricity in a Circular Conductor Left to Itself After an Arbitrary Disturbance

The theory of the motion of the electricity left to itself after an arbitrary disturbance in a conductor comprises the important science of *propagation*, in particular the [following] questions, whether the propagation of motion in conductors by electricity is mediated by *waves*, just as the propagation [of light] in the luminiferous ether or [the propagation of sound] in air, furthermore what is the *velocity* of these waves and finally which *laws* at all are valid for this propagation of waves. The initial disturbance may in fact be restricted to a small part of the conductor, and if subsequently *similar* disturbances of the equilibrium occur without external influence *successively* in all other parts of the conductor all by themselves, this transmission is given the name *propagation*, and the disturbance is given the name *wave*.

If the electricity in the conductor is to be left to itself, *all external forces* that would act on the electricity in the conductor have to be put equal $= 0$. Hence for this case one obtains the equations of motion, putting

$$f_n = 0 \qquad \text{and} \qquad g_n = 0 \ ,$$

in the equations at the end of Section 18.10, [obtaining] the following [equations:]

$$\frac{d^2 a_n}{dt^2} + \frac{c^2}{8\pi\alpha^2 k M''(1+\lambda)} \cdot \frac{da_n}{dt} + \frac{n^2 c^2 N''}{2a^2 M''(1+\lambda)} \cdot a_n = 0 \ ,$$

$$\frac{d^2 b_n}{dt^2} + \frac{c^2}{8\pi\alpha^2 k M''(1+\lambda)} \cdot \frac{db_n}{dt} + \frac{n^2 c^2 N''}{2a^2 M''(1+\lambda)} \cdot b_n = 0 \ .$$

Setting

$$\frac{c^2}{16\pi\alpha^2 k M''(1+\lambda)} = \varepsilon \ ,$$

[and]

$$\frac{n^2 c^2 N''}{2a^2 M''(1+\lambda)} = m^2 + \varepsilon^2 \ ,$$

then one obtains from these two equations by integration

$$a_n = Ae^{-\varepsilon t} \cdot \sin m(t - A') \ ,$$

$$b_n = Be^{-\varepsilon t} \cdot \sin m(t - B') \ ,$$

where A, A', B, B' are constants of integration, to be determined from the given initial disturbance.

If the original distribution of free electricity in the conductor is given by the following equation, where E_0 denotes the value of the [linear charge] density E for $t = 0$, namely

$$E_0 = \sum \left(a_n^0 \sin \frac{ns}{a} + b_n^0 \cos \frac{ns}{a} \right) \ ,$$

and if the original currents in all parts of the conductor, where i_0 denotes the value of the current intensity i for $t = 0$, [is given] by the following equation

$$i_0 = -\frac{a}{2} \sum \frac{1}{n} \left(\frac{db_n^0}{dt} \sin \frac{ns}{a} - \frac{da_n^0}{dt} \cos \frac{ns}{a} \right) \ ,$$

where a_n^0, b_n^0, da_n^0/dt, db_n^0/dt have known values, inserting these values for $t = 0$ into the above equation yields

$$a_n^0 = -A \sin mA' \ ,$$

$$b_n^0 = -B \sin mB' \ ,$$

and, after differentiating the above equations,

$$\frac{da_n^0}{dt} = mA \cos mA' - \varepsilon a_n^0 \ ,$$

$$\frac{db_n^0}{dt} = mB \cos mB' - \varepsilon b_n^0 \ .$$

These four equations yield the following values of the constants of integration:

$$A = \sqrt{a_n^{0\,2} + \frac{1}{m^2}\left(\varepsilon a_n^0 + \frac{da_n^0}{dt}\right)^2} \ ,$$

$$B = \sqrt{b_n^{0\,2} + \frac{1}{m^2}\left(\varepsilon b_n^0 + \frac{db_n^0}{dt}\right)^2} \ ,$$

$$A' = -\frac{1}{m} \arcsin \frac{a_n^0}{A} \ ,$$

$$B' = -\frac{1}{m} \arcsin \frac{b_n^0}{B} \ .$$

Inserting the latter two values into the above equations one gets

$$a_n = Ae^{-\varepsilon t} \sin\left(mt + \arcsin \frac{a_n^0}{A}\right) \ ,$$

$$b_n = Be^{-\varepsilon t} \sin\left(mt + \arcsin \frac{b_n^0}{B}\right) \ ,$$

and hence the *distribution law of free electricity* in the conductor [is given by]:

$$E = \sum e^{-\varepsilon t} \cdot \left[A \sin \frac{ns}{a} \sin\left(mt + \arcsin \frac{a_n^0}{A}\right) \right. \\ \left. + B \cos \frac{ns}{a} \sin\left(mt + \arcsin \frac{b_n^0}{B}\right)\right] \ ,$$

or, expanding the sine of the sum of two arcs:

$$E = \sum e^{-\varepsilon t} \cdot \left(a_n^0 \sin \frac{ns}{a} \cos mt + \sqrt{B^2 - b_n^{0\,2}} \cos \frac{ns}{a} \sin mt \right. \\ \left. + b_n^0 \cos \frac{ns}{a} \cos mt + \sqrt{A^2 - a_n^{0\,2}} \sin \frac{ns}{a} \sin mt\right) \ .$$

Now putting

$$a_n^0 = p + q \; , \qquad b_n^0 = p' + q' \; ,$$

$$\sqrt{B^2 - {b_n^0}^2} = p - q \; , \qquad \sqrt{A^2 - {a_n^0}^2} = p' - q' \; ,$$

whereby p, q, p', q' are determined, namely,

$$p = \frac{1}{2}\left(a_n^0 + \frac{1}{m}\left(\varepsilon b_n^0 + \frac{db_n^0}{dt}\right)\right) \; ,$$

$$q = \frac{1}{2}\left(a_n^0 - \frac{1}{m}\left(\varepsilon b_n^0 + \frac{db_n^0}{dt}\right)\right) \; ,$$

$$p' = \frac{1}{2}\left(b_n^0 + \frac{1}{m}\left(\varepsilon a_n^0 + \frac{da_n^0}{dt}\right)\right) \; ,$$

$$q' = \frac{1}{2}\left(b_n^0 - \frac{1}{m}\left(\varepsilon a_n^0 + \frac{da_n^0}{dt}\right)\right) \; ,$$

thus one obtains

$$E = \sum e^{-\varepsilon t} \cdot \left(q \sin\left(\frac{ns}{a} - mt\right) + p' \cos\left(\frac{ns}{a} - mt\right)\right)$$
$$+ \sum e^{-\varepsilon t} \cdot \left(p \sin\left(\frac{ns}{a} + mt\right) + q' \cos\left(\frac{ns}{a} + mt\right)\right) \; ,$$

or, alternatively,

$$E = \sum \sqrt{p'^2 + q^2} \cdot e^{-\varepsilon t} \sin\left(\frac{ns}{a} - mt + \arctan\frac{p'}{q}\right)$$
$$+ \sum \sqrt{p^2 + q'^2} \cdot e^{-\varepsilon t} \sin\left(\frac{ns}{a} + mt + \arctan\frac{q'}{p}\right) \; .$$

Similarly one finds the law of the *current of electricity* in the conductor, namely:

$$i = \sum \sqrt{P'^2 + Q^2} \cdot e^{-\varepsilon t} \sin\left(\frac{ns}{a} - mt + \arctan\frac{P'}{Q}\right)$$
$$+ \sum \sqrt{P^2 + Q'^2} \cdot e^{-\varepsilon t} \sin\left(\frac{ns}{a} + mt + \arctan\frac{Q'}{P}\right) \; ,$$

where P, Q, P', Q' have the following values:

$$P = -\frac{a}{4n}\left(\frac{db_n^0}{dt} + \frac{1}{m}\left(\left(m^2 + \varepsilon^2\right) a_n^0 + \varepsilon\frac{da_n^0}{dt}\right)\right) \; ,$$

$$Q = -\frac{a}{4n}\left(\frac{db_n^0}{dt} - \frac{1}{m}\left(\left(m^2 + \varepsilon^2\right) a_n^0 + \varepsilon\frac{da_n^0}{dt}\right)\right) \; ,$$

$$P' = +\frac{a}{4n}\left(\frac{da_n^0}{dt} + \frac{1}{m}\left(\left(m^2 + \varepsilon^2\right) b_n^0 + \varepsilon\frac{db_n^0}{dt}\right)\right) \; ,$$

$$Q' = +\frac{a}{4n}\left(\frac{da_n^0}{dt} - \frac{1}{m}\left(\left(m^2 + \varepsilon^2\right) b_n^0 + \varepsilon\frac{db_n^0}{dt}\right)\right) \; .$$

18.14 Comparison with Ohm's Law

It has already been discussed in Section 18.6 on which grounds Ohm's law, formulated for *steady* currents, may also be applied to *variable* currents. This [application] depends on the value of

$$\lambda = \frac{c^2}{8r\mathfrak{E}\log\frac{l}{\alpha}} \ .$$

Wherever this quantity comes into question and its value does not vanish, Ohm's law does not apply at all or only approximately. This magnitude λ has been considered in more detail in Section 18.8, with respect to the influence of more distant parts of the circuit not yet considered in Section 18.6, namely

$$\lambda = \frac{c^2}{4M''r\mathfrak{E}} \ ,$$

where the value M'' put for $2\log[l/\alpha]$ in Section 18.6 has been defined exactly and determined for a circular conductor in Section 18.10. The magnitude λ, or, as the value of the factor $c^4/[4M'']$ may be considered as constant, the value of the product $r\mathfrak{E}$ decides on the applicability of Ohm's law, thereby gaining particular importance for the theory of motion of the electricity in conductors, whose reason is easily seen from the *physical meaning* of the product $r\mathfrak{E}$.

In particular, the amount of positive electricity contained in the unit length of the conductor has been denoted by $\mathfrak{E}$, expressed in the unit of measure determined from the electrostatic law, and its *mass in milligrams* has been put equal to $[1/r]\mathfrak{E}$. From the definition of the unit of measure established for the electrostatic law (where the amount of electricity is taken as the unit of measure which exerts on an equal [amount of electricity] the unit force at unit distance according to the electrostatic law, that is, a force that produces the unit velocity during the unit time on one milligram), it follows that r^2 is the force exerted by *one milligram* of positive or negative electricity on an equal milligram of electricity at the unit distance. Whence it follows that the product $r\mathfrak{E}$ means the force that the *positive electricity contained in the unit length of the conductor*, if concentrated in one point, would exert on *one milligram* of positive electricity at unit distance.

Now the influence of this magnitude λ or that of the product $r\mathfrak{E}$ will be determined in more detail on the basis of the development of the laws of motion of the electricity in a closed conductor as given in Sections 18.8 and the following. To begin with, from Sections 18.11 and 18.12 it follows that the laws of *equilibrium* and of *steady currents* of the electricity in conductors are in complete agreement with Ohm's law because the magnitude λ or $r\mathfrak{E}$ does not come into question here, while it follows from Section 18.13 that the laws of *propagation*, or generally the laws of all *changes of motion* effective after a disturbance of equilibrium, vitally depend above all on the values of m and ε and hence indirectly on λ or $r\mathfrak{E}$. Therefore it follows that the magnitude λ or the product $r\mathfrak{E}$ (hence indirectly the total *mass* of the amount of electricity, in milligrams, contained in the conductor if the amount of electricity per unit length of a conductor were known in *electrostatic measure*) can only be known by means of observations which disclose certain *deviations from Ohm's law* in the *changes of motion* of the electricity in conductors after a disturbance of the equilibrium.

The importance thereby gained due to *more detailed observations on the changes of motion or on the propagation of motion through electricity in conductors* is clear; if these observations really allowed to detect any deviation *from Ohm's law*, then this result would disclose the value of the product $r\mathfrak{E}$, that is, the number of electrostatic units of measure which make *one milligram* of electricity when the *number of electrostatic units of measure* per unit length of the conductor is known.

To begin with, the laws of *electric wave motions* in circular conductors according to Section 18.13 shall be developed in more detail in order to test whether it could yield a certain guide on how *to conduct such observations*; then, if this were not the case, the reason for this shall be searched and if there were other motions in circular conductors which are better suited than *wave motion*.

18.15 Electric Wave Motions in a Circular Conductor

From the laws developed in Section 18.13 it follows that all motions of the electricity left to itself in a circular conductor after an arbitrary disturbance turn out to be a series of *wave trains* propagating *forward* and a series of *wave trains* propagating *backwards*, whose *first wave train* consists of two waves in each series, namely, one positive and one negative, which together cover the whole circular periphery; the *second wave train* of each series consists of four alternately positive and negative waves which together fill out the whole circle; the *third wave train* consists of six waves and so on.

Breaking up into their terms the sums, which in Section 18.13 represented the [linear] density of the free electricity E and the current intensity i, and denoting these terms by E_n and i_n according to their place number, n, then one gets

$$\begin{aligned}E_1 &= \sqrt{p'^2+q^2}\cdot e^{-\varepsilon t}\sin\left(\frac{s}{a}-mt+\arctan\frac{p'}{q}\right)\\ &+\sqrt{p^2+q'^2}\cdot e^{-\varepsilon t}\sin\left(\frac{s}{a}+mt+\arctan\frac{q'}{p}\right)\ ,\end{aligned}$$

$$\begin{aligned}i_1 &= \sqrt{P'^2+Q^2}\cdot e^{-\varepsilon t}\sin\left(\frac{s}{a}-mt+\arctan\frac{P'}{Q}\right)\\ &+\sqrt{P^2+Q'^2}\cdot e^{-\varepsilon t}\sin\left(\frac{s}{a}+mt+\arctan\frac{Q'}{P}\right)\ ,\end{aligned}$$

wherein the first parts, containing the sine of an arc that change in proportion to $(s-amt)$, represent the *first wave train propagating forward*, [and] the latter parts containing the sine of an arc that change in proportion to $(s+amt)$, [represent] the *first wave train propagating backward*. But the first wave train propagating forward consists of a *positive wave* which extends from $s=0$ to $s=\pi a$ at the moment $t=[1/m]\arctan[p'/q]$, where the wave produces a charge of the conductor with positive free electricity, and [consists] of a *negative wave* which extends from $s=\pi a$ to $s=2\pi a$ at the same moment, where the wave produces a charge of the conductor with negative free electricity. But both waves together cover the total circular periphery. The same holds for the first wave train propagating backward, consisting of a *positive wave* extending from $s=0$ to $s=\pi a$ at the moment $t=-[1/m]\arctan[q'/p]$, and of a *negative wave* extending from $s=\pi a$ to $s=2\pi a$ at the same moment.

Furthermore we have

$$E_2 = \sqrt{p'^2 + q^2} \cdot e^{-\varepsilon t} \sin\left(\frac{2s}{a} - mt + \arctan\frac{p'}{q}\right)$$
$$+ \sqrt{p^2 + q'^2} \cdot e^{-\varepsilon t} \sin\left(\frac{2s}{a} + mt + \arctan\frac{q'}{p}\right) \ ,$$

$$i_2 = \sqrt{P'^2 + Q^2} \cdot e^{-\varepsilon t} \sin\left(\frac{2s}{a} - mt + \arctan\frac{P'}{Q}\right)$$
$$+ \sqrt{P^2 + Q'^2} \cdot e^{-\varepsilon t} \sin\left(\frac{2s}{a} + mt + \arctan\frac{Q'}{P}\right) \ ,$$

where the first parts, containing the sine of an arc that changes in proportion to $(s - amt/2)$, represent the *second wave train propagating forward*, [and] the latter parts, containing the sine of an arc that changes in proportion to $(s + amt/2)$, represent the *second wave train propagating backward*. This wave train propagating forward consists of 4 waves whose first positive one extends from $s = 0$ to $s = \pi a/2$, the second negative from $s = \pi a/2$ to $s = \pi a$, the third positive from $s = \pi a$ to $s = 3\pi a/2$, and the fourth negative from $s = 3\pi a/2$ to $s = 2\pi a$ at the moment $t = [1/m]\arctan[p'/q]$. The same holds for the 4 waves of the wave train propagating backward at the moment $t = -[1/m]\arctan[q'/p]$.

Similarly, the third wave trains of both series result from E_3 and i_3, and so on.

The *intensities* of the various wave trains, which equal i^2 according to the rules of wave theory, decrease while propagating, in fact by a factor of

$$1 : e^{-2\varepsilon t} \ ,$$

during the time t. Because the value of ε changes with the value of n, this decrease varies with the place number n of the wave trains; for we had

$$\varepsilon = \frac{c^2}{16\pi\alpha^2 k M''(1+\lambda)} \ ,$$

$$\lambda = \frac{c^2}{4M'' r \mathfrak{E}} \ ,$$

wherein, according to Section 18.10,

$$M'' = -4\left(\cos\frac{1}{4}\sqrt{\frac{\alpha}{a}} + \frac{1}{3}\cos\frac{3}{4}\sqrt{\frac{\alpha}{a}} + ... + \frac{1}{2n-1}\cos\frac{2n-1}{4}\sqrt{\frac{\alpha}{a}}\right)$$
$$+ 2\log\frac{8a}{e\alpha} - \frac{1}{8}\frac{n^2\alpha}{a} + \frac{1}{2n+1}\cos\frac{2n+1}{4}\sqrt{\frac{\alpha}{a}} - \frac{1}{2n-1}\cos\frac{2n-1}{4}\sqrt{\frac{\alpha}{a}} \ ,$$

whence if α/a is very small

for $n = 1$, $\qquad M'' = 2\log\frac{8a}{\alpha} - 6.666... \ ,$

for $n = 2$, $\qquad M'' = 2\log\frac{8a}{\alpha} - 7.466...$

and so on.

Letting $w' = 1/[\pi\alpha^2 k]$ denote the resistance of the unit length of the conductor, and putting $\lambda = 0$, that is, restricting to those cases in which Ohm's law applies, we get a decrease of intensity in the unit of time in the proportion

$$= 1 : e^{-\frac{w'c^2}{16 \log \frac{8a}{\alpha} - 53.33\ldots}}$$

for the first wave trains with $n = 1$, [and in the proportion]

$$= 1 : e^{-\frac{w'c^2}{16 \log \frac{8a}{\alpha} - 59.733\ldots}}$$

for the second wave trains with $n = 2$, and so on.

Hence one sees that the decrease is the faster, the greater the resistance per unit length of the conductor, the thicker the conductor compared to its length, and the larger the place number n of the wave train, that means, the shorter the waves.

18.16 Propagation Velocity of the Wave Trains in a Circular Conductor

From Section 18.13 it follows, as shown above, that, after each disturbance of the equilibrium, the motions of electricity in a circular conductor can be split into wave trains whose propagation is determined by simple laws, as is the case for many other bodies. For some bodies like air in a circular tube, in addition, these wave trains are not altered at all by the propagation, that specifically no decrease of the intensity takes place, and that furthermore all wave trains are propagated at *equal velocity*, whence it follows that all wave trains propagating forward *combine* to a single wave train which in turn is propagated unaltered and at the same velocity like the single wave trains of which it consists. Such a combined wave train, however, consists of combined waves which can largely differ in size, form, and intensity. Such *combined waves*, remaining coherent due to the same velocity of all its constituents, have a particular physical meaning as *objects of observation* and are called *waves in the strict sense of the word*.

Thus in this more *strict* sense electric waves in a circular conductor where electric equilibrium has been disturbed would not exist, already because of the different decreases of the intensities of the various elementary wave trains, even less, however, if the various elementary wave trains had *different propagation velocities*.

Where waves exist in the more strict sense, the *propagation velocity* is of utmost importance for the knowledge about the *medium of propagation*, therefore this question concerning *electricity* has awakened particular interest and therefore the respective results from Section 18.13 shall be considered more closely.

The *propagation velocities* of the various elementary wave trains from the formulas developed in Section 18.13 were found to be equal to the increase or decrease which s must get if, when t increases by 1 [unit] in the values of E_n and i_n, the values of the arcs under the sines shall remain unaltered, that is

$$= \frac{ma}{n} \, ,$$

or, inserting the value of m from Section 18.13

$$m = \sqrt{\frac{n^2c^2N''}{2a^2M''(1+\lambda)} - \left(\frac{c^2}{16\pi\alpha^2kM''(1+\lambda)}\right)^2} ,$$

[we get]

$$= \frac{c}{\sqrt{2}} \cdot \sqrt{\frac{N''}{M''(1+\lambda)} - \frac{a^2c^2w'^2}{128n^2M''^2(1+\lambda)^2}} ,$$

wherein $w' = 1/[\pi\alpha^2k]$ is put as above. Restricting ourselves to the cases where we may put $\lambda = 0$, that is, where Ohm's law applies, then the expression for this propagation velocity reduces to

$$= \frac{c}{\sqrt{2}} \cdot \sqrt{\frac{N''}{M''} - \frac{a^2c^2w'^2}{128n^2M''^2}} ,$$

wherein the values of N'' and M'' are determined as follows

$$N'' = 2\log\frac{8a}{\alpha} - 4\left(\cos\frac{1}{4}\sqrt{\frac{\alpha}{a}} + \frac{1}{3}\cos\frac{3}{4}\sqrt{\frac{\alpha}{a}} + ... + \frac{1}{2n-1}\cos\frac{2n-1}{4}\sqrt{\frac{\alpha}{a}}\right) - \frac{n^2\alpha}{8a} ,$$

$$M'' = 2\log\frac{8a}{\alpha} - 4\left(\cos\frac{1}{4}\sqrt{\frac{\alpha}{a}} + \frac{1}{3}\cos\frac{3}{4}\sqrt{\frac{\alpha}{a}} + ... + \frac{1}{2n-1}\cos\frac{2n-1}{4}\sqrt{\frac{\alpha}{a}}\right) - \frac{n^2\alpha}{8a} - 2 - \frac{1}{2n-1}\cos\frac{2n-1}{4}\sqrt{\frac{\alpha}{a}} + \frac{1}{2n+1}\cos\frac{2n+1}{4}\sqrt{\frac{\alpha}{a}} .$$

Thus it follows that the propagation velocity is different for the various wave trains according to their different place numbers n, and it only remains the question whether, under certain conditions, the differences of the various propagation velocities would not be so small as to consider them approximately as vanishing, and what would then be the limit to be approached by all these propagation velocities.

From the values presented it follows indeed that, as long as the place number, n, does not exceed those values for which $n^2\alpha/a$ may be considered as vanishing compared to 1, we may put

$$\frac{N''}{M''} = 1 + \frac{8n^2}{(4n^2-1)M''} .$$

For large values of M'' for which the fraction $8n^2/(4n^2-1)M''$ vanishes compared to 1, and for small values of the resistance for which the fraction $a^2c^2w'^2/[128n^2M''^2]$ vanishes compared to 1,[432] then $c/\sqrt{2}$ is the desired limit which is approached by all propagation velocities, and, for the given value[433] $c = 439\,450 \cdot 10^6$ millimeter/second, this limit equals

[432][Note by WW:] The fraction $a^2c^2w'^2/[128n^2M''^2]$ can be considered as vanishing compared with 1 when, for large values of M'', that velocity expressing the resistance of the whole conductor in *absolute magnetic measure of resistance*, that is, $[\pi c/4]acw'$, is very small compared to the velocity c.

[433][Note by AKTA:] See [KW57, Section 17, p. 652 of Weber's *Werke*] with English translation in [KW21, Section 17, p. 52]. See also page 179 on Section 7.17.

$$\frac{c}{\sqrt{2}} = 310\,740 \cdot 10^6 \ \frac{\text{millimeter}}{\text{second}} \ ,$$

that is, a velocity of 41 950 miles/second.

Already Kirchhoff has found this velocity and remarked:[434]

> "that it is independent of the cross section, of the conductivity of the wire, also, finally, of the density of the electricity: its value is 41 950 German miles in a second, hence very nearly equal to the velocity of light *in vacuo*."

Could this close coincidence of the propagation velocity of electric waves with that of the light be considered as a hint to the inner relationship of both theories, then it would demand the greatest interest considering the great importance of the investigation of such a relationship. But it is clear that above all, the true meaning of this velocity with respect to electricity must be considered, but it is not of the kind encouraging great expectations.

For the approximation of the true propagation velocity to this limiting value which coincides with the velocity of light presupposes, as just demonstrated, a conducting wire not only very thin compared to its length, but also that this long and thin conducting wire would have a very small resistance. Hence it is clear that the close approach to this limit will occur only rarely, larger deviations from it very frequently. A corresponding survey is best obtained giving examples.

As examples we choose three circular copper wires with respective radii

$$a = 1000 \ , \qquad 1\,000\,000 \ , \qquad 1\,000\,000 \text{ millimeter} \ ,$$

and respective cross sections

$$\pi\alpha^2 = 1 \ , \qquad 1 \ , \qquad \frac{1}{10} \text{ square millimeter} \ .$$

The resistance of these wires, as found by measurement *in absolute magnetic measure of resistance*, can be put in rounded figures equal to

$$W = \frac{2\pi a}{\pi\alpha^2} \cdot 2 \cdot 10^6 \ ,$$

(see the *Abhandlungen der Königl. Gesellschaften der Wissenschaften zu Göttingen*, Vol, 5, Section 9).[435,436] But, according to the known relation between magnetic and mechanical measures of resistivity, we have $W = \pi c^2 a w'/4$ or $a^2c^2w'^2/128 = W^2/[8\pi^2c^2]$, after what[437]

$$\frac{c}{\sqrt{2}}\sqrt{\frac{N''}{M''} - \frac{a^2c^2w'^2}{128n^2M''^2}} = \frac{c}{\sqrt{2}}\sqrt{\frac{N''}{M''} - \frac{W^2}{8\pi^2c^2n^2M''^2}} = \frac{c'}{\sqrt{2}} \ .$$

The following Table is calculated on this basis.

[434][Note by AKTA:] [Kir57b, pp. 209-210] with English translation in [Kir57a, p. 406]. See page 214 on Chapter 8.

[435][Note by HW:] Wilhelm Weber's *Werke*, Vol. II, p. 319.

[436][Note by AKTA:] [Web53e, Section 9, pp. 315-319 of Weber's *Werke*], see also [Web53a] and [Web53c].

[437][Note by AKTA:] Weber is defining the magnitude c' by the equation following immediately after this footnote.

n	First wire $a = 1000$ $\pi\alpha^2 = 1$ $W = 4 \cdot 10^9 \cdot \pi$	Second wire $a = 1\,000\,000$ $\pi\alpha^2 = 1$ $W = 4 \cdot 10^{12} \cdot \pi$	Third wire $a = 1\,000\,000$ $\pi\alpha^2 = 1/10$ $W = 4 \cdot 10^{13} \cdot \pi$
1	$N'' = 15.119$ $M'' = 12.452$ $N''/M'' = 1.214$ $\frac{W^2}{8\pi^2c^2n^2M''^2} = \frac{1}{14\,970\,000}$ $c'^2/c^2 = 1.214$	$= 28.935$ $= 25.268$ $= 1.145$ $= 0.0166$ $= 1.128$	$= 31.605$ $= 28.938$ $= 1.092$ $= 1.2364$ $= -0.0443$
2	$N'' = 13.786$ $M'' = 11.652$ $N''/M'' = 1.183$ $\frac{W^2}{8\pi^2c^2n^2M''^2} = \frac{1}{52\,450\,000}$ $c'^2/c^2 = 1.183$	$= 27.601$ $= 25.468$ $= 1.084$ $= 0.004\,08$ $= 1.080$	$= 31.062$ $= 28.928$ $= 1.074$ $= 0,3093$ $= 0.7644$
3	$N'' = 12.986$ $M'' = 10.929$ $N''/M'' = 1.188$ $\frac{W^2}{8\pi^2c^2n^2M''^2} = \frac{1}{103\,800\,000}$ $c'^2/c^2 = 1.188$	$= 26.801$ $= 24.747$ $= 1.083$ $= 0.001\,92$ $= 1.081$	$= 30.262$ $= 28.205$ $= 1.073$ $= 0,1446$ $= 0.9283$
4	$N'' = 12.414$ $M'' = 10.383$ $N''/M'' = 1.196$ $\frac{W^2}{8\pi^2c^2n^2M''^2} = \frac{1}{166\,200\,000}$ $c'^2/c^2 = 1.197$	$= 26.230$ $= 24.198$ $= 1.084$ $= 0.001\,13$ $= 1.083$	$= 29.690$ $= 27.659$ $= 1.073$ $= 0,0846$ $= 0.9889$
5	$N'' = 11.970$ $M'' = 9.950$ $N''/M'' = 1.203$ $\frac{W^2}{8\pi^2c^2n^2M''^2} = \frac{1}{239\,000\,000}$ $c'^2/c^2 = 1.203$	$= 25.785$ $= 23.765$ $= 1.085$ $= 0.000\,75$ $= 1.084$	$= 29.246$ $= 27.226$ $= 1.074$ $= 0,0559$ $= 1.0183$

The values of c'^2/c^2 in the above Table which give the squares of the propagation velocities $c'/\sqrt{2}$ in parts of the square of the limiting value $c/\sqrt{2}$ disclose essential differences already among the first five wave trains to which the Table is restricted; for the third wire c'^2/c^2 even has a *negative* value for $n = 1$; hence here the expression for the propagation velocity of the first wave train becomes *imaginary* and therefore the laws of the changes of motion in this wire after a disturbance of the equilibrium cannot at all be interpreted in terms of propagating waves trains, but they require a different form which represents the changes of motion as a pure approximation to the state of equilibrium which may be called *absorption*, and which deserves particular attention because of its special importance for long and thin conducting wires with large resistance, namely telegraph wires.

18.17 Damping of Electric Motions in a Circular Conductor

In Section 18.13, in the integration of the two partial differential equations for the motion of the electricity left to itself in a circular conductor, namely the equations

$$\frac{d^2a_n}{dt^2} + 2\varepsilon\frac{da_n}{dt} + (m^2 + \varepsilon^2)a_n = 0 \ ,$$

$$\frac{d^2b_n}{dt^2} + 2\varepsilon\frac{db_n}{dt} + (m^2 + \varepsilon^2)b_n = 0 \ ,$$

it has been assumed for the two expressions set up for a_n and b_n

$$a_n = Ae^{-\varepsilon t}\sin m(t - A') \ ,$$

$$b_n = Be^{-\varepsilon t}\sin m(t - B') \ ,$$

that m would be a real value which, however, is not always the case. Because, putting $1/[\pi\alpha^2 k] = w'$, we had

$$m = \frac{n}{a}\cdot\frac{c}{\sqrt{2}}\sqrt{\frac{N''}{M''(1+\lambda)} - \frac{a^2c^2w'^2}{128n^2M''^2(1+\lambda)^2}} \ ,$$

namely this assumption can also be formulated so that

$$\frac{a^2c^2w'^2}{128n^2M''^2(1+\lambda)^2} < \frac{N''}{M''(1+\lambda)} \ ,$$

should hold, or, if $\lambda = 0$,

$$\frac{a^2c^2w'^2}{128n^2M''^2} < \frac{N''}{M''} \ .$$

On the other hand, the example of the third wire in the preceding Section shows that with long and thin conducting wires also the case may occur in which

$$\frac{a^2c^2w'^2}{128n^2M''^2} > \frac{N''}{M''} \ ,$$

whence it becomes clear that the integration of the above differential equations becomes illusive and hence must be sought in a different form.

Therefore, putting for this purpose

$$m = \frac{n}{a}\cdot\frac{c}{\sqrt{2}}\sqrt{\frac{a^2c^2w'^2}{128n^2M''^2(1+\lambda)^2} - \frac{N''}{M''(1+\lambda)}} \ ,$$

then the two differential equations get the following form, namely

$$\frac{d^2a_n}{dt^2} + 2\varepsilon\frac{da_n}{dt} + (\varepsilon^2 - m^2)a_n = 0 \ ,$$

$$\frac{d^2 b_n}{dt^2} + 2\varepsilon\frac{db_n}{dt} + (\varepsilon^2 - m^2)b_n = 0 \ ,$$

from which by integration we get

$$a_n = Ae^{-\varepsilon t} \cdot \left(e^{m(t-A')} - e^{-m(t-A')}\right) \ ,$$

$$b_n = Be^{-\varepsilon t} \cdot \left(e^{m(t-B')} - e^{-m(t-B')}\right) \ .$$

As performed in Section 18.13, the constants of integration A, A', B, B' will be found from the values of a_n^0, b_n^0, da_n^0/dt, db_n^0/dt given for $t = 0$ by means of which the original distribution of the free electricity in the conductor and the original currents are expressed. In this way one obtains

$$Ae^{-mA'} = \frac{1}{2m}\left((\varepsilon + m)a_n^0 + \frac{da_n^0}{dt}\right) \ ,$$

$$Ae^{+mA'} = \frac{1}{2m}\left((\varepsilon - m)a_n^0 + \frac{da_n^0}{dt}\right) \ ,$$

$$Be^{-mB'} = \frac{1}{2m}\left((\varepsilon + m)b_n^0 + \frac{db_n^0}{dt}\right) \ ,$$

$$Be^{+mB'} = \frac{1}{2m}\left((\varepsilon - m)b_n^0 + \frac{db_n^0}{dt}\right) \ .$$

Substituting these values, one obtains the following two equations

$$a_n = \frac{1}{2m}\left[\left((\varepsilon + m)a_n^0 + \frac{da_n^0}{dt}\right)e^{-(\varepsilon - m)t} - \left((\varepsilon - m)a_n^0 + \frac{da_n^0}{dt}\right)e^{-(\varepsilon + m)t}\right] \ ,$$

$$b_n = \frac{1}{2m}\left[\left((\varepsilon + m)b_n^0 + \frac{db_n^0}{dt}\right)e^{-(\varepsilon - m)t} - \left((\varepsilon - m)b_n^0 + \frac{db_n^0}{dt}\right)e^{-(\varepsilon + m)t}\right] \ .$$

Finally inserting these values of a_n and b_n into the equations

$$E = \sum \left(a_n \sin\frac{ns}{a} + b_n \cos\frac{ns}{a}\right) \ ,$$

$$i = -\frac{a}{2}\sum \frac{1}{n}\left(\frac{db_n}{dt}\sin\frac{ns}{a} - \frac{da_n}{dt}\cos\frac{ns}{a}\right) \ ,$$

one finds the laws of distribution of the free electricity and of the currents in the circular conductor for the cases considered here.

Now such a case occurs with each circular conductor, namely when the given original distribution of the free electricity and of the currents is such that the value of b_n^0 or $[1/n]\cdot[da_n^0/dt]$ is not zero for $n = 0$, and therefore this case has been excluded from the consideration in Section 18.15, where only those values of E_n and i_n have been discussed which are valid for $n = 1,\ 2,\ 3$ ... Is in fact $n = 0$, then it becomes clear that

$$\frac{a^2c^2w'^2}{128n^2M''^2(1+\lambda)^2} > \frac{N''}{M''(1+\lambda)}$$

holds and that consequently we have to put

$$m = \frac{1}{a} \cdot \frac{c}{\sqrt{2}} \sqrt{\frac{a^2c^2w'^2}{128M''^2(1+\lambda)^2}} \ .$$

But we had

$$\varepsilon = \frac{c^2w'}{16M''(1+\lambda)} \ ,$$

whence it follows that, for $n = 0$, we have to put

$$m = \varepsilon \ .$$

Now substituting this value of m into the above values of a_n and b_n, we get

$$a_0 = a_0^0 + \frac{1}{2\varepsilon}\frac{da_0^0}{dt} - \frac{1}{2\varepsilon}\frac{da_0^0}{dt} \cdot e^{-2\varepsilon t} \ ,$$

$$b_0 = b_0^0 + \frac{1}{2\varepsilon}\frac{db_0^0}{dt} - \frac{1}{2\varepsilon}\frac{db_0^0}{dt} \cdot e^{-2\varepsilon t} \ ,$$

whence by differentiation

$$\frac{da_0}{dt} = \frac{da_0^0}{dt} \cdot e^{-2\varepsilon t} \ ,$$

$$\frac{db_0}{dt} = \frac{db_0^0}{dt} \cdot e^{-2\varepsilon t} \ .$$

Now inserting these values into the equations

$$E_n = a_n \sin\frac{ns}{a} + b_n \cos\frac{ns}{a} \ ,$$

$$i_n = -\frac{a}{2n}\left(\frac{db_n}{dt}\sin\frac{ns}{a} - \frac{da_n}{dt}\cos\frac{ns}{a}\right) \ ,$$

for $n = 0$, one finds

$$E_0 = b_0^0 + \frac{1}{2\varepsilon}\frac{db_0^0}{dt}\left(1 - e^{-2\varepsilon t}\right) \ ,$$

$$i_0 = -\frac{s}{2}\frac{db_0^0}{dt}e^{-2\varepsilon t} + \frac{a}{2}\left(\frac{1}{n}\frac{da_n}{dt}\right)_0 \ ,$$

where

$$\left(\frac{1}{n}\frac{da_n}{dt}\right)_0$$

denotes the value of $([1/n] \cdot [da_n/dt])$ for $n = 0$; hence, as

$$\left(\frac{1}{n}\frac{da_n}{dt}\right)_0 = \left(\frac{1}{n}\frac{da_n^0}{dt}\right)_0 \cdot e^{-2\varepsilon t} \ ,$$

and as the coefficients of $\sin(ns/a)$ and $\cos(ns/a)$ are to have finite values in the equation

$$i_n = -\frac{a}{2n}\left(\frac{db_n}{dt}\sin\frac{ns}{a} - \frac{da_n}{dt}\cos\frac{ns}{a}\right) ,$$

whence, for $n = 0$, we should have

$$\frac{da_0}{dt} = 0 , \qquad \text{and} \qquad \frac{db_0}{dt} = 0 ,$$

[therefore,]

$$E_0 = b_0^0 ,$$

$$i_0 = \frac{a}{2}\left(\frac{1}{n}\frac{da_n^0}{dt}\right)_0 \cdot e^{-2\varepsilon t} .$$

Whence it follows that, if a circular conductor is originally homogeneously charged with free electricity along its total length, so that each unit length contains the same amount of free electricity $= b_0^0$, then this charge does not change with time t, which becomes clear all by itself. But in addition to that, if there is originally the same current in all parts of this conductor, that this current does not vanish at the moment after which the electricity is left to itself, but gradually decreases following the law of a geometric series as time t increases arithmetically. If here also the necessity of gradual vanishing becomes clear a priori, it can not be easily seen a priori how fast this should happen and what differences in this speed should take place between different conductors.

If a current with certain intensity i flows in a closed conductor at the very moment after which the electricity is left to itself because no external electromotive force acts on it, as is for example the case when an inductive magnet moving with respect to a conductor is suddenly stopped in this motion by pushing against it — then it is interesting for some practical questions to determine the amount of positive or negative electricity which still passes after this moment through each cross section of the conductor; and then further to determine the time that has to pass after the same moment until the current intensity i has decreased to $i/2$.

If $i = (a/2)([1/n] \cdot [da_n^0/dt])_0$ is given for that moment $t = 0$, then the current intensity after time t [is given by]

$$= i \cdot e^{-2\varepsilon t} ,$$

which, expressed in mechanical measure, denotes the amount of positive electricity that would pass through the cross section of the conductor in the unit of time for this current intensity. Hence, the amount of positive electricity passing through the cross section of the conductor during the time element, dt, equals

$$= i \cdot e^{-2\varepsilon t} dt ,$$

and the integral value hereof, taken from $t = 0$ to $t = \infty$, yields the total amount of positive electricity which at all passes through any cross section of the conductor after the moment considered, namely

$$i \int_0^\infty e^{-2\varepsilon t} dt = \frac{1}{2\varepsilon} \cdot i .$$

The amount of negative electricity passing through the cross section in opposite direction is equally large.

Furthermore, the following equation yields the time t during which the current intensity decays to half of its value:

$$e^{-2\varepsilon t} = \frac{1}{2} \ ,$$

hence

$$t = \frac{1}{2\varepsilon} \log \text{nat } 2 \ .$$

Now we had $\varepsilon = c^2 w'/[16M''(1+\lambda)]$, wherein we have to put $M'' = 2\log(8a/\alpha)$ for $n = 0$; hence, taking $\lambda = 0$, that amount of electricity passing through the cross section of the conductor equals

$$\frac{1}{2\varepsilon} \cdot i = \frac{16}{c^2 w'} \cdot \log \frac{8a}{\alpha} \cdot i = \frac{2}{W'} \log \frac{8a}{\alpha} \cdot i \ ,$$

when $W' = [c^2/8] \cdot w'$ denotes the resistance in magnetic measure in unit length of the conductor.

The time during which the current intensity decays to half of its value is then, expressed in seconds

$$\frac{1}{2\varepsilon} \cdot \log 2 = \frac{16}{c^2 w'} \cdot \log \frac{8a}{\alpha} \cdot \log 2 = \frac{2}{W'} \cdot \log \frac{8a}{\alpha} \cdot \log 2 \ .$$

Hence we get the following values for the wires exemplified in Section 18.16:

	First wire	Second wire	Third wire
$\frac{1}{2\varepsilon}$	$\frac{1}{104\,607}$	$\frac{1}{60\,726}$	$\frac{1}{567\,581}$
$\frac{\log 2}{2\varepsilon}$	$\frac{1}{150\,916}$	$\frac{1}{87\,609}$	$\frac{1}{818\,846}$.

However small may be here the fraction $1/2\varepsilon$ of the amount which would pass through the cross section of the conductor with the original current, produced by the positive electricity transported by the vanishing current through the cross section in the unit of time of the conductor, yet this amount of electricity could produce a very strong *charge* of the conductor if it were used for this purpose. If, for example, the current intensity originally present were equal to the *magnetic unit of measure* (which decomposes 1 milligram of water during $106\frac{2}{3}$ seconds),[438] then the positive amount of electricity passing through the cross section of the conductor in the unit of time at this current would amount to $155\,370 \cdot 10^6$ *electrostatic units of measure* and, as the current disappeared in the first wire, still $[155\,370/104\,607] \cdot 10^6$, that is almost $1\frac{1}{2}$ million *electrostatic units of measure* of positive electricity would be carried through any cross section of the conductor, that is about the 24th part of the weakest, or the 33rd part of the strongest charge of the small *Leyden jar* which have been used for the

[438][Note by AKTA:] See footnote 146 on page 134.

experiment described in Vol. 5 of the previous *Abhandlung*, where these charges have been determined in more detail on p. 254.[439,440]

It is easily seen that a similar vanishing of the current flowing in a closed conductor occurs at the moment when the circuit of a galvanic current is disrupted and that the positive amount of electricity then carried through the center cross section by the decaying current in fact contributes to *charging* the first half of the conductor, and likewise the amount of negative electricity oppositely carried through the same cross section contributes to charging the second half of the conductor, and that the opposite charges produce the *spark of disruption* at the place where the circuit has been disrupted, where it is interesting to learn about the *amounts of electricity* discharged by the *spark of disruption*.

Likewise the importance is clear to further develop the laws of the current decay for the determination of the inductive forces thus exerted on other conductors, especially for the theory of the *Rühmkorff* type and other similar *inductive machines* which hereby is given its foundation.[441]

18.18 Reference to Heat Conduction

For increasing values of t where eventually e^{-2mt} vanishes in comparison with 1, the two equations found for a_n and b_n in the preceding Section, namely

$$a_n = \frac{1}{2m}\left[\left((\varepsilon+m)a_n^0 + \frac{da_n^0}{dt}\right)e^{-(\varepsilon-m)t} - \left((\varepsilon-m)a_n^0 + \frac{da_n^0}{dt}\right)e^{-(\varepsilon+m)t}\right] ,$$

$$b_n = \frac{1}{2m}\left[\left((\varepsilon+m)b_n^0 + \frac{db_n^0}{dt}\right)e^{-(\varepsilon-m)t} - \left((\varepsilon-m)b_n^0 + \frac{db_n^0}{dt}\right)e^{-(\varepsilon+m)t}\right] ,$$

turn into the simpler equations:

$$a_n = \frac{1}{2m}\left((\varepsilon+m)a_n^0 + \frac{da_n^0}{dt}\right)e^{-(\varepsilon-m)t} ,$$

$$b_n = \frac{1}{2m}\left((\varepsilon+m)b_n^0 + \frac{db_n^0}{dt}\right)e^{-(\varepsilon-m)t} ,$$

and inserting these values of a_n and b_n into the equations

$$E = \sum\left(a_n \sin\frac{ns}{a} + b_n\cos\frac{ns}{a}\right) ,$$

$$i = -\frac{a}{2}\sum\frac{1}{n}\left(\frac{db_n}{dt}\sin\frac{ns}{a} - \frac{da_n}{dt}\cos\frac{ns}{a}\right) ,$$

we get the following distribution laws of the free electricity and the currents in the circular conductor:

[439] [Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 641.

[440] [Note by AKTA:] [KW57, Section 12, p. 641 of Weber's *Werke*] with English translation in [KW21, Section 12, pp. 36-40]. See page 170 of Section 7.12.

[441] [Note by AKTA:] See footnote 392 on page 261.

$$E = \sum \frac{1}{2m} \left[\left((\varepsilon + m) a_n^0 + \frac{da_n^0}{dt} \right) \sin \frac{ns}{a} \right.$$
$$\left. + \left((\varepsilon + m) b_n^0 + \frac{db_n^0}{dt} \right) \cos \frac{ns}{a} \right] e^{-(\varepsilon - m)t} ,$$

$$i = \frac{a}{4} \sum \frac{\varepsilon - m}{mn} \left[\left((\varepsilon + m) b_n^0 + \frac{db_n^0}{dt} \right) \sin \frac{ns}{a} \right.$$
$$\left. - \left((\varepsilon + m) a_n^0 + \frac{da_n^0}{dt} \right) \cos \frac{ns}{a} \right] e^{-(\varepsilon - m)t} .$$

Here it is easily seen that in all cases where $\varepsilon - m/[n^2] = \beta$ is a coefficient independent of n we get

$$i = -\frac{a^2 b}{2} \cdot \frac{dE}{ds} ,$$

$$\frac{di}{ds} = -\frac{1}{2} \frac{dE}{dt} ,$$

whence it follows by eliminating i

$$\frac{dE}{dt} = a^2 \beta \frac{d^2 E}{ds^2} ,$$

an equation having the same form as the equation for the heat conduction in solid bodies.

But in the preceding Section we had put

$$m = \frac{n}{a} \cdot \frac{c}{\sqrt{2}} \sqrt{\frac{a^2 c^2 w'^2}{128 n^2 M''^2 (1+\lambda)^2} - \frac{N''}{M''(1+\lambda)}} ,$$

wherein $c^2 w'/[16 M''(1+\lambda)] = \varepsilon$, thus

$$m = \varepsilon \sqrt{1 - \frac{128 N'' M'' (1+\lambda)}{a^2 c^2 w'^2} \cdot n^2} .$$

Now in all cases where the values of $n^2/[a^2 c^2 w'^2]$ and α/a are very small, we may put instead

$$m = \varepsilon \left(1 - \frac{256 \left(\log \frac{8a}{\alpha} \right)^2 \cdot (1+\lambda)}{a^2 c^2 w'^2} \cdot n^2 \right) ,$$

from which $\varepsilon - m = [8/a^2 w'] n^2 \log(8a/\alpha)$, therefore

$$\beta = \frac{8}{a^2 w'} \cdot \log \frac{8a}{\alpha}$$

is a coefficient independent of n.

Hence for the changes of motion of electricity in the cases just described, this yields laws similar to those for the heat conduction in solids as has already been demonstrated by

Thomson and Kirchhoff.[442] Even if the expression for the propagation velocity of the longer wave trains, that is for smaller values of n, become imaginary and hence demand other laws for this part of the motion which approach the laws of heat conduction in solid bodies, a still remaining part of the motion deserves particular attention which yields shorter wave trains for which greater values of n are valid, for which the expression for the propagation velocity stays real and thus the laws developed in Section 18.13 remain valid. After a disturbance of the equilibrium, thus there are always *wave trains* in such a conductor propagating at certain *velocities*, however, there is no *pure wave motion*, but it is mixed with other motions which are governed by laws similar to those when heat is conducted.

Now considering all conditions resulting from such a mixture of motions subject to quite different laws, it becomes immediately clear that Wheatstone's observation[443] of the *non-synchronicity of the sparks* at two mutually very distant places where the long conducting wire is disrupted does not at all allow to conclude a definite propagation velocity, and that Wheatstone's way of observation, as meaningful it may be and as valuable its results for other contexts may be, if they could really be guaranteed exactly, yet it is not suited directly for the purpose in question, like it will generally be not possible at all to succeed in finding such ways of observation by means of which the laws of all changes of motion of electricity in a conductor after a disturbance of the equilibrium may be founded on *pure experience*. Hence the aim of the *observations* will here be restricted *to test* the laws derived up to now from our previous knowledge about electricity. Therefore it was necessary, as has been tried in the previous Sections, to treat the derivation of the laws before the observations to be carried out for their test, even more because the laws thus formulated have to be used as a *guide* in the search of the *most suitable ways of observation* to be applied for the test.

18.19 Oscillations of Electricity in a Circular Conductor

As regards the *most appropriate ways of observation* to test the laws of electric motions, it becomes automatically clear from the laws developed so far that, considering the extremely high *velocity* of most of the *electric wave trains* in good conductors according to these laws and the quick *damping* of these wave trains resulting from the same laws, with the limits imposed on all observations by the sensory tools, it would barely be possible *to conduct exact observations and measurements for direct testing of these laws*. An exact performance of measurements always demands a certain expenditure of time which is appropriate for such non-persistent phenomena. Considering therefore that the finest measurements of physics are those concerning either *equilibrium phenomena*, or *uniform motions*, or *periodically recurring phenomena*, as for example oscillations of a pendulum, then this suggests to base a testing method also for these laws on observations of the motion of electricity in conductors, apart from *constant currents*, on *periodically recurring phenomena*, supposing that methods will be found for the fine performance of such observations.

But periodically occurring motions of the electricity in a conductor cannot exist all by themselves, but only due to continuous excitation by external electromotive forces and their production suggests the fast rotation of a small magnet about an axis perpendicular to its magnetic axis as the simplest and, for finer observations and measurements, most practi-

[442][Note by AKTA:] See footnote 387 on page 259.
[443][Note by AKTA:] See footnote 138 on page 129.

cable method. In order to obtain a guide to practical equipments for exact observations of periodically occurring motions or *oscillations* of the electricity in a conductor thus produced, we shall first try to develop the laws of such electric oscillations in a circular conductor from the partial differential equations formulated in Section 18.10.

18.20 Oscillations Due to Induction by a Rotating Magnet

The electromotive force exerted by fast rotation of a small magnet in the vicinity of the circular conductor on any point, s, of the conductor *in a certain moment of time* may be represented, if a denotes the radius [of the conductor], by

$$\sum\left(f_m \sin\frac{ns}{a} + g_n \cos\frac{ns}{a}\right) ,$$

where f_n and g_n depend only on the place number, n. But due to the uniform rotation of the magnet, all these forces acting on different points, s, of the conductor are subjected to a regular periodic change, and in effect, with a suitable set up, they are proportional to the sine of an angle uniformly growing with time. For an *arbitrary moment*, all these forces may be represented at the end of time t by

$$\sin\mu t \cdot \sum\left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a}\right) .$$

Inserting $f_n \sin\mu t$ and $g_n \sin\mu t$ into the two partial differential equations at the end of Section 18.10 in place of f_n and g_n which denoted arbitrary functions of the time there, where now f_n and g_n have values independent of time, one gets the following two partial differential equations[444]

$$\frac{d^2a_n}{dt^2} + \frac{c^2}{8\pi\alpha^2kM''(1+\lambda)} \cdot \frac{da_n}{dt} + \frac{n^2c^2N''}{2a^2M''(1+\lambda)} \cdot a_n - \frac{nc^2}{4aM''(1+\lambda)} \cdot g_n \sin\mu t = 0 ,$$

$$\frac{d^2b_n}{dt^2} + \frac{c^2}{8\pi\alpha^2kM''(1+\lambda)} \cdot \frac{db_n}{dt} + \frac{n^2c^2N''}{2a^2M''(1+\lambda)} \cdot b_n + \frac{nc^2}{4aM''(1+\lambda)} \cdot f_n \sin\mu t = 0 .$$

Now one sees easily that, putting

$$a_n = p\sin(\mu t + \rho) ,$$

$$b_n = q\sin(\mu t + \rho) ,$$

[then] p, q and ρ can be determined so that the values of a_n and b_n thus obtained satisfy the two partial differential equations. Inserting the above values, namely a_n and b_n, and the values derived from them,

$$\frac{da_n}{dt} = p\mu\cos(\mu t + \rho) ,$$

[444][Note by PM and AKTA:] The following equations are ordinary differential equations.

$$\frac{db_n}{dt} = q\mu\cos(\mu t + \rho) \ ,$$

$$\frac{d^2 a_n}{dt^2} = -p\mu^2 \sin(\mu t + \rho) \ ,$$

$$\frac{d^2 b_n}{dt^2} = -q\mu^2 \sin(\mu t + \rho) \ ,$$

into the equations above, one obtains, putting $1/[\pi\alpha^2 k] = w'$ for brevity, and either $\lambda = 0$ according to Ohm's law, or M'' for $M''(1+\lambda)$,

$$-p\mu^2 \sin(\mu t + \rho) + \frac{p\mu c^2 w'}{8M''}\cos(\mu t + \rho) + \frac{pn^2 c^2 N''}{2a^2 M''} \cdot \sin(\mu t + \rho) - \frac{c^2 n}{4aM''} \cdot g_n \sin\mu t = 0 \ ,$$

$$-q\mu^2 \sin(\mu t + \rho) + \frac{q\mu c^2 w'}{8M''}\cos(\mu t + \rho) + \frac{qn^2 c^2 N''}{2a^2 M''} \cdot \sin(\mu t + \rho) + \frac{c^2 n}{4aM''} \cdot f_n \sin\mu t = 0 \ .$$

Expanding the sine and cosine of the sum in terms of sine and cosine of the parts, one gets

$$\begin{aligned}&\left(\frac{\mu c^2 w'}{8M''} \cdot p\sin\rho + \left(\mu^2 - \frac{n^2 c^2 N''}{2a^2 M''}\right) \cdot p\cos\rho + \frac{c^2 n}{4aM''} \cdot g_n\right)\sin\mu t \\ &\quad + \left(\left(\mu^2 - \frac{n^2 c^2 N''}{2a^2 M''}\right) \cdot p\sin\rho - \frac{\mu c^2 w'}{8M''} \cdot p\cos\rho\right)\cos\mu t = 0 \ ,\end{aligned}$$

$$\begin{aligned}&\left(\frac{\mu c^2 w'}{8M''} \cdot q\sin\rho + \left(\mu^2 - \frac{n^2 c^2 N''}{2a^2 M''}\right) \cdot q\cos\rho - \frac{c^2 n}{4aM''} \cdot f_n\right)\sin\mu t \\ &\quad + \left(\left(\mu^2 - \frac{n^2 c^2 N''}{2a^2 M''}\right) \cdot q\sin\rho - \frac{\mu c^2 w'}{8M''} \cdot q\cos\rho\right)\cos\mu t = 0 \ .\end{aligned}$$

If these equations are to be valid for any value of t, one obtains for $\cos\mu t = 0$ the two equations

$$\frac{\mu c^2 w'}{8M''} \cdot p\sin\rho + \left(\mu^2 - \frac{n^2 c^2 N''}{2a^2 M''}\right) \cdot p\cos\rho + \frac{c^2 n}{4aM''} \cdot g_n = 0 \ ,$$

$$\frac{\mu c^2 w'}{8M''} \cdot q\sin\rho + \left(\mu^2 - \frac{n^2 c^2 N''}{2a^2 M''}\right) \cdot q\cos\rho - \frac{c^2 n}{4aM''} \cdot f_n = 0 \ ,$$

and for $\sin\mu t = 0$ also the third equation, namely

$$\left(\mu^2 - \frac{n^2 c^2 N''}{2a^2 M''}\right)\sin\rho - \frac{\mu c^2 w'}{8M''} \cdot \cos\rho = 0 \ ,$$

from which p, q and ρ are be determined so that the two partial differential equations are satisfied by the values of a_n and b_n thus determined. One gets in fact

$$\rho = \arctan \frac{\mu a^2 c^2 w'}{4(2\mu^2 a^2 M'' - n^2 c^2 N'')} \ ,$$

$$\begin{aligned} p &= -\frac{ac^2 n}{2(2\mu^2 a^2 M'' - n^2 c^2 N'')} \cdot g_n \cos\rho \\ &= -\frac{2ac^2 n}{\sqrt{16(2\mu^2 a^2 M'' - n^2 c^2 N'')^2 + \mu^2 a^4 c^4 w'^2}} \cdot g_n \ , \end{aligned}$$

$$\begin{aligned} q &= +\frac{ac^2 n}{2(2\mu^2 a^2 M'' - n^2 c^2 N'')} \cdot f_n \cos\rho \\ &= +\frac{2ac^2 n}{\sqrt{16(2\mu^2 a^2 M'' - n^2 c^2 N'')^2 + \mu^2 a^4 c^4 w'^2}} \cdot f_n \ . \end{aligned}$$

Adding the values of a_n and b_n, found in Section 18.13 for the case where $f_n = 0$ and $g_n = 0$, to these special values of a_n and b_n which satisfy the partial differential equations, then the two sums yield the complete integral values of a_n and b_n, namely

$$a_n = p\sin(\mu t + \rho) + Ae^{-\varepsilon t} \cdot \sin\left(mt + \arcsin\frac{a_n^0}{A}\right) \ ,$$

$$b_n = q\sin(\mu t + \rho) + Be^{-\varepsilon t} \cdot \sin\left(mt + \arcsin\frac{b_n^0}{B}\right) \ ,$$

wherein A and B as well as ε and m have the meaning as given in Section 18.13. If m has an imaginary value, then the values of a_n and b_n developed in Section 18.17 replace the added terms. But it becomes clear that the added terms decrease for increasing values of t and that they, as shown in Section 18.17, may be considered as vanishing already after a very small portion of a second has passed, thus from then on the motion of electricity in the circular conductor becomes uniform and periodic, the production of which was the aim of the described method with the rotating magnet.

Omitting the terms vanishing with time and inserting these values of a_n and b_n into the equations

$$E = \sum \left(a_n \sin\frac{ns}{a} + b_n \cos\frac{ns}{a}\right) \ ,$$

$$i = -\frac{a}{2}\sum \frac{1}{n}\left(\frac{db_n}{dt}\sin\frac{ns}{a} - \frac{da_n}{dt}\cos\frac{ns}{a}\right) \ ,$$

yields the following laws of the distribution of free electricity and of the currents in the circular conductor for the regularly ongoing electric oscillation:

$$E = \sum \sin(\mu t + \rho)\left(p\sin\frac{ns}{a} + q\cos\frac{ns}{a}\right) \ ,$$

$$i = -\frac{a\mu}{2}\sum \frac{1}{n}\cos(\mu t + \rho)\left(q\sin\frac{ns}{a} - p\cos\frac{ns}{a}\right) \ ,$$

where p, q and ρ have the above values. From these values, however, we get

$$p = -\frac{2n}{\mu a w'} \sin\rho \cdot g_n \ ,$$

$$q = +\frac{2n}{\mu a w'} \sin\rho \cdot f_n \ .$$

Substituting these values of p and q in both equations, we get

$$E = \frac{2}{\mu a w'} \sum n \sin\rho \sin(\mu t + \rho) \left(f_n \cos\frac{ns}{a} - g_n \sin\frac{ns}{a} \right) \ ,$$

$$i = -\frac{1}{w'} \sum \sin\rho \cos(\mu t + \rho) \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a} \right) \ ,$$

or, expanding $\sin(\mu t + \rho)$ and $\cos(\mu t + \rho)$[445]

$$\begin{aligned} E = & \frac{2}{\mu a w'} \sin\mu \cdot \sum n \sin\rho \cos\rho \left(f_n \cos\frac{ns}{a} - g_n \sin\frac{ns}{a} \right) \\ & + \frac{2}{\mu a w'} \cos\mu t \cdot \sum n \sin^2\rho \left(f_n \cos\frac{ns}{a} - g_n \sin\frac{ns}{a} \right) \ , \end{aligned}$$

$$\begin{aligned} i = & \frac{1}{w'} \sin\mu t \cdot \sum \sin^2\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a} \right) \\ & - \frac{1}{w'} \cos\mu t \cdot \sum \sin\rho \cos\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a} \right) \ . \end{aligned}$$

Finally, herein putting

$$\frac{\sum \sin^2\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a} \right)}{\sum \sin\rho \cos\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a} \right)} = \tan\gamma \ ,$$

$$\frac{\sum n \sin^2\rho \left(f_n \cos\frac{ns}{a} - g_n \sin\frac{ns}{a} \right)}{\sum n \sin\rho \cos\rho \left(f_n \cos\frac{ns}{a} - g_n \sin\frac{ns}{a} \right)} = \tan\gamma' \ ,$$

$$\begin{aligned} & \left(\sum \sin^2\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a} \right) \right)^2 \\ + & \left(\sum \sin\rho \cos\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a} \right) \right)^2 = k^2 \ , \end{aligned}$$

$$\begin{aligned} & \left(\sum n \sin^2\rho \left(f_n \cos\frac{ns}{a} - g_n \sin\frac{ns}{a} \right) \right)^2 \\ + & \left(\sum n \sin\rho \cos\rho \left(f_n \cos\frac{ns}{a} - g_n \sin\frac{ns}{a} \right) \right)^2 = k'^2 \ , \end{aligned}$$

we get

$$E = \frac{2}{\mu a w'} \cdot k' \sin(\mu t + \gamma') \ ,$$

[445][Note by AKTA:] In the next equations I replaced Weber's notation $\sin\rho^2$ by $\sin^2\rho$.

$$i = -\frac{1}{w'} \cdot k' \cos(\mu t + \gamma) \ .$$

But putting

$$\sum \sin^2 \rho \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a} \right) = f \ ,$$

$$\sum \sin \rho \cos \rho \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a} \right) = g \ ,$$

$$\frac{df}{ds} = f' \ ,$$

$$\frac{dg}{ds} = g' \ ,$$

we get

$$E = \frac{2}{\mu w'} \sqrt{f'^2 + g'^2} \cdot \sin \left(\mu t + \arctan \frac{f'}{g'} \right) \ ,$$

$$i = -\frac{1}{w'} \sqrt{f^2 + g^2} \cdot \cos \left(\mu t + \arctan \frac{f}{g} \right) \ ,$$

whence it is easy to derive the equation

$$\frac{di}{ds} = -\frac{1}{2} \frac{dE}{dt} \ .$$

18.21 Equality of Phases and Amplitudes of Electric Oscillations in Circular Conductors

Considering that the electromotive force exerted on the whole conducting wire by the rotating magnet is represented by

$$\sin \mu t \cdot \int ds \cdot \sum \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a} \right) \ ,$$

and that g_0 must have a specific finite value if this whole force should not be zero, then the value found for i can be presented more clearly if, in the given values of $\tan \gamma$ and k^2, the first terms of the series, namely the terms corresponding to the place number $n = 0$, are separated in the following way, denoting the value of ρ for $n = 0$ by ρ_0:

$$\tan \gamma = \frac{g_0 \sin^2 \rho_0 + \sum_1^\infty \sin^2 \rho \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a} \right)}{g_0 \sin \rho_0 \cos \rho_0 + \sum_1^\infty \sin \rho \cos \rho \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a} \right)} \ ,$$

$$k^2 = g_0^2 \sin^2\rho_0 + 2g_0 \sin\rho_0 \cos\rho_0 \cdot \sum_1^\infty \sin\rho\cos\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a}\right)$$
$$+ 2g_0 \sin^2\rho_0 \cdot \sum_1^\infty \sin^2\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a}\right)$$
$$+ \left(\sum_1^\infty \sin\rho\cos\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a}\right)\right)^2$$
$$+ \left(\sum_1^\infty \sin^2\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a}\right)\right)^2 .$$

As now herein we had

$$\sin\rho = \frac{\mu a^2 c^2 w'}{\sqrt{16(2\mu^2 a^2 M'' - n^2 c^2 N'')^2 + \mu^2 a^4 c^4 w'^2}} ,$$
$$\cos\rho = \frac{4(2\mu^2 a^2 M'' - n^2 c^2 N'')}{\sqrt{16(2\mu^2 a^2 M'' - n^2 c^2 N'')^2 + \mu^2 a^4 c^4 w'^2}} ,$$

and, denoting the value of M'' for $n = 0$ by M''_0, we get the values of $\sin\rho_0$ and $\cos\rho_0$ [as given by]

$$\sin\rho_0 = \frac{c^2 w'}{\sqrt{64\mu^2 M''^2_0 + c^4 w'^2}} ,$$
$$\cos\rho_0 = \frac{8\mu M''_0}{\sqrt{64\mu^2 M''^2_0 + c^4 w'^2}} .$$

Considering furthermore that the ratios $\mu a^2 w'/N''$ and $\mu a/c$ are very small fractions also for very long and thin conductors and for the greatest accessible velocity of rotation of the small magnet, then it is clear that we may put with sufficient approximation for all values $n > 0$

$$\sin\rho = \frac{\mu a^2 w'}{4n^2 N''} ,$$

$$\cos\rho = 1 .$$

Hence it is clear that already $\mu a^2 w'/N''$ being a very small fraction, $\sin\rho = \mu a^2 w'/[4n^2 N'']$ all the more may be considered as vanishingly small the larger the place number n. Therefore also for very long and thin conductors and for very rapid rotation of the small magnet we may approximately assume

$$\gamma = \rho_0 \qquad \text{and} \qquad k = g_0 \sin\rho_0 ,$$

whence we find

$$i = -\frac{g_0}{w'} \sin\rho_0 \cos(\mu t + \rho_0) .$$

As g_0/w' and ρ_0 have values independent of s, it follows that the electric oscillations have equal *phase* and oscillation *amplitude* in all parts of a circular conductor, even if the electromotive forces exerted by the rotating magnet are very unevenly distributed along the different parts of the conductor.

From the evenness of oscillation phases and amplitudes in all parts of the circular conductor, it follows that the current intensity at any point always equals the average current intensity in the whole conductor. But we had derived the law for the averages of the current intensities in closed conductors already in Section 18.9 where, denoting the average of the external electromotive force by $[1/2\pi a] \cdot S$ and putting

$$\frac{8}{c^2}\int M_0'' ds + \frac{4\pi a}{r\mathfrak{E}} = p \ ,$$

$$\frac{2\pi a}{\pi\alpha^2 k} = w = 2\pi a w' \ ,$$

we had the result

$$i = \frac{1}{p}e^{-wt/p} \cdot \int e^{wt/p} \cdot S dt \ .$$

Now applying this law to our case where the oscillations in a conductor are produced by a rotating magnet and where the average of the electromotive forces exerted by the rotating magnet on the conductor was equal to

$$\frac{1}{2\pi a} \cdot S = g_0 \sin \mu t \ ,$$

we get

$$\begin{aligned} i &= \frac{2\pi a g_0}{p}e^{-wt/p} \cdot \int e^{wt/p} \cdot \sin\mu t \cdot dt = \frac{2\pi a g_0}{p} \cdot \frac{\frac{w}{p}\sin\mu t - \mu\cos\mu t}{\frac{w^2}{p^2} + \mu^2} \\ &= -\frac{2\pi a g_0}{\mu p\sqrt{\left(\frac{w}{\mu p}\right)^2 + 1}} \cdot \cos\left(\mu t + \arctan\frac{w}{\mu p}\right) \ . \end{aligned}$$

As now

$$p = \frac{8}{c^2} \cdot \int M_0'' ds + \frac{4\pi a}{r\mathfrak{E}} = \frac{8}{c^2} \cdot \int M_0''(1+\lambda) ds \ ,$$

and

$$w = 2\pi a w' \ ,$$

one gets, putting M_0'' instead of $M_0''(1+\lambda)$ for simplification as in Section 18.20,

$$\frac{w}{\mu p} = \frac{\pi a c^2 w'}{4\mu\int M_0'' ds} = \tan\rho_0 \ ,$$

$$\frac{w}{\mu p\sqrt{\left(\frac{w}{\mu p}\right)^2 + 1}} = \frac{2\pi a w'}{\mu p\sqrt{\left(\frac{w}{\mu p}\right)^2 + 1}} = \sin\rho_0 \ ,$$

hence agreeing with the above result found for *circular* conductors

$$i = -\frac{g_0 \sin \rho_0}{w'} \cdot \cos(\mu t + \rho_0) \ .$$

As the above law for the averages of the current intensities in closed conductors depending on the averages of the electromotive forces in Section 18.9 was found not to be restricted to *circular* conductors only, but also to be independent of the consideration of the shape of the closed conductor, this yields that the law, derived for the case where the electromotive forces originating from a rotating magnet are given, equally holds for closed conductors of any shape.

The presented result that the *phases* and *amplitudes* of electric oscillations in circular conductors be equal everywhere, is based on the assumption that the ratios $\mu a^2 w'/N''$ and $\mu a/c$ are very small fractions. As now these fractions increase with the length and the fineness of the conductor and with the rotational velocity of the magnet, it is interesting to calculate their actual values for some examples of long and thin wires at great rotational velocities. Choosing the three conducting wires already exemplified in Section 18.16, we get the values presented in the following Table.

	First wire	Second wire	Third wire
a	1000	1 000 000	1 000 000
w'	$\frac{1}{120\,697 \cdot 10^{12}}$	$\frac{1}{120\,697 \cdot 10^{12}}$	$\frac{1}{12\,070 \cdot 10^{12}}$
N'' (for $n = 1$)	15.119	28.935	31.237
$100 \frac{a^2 w'}{N''}$	$\frac{1}{18\,248 \cdot 10^6}$	$\frac{1}{34\,939}$	$\frac{1}{3770}$
$100 \frac{a}{c}$	$\frac{1}{4\,394\,500}$	$\frac{1}{4394}$	$\frac{1}{4394}$

The two bottom rows of this Table contain the values of the two ratios for the three exemplified wires if $\mu = 100$, that is at 15.965 turns of the magnet per second. We see that in all these cases the values of these ratios are very small fractions, while we also see that, as these values may be 10 times larger at 159.65 turns per second and 100 times larger at 1596.5 turns per second, indeed cases may occur where these ratios become considerably large quantities and where hence the law of equality of the *phases* and *amplitudes* in the conductor would not hold any more.

18.22 Distribution of the Free Electricity in a Circular Conductor During the Electric Oscillation

The law of the distribution of the free electricity in a circular conductor during the electric oscillation is contained in the expression for the [linear] density, E, found in Section 18.20, namely

$$E = \frac{2}{\mu a w'} \cdot k' \sin(\mu t + \gamma') \ ,$$

where the coefficient k' was determined by the equation

$$k'^2 = \left(\sum n \sin^2 \rho \left(f_n \cos \frac{ns}{a} - g_n \sin \frac{ns}{a}\right)\right)^2 + \left(\sum n \sin\rho \cos\rho \left(f_n \cos \frac{ns}{a} - g_n \sin \frac{ns}{a}\right)\right)^2 .$$

Hence we see that also the amount of the charge with free electricity in each point of the circular conductor changes in proportion to the sine of an arc increasing in proportion to t, but that the maximum charge $= 2k'/[\mu a w']$ which takes place when the sine $= 1$, is different in different points of the conductor and that in fact the change from element to element is greatest approximately in those points where the electromotive force exerted by the rotating magnet deviates most from its average; where this electromotive force equals its average, also the charge is approximately constant, in fact it equals zero. Hence in the whole conductor there would be no free electricity anywhere if the rotating magnet acted equally on all points of it, where it is assumed that the circular conductor would have no charge from free electricity independent of the rotating magnet.

Since $\sin\rho$ and $\cos\rho$ retain finite values for $n = 0$ according to the preceding Section, it is clear that the above value for k'^2 can be written as

$$k'^2 = \left(\sum_1^\infty \sin^2 \rho \left(f_n \cos \frac{ns}{a} - g_n \sin \frac{ns}{a}\right)\right)^2 + \left(\sum_1^\infty n \sin\rho \cos\rho \left(f_n \cos \frac{ns}{a} - g_n \sin \frac{ns}{a}\right)\right)^2 .$$

Furthermore, putting

$$\sin\rho = \frac{\mu a^2 w'}{4n^2 N''} ,$$

$$\cos\rho = 1 ,$$

under the assumptions made in the previous Section and when the value of $\sin\rho$ is very small, the first part of k'^2, containing the factor $\sin^2\rho$ in the sum, may be neglected compared to the second term, whence we thus get

$$k' = \frac{\mu a^2 w'}{4} \sum_1^\infty \frac{1}{nN''} \left(f_n \cos \frac{ns}{a} - g_n \sin \frac{ns}{a}\right) .$$

This now yields

$$\frac{dk'}{ds} = -\frac{\mu a w'}{4} \sum_1^\infty \frac{1}{N''} \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a}\right) .$$

If finally $\log[8a/\alpha]$ is a very large number and if furthermore the series

$$\sum \left(f_n \sin \frac{ns}{a} + g_n \cos \frac{ns}{a}\right)$$

converges so fast that all terms of the series may be neglected for $n > \nu$, while

$$2\left[1+\frac{1}{3}+...+\frac{1}{2\nu-1}\right]+\frac{\nu^2\alpha}{8a}$$

vanishes in comparison with $\log[8a/\alpha]$, then we may put $N'' = 2\log[8a/\alpha]$ and

$$\frac{dk'}{ds} = -\frac{\mu a w'}{8\log\frac{8a}{\alpha}}\left(\sum_0^\nu\left(f_n\sin\frac{ns}{a}+g_n\cos\frac{ns}{a}\right)-g_0\right) .$$

Now the factor

$$\sum_0^\nu\left(f_n\sin\frac{ns}{a}+g_n\cos\frac{ns}{a}\right)-g_0$$

equals the difference between the electromotive force exerted at the point s by the rotating magnet and its average along the whole length of the conductor; hence dk'/ds or the change of k' with respect to the change of s is proportional to this difference.

As is easily seen, the discharge of sparks[446] and the necessary degree of *insulation* of the conductor depend on the amount of these charges, if the flash over is to be avoided, a topic to be treated in more detail only when the conductors in question are not just circular but constitute a system of closely spaced windings, a case that has been excluded here.

18.23 Guide to the Observations

It remains to use the results of the previous development as a guide to the *observations* by means of which those results shall be checked with experience. Such a guide is particularly necessary if there are no analogies of other motion phenomena which may be used for this purpose, and from what was previously said it follows that here such analogies are missing in many respects.

In the absence of analogies with other already known and investigated motion phenomena, above all the question is to know the determination of *objects of observation*, which are particularly important and suited for *more detailed determination by observations*. Furthermore the closer knowledge of the *conditions* is required under which the most exact determinations can be obtained via these objects of observation. Now it is clear that the more detailed discussion of these *conditions* is best combined with the discussion of the *tools* for its effective representation and with the very *execution of the observations*, which will be the subject of the following Section of this treatise. Therefore, at the end of this Section the *objects of observation* shall be only briefly indicated which, according to the preceding development, seem to be particularly *important and suited for a more detailed determination by observations*.

The *velocity of propagation* which is so important for other motion phenomena, seemingly has not to be included here, as already mentioned in Section 18.18, but instead other different topics are available for the observation.

According to the developed laws, there are essentially *three* topics which prove to be particularly suited as topics of observation to test the formulated laws, namely *first* the comparison of the phases and the amplitudes of the oscillations of the electricity at various

[446][Note by AKTA:] In German: *Das Ueberspringen elektrischer Funken.*

places of a long closed conductor on which a rotating magnet acts inductively; *second* the law of the dependence of the oscillation amplitude on the rotation velocity of the magnet; finally *third* another important topic suggests the observations of the *dependence of the oscillation amplitude* produced by a rotating magnet in a closed conducting wire on the *shape* of this wire.

The *equality of the phase and of the amplitude of the oscillation* which, according to the formulated laws, should occur in all parts even in a very long closed circuit and at a high velocity of rotation, is a topic all the more suited for an experimental test the more unexpected this result would seem. For without a more detailed definition of the conditions it would be expected in a very long circuit, where all motions start in one place and are subject to a very strong damping or absorption while propagating, that all motions would arrive only very faint at the most distant parts of the circuit even if the oscillations were continuously excited. As further the propagation from the place of excitation occurs in both directions, one would expect that the encounter of interchanging positive and negative oscillations from opposite sides would result in *amplification* at some places, and in *cancellation* at other places, as with interference phenomena. Finally, even if oscillations, due to such an encounter which are perfectly synchronous in all parts of the circuit, were *possible*, one would indeed expect that this *possible* case were tied to special conditions, for instance certain velocities of rotation, but not that such synchronous oscillations would arise in all parts of the circuit *at any velocity of rotation*. Hence the presented result is highly unexpected as regards all analogies offered by the propagation of motion in other known cases and therefore is especially suited for an experimental test of the results of the theory built on our hitherto existing knowledge about electricity.

Further, the *dependence of the oscillation amplitude on the velocity of rotation* of the magnet is suitable from a different perspective, namely, the *quantitative test* of the formulated law by observations and measurements which are arranged in a sequence of increasing velocity of rotation.

Finally, if one succeeded in addition to gain more detailed determinations of the *dependence of the oscillation amplitude on the shape of the circuit* by exact observations and measurements, one would not only obtain a new test of the formulated laws, but also an essential supplement of our knowledge of the very electricity from which these laws were derived. From our hitherto existing knowledge, the electricity as a body must be attributed with a *mass*, and this mass exerts a *force* on another similar mass; still missing is the knowledge of the *ratio* of that mass to this force. Now the knowledge of this ratio was also not required as long as we dealt with *equilibrium phenomena* or with *steady motions*, where it was sufficient to know the *forces*; here the different amounts of electricity could be distinguished according to the strength of the *forces* they exert on the *same* amount of electricity at the unit distance, instead of their *masses*, and the latter amount of electricity could be determined by means of the *force* it exerts on an *identical* amount of electricity at the unit distance. Indeed such a specific quantity of electricity thus determined was the so called *electrostatic unit of measure*. If we do not deal with a mere *equilibrium* or a mere *maintenance of an already existent motion* but, instead, if an amount of electricity is to receive a *new motion* not existing before, then the pure knowledge of the forces is not sufficient but also the knowledge of the *mass* of the electricity to be set in motion is required, or of the ratio of this mass and of the force it exerts on the electrostatic unit of measure at the unit distance, that is the knowledge of the *number of electrostatic units of measure*, *contained* *in the unit of mass* (milligram) *of electricity*. Above,[447] this number has been denoted by r and hence the *mass* of each specific amount of electricity, $\mathfrak{E}$, determined in *electrostatic measure*, is thus found equal to $[1/r] \cdot \mathfrak{E}$. Now if any *force*, f, acts on this *mass*, it is clear that the *ratio* of this force through the mass $[1/r] \cdot \mathfrak{E}$ on which it acts, yields the *velocity* of the motion imparted by the force to this mass per unit time, $= fr/\mathfrak{E}$.[448]

Now our knowledge of the existing amounts of electricity in *electrostatic units of measure* is indeed limited to the *free* amounts of electricity contained in the bodies obtained by the observations and does not cover the amounts of electricity contained in the *neutral* fluid. Likewise, our knowledge of the forces, f, is limited to those acting on *free* amounts of electricity, while the observations yield only the knowledge of the coefficient, f', denoted by the name *electromotive force*, which has to be multiplied by the unknown *number of electrostatic units of measure contained in the neutral fluid*, $\mathfrak{E}$, in order to obtain $f = f' \cdot \mathfrak{E}$. On the other hand, in the whole electrodynamics we do not have to investigate the *velocity* itself, but only the *current density* and its changes, that is, the product of the *number of electrostatic units of measure*, $\mathfrak{E}$, contained in the flowing electricity, and that *velocity* $rf/\mathfrak{E}$, that is, $rf = f' \cdot r\mathfrak{E}$, where the *electromotive force*, f', is already known, hence where only the product $r\mathfrak{E}$ remains to be determined.

If hence, in agreement with the previous development of the determination of the *current densities* and of their changes, we do not need the very knowledge of the number of electrostatic units of measure, r, contained in the unit of mass (milligram), but only the knowledge of the product $r\mathfrak{E}$, it is clear that, on the other hand, *from the observation of the current densities* and their changes, also only the knowledge of the product $r\mathfrak{E}$ can be obtained; but the importance of the knowledge of $r\mathfrak{E}$ is immediately clear, the tentative to know it by means of detailed and exact observations on the *dependence of the oscillation amplitude on the shape of the circuit*, according to the guide based on the derived laws, turns out to be the most appropriate.

To this end, the most exact knowledge of the *conditions* under which definite statements about this dependence can be obtained is necessary, the discussion of which, as already mentioned, shall be combined with the discussion of the tools for the practical presentation in the following Sections of this treatise.

[447][Note by AKTA:] See page 288 on Section 18.5.

[448][Note by AKTA:] That is, $fr/\mathfrak{E}$ represents the acceleration acquired by this mass $[1/r] \cdot \mathfrak{E}$ through the action of the force f.

II - Observations of Oscillations

18.24 Method of Observation

According to the guide for the observations presented in the previous Section mainly the *differences of the phases and amplitudes of electric oscillations* shall be observed and measured in closed conductors. But no *galvanometers* are suited for these observations and measurements, as for the observation and measurement of the *intensities of steady currents*. For if an *electric oscillation* is present in the *multiplier* of a galvanometer, instead of a steady current, the needle of the galvanometer cannot stay at rest and in equilibrium, but must also perform oscillations that become the weaker the smaller the fraction of the period of electric oscillation compared with the period of oscillation of the magnetometer needle; if these oscillations were becoming vanishingly weak, the galvanometer needle would exactly behave as if there were no electric oscillation in the multiplier, it would stay fixed in the equilibrium position without any deflection, so that nothing could be determined. Therefore, the observation of *electric oscillations* and in particular the measurement of its differences of amplitudes and phases demand that the closed conductor in which the oscillations take place constitutes not only a multiplier as part of a galvanometer, but also a solenoid supported as a torsion balance which, together with the *multiplier*, makes an *electrodynamometer*, whose construction has already been described in the first treatise on *Electrodynamic Measurements* (*Abhandlung bei Begründung der Königl. Sächs. Gesellschaft der Wissenschaften*, Leipzig 1846)[449,450] and the use of which for the observation of *electric oscillations* has been discussed in general and analyzed with an example at this very place.

Now the method how to determine the *differences of phases* and *amplitudes of electric oscillations* in closed conductors from observations by means of a dynamometer shall be considered more closely here, where it may be assumed for simplification that the multiplier formed by the conductor carrying the electric oscillations be formed by a vertical ring having a quite large diameter, similar to that of a tangent galvanometer,[451] with the *solenoid* formed by the same conductor concentrated in a volume as small as possible supported in its center, replacing the pivoted *needle* of the galvanometer there.

There is a fundamental difference between this *needle* and the *solenoid* formed by the conductor that carries the oscillation, [namely,] that the *needle* has a *constant magnetic moment* on which the steady current in the multiplier acts, while the *solenoid* has a *galvanic moment* which, according to Ampère's law, indeed would be completely *equivalent* to the magnetic moment of the needle, which, however, would not be constant for an *electric oscillation* in the solenoid, but varies with the phase of the electric oscillation. Furthermore, it is not a steady current from the multiplier that acts on this *variable galvanic moment* of the solenoid, but the *electric oscillation* existing in the multiplier, whose action on the solenoid is also *variable* with the phase of the oscillation.

Let a and n be the average radius and the number of windings, respectively, of the *multiplier* and likewise a' and n' for the *solenoid*, further i and i' the current intensities in multiplier and solenoid, respectively, expressed in *absolute magnetic measure* as used in galvanometry, whence $[c/\sqrt{8}]\cdot idt$ and $[c/\sqrt{8}]\cdot i'dt$ represent the amount of positive electricity

[449][Note by HW:] Wilhelm Weber's *Werke*, Vol. III, pp. 35 and 123.

[450][Note by AKTA:] [Web46, Sections 1 and 16, pp. 35 and 123 of Weber's *Werke*] with partial French translation in [Web87] and a complete English translation in [Web07].

[451][Note by AKTA:] See footnote 12 on page 22.

passing through the cross section of the conductors during the time element dt; then $n\pi a^2 i$ and $n'\pi a'^2 i'$ represent the *galvanic moments* of the multiplier and the solenoid. Twice the product of these two galvanic moments divided by the third power of the distance a between the solenoid concentrated at the center and the ring of the multiplier yields the *directive force*[452] exerted on the solenoid by the multiplier which, multiplied by the sine of the angle formed between the solenoid axis and the multiplier axis, or, equivalently, multiplied by the cosine of the deflection angle φ between the solenoid axis and the plane of the multiplier ring, yields the *torque*[453] exerted by the multiplier on the solenoid, namely

$$= 2\frac{n\pi a^2 i \cdot n'\pi a'^2 i'}{a^3} \cdot \cos\varphi \ .$$

With this complete analogy between the theory of the *electrodynamometer* and that of the *galvanometer*, no further explication is needed, but instead, considering how to use the instrument, we can immediately pass over to the case when the conducting wire which includes multiplier and solenoid carries *electric oscillations*, where thus the *current intensities* i and i' vary with the sine of an angle, which increase in proportion with time t.

If in this case i and i' represent the maximum current intensities corresponding to the maximum values of the sine, then the current intensities can be represented by $i\sin(\mu t+\gamma)$ and $i'\sin(\mu t+\gamma')$ for any moment at the end of the time t. If $\mathfrak{E}$ denotes the amount of positive electricity contained in the unit length of the conductor, the distance between an oscillating particle in the multiplier or solenoid from its equilibrium position during this oscillation for this moment will be represented by $[i/\mu\mathfrak{E}]\cos(\mu t+\gamma)$ and $[i'/\mu\mathfrak{E}]\cos(\mu t+\gamma')$, where $i/[\mu\mathfrak{E}]$ and $i'/[\mu\mathfrak{E}]$ is the *amplitude of oscillation* to be determined here. — As, however, with currents one renounces the knowledge of the drift velocity itself,[454] being satisfied with the product of this drift velocity and the unknown factor $\mathfrak{E}$, similarly one contents oneself here with the determination of the product of this oscillation amplitude and this very factor $\mathfrak{E}$, because the observations will only permit us to express this product in *absolute measure*.

In this case, with these new specifications of the current intensities, one now obtains the torque exerted by the multiplier on the solenoid:

$$= 2\frac{n\pi a^2 i \cdot n'\pi a'^2 i'}{a^3} \sin(\mu t+\gamma)\sin(\mu t+\gamma') \cdot \cos\varphi \ ,$$

wherein i and i' have constant values independent of the time t.

Under the influence of this torque whose magnitude changes incessantly with time t, the mobile solenoid clearly cannot get to rest at all; therefore the question arises what kind of *observations* can be performed with this incessant motion of the solenoid and what can be determined from these observations. In order to answer this question *we have to develop the laws of motion of the solenoid under the influence of such a variable torque*.

To simplify this development we may at first stick to the case where the *current intensities in the multiplier and the solenoid are always the same*, where hence we may put

$$i = i' \qquad \text{and} \qquad \gamma = \gamma' = 0 \ .$$

This case yields the variable torque acting on the solenoid equal to

[452][Note by AKTA:] See footnote 66 on page 77.

[453][Note by AKTA:] See footnote 10 on page 18.

[454][Note by AKTA:] See footnote 52 on page 61. Weber is referring here to the velocity of the electrified particles relative to the matter of the conductor.

$$= 2\frac{\pi^2 nn' a'^2}{a} \cdot i^2 (\sin \mu t)^2 \cos \varphi \ ,$$

which may be written as

$$= \frac{\pi^2 nn' a'^2}{a} \cdot i^2 (1 - \cos 2\mu t) \cos \varphi \ .$$

But from the construction of the *electrodynamometer* we know that the solenoid is supported in a *bifilar* fashion, whence with the given length and distance of the two supporting wires there is a *static directive force* which can easily be determined and shall be denoted by S. If now this bifilar suspension of the solenoid is *normally* regulated, so that the torque resulting from the *static directive force* equals zero, when the axis of the solenoid is parallel to the plane of the ring of the multiplier, or when the *angle of deflection* $\varphi = 0$, then we get the *static torque* acting on the solenoid for any value of the angle φ equal to

$$= -S \sin \varphi \ .$$

Adding this *static* torque to the above *electrodynamic* torque, the sum of both torques acting on the solenoid divided by the moment of inertia K of the solenoid, yields the rotational acceleration of the solenoid, $d^2\varphi/dt^2$, at the end of the time t, whence follows the *equation of motion of the solenoid*, namely

$$\frac{\pi^2 nn' a'^2}{a} \cdot i^2 (1 - \cos 2\mu t) \cos \varphi - S \sin \varphi = K \frac{d^2 \varphi}{dt^2} \ .$$

Putting herein

$$\varphi = v + \alpha \ ,$$

in which one assumes the *constant* value for v determined by the following equation

$$\tan v = \frac{\pi^2 nn' a'^2}{aS} \cdot i^2 \ ,$$

hence $d^2\varphi/dt^2 = d^2\alpha/dt^2$, so that one gets

$$\frac{d^2\alpha}{dt^2} + \frac{S}{K}\left[(1 + (1 - \cos 2\mu t) \tan^2 v) \cos v \sin \alpha + \cos 2\mu t \cdot \sin v \cos \alpha\right] = 0 \ .$$

Under the assumption that v and α have small values (which, as a rule, is possible because the solenoid, equipped with a mirror, is to be observed quite like a magnetic needle, where the deflection of the solenoid shall always stay within narrow limits given by the length of the scale), we can write

$$\frac{d^2\alpha}{dt^2} + \frac{S}{K}(\alpha \sec v + \cos 2\mu t \cdot \sin v) = 0 \ ,$$

whence we get by integration:

$$\alpha = \frac{\sin v}{4\mu^2 \frac{K}{S} - \sec v} \cdot \cos 2\mu t + A \sin(t - B)\sqrt{\frac{S \sec v}{K}} \ ,$$

with A and B the two constants of integration. Now denoting by τ the period of oscillation of the solenoid,[455] which corresponds to the directive force S and the moment of inertia K, and by ϑ the period of oscillation of the electricity in the conducting wire, we get

$$\frac{K}{S} = \frac{\tau^2}{\pi^2} \qquad \text{and} \qquad \mu = \frac{\pi}{\vartheta} \ ;$$

consequently

$$\alpha = \frac{\sin v}{4\frac{\tau^2}{\vartheta^2} - \sec v} \cdot \cos\frac{2\pi}{\vartheta}t + A \sin\frac{\pi}{\tau}(t - B)\sqrt{\sec v} \ ,$$

or, for the assumed small value of v and putting $A = 0$, that means apart from that small oscillation which would be performed by the solenoid if solely under the influence of the static directive force S and the electrodynamic [directive force] $nn'\pi^2 a'^2 i^2/a$ (as this oscillation is easily suppressed during the observation by standard damping devices), it follows

$$\alpha = \frac{\vartheta^2}{4\tau^2 - \vartheta^2} \cdot v \cos\frac{2\pi}{\vartheta}t \ .$$

As an example we choose the case which will occur in the following observations where we had, expressed in seconds

$$\tau = 15 \ , \qquad \vartheta = \frac{1}{520} \ ,$$

which hence yields

$$\alpha = \frac{1}{243 \cdot 10^6} \cdot v \cos\frac{2\pi}{\vartheta}t \ ,$$

that means where α vanishes completely compared to v. The same is valid for all observations to be treated here.

When α vanishes, the *constant deflection of the solenoid* v can now be observed directly with utmost precision and one finds

$$i = \frac{1}{\pi a'}\sqrt{\frac{aS \tan v}{nn'}} \ ,$$

whence the electric oscillation in the closed conductor is determined completely if the period of oscillation, ϑ, were known from counting the revolutions of the rotating magnet, namely

$$i \sin\frac{\pi}{\vartheta}t = \frac{\sin\frac{\pi}{\vartheta}t}{\pi a'} \cdot \sqrt{\frac{aS \tan v}{nn'}} \ .$$

If, instead of the electric oscillation, there were a *constant current* having intensity $i\sqrt{\frac{1}{2}}$, then the torque exerted by the multiplier on the solenoid would be

$$= \frac{\pi^2 nn' a'^2}{a} i^2 \cos\varphi \ ,$$

and this torque, together with the static torque, $-S\sin\varphi$, would amount to zero for equilibrium, whence the *deflection* φ *of the solenoid for equilibrium* would be given by

[455][Note by AKTA:] In German: *Die Schwingungsdauer des Solenoids*. See also footnote 199 on page 164.

$$\varphi = v \ .$$

Therefore the result from the above consideration can be expressed as follows:

> *If the period of oscillation of the electricity in the closed conductor makes a very small fraction of the static period of oscillation of the solenoid, the solenoid behaves as if there were a constant current in the conductor whose intensity is to the maximum intensity, i, of the currents due to the electric oscillation as* $1 : \sqrt{2}$.

There is then a deflection of the solenoid which can be observed similarly as if there were a *constant current* in the closed conductor and if from this observed deflection (according to the same law as with galvanometers) the intensity of the *constant current* is calculated which could cause it, then this intensity has only to be multiplied by $\sqrt{2}$ in order to obtain the *maximum intensity*, i, of the currents occurring in the electric oscillation, or multiplied by $c/[2\mu\mathfrak{E}]$ in order to obtain the *amplitude* of the electric oscillation in the closed conductor, whereby, however, as already remarked, $\mathfrak{E}$ must be left indefinite as an unknown coefficient and where only $ci/[2\mu]$ can be expressed in *absolute units of measure*. Hereby the problem is solved how *to observe and to determine with an electrodynamometer the electric oscillation caused in a closed conductor by a magnet rotating at a known speed.*

The solution of the problem, however, has here been limited to the case where the multiplier and solenoid belong to adjacent parts of the closed conductor where there is no noticeable difference of the oscillation amplitude and the oscillation phase of the electricity. If the multiplier and solenoid belonged to two parts of the closed conductor where the period of oscillation of the electricity were indeed the same, but where the current maxima i and i', as well as the phases of the oscillation λ and λ', would have to be distinguished; then the starting point of the time t may always be chosen so that the arithmetic mean $(\lambda + \lambda')/2$ of both phases of the oscillation is equal to zero. Then the current intensities related to the electric oscillation can be represented in these two parts of the closed conductor by

$$i \sin \frac{\pi}{\vartheta}(t + \lambda) \qquad \text{and} \qquad i' \sin \frac{\pi}{\vartheta}(t - \lambda) \ .$$

One then observes *first* the deflection of the solenoid, v, when both multiplier and solenoid belong to the first part of the closed conductor. According to the established rules, the maximum intensity, i, of the currents of the electric oscillation in this part can be determined from this observation, namely

$$i = \frac{1}{\pi a'} \sqrt{\frac{aS \tan v}{nn'}} \ .$$

Second, one observes the deflection of the solenoid, v', when both multiplier and solenoid belong to the other part of the closed conductor and finds the maximum intensity, i', of the currents of the existing electric oscillation in this part

$$i' = \frac{1}{\pi a'} \sqrt{\frac{aS \tan v'}{nn'}} \ .$$

Third finally, one observes the deflection of the solenoid, v'', when the multiplier belongs to the first part and the solenoid to the latter part of the closed conductor. Then from the third observation also the *difference of the phases of the oscillation*, 2λ, in both parts of the closed conductor can be determined. According to the previous details we then have the *equation of motion of the solenoid*, namely:

$$2\frac{\pi^2 nn'a'^2}{a}\cdot ii'\sin(\mu t+\lambda)\sin(\mu t-\lambda)\cdot\cos\varphi - S\sin\varphi = K\frac{d^2\varphi}{dt^2}\ ,$$

for which, as

$$\sin(\mu t+\lambda)\sin(\mu t-\lambda)=\sin^2\mu t-\sin^2\lambda=\frac{1}{2}\left(1-\cos 2\mu t-2\sin^2\lambda\right)$$

holds, we can write:

$$\frac{\pi^2 nn'a'^2}{a}\cdot ii'(1-\cos 2\mu t-2\sin^2\lambda)\cos\varphi - S\sin\varphi = K\frac{d^2\varphi}{dt^2}\ .$$

Let

$$\varphi = u+\alpha\ ,$$

where we take

$$\tan u = \frac{\pi^2 nn'a'^2}{aS}\cdot ii'\ .$$

As after that we have $d^2\varphi/dt^2 = d^2\alpha/dt^2$, we obtain

$$\frac{d^2\alpha}{dt^2}+\frac{S}{K}\Big[\left(1+(1-\cos 2\mu t-2\sin^2\lambda)\tan^2 u\right)\cos u\sin\alpha \\ + \left(\cos 2\mu t+2\sin^2\lambda\right)\sin u\cos\alpha\Big]=0\ .$$

Under the assumption that u and α have small values we get

$$\frac{d^2\alpha}{dt^2}+\frac{S}{K}\left[\left(1-2\sin^2\lambda\sin^2 u\right)\frac{\alpha}{\cos u}+\left(\cos 2\mu t+2\sin^2\lambda\right)\sin u\right]=0\ ,$$

or, when

$$\beta = \left(1-2\sin^2\lambda\sin^2 u\right)\alpha\ ,$$

and

$$S' = \left(1-2\sin^2\lambda\sin^2 u\right)S\ ,$$

[we get]

$$\frac{d^2\beta}{dt^2}+\frac{S'}{K}\left[\sec u\cdot\beta+\left(\cos 2\mu t+2\sin^2\lambda\right)\sin u\right]=0\ .$$

By integration we get from this

$$\beta = \frac{\sin u}{4\mu^2\frac{K}{S'}-\sec u}\cdot\cos 2\mu t-\sin 2u\sin^2\lambda+A\sin(t-B)\sqrt{\frac{S'\sec u}{K}}\ ,$$

$$\alpha = \frac{S \sin u}{4\mu^2 K - S' \sec u} \cdot \cos 2\mu t - \frac{\sin 2u \cdot \sin^2 \lambda}{1 - 2\sin^2 u \sin^2 \lambda} + A' \sin(t - B)\sqrt{\frac{S' \sec u}{K}} .$$

If now, with fast oscillations of the electricity and after the solenoid has settled to rest, the first and second parts of α vanish, one obtains the constant value of the deflection φ, denominated by v'', namely

$$v'' = u - \frac{\sin 2u \cdot \sin^2 \lambda}{1 - 2\sin^2 u \sin^2 \lambda} ,$$

whence we get

$$\sin^2 \lambda = \frac{u - v''}{2 \sin u(\cos u + [u - v''] \sin u)} .$$

As u is already known from the values of i and i' determined by the previous observations by means of the equation

$$\tan u = \frac{\pi^2 n n' a'^2}{a} \cdot i i' ,$$

we *solved the problem to determine the phase difference* 2λ *of the electric oscillations at two different places of the closed circuit from the observed deflection* v''.

18.25 The Commutators

To meet the aim of an exact comparison of the *amplitudes and phases of oscillations* at two places of a closed conductor, if they were only slightly different, the execution of the described observations after the method described in the preceding Section would demand a very detailed fineness and exactness, hardly attainable if they had to be performed *separately and independently*. But the achievement of this aim can be extremely simplified if these observations can be combined *pairwise* and performed *simultaneously* with the same closed conductor and the same rotation of the magnet. For this aim, *two as equally as possible constructed electrodynamometers* are required with their multipliers and solenoids making part of the same closed circuit. If a system of exactly corresponding observations is to be performed by means of two such *electrodynamometers* belonging to the same circuit, the most essential condition to be fulfilled is that the *period of oscillation* of the solenoids of the two electrodynamometers, suspended in a bifilar manner, be identical, which is very easily realized if the construction of the electrodynamometer provides the possibility to control at will the distance of the suspending wires of one solenoid or of both, whereby the period of oscillation of one solenoid can be exactly matched with that of the other solenoid. If the two solenoids are at *complete rest* before an observation series is started, then a more expanded observation series may be performed in such a way that all of the *observed elongations* of *both* solenoids set in motion by electric oscillations in the circuit are pairwise valid for *equal moments*.

The complete correspondence of both electrodynamometers in other respects is considerably much less taken into consideration as with the period of oscillation. For it is easily seen that if both electrodynamometers in the closed circuit are closely spaced in series, so that both belong to the same part of the circuit where there are no noticeable differences of the oscillation amplitudes and phases, then a very exact comparison of both instruments is possible by means of simultaneous observations with both instruments in correspondence which can be continued over a longer duration at equal period of oscillation of the solenoids, whence all observations performed with one of the instruments can be reduced exactly so as to yield the same results that would be obtained with the other instrument if this instrument were identical.

Under this assumption both electrodynamometers, exactly adjusted with each other, can be applied at two different very distant places of one and the same conductor and then, by simultaneously observing both instruments with one electric oscillation in the conductor, a much finer comparison of the *oscillation amplitudes* at both places of the circuit can be gained than would be possible if one and the same electrodynamometer would be applied and observed at both places and different times, whereby it would have to be assumed that the rotation of the magnet were identical at both times, an assumption that never can be fulfilled in reality and which can be completely economized with these synchronized corresponding observations.

Furthermore, after mutually comparing them exactly, both electrodynamometers can also serve to place the *solenoid* of one electrodynamometer at another distant part of the closed conductor, while the *multiplier* of the same instrument stays at its former place, and then to perform simultaneous corresponding observations by means of this electrodynamometer and the other one, fixed at its place, by means of which any ever so minute *phase difference* of the electric oscillation is recognized at the two very distant places of the closed conductor without the necessity to assume an identical rotation of the magnet at different times.

Finally, it is now of great importance for the exactness and the reliability of the results derived from these observations that the different series of observation, namely *first* those for the comparison of the instruments and *second* those for the comparison of the oscillation amplitudes and phases, be performed alternately in direct succession and repeated while the magnet is continuously rotated in an utmost uniform fashion, where it is required to be able to replace either the whole electrodynamometer or one of its parts, for example the solenoid, *momentarily* between two observations, which is easily realized by means of suitably designed *commutators*.

These *commutators*, as they will be applied in the following experiments, consist of a number of twin cells, that is, cells pairwise connected by a conductor and connected to the ends of the various parts of the conducting wire. These twin cells then can be connected again pairwise with each other, namely, combined together in two different ways, by distinguishing the front cell from the rear cell. One of these methods of pairwise combination of the twin cells can indeed be obtained by means of a fixed system of connective wires which are simultaneously immersed in all front cells; the other method of pairwise combination of the twin cells can be obtained by another fixed system of connective wires which are simultaneously immersed in all rear cells. And these two different systems of connective wires can behave like the two arms of a lever, so that immersing one of the systems causes the other to emerge and vice versa. It is immediately clear that by means of such a commutator with 6 *twin cells* one part of the conducting wire may be disconnected from its connection with two other parts of the conducting wire, thus connecting the two latter parts among themselves, and finally the hitherto connected two parts may be disconnected and the previously disconnected part may be inserted between them. All this is performed by means of a simultaneously operated *interchange*, namely by turning a lever whereby the system of connecting wires is immersed into the front cells, while the other system emerges from the rear cells, or vice versa. — Moreover, one needs commutators with 4 *twin cells*, if not during the observations, but beforehand. In fact, before the observations the solenoids of the two electrodynamometers are to be damped, wherefore *first* a current is needed which passes through the solenoid and the dynamometer to be damped, *second* a commutator with 4 twin cells, two of which are connected to the ends of the multiplier wire and the other two of which to the ends of the solenoid wire. By means of this commutator the multiplier can be connected at will now *in parallel*, now *crosswise*. With one kind of connection the current through the multiplier exerts a *positive* torque on the solenoid carrying the same current, with the other kind a *negative* torque, and the solenoid is damped if the former kind of connection is established during the *backward oscillation*, the latter kind during the *forward oscillation*. As the effect of a current simultaneously present in multiplier and solenoid is completely independent of the *direction* of the current, the *alternating current induced in the circuit by the rotating magnet* can be used, instead of a steady current, whereby it is possible to let the damping of the dynamometer, after the rotation of the magnet has started, *immediately precede the observations*.

If now all these operations, namely, the damping of the dynamometer and then all observations required at different places of the circuit for the comparison of the oscillation amplitudes and phases, while the magnet is continuously rotated, are executed successively without interruption, this requires five commutators in total which have to be connected with the various parts of the circuit in a systematic way to be explained in more detail.

For an easier overview *first* the 5 commutators, *second* the various parts of the circuit to be connected with the commutators are now to be denoted exactly and distinguished. Then Figure 1 will serve to give the overview of the set up as a whole and of all connections in detail.

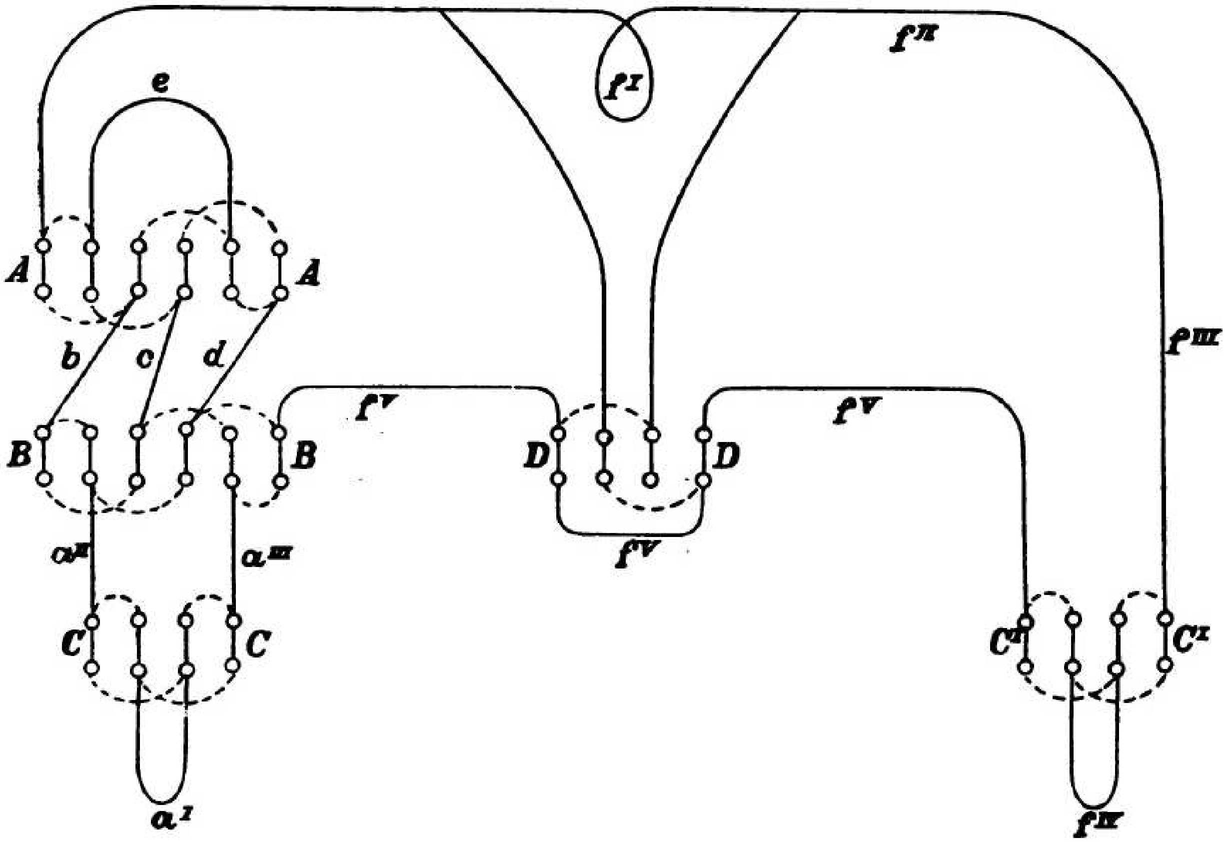


Fig. 1.

The *first* commutator, denoted by A, is required either in order to put one of the *electrodynamometers* (multiplier and solenoid in one) alternately into two different places of the conducting wire, or in order to put the *solenoid* of this electrodynamometer into two different places of the conducting wire while the respective multiplier stays in its place. This requires a commutator with 6 twin cells, two of which are required for both ends of the electrodynamometer to be replaced, two for the ends of the conducting wire at one of the switch-on locations, and finally two for the ends of the conducting wire at the other switch-on location.

The *second* commutator, denoted by B, is required as *auxiliary commutator*, the setting of which determines whether the *multiplier including solenoid* or the *solenoid* of one of the electrodynamometers alone is put into two different places of the conducing wire by alternative operation of commutator A, which equally requires a commutator with 6 twin cells.

The *third* and *fourth* commutator, namely C and C', respectively, are used to dampen the solenoids before the observations start. This requires commutators with 4 twin cells, two of which for the two ends of the leads to the solenoid and two for the leads to the respective multiplier including the remaining conducting wire from which, however, the part belonging to the other electrodynamometer has to be excluded, so that the solenoid at rest is not disturbed while the other one is damped.

Finally, the *fifth* commutator, denoted by D, is required in order to establish the connection between the dynamometers with the commutators C and C', respectively, now with one, now with the other solenoid. To this purpose, a commutator with 4 twin cells is needed, two of which for the ends of the conducting wire at the switch-on location of the commutator, the other two for two connecting wires through which the current can be led to the conducting wire by-passing one or the other electrodynamometer.

The following parts of the *closed conducting wire* have to be distinguished which can be connected by means of the commutators in various ways.

The *first wire*, denoted by a, is the *multiplier wire of the first dynamometer* whose ends lead to two twin cells of the commutator C, besides *two short connecting wires* of the two other twin cells of this commutator [leading] to two twin cells of commutator B. These various parts of wire a, which always stay connected in the same way during the observations, shall be distinguished by symbols a^{I}, a^{II}, a^{III}.

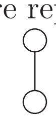
The *second* wire, denoted by b, is the *solenoid wire of the first dynamometer*, whose ends are connected to a twin cell of commutator B and to a twin cell of commutator A.

The *third* and *fourth* wires, denoted by c and d, are two *short connecting wires* of two twin cells of commutator B [leading] to two twin cells of commutator A, whose resistance may be considered as vanishingly small.

The *fifth* wire, denoted by e, is one of the two very long pieces of wire which are needed during the observations in order to either remove both dynamometers from each other or to remove the solenoid of the first dynamometer from its multiplier by connecting either the two wire endings of one dynamometer with those of the other one, or the two wire endings of the multiplier and the wire endings of the solenoids by a long piece of wire. Both ends of the long piece of wire, e, are connected to two twin cells of commutator A.

The *sixth* wire, denoted by f, is the whole rest of the conducting circuit and comprises the *inductor ring* of the rotating magnet, further the *second long piece of wire just mentioned before*, then the *wire of the second dynamometer*, of the solenoid as well as the multiplier, and finally *a connecting wire leading back to the first wire*. These various parts of the wire f, which stay connected in the same way during the observations, are to be distinguished by the symbols f^{I}, f^{II}, f^{III}, f^{IV}, f^{V}. The commutator C' is plugged in between the solenoid wire f^{III} and the multiplier wire f^{IV} of the second dynamometer, the latter, however, will not be used during the observations. Likewise, there is a plug in the connecting wire f^{V} for the commutator D which, however, stays closed because also this commutator is not needed during the observations.

Hence now Figure 1 has been sketched for a better illustration where the various twin cells of the commutators A, B, C, C', D are represented by the symbol



and one of the two ways of connection is sketched by the upper dashed arcs and the other way is sketched by the lower dashed arcs.

The commutators C and C', keeping the *top* setting, and D, being excluded from the circuit by means of a wire connecting its first and last cell after the solenoid is damped, are not used to displace the *first dynamometer*, consisting of the multiplier a^{I} and the solenoid b, from the first switching place to the other one during the observations, but the displacement is performed simply by a change of the setting of the commutator A after the connections, sketched by dashed arcs on *top*, are established by means of commutator B. Because the setting of commutator A, sketched by means of dashed *top* arcs, completes a closed circuit with the shown setting of commutator B, where the wires denoted follow in the following sequence:

$$abefdca;$$

the setting of commutator A, sketched by means of the *bottom* dashed arcs, completes a closed circuit with the following sequence of wires:

$$abfdeca.$$

Breaking up f into its parts f^I, f^{II}, f^{III}, f^{IV}, f^V, and representing the whole circuit by means of the four sides of a rectangle with its long sides symbolizing the long connecting wires, denoted by e and f^{II}, then Figure 2 sketches the *former* case and Figure 3 the *latter* case.

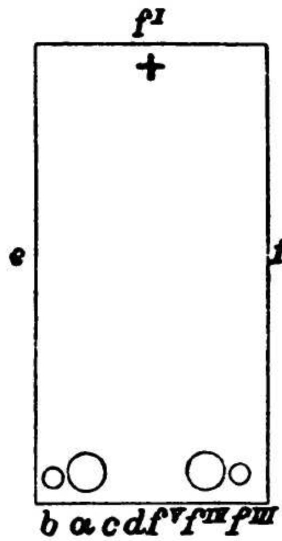


Fig. 2.

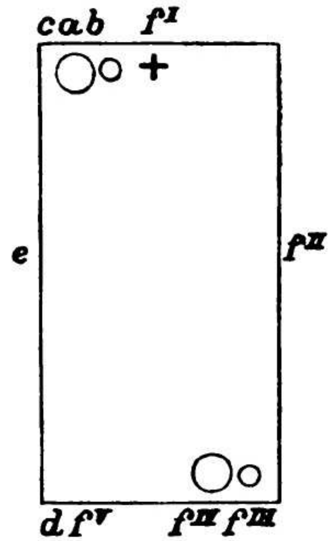


Fig. 3.

In addition, the place of the inductor, f^I, with the rotating magnet has been marked with +, the two places of the multipliers, a and f^{IV}, with larger circles, the two places of the solenoids b and f^{III} by smaller circles. The inductor, f^I, with the rotating magnet is always at the top side of the rectangle, the dynamometer, $f^{III}f^{IV}$, is always at the opposite bottom side. The place of the other dynamometer, *ab*, is alternated and, in the *former* case, is situated at the bottom side besides the first dynamometer, $f^{III}f^{IV}$, opposite to the inductor, f^I, in the *latter* case at the top side besides the inductor, f^I, opposite to the dynamometer, $f^{III}f^{IV}$. Hence by operating the commutator A, the dynamometer *ab* is switched now at a place of the circuit very far from the inductor, f^I, now at a place very close to it as was demanded for the *first series of observations.*

Likewise, the commutators C, C' and D are not used to displace the *solenoid* b of the first dynamometer from one switching place to the other one, but the displacement is performed by a simple change of the setting of commutator A after the connections, symbolized by the lower dashed arcs, have been established by means of commutator B. Because the setting of commutator A, sketched by means of dashed *top* arcs, completes a closed circuit where the wires denoted follow in the following sequence:

$$adcbefa;$$

the setting of commutator A, sketched by means of the *bottom* dashed arcs, completes a closed circuit with the following sequence of wires:

$$adecbfa.$$

The *former* case is represented by Figure 4, the *latter* case by Figure 5.

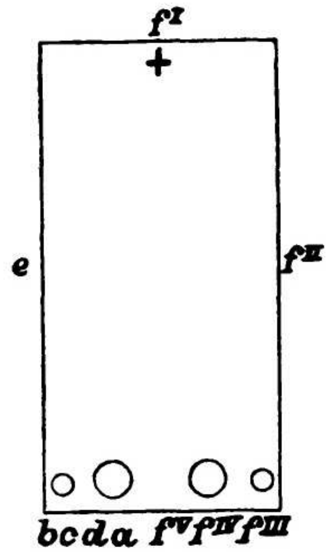


Fig. 4.

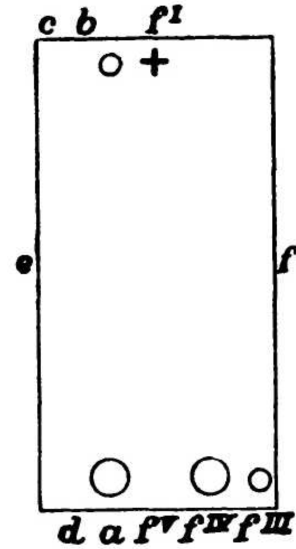


Fig. 5.

We see that the places of the inductor f^I of the dynamometer, $f^{III}\,f^{IV}$, and also of the multiplier, a, always stay unchanged and that simply the place of the solenoid b alternates, which in the *former* case, is situated at the bottom side of the rectangle besides the multiplier, a, opposite to the inductor, f^I, and in the *latter* case is situated at the top side of the rectangle besides the inductor, f^I, opposite to the multiplier, a. Hence by operating the commutator A, the solenoid b is switched now at a place of the circuit very far from the multiplier, a, now at a place very close to it as was demanded for the *second series of observations*.

In order to dampen the solenoids before starting the observations, the piece of the wire, f^V, connecting the first and fourth twin cell of the commutator, D, is taken out. Then, in order to dampen the solenoid of the *first* dynamometer, b, the commutator, D, is given the setting symbolized by the upper dashed arc, whereby the wire, f^{II}, together with the second dynamometer, $f^{III}\,f^{IV}$, are connected and whereby, depending on the setting of commutator C, a circuit is completed with the following sequence of wires:

$$\begin{aligned} \text{with } \textit{top} \text{ setting of } C: &\quad a^I a^{II} bef^I f^V a^{III} a^I, \\ \text{with } \textit{bottom} \text{ setting of } C: &\quad a^I a^{III} cdf^V f^I eba^{II} a^I, \end{aligned}$$

where the top settings have been assumed for the commutators A and B. Hence we see that, with the given direction of the current through the multiplier a^I, the direction of the current through the solenoid b with the *top* setting of C is given by $a^{II}be$, while it is given by eba^{II} with the *bottom* setting, being hence opposite to the former, whereby in both cases opposite equal torques are exerted on the solenoid b, one of which can always be used to *dampen* the motion of the solenoid.

In order to dampen the solenoid of the second dynamometer, f^{III}, the commutator D is given the *bottom* setting symbolized by a dashed arc whereby the wires $eba^{II}a^Ia^{III}$, including the first dynamometer, are connected and whereby, depending on the setting of commutator

C^I, a circuit with the following sequence of wires is completed:

$$\text{with } \textit{top} \text{ setting of } C^I\text{:} \quad f^{IV} f^V f^I f^{II} f^{III} f^I,$$
$$\text{with } \textit{bottom} \text{ setting of } C^I\text{:} \quad f^{IV} f^{III} f^{II} f^I f^V f^{IV}.$$

We see that, given the current direction through multiplier f^{IV}, the current direction through the solenoid f^{III} with the *top* setting of C^I is given by $f^{II} f^{III} f^{IV}$, while by the *bottom* setting it is given by $f^{IV} f^{III} f^{II}$, hence opposite, whereby in both cases opposite torques are exerted on the solenoid f^{III}, one of which can always be used to *dampen* the motion of the solenoid.

After damping both solenoids, the commutator D is opened and the removed piece of wire f^V is again inserted to connect the first and last cell.

18.26 The Long Conducting Wires

For the displacement of the *solenoid* of an electrodynamometer or for the displacement of the *whole electrodynamometer* (solenoid and multiplier) from one switching place of the closed circuit to the other one, it is a matter of great importance for the observations that the two conducting wires connecting the two switching places be of almost equal and great length. Therefore two parts of the closed circuit, namely the wires e and f^{II}, have been explicitly mentioned for serving this purpose in the previous Section. In the circuit used in the following experiments each of these two wires had a length of 36 600 meters or almost 5 miles.

Considering the great length of the whole circuit containing these two long wires, it is immediately clear that it is practically impossible to give them the exact shape of a circle as has been assumed for simplification in the previous Section when developing the laws. But even apart from this great length to be associated with the closed conductor, the simple shape of a circle could not be applied in a circuit that must contain an *inductor ring* for the rotating magnet and *two dynamometers* for the purpose of the observations, because pieces of the conducting wire have to be employed whose shape and position are determined by the rules valid for the construction of these instruments.

Obviously this in practice unavoidable deviation of the shape of the closed conductor from a circle has an influence on the electric oscillations caused by the rotating magnet in the conductor, and thereby the law of the dependence of the amplitude of the electric oscillations on the rotation velocity of the magnet is essentially changed. If, however, it is not a question of observations by which the amplitude is exactly determined and measured, but only of those to compare the amplitudes at two different places of the conductor (or to determine only the phase difference at both places), then the deviation from the circular shape is of minor importance. Because if, according to the laws developed in the previous Section, there were really no noticeable difference of oscillation amplitude and phase at two very distant places of *a circular conductor* even at high rotation velocities, there would be no reason to assume that such a difference would be brought forth simply by a deviation of the conductor from the circular shape; still more so, if the observations show that in a closed conductor with *an arbitrary shape entirely different from a circle* that there are no noticeable differences of oscillation amplitude and phase, it might be allowed conversely to consider this result as generally valid, also for differences of oscillation amplitudes and phases being unnoticeable for a perfect circular shape.

It is of special importance for these two 5 mile wires making part of the closed conductor that, if no telegraph wires are used but if these long wires shall instead be housed in the closed laboratory where the observations are made as is necessary to completely control all essential external conditions of the observations, these long wires have to be wound on *spools* to save space. Now it is clear, however, that in the case of electric currents now abruptly arising and now dying off again as happens due to the electric oscillations caused by the fast rotation of the magnet, all windings of the conducting wire on the spool must mutually exert electromotive forces according to the laws of *Volta-induction*[456] which sum up to a strong *damping force*, thereby essentially reducing the amplitude of the electric oscillations so that the latter could not any more be observed even with the most sensitive dynamometers at faster rotations of the magnet. In order to perform the observations it is therefore utterly important to find a method to wind the long wires on *spools* so that such a *mutual induction between the wire windings* is avoided.

To achieve this aim, provided the wires are well braid,[457] the simplest and most perfect way is to combine the two *halves* of each piece to a *twin wire* before winding it on *one coil*. This combination is best obtained by *braiding* the two halves, each of which is already braid and thus kept isolated from each other by means of this double braiding, *wound once again together with cotton or silk*. Then, connected at *one of the ends*, the two halves form a conductor through which a current entering by the other open end and passing through one of the wire halves is led back to the open end passing through the other half wire *on almost the same path*. Then the end where the two wire halves are connected is fixed at the spool intended for the braiding and subsequently the whole *twin wire is braided on this spool*, so that the end which leaves the two wire halves non-isolated are freely exposed on top and the whole twin wire can be connected to the remainder of the circuit by means of these two non-isolated ends of the two wire halves.

In this way all current elements pertaining to such a twin wire are ordered *pairwise*, so that only oppositely equal current elements are close neighbors. It is clear that, even at the fastest changes of intensity, such pairs of current elements cannot exert an electromotive force on any other more distant conductor element and that hence this twin wire, in whatever way it may be wound by braiding it on the spool, is not subject to any damping force as a consequence of these windings which otherwise the electric oscillations would have experienced by the rotating magnet, as would have been the case had the wire simply been wound in a unidirectional fashion alongside its whole length.

Without the above method fast electric oscillations in such a long conducting circuit would become vanishingly weak, though not in consequence of the great resistance of the circuit, but in consequence of the mutual induction between all the windings, making their observation practically impossible. Here it is barely necessary to remark that the same method may find fruitful applications also in other cases where similar conditions are met under which this very method will perform in a similar way.

This is valid, in particular, for long distance *electric telegraphs*, where for the purpose of [sending] telegraph signals, electric currents in a very long circuit arise and vanish very rapidly. We have indeed already mentioned that the damping forces, exerted hereby by the electricity in the various wire elements (and suffered especially under the influence of an enclosing conductor), cause great impediments due to delays in signaling which threaten to thwart the further expansion especially of undersea telegraph lines. For example from Europe to America, quite independent of the technical difficulties in connection with the laying and

[456][Note by AKTA:] See footnote 402 on page 267.

[457][Note by AKTA:] In German: *Wenn die Drähte gut umsponnen sind*. A braided wire here means silk or cotton woven around the conductor for insulating purposes.

maintenance. Hereby, not only the forces mutually exerted between the electricities in the various wire elements come into play, but also those forces exerted by the electricity on the *neighboring conductors* and sustained by them, and even the forces exerted by the *magnetism of the earth* and its variability on the electricity of the various wire elements. All impediments arising from this for fast signal processing and great extension of the circuit can be totally or almost totally avoided by applying the above method, always placing two wire elements closely together in which the electric current and charge are practically *equal but opposite*. Hence it is very clear that, as a rule for further extension of the telegraph line, a cable must be designed so as to have a wire with current passing in one direction very close to a second wire leading the current back, where hence the return of the current via the earth must be given up. That the insulation between closely spaced wires does not cause difficulties seems to be shown by the example of our circuit where the two wires packed closely together by means of a common braiding are insulated from each other only by covering each individual wire with silk before joining them. The thickness of the insulator coating here was less than 1/10 of a millimeter and yet the insulation was to be considered perfect for currents so strong that the scale range barely sufficed for the dynamometer deviations caused by them, as will be demonstrated by the corresponding observations to be described below.

18.27 Observations to Compare the Amplitude of Electric Oscillations at Two Different Places of a Long Closed Circuit

According to the arrangement discussed in the previous Sections now four series of observations have been performed, all on one day, September 28, 1860, *alternately* to compare the *amplitude* and to determine the *phase difference* at two distant places of the above long closed circuit while electric oscillations were excited therein by a fast rotating magnet. However, while not performed in immediate succession, the two series of observations to compare the *amplitude* shall *both* be considered *together* in the present Section, likewise *both* series of observations to determine the *phase difference* in the following Section.

The corresponding observations at both electrodynamometers were made by Mr. Schering and myself, while Mr. Klinkerfues and Mr. H. Weber performed the uniform rotation of the magnet in the inductor coil and determined its velocity.[458] This velocity was kept as close as possible to 260 turns per second whereby only slight deviations occurred which were noticeable as small variations of the deviations of the solenoids of both dynamometers.

The *duration of the periods of the solenoids* was regulated in such a way that it was equal in size and lasted almost exactly 15 seconds. In doing so, the sensitivity of both instruments, however, was very different as a consequence of using indeed equal spools for both solenoids, but having different wire thickness and hence different numbers of windings. The more *sensitive* dynamometer, the one with the solenoid having a greater number of windings, was used for those observations by means of which the oscillation amplitude was to be determined alternately at two different places of the conducting circuit, while the *less sensitive* dynamometer served for the corresponding observations, in order to account for the influence of small variations of the rotation velocity, to which purpose the same had to stay

[458][Note by AKTA:] Ernst Christian Julius Schering (1824-1897), Ernst Friedrich Wilhelm Klinkerfues (1827-1884) and Heinrich Weber (1839-1928).

fixed at a certain place of the conducting wire.

First Series.

According to the setup prescribed in Section 18.24, the first series of observations was performed in order to compare the *intensity* or oscillation amplitude of the electric oscillations at two different places of the long closed circuit.

All *observations* are expressed in parts of a millimeter scale, the image of which, 2 100 divisions away from it and attached to the solenoid, plane mirror on the magnetometer, was observed in the usual way with a telescope. In order to exploit the whole expansion of the scale, the telescopes including their scales were placed before the mirrors in such a way that the solenoid position at rest with the magnet at rest or with the open circuit did not correspond, as usual, to the center of the scale above the telescope, but to a point near the beginning of the scale, because the solenoid was always deflected towards one and the same side from its position at rest.

During the whole series of observations the *magnet* was kept in continuous uniform *rotation*. Between the various sets of observation distinguished by number the *commutator* A described in Section 18.25 was operated, namely the first time, when it had previously been open, it was closed, and afterward the *top* and *bottom* settings were simply exchanged. During this the *commutator* B was kept closed in the *top setting*, likewise the two *commutators* C and C' with 4 cells each which had been used for the damping of the solenoids before the beginning of the observations, while the commutator D with 4 cells was opened and completely taken out of the circuit by re-inserting the piece of wire connecting the first and fourth cell which had been removed while the two solenoids were damped.

Before starting the observations the solenoids of both dynamometers, as explained in Section 18.25, were damped as much as possible. — As the setting of the three commutators B, C, C' was kept fixed during the whole series of observations, it sufficed to remark in the headlines of the various sets whether the circuit was open or closed and whether, in the latter case, the *top* or *bottom setting of commutator* A was employed according to the scheme in Section 18.25. — With the *closed circuit*, where the solenoids moved more vividly, the *second averages* have been chosen from the subsequently observed elongations to determine the *state of rest*.

Top Setting of Commutator B.							
Set number 1, open circuit				Set number 2, closed circuit Top setting of commutator A			
Dynamometer no. 1		Dynamometer no. 2		Dynamometer no. 1		Dynamometer no. 2	
Observed elongations	State of rest	Observed elongations	State of rest	Observed elongations	State of rest	Observed elongations	State of rest
52.9		+ 11.4		901.7		638.2	
	34.55		−0.30				
16.2		−12.0		854.7	876.55	606.8	621.38
	34.40		−0.40				
52.6		+ 11.2		895.1	877.93	633.7	622.35
	34.60		−0.35				
16.6		−11.9		866.8	880.82	615.2	624.52
	34.55		−0.45				
52.5		+11.0		894.6	882.70	634.0	625.83
	34.70		−0.20				
16.9		−11.4		874.8		620.1	
Average 34.56		Average −0.34		Average 879.50		Average 623.52	
Set number 3, closed circuit Bottom setting of commutator A				Set number 4, closed circuit Top setting of commutator A			
843.1		596.2		811.4		574.2	
908.8	872.35	644.9	617.95	941.8	880.90	668.5	624.65
828.7	870.45	585.8	616.60	828.6	890.82	587.4	631.87
915.6	873.72	649.9	619.10	964.3	887.95	684.2	629.68
835.0	879.35	590.8	623.20	794.6	872.93	562.9	619.13
931.8		661.3		838.2		666.5	
Average 873.97		Average 619.21		Average 883.15		Average 626.33	
Set number 5, closed circuit Bottom setting of commutator A				Set number 6, closed circuit Top setting of commutator A			
796.7		564.7		748.3		531.5	
978.0	885.60	694.1	628.20	1007.5	877.80	714.5	622.93
789.7	887.47	559.9	629.45	747.9	879.30	531.2	623.97
992.5	884.35	704.1	627.15	1013.9	880.67	719.0	624.97
762.7	876.35	540.5	621.37	747.0	876.65	530.7	622.13
987.5		700.4		998.7		708.1	
Average 883.44		Average 626.55		Average 878.61		Average 623.50	

Set number 7, closed circuit Bottom setting of commutator A			
Dynamometer no. 1		Dynamometer no. 2	
Observed elongations	State of rest	Observed elongations	State of rest
742.8		527.6	
1011.1	880.28	716.2	624.38
756.1	883.47	537.5	626.77
1010.6	882.45	715.9	626.00
752.5	879.43	534.7	623.80
1002.1		709.9	
Average 881.41		Average 625.24	

Comparing the corresponding deflections of both simultaneously observed dynamometers which we obtain by subtracting the state of rest observed with *open* circuit (set number 1) from that observed with *closed* circuit and hereby restricting ourselves at first to those cases, sets numbers 2, 4, 6, where both dynamometers have been positioned symmetrically and close together and separated from either side from the inductor of the rotating magnet by means of the long conducting wires, where hence the oscillation amplitude and phase should always be equal in both dynamometers; we get *the ratio of their sensitivities* from the ratio of the observed deflections of both dynamometers. Hence we obtain the sensitivity of the *first* dynamometer expressed in parts of that of the *second* dynamometer:

from sets number 1 and number 2: $844.94/623.86 = 1.3544$,
from sets number 1 and number 4: $848.59/626.67 = 1.3541$,
from sets number 1 and number 6: $844.05/623.84 = 1.3530$,

hence the average *ratio of the sensitivity of dynamometer number 1 and of dynamometer number 2* behaves as

$$1.3538 : 1 .$$

After this mutual comparison of the sensitivities of both dynamometers the number of readings of one dynamometer alternatively switched onto two different places of the circuit may be reduced by making use of the observed deflections of the other dynamometer, always kept fixed at the same place of the circuit, as if the deflections were observed *simultaneously* at the two places of the circuit *by means of identical dynamometers*. Namely from the corresponding deflections of the auxiliary dynamometer it is now always possible to calculate the deflections of the main dynamometer as would have been observed if the main dynamometer had stayed in its original place, for which the comparison of its sensitivity with that of the other dynamometer is valid, and this *comparison calculated for the first position* of the main dynamometer in the circuit may then be compared with the *deflection really observed* for the second position of the main dynamometer.

That is, if the obtained ratio of 1.3538 is multiplied by the deflections of the auxiliary dynamometer observed in the 3rd, 5th, and 7th set, after subtracting the state of rest found in set number 1,

$$619.55, \qquad 626.89, \qquad 625.58 ,$$

one gets the deflections which would have been observed at the main dynamometer if the latter had kept its place in the circuit like it has been in the 2nd, 4th, and 6th set.

The following Table contains the values of the *calculated deflections* in the *second* column; the *third* column contains the *actually observed deflections at position II of the main dynamometer* for which these *calculated deflections of the main dynamometer at position I* were valid; the *fourth* column finally lists the differences between both.

Set number	Calculated deflection for position I	Observed deflection for position II	Difference
3	838.75	839.41	+0.66
5	848.69	848.88	+0.19
7	846.92	846.85	−0.07
Average	844.78	845.05	+0.26

These deflections, observed in scale units, divided by the distance, 2 100 scale units, of the mirror from the scale, now yield the tangents of twice the angles designated by v and v' in Section 18.24. Hence we have for position I:

$$\tan v = \tan \frac{1}{2} \arctan \frac{844.78}{2100} = 0.193\,601 \ ,$$

and for position II:

$$\tan v' = \tan \frac{1}{2} \arctan \frac{845.05}{2100} = 0.193\,656 \ .$$

But now, according to Section 18.24, the ratio of the squares of the intensities i and i', or *the squares of the amplitudes of the electric oscillations* at the two positions I and II in comparison where the main dynamometer has been placed by means of the top and bottom settings of commutator A, behaves as

$$i^2 : i'^2 = \tan v : \tan v' \ ;$$

hence one gets

$$i' = 1.000\,142 \cdot i \ .$$

Position I in the circuit, however, is almost 5 miles away from the inductor which houses the rotating magnet, while position II is very close to the inductor. It seems that this indeed means that the amplitude of the electric oscillations, produced by the rotating magnet in the whole circuit are somewhat weaker at the great distance from the inductor where the excitation started, namely at the position denoted by I, than very close to the inductor, at position II; the difference found, however, is exceedingly small, so that it cannot be safely established even by means of the most exact observations, it is in fact less than 1/7000 of the total oscillation amplitude corresponding to the full deflection of the dynamometer. In fact these observations hence show that at two positions of the circuit at a mutual distance of almost 5 miles *practically no difference of the amplitudes of the electric oscillations can be detected* even by the most exact observations.

Concerning the *exactness of the observations* it is indeed clear that its closer determination cannot yet be gained from just so few repetitions as in this first series of observations; yet, as no deviation from the average surpasses 0.40 scale units, one may consider this average obtained from all 3 observations as reliable, which corresponds to one part in 845 of the total oscillation amplitude. — Such an accuracy of the intensity measurements of *electric oscillations* surpasses the precision that could be obtained hitherto by means of intensity measurements of almost any other kind of oscillation. In *acoustics* and *optics* the intensity of sound and light depends on the oscillation amplitude and it is known how far the intensity measurements of sound and light stay behind this precision. Only the observations of the oscillation amplitude of a magnetic needle or generally by means of a torsion balance in unifilar or bifilar suspension after Gauss' method allow equal or, under favorable conditions, a still somewhat higher precision.[459] — It is worthwhile to remark that the same precision by means of the same inductor and the same dynamometers, which served for the production and observation of electric oscillations, 520 of which took place every second, would have been obtained equally easily in a circuit almost 10 miles long even if the frequency of the

[459] [Note by AKTA:] See [Web38a] with English translation in [Web41a] and [Web66a]; and [Web94a].

electric oscillations were increased to more than 1000 per second and the length of the circuit to more than 30 miles without having to reinforce the wire of the prolonged circuit; for the electric oscillation and its effect were deliberately weakened during the above experiments; namely *first* by excluding one half of the inductor on which the rotating magnet acted; *second* by increasing the static directive force of the solenoids of both dynamometers; otherwise the length of the scale would not have sufficed to perform the observations. The observed effects would have been equally strong in a much longer circuit making use of the whole inductor and decreasing the static directive force of the solenoids, whereby the duration of their oscillation would have been increased from 15 to 20 seconds.

In order to eliminate any doubt that this precision be just an apparent one and that the coincidence of the observations repeated only 3 times in the above series of observations be just accidental, finally a *second series* of observations were performed by means of the identical set up and on this very day, the results of which are compiled in the same way for comparison with the preceding series in the following Table.

Second Series.

The remarks preceding the first series are equally valid for the second series.

Top Setting of Commutator B.							
Set number 1, open circuit				Set number 2, closed circuit Top setting of commutator *A*			
Dynamometer no. 1		Dynamometer no. 2		Dynamometer no. 1		Dynamometer no. 2	
Observed elongations	State of rest	Observed elongations	State of rest	Observed elongations	State of rest	Observed elongations	State of rest
30.3		−8.5		858.7		615.7	
	32.35		−1.65				
34.4		+5.2		904.2	879.00	636.1	623.83
	32.35		−1.55				
30.3		−8.3		848.9	879.68	607.4	624.47
	32.20		−1.65				
34.1		+5.0		916.7	880.52	647.0	625.15
	32.20		−1.50				
30.3		−8.0		839.8	876.00	599.2	621.90
	32.25		−1.55				
34.2		+4.9		907.7		642.2	
Average 32.27		Average −1.58		Average 878.80		Average 623.84	
Set number 3, closed circuit Bottom setting of commutator *A*				Set number 4, closed circuit Top setting of commutator *A*			
837.0		596.0		792.5		554.0	
918.8	880.60	649.0	624.70	964.0	881.38	683.1	623.30
847.8	885.58	604.8	628.67	805.0	884.72	573.0	628.07
927.9	881.70	656.1	625.75	964.9	880.70	683.2	625.22
823.2	880.00	586.0	624.38	788.0	879.35	561.5	624.25
945.7		669.4		976.5		690.8	
Average 881.97		Average 625.87		Average 881.54		Average 625.21	
Set number 5, closed circuit Bottom setting of commutator *A*				Set number 6, closed circuit Top setting of commutator *A*			
794.0		566.4		783.2		559.9	
962.4	875.90	678.5	621.10	961.1	877.62	679.0	623.15
784.8	876.73	561.0	621.93	805.1	875.53	574.7	621.42
974.9	879.92	687.2	623.95	930.8	869.10	657.3	617.10
785.1	874.35	560.4	620.02	809.7	872.52	579.1	619.68
952.3		672.1		939.9		663.2	
Average 876.73		Average 621.75		Average 873.69		Average 620.34	

Set number 7, closed circuit Bottom setting of commutator *A*			
Dynamometer no. 1		Dynamometer no. 2	
Observed elongations	State of rest	Observed elongations	State of rest
783.5		560.8	
969.6	878.25	682.9	623.13
790.3	881.72	565.9	625.67
976.7	882.28	688.0	626.08
785.4	882.12	562.4	625.95
981.0		691.0	
Average 881.09		Average 625.21	

The ratios of the sensitivity of the first dynamometer, expressed in parts of [the sensitivity] of the second [dynamometer], from the observations listed in the second column yield the following values:

from sets number 1 and number 2: $846.53/625.42 = 1.3535$,
from sets number 1 and number 4: $849.27/626.79 = 1.3549$,
from sets number 1 and number 6: $841.42/621.92 = 1.3529$,

hence the average ratio of the sensitivity of the first and the second dynamometer behaves as

$$1.3538 : 1 \; .$$

Now multiplying this ratio 1.3538 with the observed deflections of the second dynamometer which was always fixed at its position in the circuit during all observations, namely the deflections resulting from the difference between the positions at rest with open circuit in set number 1 and with closed circuit in sets numbers 3, 5 and 7, yield:

$$627.45, \qquad 623.33, \qquad 626.79 \; ,$$

so the products

$$849.45, \qquad 843.86, \qquad 848.55 \; ,$$

yield the values of the deflections which would have been observed at the first dynamometer if the latter had kept its position as in sets numbers 2, 4 and 6, while the deflections of the positions at rest in sets numbers 3, 5 and 7 correspond to the changed position of the dynamometer. The following Table lists the deflections from the initial position together with the corresponding deflections of the first dynamometer in the changed position.

Set number	Calculated deflection for position I	Observed deflection for position II	Difference
3	849.45	849.70	+0.25
5	843.86	844.46	+0.60
7	848.55	848.82	+0.27
Average	847.29	847.66	+0.37

Herewith, like with the previous series of observations, we get the comparison of the oscillation amplitude or of the current intensity at position I and II, [respectively]. For position I:

$$\tan v = \tan \frac{1}{2} \arctan \frac{847.29}{2100} = 0.194\,134 \; ,$$

and for position II:

$$\tan v' = \tan \frac{1}{2} \arctan \frac{847.66}{2100} = 0.194\,212 \; ,$$

hence, as according to Section 18.24 [one has]

$$i^2 : i'^2 = \tan v : \tan v' \; ,$$

it follows that

$$i' = 1.000\,201 \cdot i \ .$$

Thus the difference between the oscillation amplitude at both positions, one of which was almost 5 miles away from the inductor of the rotating magnet while the other one was close to the inductor, makes barely 1/5000 of the total oscillation amplitude corresponding to the total deflection of the dynamometer. It is clear that also this difference is too small in order to be safely declared even for the most exact observations and therefore also this second series of observations will confirm that *there is practically no difference of the amplitudes of the electric oscillations that can be safely established even for two positions almost 5 miles apart*.

18.28 Observations to Determine the Difference of the Phase of Electric Oscillations at Two Different Places of a Long Closed Circuit

On the basis of an equal setup as described for the two foregoing series of observations, a *third* series of observations was performed, however, not for the comparison of the oscillation amplitudes, but for the determination of the *phase difference* of the electric oscillations at two different places of a long circuit. To this aim, like with the previous series of observations, the commutator A described in Section 18.25, was closed, if previously open, between the various sets of observation distinguished by number, or it was opened if previously closed, that means, the *top* and *bottom* setting were exchanged. *Commutator* B, on the other hand, was kept closed, but in the *bottom setting* (instead of in the top setting as before). Finally, the two *commutators* C and C', equipped with 4 cells each, used to dampen the solenoids before starting the observations, were again kept closed and set exactly as before during the observations. After damping the solenoids, the commutator D was opened before starting the observations and completely excluded from the circuit by means of a wire connecting its first and last cell.

Third Series.

Bottom Setting of Commutator B.							
Set number 1, open circuit				Set number 2, closed circuit Top setting of commutator A			
Dynamometer no. 1		Dynamometer no. 2		Dynamometer no. 1		Dynamometer no. 2	
Observed elongations	State of rest	Observed elongations	State of rest	Observed elongations	State of rest	Observed elongations	State of rest
33.9		−3.5		849.5		995.0	
	35.95		−3.70				
38.0		−3.9		917.7	883.25	263.1	626.67
	35.95		−3.65				
33.9		−3.4		848.1	889.37	985.5	630.87
	35.95		−3.20				
38.0		−3.0		943.6	889.15	289.4	629.60
	35.95		−3.40				
33.9		−3.8		821.3	875.85	954.1	619.50
	36.00		−3.45				
38.1		−3.1		917.2		280.4	
Average 35.96		Average −3.48		Average 884.41		Average 626.66	
Set number 3, closed circuit Bottom setting of commutator A				Set number 4, closed circuit Top setting of commutator A			
812.8		940.0		943.2		363.1	
956.7	882.55	317.6	625.27	809.5	875.45	873.8	619.30
804.0	876.10	925.9	620.57	939.6	873.17	366.5	617.70
939.7	873.35	312.9	618.70	804.0	875.20	864.0	618.62
810.0	878.10	923.1	622.32	953.2	880.35	380.0	622.13
952.7		330.2		811.0		864.5	
Average 877.52		Average 621.72		Average 876.04		Average 619.44	
Set number 5, closed circuit Bottom setting of commutator A				Set number 6, closed circuit Top setting of commutator A			
925.7		366.2		919.8		369.0	
836.3	881.82	879.8	625.35	845.1	886.12	881.2	627.52
929.0	877.45	375.6	622.35	934.5	885.68	378.7	627.53
815.5	875.10	858.4	618.15	828.6	880.35	871.5	624.62
940.4	879.10	380.2	621.40	929.7	878.32	376.8	622.53
820.1		866.8		825.3		865.0	
Average 878.37		Average 621.81		Average 882.62		Average 625.55	

Set number 7, closed circuit Bottom setting of commutator A			
Dynamometer no. 1		Dynamometer no. 2	
Observed elongations	State of rest	Observed elongations	State of rest
898.0		367.5	
850.5	875.77	870.9	619.52
904.1	872.63	368.8	619.13
831.8	870.10	868.0	618.45
912.7	868.52	369.0	615.77
816.9		857.1	
Average 871.76		Average 618.22	

For the ratio of the sensitivity of the first dynamometer, expressed in parts of the second one, the observations of this third series yield the following values:

from sets number 1 and number 2:	$848.45/630.14 = 1.3464$,
from sets number 1 and number 4:	$840.08/622.92 = 1.3486$,
from sets number 1 and number 6:	$846.66/629.03 = 1.3460$,

hence the average ratio of the respective first and second dynamometer sensitivities equals

$$1.3740 : 1 \ .$$

Now multiplying this ratio 1.3740 by the observed deflections of the second dynamometer which has kept its position in the circuit during all observations, namely the deflections resulting from the differences of the positions of rest with open circuit in set number 1 and with closed circuit in sets numbers 3, 5 and 7, one obtains:

$$625, 20, \qquad 625.29, \qquad 621.70 \ ,$$

so that one gets the products

$$842.15, \qquad 842.26, \qquad 837.44 \ ,$$

which yield the values of the deflections which would have been observed at the first dynamometer if the solenoid had kept its positions during the observations of sets numbers 2, 4 and 6, while the deflections taken from the positions of rest of sets numbers 3, 5 and 7 correspond to the changed position of the solenoid in the circuit. The following Table lists the deflections of the solenoid of the original position and the corresponding deflections of the changed position.

Set number	Calculated deflection for position I of the solenoid	Observed deflection for position II of the solenoid	Difference
3	842.15	841.56	−0.59
5	842.26	842.41	+0.15
7	837.44	835.80	−1.64
Average	840.62	839.92	−0.70

Also this series of observations has been repeated again in order to test the exactness attributed to these observations and we let this *fourth* series of observations follow immediately.

Fourth Series.

Bottom Setting of Commutator B.							
Set number 1, open circuit				Set number 2, closed circuit Top setting of commutator A			
Dynamometer no. 1		Dynamometer no. 2		Dynamometer no. 1		Dynamometer no. 2	
Observed elongations	State of rest	Observed elongations	State of rest	Observed elongations	State of rest	Observed elongations	State of rest
44.8		+4.3		840.9		592.2	
	36.50		−1.25				
28.2		−6.8		917.6	879.82	650.3	621.70
	36.35		−1.25				
44.5		+4.3		843.2	880.80	594.0	622.37
	36.20		−1.15				
27.9		−6.6		919.2	882.23	651.2	623.40
	36.25		−1.20				
44.6		+4.2		847.3	884.05	597.2	624.70
	36.35		−1.30				
28.1		−6.8		922.4		653.2	
Average 36.33		Average −1.23		Average 881.72		Average 623.04	
Set number 3, closed circuit Bottom setting of commutator A				Set number 4, closed circuit Top setting of commutator A			
794.5		559.9		772.9		543.9	
970.1	882.67	687.4	623.90	1005.7	886.77	712.1	626.20
796.0	882.95	560.9	624.00	762.8	883.15	536.7	623.57
969.7	885.48	686.8	625.82	1001.3	882.50	708.8	623.15
806.5	887.97	568.8	627.70	764.6	882.25	538.3	623.00
969.2		686.4		998.5		706.6	
Average 884.77		Average 625.36		Average 883.67		Average 623.98	
Set number 5, closed circuit Bottom setting of commutator A				Set number 6, closed circuit Top setting of commutator A			
744.1		524.2		988.3		698.9	
1023.3	883.92	724.3	624.45	779.0	883.07	550.1	623.95
745.0	883.75	525.0	624.32	986.0	879.20	696.7	621.00
1021.7	885.80	723.0	625.80	765.8	878.93	540.5	620.75
754.8	883.37	532.2	624.15	998.1	884.37	705.3	624.72
1002.2	879.68	709.2	621.60	775.5		547.8	
795.5		535.8					
Average 883.30		Average 624.06		Average 881.39		Average 622.61	

Set number 7, closed circuit Bottom setting of commutator A			
Dynamometer no. 1		Dynamometer no. 2	
Observed elongations	State of rest	Observed elongations	State of rest
960.0		678.2	
805.4	884.25	570.5	625.37
966.2	882.20	682.3	623.75
791.0	876.15	559.9	619.38
956.4	875.35	675.4	618.87
797.6		564.8	
Average 879.49		Average 621.84	

From the observations of this fourth series we obtain the following values for the sensitivity of the first dynamometer in parts of that of the second dynamometer:

from sets number 1 and number 2: $845.39/624.27 = 1.3542$,
from sets number 1 and number 4: $847.34/625.21 = 1.3553$,
from sets number 1 and number 6: $845.06/623.84 = 1.3546$,

hence the average ratio of the respective first and second dynamometer sensitivities equals

$$1.3547 : 1 \, .$$

Now multiplying this ratio 1.3547 by the observed deflections of the second dynamometer which has kept its position in the circuit during all observations, namely the deflections resulting from the differences of the positions of rest with open circuit in set number 1 and with closed circuit in sets numbers 3, 5 and 7, one obtains:

$$626.59, \qquad 625.29, \qquad 623.07 \, ,$$

so we get the products

$$848.84, \qquad 847.10, \qquad 844.10 \, ,$$

which yield the values of the deflections which would have been observed at the first dynamometer if the solenoid had kept its positions during the observations of sets numbers 2, 4 and 6, while the deflections taken from the positions of rest of sets numbers 3, 5 and 7 correspond to the changed position of the solenoid in the circuit. The following Table lists the deflections of the solenoid of the original position and the corresponding deflections of the changed position.

Set number	Calculated deflection for position I of the solenoid	Observed deflection for position II of the solenoid	Difference
3	848.84	848.44	−0.40
5	847.10	846.97	−0.13
7	844.10	843.16	−0.94
Average	846.68	846.19	−0.49

Comparing these averages found in the *fourth* series of observations with those obtained from the *third* series of observations, we find the ratio of the deflections at both positions in such an agreement, that it obviously suffices for all further considerations to take into account the averages of both series, namely for the deflections of the solenoid at position I:

$$843.65 \, ,$$

and at position II:

$$843.055 \, .$$

Now this yields the following values for the determination of the difference of the oscillation phases at position I and II according to Section 18.24:

$$u = \frac{1}{2} \arctan \frac{843.65}{2100} = 10°56'37.0'' \ ,$$

$$v'' = \frac{1}{2} \arctan \frac{843.055}{2100} = 10°56'11.7'' \ .$$

The latter task would need a minor correction if, according to the result of the previous Section, one were to take into account not the oscillation amplitudes at positions I and II considered as equal, but instead the little difference, although it cannot be guaranteed in any way. In that case, we would have to put $v'' = 10°56'13.2''$. Here however we will keep to the first task because there is no reason to assume such an inequality of the oscillation amplitude because in practice it cannot be safely demonstrated at all.

This finally yields the determination of the phase difference 2λ at both positions I and II according to Section 18.24

$$\sin^2 \lambda = \frac{u - v''}{2 \sin u(\cos u + [u - v''] \sin u)} = 0.000\,329 \ ,$$

hence

$$2\lambda = 2°4'43'' \ ,$$

which corresponds to about 1/87 of the period of oscillation.

Also this determination of the *phase difference* is based on such a small difference of the observed deflections, being equal to only 3/5 of a scale unit, that it *can be just as little considered as safely established by experience, as the small difference of the oscillation amplitude in the previous Section*.

18.29 Result of the Test

The observations described in Sections 18.27 and 18.28 serve to test the laws developed on the previous Section in what concerns the behavior of the *oscillation amplitudes* and *oscillation phases* at various positions of a closed conductor and thereby the identity of amplitudes and phases has been established even for very fast oscillations in a very long closed conducting wire. The method used for these observations of simultaneous corresponding observations by means of two dynamometers with exactly corresponding periods of oscillation allowed thereby a very high exactness, and this test could be carried out much further if the means for a still faster rotation of the magnet and for the production of still longer circuits were available. Such a further expansion of the test, however, would not suffice, as it seems, to justify the effort and the expenditure to be invested, and their expansion up to 520 oscillations per second and up to a length of 10 miles of the circuit would already suffice. Even if amplitudes and phases of electric oscillations in closed conductors were not generally equal everywhere, it is indeed clear that yet their differences should have to become the smaller, the *longer* the *period of oscillation* depending on the velocity of rotation, and the *shorter* the *conducting wire* would be, so that finally these differences should have to become unnoticeable with increasing period of oscillation and decreasing length of the conducting wire. Hence the intended test can only arrive at its aim if it were considerably extended beyond the limits within which such a compensation would have to occur in any case, and the question therefore arises whether a length of conductor of 10 miles and a period of oscillation of 1/520 of a second would serve this purpose.

Let a *simple wave train* start from a point and consider the same during the *first cycle*. Let the wave period be 1/520 of a second and the propagation velocity be equal to the usual $c\sqrt{\frac{1}{2}} = 41\,950$ miles [per second]. In this case, if it were realizable, a *decrease of the amplitude* of the electric oscillation would take place as the distance increases from the starting position of the wave train together with an increase of the *phase difference* with the same distance for any moment, both of which are easy to determine.

In fact, for such a *simple wave train* the displacement of an electric particle, σ, can be represented by the following equation:

$$\sigma = Ae^{-\varepsilon t} \cdot \sin 520\pi \left(t - \frac{s}{c}\sqrt{2}\right) ,$$

where, according to Section 18.15, we may approximately put

$$\varepsilon = \frac{c^2}{16\pi\alpha^2 kM''} ,$$

and

$$M'' = 2\log\frac{8a}{\alpha} .$$

As now further, according to Section 18.16,

$$\frac{1}{\pi\alpha^2 k} = w' = \frac{16 \cdot 10^6}{\pi\alpha^2 \cdot c^2} ,$$

hence

$$\varepsilon = \frac{10^6}{2\pi\alpha^2 \log\frac{8a}{\alpha}} ,$$

and with $2\pi a = 76 \cdot 10^6$ millimeter and $\alpha = 1/8$ millimeter for our circuit, we obtain

$$\varepsilon = 477\,000 .$$

The distance of $s = 30 \cdot 10^6$ millimeter (about 5 miles) now corresponds to a duration of

$$t = \frac{s}{c} \cdot \sqrt{2} = \frac{1}{8177} \text{ second,}$$

hence the ratio of the amplitudes at the starting point and at a distance of 5 miles (equals):

$$1 : e^{-\varepsilon t} = 1 : e^{-54.7} = 573 \cdot 10^{21} : 1 ,$$

whence the *amplitude* has become so small at a distance of 5 miles that it completely vanishes compared to that of the wave train at its starting point.

At a given moment the *phase difference* at the starting point of the wave train and at a distance $= s$ from there is represented by

$$520\frac{\pi s}{c} \cdot \sqrt{2} ,$$

thus equals $0.0636 \cdot \pi = 10°27'$ for $s = 38 \cdot 10^6$, a phase difference which can by no means be considered unnoticeable in view of the exactness allowed by the observations according to the previous Section.

According to these rough figures taken from the consideration of the elementary waves, the experiments described in the previous Sections may be considered as sufficient for the test of the laws established in the preceding Section for the ratios of the amplitudes and phases in closed conductors.

Finally let us remark that the identity of the oscillation amplitude at different positions at a large mutual distance of the closed conducting wire serves as proof, too, that the wire braided with silk may be considered as sufficiently insulated for electric currents like those produced by the rotating magnet; for with an insufficient insulation the currents would have been weaker far away from the inductor than closer to it.

18.30 Observations of the Dependence of the Oscillation Amplitude on the Rotation Velocity of the Magnet

After having confirmed the *identity of the oscillation amplitudes and oscillation phases* for the longest circuit and the fastest rotation velocity of the magnet possible with the present means, whence it is clear that this identity holds all the more for shorter circuits and lower rotational velocities, essentially only the test of the law of the *dependence of the oscillation amplitude on the rotation velocity of the magnet* for the circuit in question remains for the *quantitative test* of the laws developed in the preceding Section.

From the identity of the oscillation amplitudes and oscillation phases in all parts of a closed conductor, it follows all by itself that the intensity of the current in any point always equals the *average* of the current intensity in the whole conductor. Now the law for the *averages of the current intensities* in closed conductors depending on the *average values of the electromotive forces* has been developed in Section 18.9, independent of the consideration of the shape of the closed conductor, whence in Section 18.21 the law of this dependence has been determined more closely for the case when this *average value of the electromotive forces* changes in proportion to the sine which increases in proportion to time, which happens when the electromotive forces are produced by *rotation of a small magnet*. Hence putting namely the *average electromotive force* equal to $g_0 \sin \mu t$, this yielded the following law for the *average current intensity*

$$i = -\frac{g_0}{w'} \cdot \sin \rho_0 \cos(\mu t + \rho_0) \ ,$$

where w' denoted the resistance of the unit length of the conductor and where we had

$$\rho_0 = \frac{\pi a c^2 w'}{4\mu \int M_0'' ds} \ .$$

But, according to this law, with a magnet of given strength and position and a given circuit, for which the resistance w' and the coefficient $\int M_0'' ds$ depending on the shape of the circuit, as well as the factor g_0 depending on the strength of the magnet and on its position in the closed circuit, have definite values, the current intensity i depends still only on the faster or slower *rotation velocity* to be determined by μ, with $\mu/[2\pi]$ designating the number of revolutions per unit time.

However, besides the law of the *dependence of the current intensity on the rotation velocity of the magnet*, also the dependence of the *absolute value of the current intensity* i on the *absolute values of the constants* w', $\int M_0'' ds$ and g_0 in addition could be subjected to a further test; but in what concerns the dependence on w' and g_0, the same has already been tested for vanishing values of μ, which yields $\rho_0 = \pi/2$ and hence

$$i = \frac{g_0}{w'} \ ,$$

which constitutes the well known Ohm's law firmly based on experience; but in what concerns the dependence on $\int M_0'' ds$, this test would be easy to perform as soon as there were analytical methods at hand to determine the value of the constant $\int M_0'' ds$ from the shape of the closed conductor. The knowledge of this value for a *circular* conductor is not sufficient, because the observations requiring one *inductor* and two *dynamometers* cannot be carried out using a circular conductor.

The performance of the demanded *quantitative test*, however, requires an exact knowledge of the instruments by means of which the observations are made, in particular an exact knowledge of the *dynamometers* in use. Provided a practical device in order to regulate the mutual position of the *multiplier* and the *solenoid* of each dynamometer and the *period of oscillation of the solenoid* is at hand, this above all is a matter of practical *positioning* and then of the *test of the instrument*. The *positioning* of the instrument is to be performed in such a way that the axis of the *solenoid is horizontal and parallel to the magnetic meridian*; the axis of the *multiplier housing the solenoid shall be equally horizontal and perpendicular to the axis of the solenoid*. The center of the multiplier is to coincide with the center of the solenoid. If this is approximately done guided by external markings, it remains to be discussed how one could test by *observations* performed with the same instrument, whether the conditions are realized exactly, or how large the remaining *deviations* still are, as well as which experiments are needed to determine also those *elements of the instrument*, the knowledge of which is required for the *exact quantitative determinations* by means of the observations performed with them.

18.31 Test of the Dynamometer

Aiming at such a test of the dynamometer, we let the current of a *constant* voltaic pile equally pass through the multiplier of a *tangent galvanometer*, used to determine the current intensity, now *forward*, now *backward*, through the *solenoid* and through the *multiplier*, while the latter is connected with the solenoid, now in parallel, now crosswise, which is easily performed by means of a *commutator* whose twin cells are connected to the ends of the solenoid and multiplier wire. In all these 4 cases the *deflection* of the solenoid from the original equilibrium position is observed in the familiar way. These observations serve to determine

1. the deviation, μ, of the solenoid axis from the magnetic meridian at the original position of equilibrium,
2. the deviation, δ, of the angle between the solenoid axis and the multiplier axis from a right angle,

3. the ratio, ε, of the directive force, exerted by terrestrial magnetism on the solenoid with given current intensity, and the static directive force of the solenoid,

4. the ratio, $\varkappa$, of the directive force, exerted by the multiplier on the solenoid with given current intensity in both of them, and the static directive force of the solenoid.

The observation yields the *deflection* of the solenoid from the original equilibrium position in scale units, and dividing this number of scale units by double the distance, R, between the mirror and the scale in scale units, we get for smaller deflections the same expressed in *arc values*, which we denote by a, b, c, d for the 4 cases discussed. Then we get

$$\varkappa = \frac{1}{2}\left(\frac{c+b}{c-b} + \frac{d+a}{d-a}\right) ,$$

$$\varepsilon = \frac{da - cb - (db - ca)\varkappa}{d + c - b - a} ,$$

$$\delta = \frac{1}{2\varepsilon}\left(\frac{c+b}{c-b} - \frac{d+a}{d-a}\right) ,$$

$$\mu = \frac{1}{\varkappa}\Big(\varepsilon - (1 + \varkappa - \delta\varepsilon)a\Big) .$$

As a proof we only have to determine the *static*, the *geomagnetic*, and the *electrodynamic* torques acting on the solenoid, whose sum is to be put equal to zero for the equilibrium at the observed deflection.

Let s be the *static* directive force, then the *static* torque, at the deflection, φ, from the static equilibrium that had existed before a current passed through the circuit, is given by

$$= -s\sin\varphi .$$

Let further i be the *current intensity*, positive if the current carrying solenoid is equivalent to a magnet pointing northward with its north pole, let mi be the *geomagnetic* directive force, with m being the product of the horizontal part of terrestrial magnetism and the area encircled by the solenoid wire; finally let μ, as already mentioned, be the angle between the solenoid axis and the magnetic meridian at static equilibrium; then the *geomagnetic* torque equals

$$= -mi\sin(\varphi + \mu) ,$$

or, when μ is very small,

$$= -mi(\sin\varphi + \mu\cos\varphi) .$$

Let finally $(\pi/2 + \delta)$ be the angle between the multiplier axis pointing eastward and the solenoid axis pointing northward; let further ei^2 be the electrodynamic directive force exerted by the multiplier on the solenoid, with positive e when multiplier and solenoid are connected in such a way that the multiplier acting at a distance with i positive is equivalent to a magnet with its south pole pointing eastward; then the *electrodynamic* torque equals

$$= ei^2\cos(\varphi - \delta) ,$$

or, if δ is very small,

$$= ei^2(\cos\varphi + \delta\sin\varphi)\ .$$

Now the equilibrium condition for the solenoid with the observed deflection, φ, demands that the sum of the three torques equals to zero, that is

$$-s\sin\varphi - mi(\sin\varphi + \mu\cos\varphi) + ei^2(\cos\varphi + \delta\sin\varphi) = 0\ .$$

Dividing this equation by $-s\cos\varphi$ and noting that $\tan\varphi$ may be replaced by the *arc value of the observed deflection*, that is by a in the first of the four cases considered, we obtain the following equation

$$a + \frac{mi}{s}(a+\mu) - \frac{ei^2}{s}(1+\delta a) = 0\ .$$

In this *first* case, the current passed the solenoid wire in the *forward* direction and the solenoid was connected in *parallel* with the multiplier. In the *second* case, with the current also passing the solenoid wire in the *forward* direction but with the solenoid connected crosswise with the multiplier, the current intensity i stays positive, but e changes sign while the arc value of the deflection b observed in this case has to replace $\tan\varphi$, whence

$$b + \frac{mi}{s}(b+\mu) + \frac{ei^2}{s}(1+\delta b) = 0\ .$$

In the *third* case, with the current passing the solenoid wire *backwards*, but with the solenoid connected in *parallel* with the multiplier as in the first case, i changes sign and e is positive as in the first case, while the arc value of the deflection, c, observed in this case has to replace $\tan\varphi$, whence

$$c - \frac{mi}{s}(c+\mu) - \frac{ei^2}{s}(1+\delta c) = 0\ .$$

Finally in the *fourth* case with the current passing the solenoid wire *backwards* as in the third case, and with the solenoid connected *crosswise* with the multiplier, as in the second case, i is negative as in the third case and e is negative as in the second case, while the arc value of the deflection, d, observed in this case, has to replace $\tan\varphi$, whence

$$d - \frac{mi}{s}(d+\mu) + \frac{ei^2}{s}(1+\delta d) = 0\ .$$

Replacing $\varkappa$ by mi/s and ε by ei^2/s we get the given values of $\varkappa$, ε, δ and μ from these four equations.

Let us take as example the dynamometer used for the following experiments for which the observations yielded in terms of scale units:

$$\begin{aligned} 2Ra &= +440.01\ , \\ 2Rb &= -443.81\ , \\ 2Rc &= +448.26\ , \\ 2Rd &= -450.68\ . \end{aligned}$$

Here we had $2R = 5075$ scale units. It now follows from this that

$$
\begin{aligned}
\varkappa &= 0.008\,484\ , \\
\varepsilon &= 0.088\,0\ , \\
\delta &= -0.039\,7\ , \\
\mu &= +0.032\,3\ .
\end{aligned}
$$

The values of $\varkappa$ and ε, which are easy to refer to the normal values valid for the unit of the current intensity after i has been measured by means of a tangent galvanometer, yield the *figures for the strength of the solenoid and for the sensitivity of the dynamometer*. The other two values, δ and μ, on the other hand, refer to the *setup* and indicate the deviations of this setup under the conditions defined for them. In fact, this yields that, instead of a right angle, the solenoid axis makes the angle

$$\frac{\pi}{2} + \delta = 87°43'31''$$

with the multiplier axis and that the solenoid axis, instead of coinciding with the magnetic meridian at the *static* equilibrium, deviates eastward from it by the angle

$$\mu = 1°51'\ .$$

One can see from this that, with the instrument being equipped with fine gradings, the errors of the setup are very easy to correct. — Even if these small errors remain uncorrected, the observations made with this instrument can still be refined and those values can be calculated that would have been obtained for an exact setup.

For the purpose of the following oscillation experiments the latter deviation, designated by μ, does not come into question because of the very rapidly alternating sign of i, but only the deviation designated by δ, and for a deviation x' observed in terms of scale units, we easily arrive at the corrected value x, namely

$$x = x' - \frac{\delta x'^2}{2R} = x' + \frac{x'^2}{127\,780}\ .$$

The observations presented in the following Sections were the beginning of a common work performed by myself and R. Kohlrausch which has been interrupted by the illness and the passing away of my dear friend.[460] The devices used for the uniform fast rotation of the magnet and for the measurement of this velocity including the respective observations have been performed by him.

18.32 First Series

The following series of observations has been performed jointly by R. Kohlrausch and myself on April 12, 1857. It concerns the *dependence of the oscillation amplitude on the rotation velocity of the magnet* and was made using four different circuits but always using the same

[460][Note by AKTA:] Rudolf Hermann Arndt Kohlrausch (1809-1858) had collaborated with Weber on the measurment of his fundamental constant c, [Web55b] with English translation in [Web21c]; [WK56] with English translation in [WK03] and Portuguese translation in [WK08]; [KW57] with English translation in [KW21]; see also [WK68]. See Chapters 5, 6 and 7.

dynamometer and the same *inductor coil*, the latter consisting of one piece of a very fine copper wire, about 1500 meters long and having a resistance of $23 \cdot 10^{12}$ in absolute units, which was the constant part of the four circuits. In addition we used

in the first circuit A a piece 2800 meters long with resistance $= 8.79 \cdot 10^{12}$,
in the second circuit B a piece 5600 meters long with resistance $= 18 \cdot 10^{12}$,
in the third circuit C a piece 8000 meters long with resistance $= 27 \cdot 10^{12}$,
in the fourth circuit D a piece 10800 meters long with resistance $= 35 \cdot 10^{12}$.

The scale was in a fixed position at a distance of 2537.5 scale units from the little plane mirror fixed at the solenoid and made a right angle with the perpendicular of the mirror at the static equilibrium of the solenoid, and the vertical plane of the perpendicular of the mirror cut it at the 800th scale mark. The telescope mounted behind the scale could be displaced in such a way that, at static equilibrium of the solenoid, now the 800th scale marking, now a higher or lower scale marking could be observed, in order to enable observing the deflection of the solenoid also when it surpassed half of the scale width.

Circuit	Oscillation number m	Position of static equilibrium	Deflected position	Deflection in scale units y	x'	x
	288.80	361.69	1475.85	+1114.16	+1111.84	+1121.51
	217.80	362.52	1242.34	+879.82	+879.80	+885.86
	141.77	363.13	892.45	+529.32	+526.91	+529.08
	106.46	363.13	710.41	+347.28	+343.61	+344.53
A	289.88	363.13	1480.61	+1117.48	+1115.01	+1124.74
	170.26	1210.44	857.52	−352.92	−349.96	−349.00
	140.46	1210.44	681.63	−528.81	−527.08	−524.90
	215.10	1210.44	328.91	−881.53	−881.42	−875.34
	281.68	1210.44	97.05	−1113.39	−1109.88	−1100.24
	106.76	800.15	630.73	−169.42	−169.23	−169.01
	142.74	800.15	561.78	−238.37	−237.84	−237.40
	216.80	800.15	463.11	−337.04	−335.55	−334.67
	290.26	800.15	410.90	−389.25	−386.95	−385.78
B	106.84	800.24	969.02	+168.78	+168.59	+168.81
	144.15	800.24	1040.27	+240.03	+239.49	+239.94
	217.90	800.24	1136.55	+336.31	+334.84	+335.72
	288.04	800.24	1185.91	+385.67	+383.44	+384.59
	107.12	800.24	897.67	+97.43	+97.39	+97.46
	144.32	800.24	931.32	+131.08	+131.00	+131.13
	214.80	800.24	988.77	+170.89	+170.69	+170.92
	289.62	800.24	988.77	+188.53	+188.26	+188.54
C	108.60	800.24	701.11	−99.13	−99.09	−99.01
	143.68	800.24	669.66	−130.58	−130.50	−130.37
	219.78	800.24	628.57	−171.67	−171.47	−171.24
	286.90	800.24	611.76	−188.48	−188.21	−187.94
	108.42	800.58	731.93	−68.65	−68.63	−68.59
	144.44	800.58	711.98	−88.60	−88.57	−88.51
	217.68	800.58	689.18	−111.40	−111.35	−111.25
	289.56	800.58	679.54	−121.04	−120.97	−120.86
D	430.80	800.58	675.17	−125.41	−125.33	−125.21
	433.06	801.30	926.27	+124.97	+124.89	+125.01
	109.06	801.30	869.69	+68.39	+68.37	+68.41
	143.50	801.30	888.91	+87.61	+87.58	+87.64
	217.12	801.30	912.31	+111.01	+110.96	+111.06
	286.92	801.30	922.00	+120.70	+120.63	+120.74

The values of x' in the penultimate column are calculated using the deflections y listed in the foregoing column by means of the formula $x' = y - y^3/5075^2$, whence we obtain $x' = 2R\tan\varphi$ with sufficient precision when the observed deflection $y = R\tan 2\varphi$. Only for the observations on circuit A where the telescope was displaced, the deflections listed in the third column from the right do not directly give the values of $R\tan 2\varphi$, but the latter must be calculated as follows. Separating the value of the deflection, y', given in this column into two parts, namely into part y'' which reaches from the scale mark observed for the static equilibrium to the 800th scale mark, and into part y''' reaching from the 800th scale mark to the scale mark observed for the deflection, we get

$$y = R \tan 2\varphi = \frac{y'}{1 - \frac{y'' y'''}{R^2}} ,$$

from which the value of $x' = 2R \tan \varphi$ is easily calculated for $R = 2537.5$.

Now the values of x' calculated in this way have finally to be corrected as indicated at the end of the previous Section, whence the values of $x = x' + x'^2/127\,780$ listed in the last column are calculated.

18.33 Calculations of the Observations

The described observations serve to test the law developed in the previous Section concerning the dependence of the oscillation amplitude on the rotation velocity of the magnet. According to Section 18.24 the oscillation amplitude is expressed by $i/[\mu\mathfrak{E}]$, where i denotes the maximum intensity of the electric currents accompanying the electric oscillations. As now further according to Section 18.30 the current intensity in each moment of the oscillation is given by the value

$$-\frac{g_0}{w'} \sin \rho_0 \cos(\mu t + \rho_0) ,$$

hence the maximum intensity of the currents accompanying the electric oscillations is given by

$$i = \frac{g_0}{w'} \sin \rho_0 ,$$

where we had

$$\tan \rho_0 = \frac{\pi a c^2 w'}{4\mu \int M_0'' ds} ,$$

then, according to Section 18.24, we get the following expression for the *oscillation amplitude*:

$$\frac{i}{\mu\mathfrak{E}} = \frac{\pi a c^2 g_0}{\mu\mathfrak{E}\sqrt{16\mu^2 (\int M_0'' ds)^2 + \pi^2 a^2 c^4 w'^2}} .$$

Further the observations yield according to Section 18.24

$$i = \frac{1}{\pi a'} \cdot \sqrt{\frac{aS \tan v}{nn'}} ,$$

where v denotes the observed deflection of the solenoid, hence the oscillation amplitude equals

$$\frac{i}{\mu\mathfrak{E}} = \frac{1}{\pi a' \mu\mathfrak{E}} \cdot \sqrt{\frac{aS \tan v}{nn'}} = \frac{\pi a c^2 g_0}{\mu\mathfrak{E}\sqrt{16\mu^2 (\int M_0'' ds)^2 + \pi^2 a^2 c^4 w'^2}} .$$

For the test of the obtained laws by means of the observations this yields the value denoted by $\pm x = 2R \tan v$ in the last column of the observation Table in the preceding Section as given by

$$\pm x = \frac{\pi^4 a a'^2 n n' c^4 g_0^2}{16\mu^2 (\int M_0'' ds)^2 + \pi^2 a^2 c^4 w'^2} \cdot \frac{2R}{S} \ .$$

But now μ/π is the oscillation number[461] denoted by m in the Table and g_0 is proportional to the rotation velocity according to the laws of magnetic induction, or $g_0 = m g_0'$, where g_0' denotes the value of g_0 for the oscillation number $m = 1$; hence we have

$$\pm x = \frac{\pi^2 a a'^2 n n' c^4 {g_0'}^2 m^2}{16 (\int M_0'' ds)^2 \cdot m^2 + a^2 c^4 w'^2} \cdot \frac{2R}{S} \ ,$$

or, putting

$$C = \frac{\pi^2 a'^2 n n' {g_0'}^2}{a w'^2} \cdot \frac{2R}{S} \ ,$$

[and]

$$P = \frac{16 (\int M_0'' ds)^2}{a^2 c^4 w'^2} \ ,$$

[one gets]

$$C - Px - \frac{x}{m^2} = 0 \ ,$$

where C and P have constant values for all observations which have been made by means of the same circuit, the same rotating magnet, and the same dynamometer. Here the value of x, ignoring its sign, is always to be taken *positive*.

Hence the observations listed in the Table of the preceding Section yield the following equations for the determination of the constants C and P for the circuit A:

$$C - 1121.51P - \frac{1121.51}{288.80^2} = 0 \ ,$$

$$C - 885.86P - \frac{885.86}{217.80^2} = 0 \ ,$$

$$C - 529.08P - \frac{529.08}{141.77^2} = 0 \ ,$$

$$C - 344.53P - \frac{344.53}{106.46^2} = 0 \ ,$$

$$C - 1124.74P - \frac{1124.74}{289.88^2} = 0 \ ,$$

$$C - 349.00P - \frac{349.00}{107.26^2} = 0 \ ,$$

$$C - 524.90P - \frac{524.90}{140.46^2} = 0 \ ,$$

[461] [Note by AKTA:] In German: *Schwingunszahl*. On page 366 of Section 18.30 Weber had designated by $\mu/(2\pi)$ the number of revolutions per unit time.

$$C - 875.34P - \frac{875.34}{215.10^2} = 0 \ ,$$

$$C - 1100.24P - \frac{1100.24}{281.68^2} = 0 \ ,$$

from which we obtain the most probable values of C and P, namely:

$$C = 0.037\,978 \ , \qquad P = 0.000\,021\,865 \ .$$

Calculating the values of C and P for the circuits B, C, and D in a similar way, one gets the results listed in the following Table.

Circuit	C	P
A	0.037 978	0, 000 021 865
B	0.022 795	0.000 047 050
C	0.015 174	0.000 068 093
D	0.011 869	0.000 087 274

Now finally calculating the values of x from the given values of m one gets, in progressive order of the values of m, the following comparison of the calculated values of x and those found by observation:

A				B			
m	Observed value of x	Calculated value of x	Difference	m	Observed value of x	Calculated value of x	Difference
106.46	344.53	344.96	−0.43	106.76	169.01	169.12	−0.11
107.26	349.00	349.11	−0.11	106.84	168.81	169.28	−0.47
140.46	524.91	523.48	+1.43	142.74	237.40	237.13	+0.27
141.77	529.08	530.29	−1.21	144.15	239.94	239.51	+0.43
215.10	875.34	873.50	+1.84	216.80	334.67	333.63	+1.04
217.80	885.86	884.34	+1.52	217.90	335.72	334.68	+1.04
281.68	1100.24	1101.83	−1.59	288.04	384.59	385.69	−1.10
288.80	1121.51	1121.81	−0.30	290.26	385.78	386.89	−1.11
289.88	1124.74	1124.77	−0.03				
C				D			
107.12	97.46	97.74	−0.28	108.42	68.59	68.87	−0.28
108.60	99.01	99.25	−0.24	109.06	68.41	69.27	−0.86
143.68	130.37	130.21	+0.16	143.50	87.64	87.38	+0.26
144.32	131.13	130.69	+0.44	144.44	88.51	87.79	+0.72
214.80	170.92	169.04	+1.88	217.12	111.06	109.41	+1.65
219.78	171.24	170.89	+0.35	217.68	111.25	109.52	+1.73
286.90	187.94	189.10	−1.16	286.92	120.74	119.38	+1.36
289.62	188.54	189.64	−1.10	289.56	120.86	119.69	+1.17
				430.80	125.21	128.09	−2.88
				433.06	125.01	128.17	−3.16

Now this first test can be tied to another test. Indeed, according to the established law the two constants C and P should be inversely proportional to the square of the average resistance of the unit length of the conducting wire, w'; but the other quantities on which their values depend according to the presented formulas:

$$C = \frac{\pi^2 a'^2 nn' g_0'^2}{aw'^2} \cdot \frac{2R}{S} ,$$

[and]

$$P = \frac{16(\int M_0'' ds)^2}{a^2 c^4 w'^2} ,$$

are not equal for all four circuits to which the observations described in the previous Section refer, but the magnitude g_0' on which the constant C depends, and the magnitude $\int M_0'' ds$ on which the constant P depends, have a special value for each circuit. But now designating the total length of the conducting wire by l which also has different values for the four circuits and minding that $g_0' \sin \mu t$ at the time t denotes the *average* electromotive force exerted by the rotating magnet at the velocity for which $m = 1$ *on any unit length of the total conducting wire*; then $lg_0' \sin \mu t$ at the time t denotes the electromotive force exerted by the rotating magnet at the velocity for which $m = 1$ *on the total conducting wire*. As now the induction by the rotating magnet acted only on the inductor wire common to all four circuits, it follows that lg_0' has the same value for all four circuits; hence the formula

$$C = \frac{\pi^2 a'^2 nn' g_0'^2}{aw'^2} \cdot \frac{2R}{S} = \frac{\pi^2 a'^2 nn' (lg_0')^2}{a} \cdot \frac{2R}{S} \cdot \frac{1}{(lw')^2} ,$$

yields that the values of the constant C must be inversely proportional to the squares of the resistances of these four circuits; for lw' designates the resistance of the whole circuit, as w' was the average resistance of the unit length.

In order to test the established law also with this respect, the values of the resistances of these four circuits presented at the beginning of the previous Section must be added to the observations considered, we note, however, that their determination was not considered the main purpose of the previous considerations, but, without claiming special exactness (they were based partly on the mere comparison in terms of wire lengths) was to serve only as a short description to distinguish the four circuits. However, we use also these determinations for a test of the established law because they, like all other observations presented, had been determined several years ago without respect to the laws developed here.

But, as mentioned at the beginning of the previous Section, according to this determination the resistances of the four circuits are roughly in proportion to

$$31.79 : 41 : 50 : 58 ,$$

while, according to the established law, dividing the number 6.2158 by $\sqrt{C}$, the same proportions are obtained as

$$31.89 : 41.17 : 50.46 : 57.06 ,$$

which agrees quite well with the above experimental results.

18.34 Second Series

The following series of observations, also jointly performed by Kohlrausch and myself on April 18 and 22, 1857, is basically a repetition of the preceding one, but other than the whole conducting wire consisting of simply wound wire spools in the former series, this was valid in the following [series] only for circuit A, while another circuit was formed by switching a *twin wire* E into circuit A. This *twin wire* E was composed of two very fine copper wires braided by silk and hence insulated, but fixed together by another common braiding, in order to reduce the damping of electric oscillations caused by mutual induction in accordance with the prescription of Section 18.26, which should thereby be tested. — This damping was in a way additionally reduced by the fact that the twin wire E was mounted on a special support keeping all windings at least 20 millimeters away from each other instead of being wound up on a spool.

According to Section 18.32 the circuit A had a length of some 4300 meters with a resistance, in absolute measure, equal to $3179 \cdot 10^{10}$. The twin wire E had a length of 1412 meters (hence the single wire was 2824 meters long) and a resistance equal to $4292 \cdot 10^{10}$.

The setup of the scale and the reading telescope was identical with what was mentioned in Section 18.32 for the previous series of observations. — For each circuit, two series of observations have been performed, the first one on April 18, the second one on April 22.

Circuit	Oscillation number m	Position of static equilibrium	Deflected position	Deflection in scale units y	x'	x
	107.22	801.98	728.42	−73.56	−73.54	−73.50
	139.66	801.98	681.37	−120.61	−120.54	−120.40
	214.10	801.98	542.95	−259.03	−258.36	−257.84
	279.44	801.98	402.21	−399.77	−397.29	−396.06
$A + E$	108.02	801.98	876.36	+74.38	+74.36	+74.40
	141.82	801.98	926.06	+124.08	+124.01	+124.13
	213.76	801.98	1059.59	+257.61	+256.95	+257.47
	282.80	801.98	1206.57	+404.59	+402.03	+403.29
	280.58	389.64	1468.13	+1078.49	+1075.80	+1084.86
	214.12	425.42	1281.09	+855.67	+855.32	+861.05
	142.58	425.42	954.80	+529.38	+528.38	+530.56
	141.66	425.42	948.80	+523.38	+522.36	+524.50
A	103.36	425.42	762.90	+337.48	+335.26	+336.14
	283.60	1193.13	86.44	−1106.69	−1102.50	−1092.99
	215.16	1193.13	323.28	−869.85	−869.06	−863.15
	139.82	1193.13	676.38	−516.75	−515.29	−513.21
	110.53	1193.13	827.49	−365.64	−363.10	−362.07
	106.10	801.52	730.24	−71.28	−71.26	−71.22
	142.88	801.52	676.72	−124.80	−124.73	−124.61
	214.80	801.52	543.31	−258.21	−257.55	−257.03
	279.92	801.52	404.02	−397.50	−395.05	−393.83
$A + E$	107.43	801.43	873.91	+72.48	+72.46	+72.50
	141.94	801.43	924.38	+122.95	+122.88	+123.00
	215.64	801.43	1060.12	+258.69	+258.03	+258.55
	279.30	801.43	1194.00	+392.57	+390.22	+391.41
	282.02	384.83	1465.47	+1080.64	+1077.20	+1086.28
	217.17	385.13	1249.53	+864.40	+864.14	+869.98
	139.59	385.13	893.92	+508.79	+506.73	+508.74
	107.15	385.13	728.92	+343.79	+340.65	+341.56
A	281.96	1202.32	106.96	−1095.36	−1090.85	−1081.54
	217.14	1202.30	329.47	−872.83	−872.43	−866.48
	144.32	1202.30	664.30	−538.00	−536.49	−534.24
	139.56	1202.30	690.03	−512.27	−510.54	−508.50
	106.64	1202.30	860.67	−341.63	−340.30	−339.39

The calculation of the x values in this Table is the same as has been discussed for the preceding Table.

Now from the corresponding values of m and x in this Table we can calculate the most probable values of the constants C and P for the circuits A and $A + E$ in the same way, as in Section 18.33 from the values of Section 18.32 for the circuits A, B, C, D. In this way, the results listed in the following Table have been obtained.

Circuit	C	P
$A+E$	0.006 672	0, 000 004 041
A	0.037 63	0.000 021 96
$A+E$	0.006 610	0.000 004 083
A	0.037 14	0.000 021 63

Finally calculating the x values from the given values of m, we obtain, in successive order of the m values, the following comparison of the values calculated for x and the values found by observation.

$A+E$				A			
m	Observed value of x	Calculated value of x	Difference	m	Observed value of x	Calculated value of x	Difference
107.22	73.50	73.29	+0.21	103.36	336.14	335.82	+0.32
108.02	74.40	74.34	+0.06	110.53	362.07	362.41	−0.34
139.66	120.40	120.63	−0.23	139.82	513.21	514.68	−1.47
141.82	124.13	124.10	+0.03	141.66	524.50	524.09	+0.41
213.76	257.47	257.34	+0.13	142.58	530.56	528.81	+1.75
214.10	257.84	258.04	−0.20	214.12	861.05	859.37	+1.68
279.44	396.06	396.02	+0.04	215.16	863.15	863.68	−0.53
282.80	403.29	402.55	+0.74	280.58	1084.86	1085.42	−0.56
				283.60	1092.99	1093.l70	−0.71
$A+E$				A			
106.10	72.22	71.14	+0.08	106.64	339.39	338.98	+0.41
107.43	72.50	72.85	−0.35	107.15	341.56	341.58	−0.02
141.94	123.00	123.04	−0.04	139.56	508.50	508.97	−0.47
142.88	124.61	124.56	+0.05	139.59	508.74	509.12	−0.38
214.80	257.03	256.62	+0.41	144.32	534.24	533.30	+0.94
215.64	258.55	258.32	+0.23	217.14	866.48	866.96	−0.48
279.30	391.41	391.07	+0.34	217.17	869.98	867.08	+2.90
279.92	393.83	392.37	+1.46	281.96	1081.54	1085.70	−4.16
				282.02	1086.28	1085.85	+0.43

Finally, as indicated at the beginning of this Section, the observed resistances of the circuits $A+E$ and A to each other are roughly in the ratio of

$$7471 : 3179\ ,$$

while this ratio, according to the law presented in the preceding Section, applied here in the same way, as the values of the constant C for both circuits should be inversely proportional to the squares of the resistances of these circuits, and this proportion is obtained dividing 611.75 by $\sqrt{C}$ as

$$7507 : 3164\ ,$$

which agrees with the proportions obtained from the observations as far as the modest exactness of the resistance measurement justifies to expect.

18.35 Proportion of the Electrostatic Force of Two Equal Amounts of Electricity to Their Mass

Finally, the third subject for more detailed observations according to Section 18.23 remains to be considered, namely *the dependence of the amplitudes of the oscillations produced by a rotating magnet in a closed conductor on the shape of the conducting wire.* Exact observations concerning this subject can not only serve to test the formulated laws but can moreover, as already discussed in Section 18.23, be used to expand our knowledge about electricity, namely, in order to determine the still unknown *ratio of the force due to the electrostatic interaction between two equal amounts of electricity to their mass.*

Let $\mathfrak{E}$ be the amount of positive electricity in the unit length of the conducting wire expressed in electrostatic measure, then it exerts the electrostatic force equal to $\mathfrak{E}^2$ at unit distance on an equal amount of electricity, while its mass has been expressed[462] by $[1/r]\mathfrak{E}$, which yields the unknown proportion of that force to this mass,

$$\frac{\mathfrak{E}^2}{\frac{1}{r}\mathfrak{E}} = \frac{r\mathfrak{E}}{1} \ .$$

If now this proportion, or the unknown quantity $r\mathfrak{E}$ has a value comparable with other determined quantities, then it can be easily shown that this value could be determined most exactly from observations of the *dependence of the oscillation amplitude on the shape of the circuit.*

From the differential equations of electric motions in closed conductors established in Sections 18.8 and 18.10, it is clear that they contain the magnitude $r\mathfrak{E}$ only in the expression

$$\frac{4M''(1+\lambda)}{c^2} = \frac{4M''}{c^2} + \frac{1}{r\mathfrak{E}} \ .$$

However, by no means not all effects determinable from the differential equations depend on this expression contained in the differential equations; because for the determination of some effects the differential equations can be simplified so that this expression does not occur at all any more. As shown in Sections 18.11 and 18.12, this holds for all equilibrium effects or for the conservation of already existing motions, whence it follows vice versa that observations of equilibrium effects or of effects due to steady currents cannot serve to determine the quantity $r\mathfrak{E}$.

The other effects, on the contrary, determined when the expression containing $r\mathfrak{E}$ does not vanish from the differential equations, comprise the *electric oscillations* produced in a closed conducting wire by induction due to a rotating magnet whose laws have been developed in Section 18.20 from these differential equations, according to which the current intensity of such an oscillation in the conducting wire was obtained as given by

$$i = -\frac{1}{w'}\sqrt{f^2+g^2} \cdot \cos\left(\mu t + \arctan\frac{f}{g}\right) \ ,$$

if we had

$$f = \sum \sin^2\rho \left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a}\right) \ ,$$

[462][Note by AKTA:] See page 288 on Section 18.5.

$$g = \sum \sin\rho\cos\rho\left(f_n \sin\frac{ns}{a} + g_n \cos\frac{ns}{a}\right) \ ,$$

$$\tan\rho = \frac{\mu a^2 c^2 w'}{4(2\mu^2 a^2 M''(1+\lambda) - n^2 c^2 N'')} = \frac{\mu a^2 w'}{2\mu^2 a^2 \left(\frac{4M''}{c^2} + \frac{1}{r\mathfrak{E}}\right) - 4n^2 N''} \ .$$

According to Section 18.21 this determination of the current intensity i is simplified in most cases, namely those cases in which all remaining terms may be considered as vanishingly small compared with the one corresponding to the place number $n = 0$. Indeed for these cases we obtain

$$i = -\frac{g_0}{\sqrt{w'^2 + 4\mu^2\left(\frac{4M_0''}{c^2} + \frac{1}{r\mathfrak{E}}\right)^2}} \cdot \cos\left(\mu t + \arctan\frac{w'}{2\mu\left(\frac{4M_0''}{c^2} + \frac{1}{r\mathfrak{E}}\right)}\right) \ ,$$

where, according to Section 18.10, we have to put

$$M_0'' = 2\log\frac{8a}{\alpha} \ .$$

Hence it follows that the magnitude $r\mathfrak{E}$ could be found out by measuring the current intensity i when we have the rotation velocity of the magnet, namely $\mu/[2\pi]$ turns per second, the length $= 2\pi a$, the thickness $= 2\alpha$, and the resistance $= 2\pi a w'$ of the conducting wire, because also the induction coefficient g_0, proportional to the rotation velocity, can be determined from the strength of the inducing magnet and from its distance and position with respect to the induced conducting wire.

Hence it would be possible to directly determine *directly* the magnitude $r\mathfrak{E}$ *without considering the dependence of the amplitude of electric oscillations on the shape of the conducting wire*; it is easily seen, however, that this *direct* way cannot practically lead to any exact result if the magnitude $1/[r\mathfrak{E}]$ is a very small fraction of the magnitude $4M_0''/c^2$, especially when considering that the determination of M_0'' and c cannot be performed very precisely. Considering in addition, however, the dependence of the amplitude of the electric oscillations on the shape of the conducting wire, the following *indirect* way allows to determine the magnitude $r\mathfrak{E}$ with much more precision.

In order to use the observation of the dependence of the amplitude of electric oscillations on the shape of the conducting wire for a more exact determination of the magnitude $r\mathfrak{E}$, it is essential to find a method to give a closed conducting wire *two different shapes*, which allow to exactly determine either the two values of M_0'', or at least their ratio $\nu : 1$.

Here it comes into question that in the first part of this treatise the development of the laws of motion of electricity had to be restricted to *circular conductors*, for which an essential simplification could be achieved because the values of the integrals denoted by M, N, M'' and N'' were equal for all points of the conductor shape. But the latter also holds for a system consisting of two equal and parallel circles, which could not be taken into consideration only because it forms two separate conductors. For practical purposes when performing the observations, however, such a system may be substituted by a closed double winding conductor with respect to almost all considerations, and hence, according to this substitution, the values of the definite integrals M, N, M'' and N'' may be put equal for all points of a closed conductor which forms two equal very close circular windings, whereby it is possible to expand the laws of motion of electricity, at first formulated for a circular conductor only, to a closed conductor consisting of two equal very close circular windings.

Such a conductor, however, as we see easily, presents a very essential *alternative* concerning the variety of the connection between both of its windings, which may *either* provide the same current passing through both windings in series in the same direction, *or* passing through the second winding in opposite direction as the first one. These two cases correspond to completely different values of M, N, M'' and N'' whose ratio can be determined exactly. In fact the same holds also for the value of M'' for $n = 0$, denoted by M_0'', and we may put the ratio of both values of M_0'' for the two above cases equal to $\nu : 1$.

Let A be the greatest current intensity of an electric oscillation in this conductor in the *first* case, and B in the *second* case, then we have, according to Section 18.23,

$$A = \frac{g_0}{\sqrt{w'^2 + 4\mu^2 \left(\frac{4M_0''}{c^2} + \frac{1}{r\mathfrak{E}}\right)^2}} \ ,$$

$$B = \frac{g_0}{\sqrt{w'^2 + \mu^2 \left(\frac{4M_0''}{\nu c^2} + \frac{1}{r\mathfrak{E}}\right)^2}} \ ,$$

and *both values*, A *and* B, *can be determined by measurement*; but here we assumed that the induction due to the rotating magnet does not extend to both windings of the closed conductor, but that the induction be limited to one of both windings or only to one part of it. In reality, this element is represented by the small *inductor* which houses the rotating magnet.

But now these values of A and B can be determined by measurement for different rotation velocities, that is, for different values of μ, and it is clear that their difference must vanish completely before long with decreasing values of μ. Now let μ_0 be the small value of μ for which this difference is not noticeable. Let then C denote the values of A and B when considered equal; then we get

$$C = \frac{\mu_0}{\mu} \cdot \frac{g_0}{w'} \ .$$

because the coefficient of induction, denoted by g_0, is proportional to the rotation velocity. *Also this third value*, C, *can be determined by measurement*.

Eliminating now g_0 from the three obtained equations where A, B, C are known by measurement, we obtain the following two equations:

$$\frac{4M_0''}{c^2} + \frac{1}{r\mathfrak{E}} = \frac{w'}{2\mu}\sqrt{\frac{\mu^2}{\mu_0^2} \cdot \frac{C^2}{A^2} - 1} \ ,$$

$$\frac{4M_0''}{\nu c^2} + \frac{1}{r\mathfrak{E}} = \frac{w'}{2\mu}\sqrt{\frac{\mu^2}{\mu_0^2} \cdot \frac{C^2}{B^2} - 1} \ ,$$

and hence follows:

$$\frac{4M_0''}{c^2} = \frac{\nu}{\nu - 1} \cdot \frac{w'}{2\mu} \left\{ \sqrt{\frac{\mu^2}{\mu_0^2} \cdot \frac{C^2}{A^2} - 1} - \sqrt{\frac{\mu^2}{\mu_0^2} \cdot \frac{C^2}{B^2} - 1} \right\} \ ,$$

$$\frac{1}{r\mathfrak{E}} = \frac{1}{\nu - 1} \cdot \frac{w'}{2\mu} \left\{ \nu \sqrt{\frac{\mu^2}{\mu_0^2} \cdot \frac{C^2}{B^2} - 1} - \sqrt{\frac{\mu^2}{\mu_0^2} \cdot \frac{C^2}{A^2} - 1} \right\} .$$

The *first* of theses equations where all quantities are known may either serve to test the theory or the observations, while the unknown quantity $r\mathfrak{E}$ is found from the *second* equation, where not even the closer determination of the quantities M_0'', g_0, and C[463] is required, but only that of the ratio $\nu : 1$.

The latter of both equations thus found can finally be given a somewhat simpler form noting that the resistance of the unit length of the conducting wire equals $w' = 1/[\pi\alpha^2\varkappa]$, with α the radius of the wire and $\varkappa$ the specific conductivity of the metal, and that further the amount of positive electricity expressed in electrostatic units of measure contained in the unit length of the conducting wire equals $\mathfrak{E} = \pi\alpha^2 \cdot \mathfrak{E}_0$, where $\mathfrak{E}_0$ denotes the positive amount of electricity *in the unit volume* of the conducting wire. Substituting these values we get

$$\frac{1}{r\mathfrak{E}_0} = \frac{1}{2(\nu - 1)\mu\varkappa} \cdot \left\{ \nu \sqrt{\frac{\mu^2}{\mu_0^2} \cdot \frac{C^2}{B^2} - 1} - \sqrt{\frac{\mu^2}{\mu_0^2} \cdot \frac{C^2}{A^2} - 1} \right\} .$$

Hence it remains to be considered how to determine the ratio denoted by $\nu : 1$.

Let $2\pi a$ be the length of the total closed conducting wire as before, each half of which, equal to πa, constitutes one winding, and consider both of these windings as parallel circles the centers of which lie at a distance, δ, perpendicular to the plane of the circle; then the value of M_0'' may be split into two parts at any point of that entire conducting wire, namely the part due to the circle containing the point under consideration and the part due to the other circle at a distance $= \delta$ from the first circle. The *former* part is immediately found equal to the value of M_0'' for a circle of radius $= \frac{1}{2}a$, namely, according to Section 18.16, equal to double the logarithm of the ratio of the 8-fold circle radius to the wire radius, $= 2\log(4a/\alpha)$. As is easily shown, the *latter* part is obtained from the former part by merely substituting the radius, α, by the distance between the two circles, δ, namely $= 2\log(4a/\delta)$. — If now the two windings are connected in such a way that the current passes them in the same direction, then the value of M_0'' of the entire closed conductor at any point of its first or second winding equals the *sum* of both parts,

$$= 2\log\frac{4a}{\alpha} + 2\log\frac{4a}{\delta} ;$$

if, on the other hand, both windings are connected so that the second winding is passed in the counter sense as the first one, then the value of M_0'' equals the *difference* of both parts,

$$= 2\log\frac{4a}{\alpha} - 2\log\frac{4a}{\delta} ;$$

Hence we get the desired ratio as given by

$$\nu : 1 = \left(2\log\frac{4a}{\alpha} + 2\log\frac{4a}{\delta}\right) : \left(2\log\frac{4a}{\alpha} - 2\log\frac{4a}{\delta}\right) = \left(\frac{2\log\frac{4a}{\alpha}}{\log\frac{\delta}{\alpha}} - 1\right) : 1 .$$

[463][Note by AKTA and PM:] In the original we have here c instead of C. We are replacing the lowercase c by the uppercase C.

18.36 Conclusion

The discussions concerning the determination of the magnitude $r\mathfrak{E}$ in the previous Section serve mainly to specifically exemplify that *the dependence of the amplitude of the oscillation produced by a rotating magnet in a closed conductor on the shape of the conducting wire*, as presented in Section 18.23, constitutes a third topic important and particularly suitable for more *exact observations*, deserving a more careful and more extensive treatment because of its multiple meaning. If the performance of exact observations on this topic is to be of practical use, it becomes clear that this has to be done in connection *first* with the extensive discussion of the dependence of the values of the integrals, denoted by N, N'', M and M'' in Section 18.8, on the shape of the conducting wire, restricted to the single case of a circular wire in Section 18.10, and *second* with a specific discussion of the value of the magnitude $r\mathfrak{E}$, the determination of which is of great interest all by itself, as has been treated in the preceding Section. Now it has indeed been shown in the preceding Section how the determination of this magnitude $r\mathfrak{E}$ would be possible from observations of a *special case* of the dependence of the oscillation amplitude on the shape of the conducting wire without entering into an extensive discussion of this dependence in general; but this, however, still demands a lot of work and observations even for the solution of the hereby essentially simplified and limited task, for which the measures described and used in this treatise are not sufficient.

As now furthermore also the determination of the magnitude $r\mathfrak{E}$ concerned less the test of the laws developed in the first part of this treatise, but rather a novel application of the theory with a particular and separate aim — appropriate as a topic for a special treatise —; it seems adequate to restrict beforehand the *performance of observations*, intended in the second part of this treatise and concerning a test of the laws, to the *observations, presented in the first section, concerning the first two topics presented in Section 18.23* — [namely, (1),] comparison of amplitudes and phases of electric oscillations at various places of a long conducting wire — [and (2), the] law of the dependence of the oscillation amplitude on the rotation velocity of the magnet —; and to reserve the performance of *all the more exact observations concerning the third topic* — namely, the law of the dependence of the oscillation amplitude on the shape of the conducting wire, including the respective special questions and tasks according to the instruction of the preceding Section — for a future treatise for which, finally, the results of the present treatise were intended here as the preparatory foundations only.

Chapter 19

The Origins and Meanings of Weber's Constant c, of Maxwell's Constant c and the Relation of these Two Different Constants with Light Velocity in Vacuum

A. K. T. Assis[464]

I present the origins and meanings of Wilhelm Weber's constant c and of Maxwell's constant c. Although these two constants are represented by the same symbol, they refer to different magnitudes which have different numerical values. Weber's constant c is equal to Maxwell's constant c times the square root of two. I also discuss the relations of these two different constants with light velocity in vacuum.

19.1 Weber's Constant c

In 1846 Wilhelm Weber unified electrostatics (Coulomb's force of 1785), electrodynamics (Ampère's force between current elements of 1822 and 1826) and Faraday's law of induction (1831). In Section 19 of his work, Weber presented the following force between two particles electrified with charges e and e' and separated by a distance r:[465]

$$\frac{ee'}{r^2}\left[1-a^2\left(\frac{dr}{dt}\right)^2+b\frac{d^2r}{dt^2}\right] . \tag{19.1}$$

In this Equation a is a constant with the inverse dimension of a velocity and b another magnitude which he showed to be given by:[466]

$$b=2ra^2 . \tag{19.2}$$

[464] Homepage: `www.ifi.unicamp.br/~assis`
[465] [Web46, Section 19, p. 142 of Weber's *Werke*] and [Web07, Section 19, p. 87].
[466] [Web46, p. 144 of Weber's *Werke*] and [Web07, p. 89].

By combining Equations (19.1) and (19.2) he expressed his fundamental law as:

$$\frac{ee'}{r^2}\left[1-a^2\left(\frac{dr}{dt}\right)^2+2a^2\cdot r\frac{d^2r}{dt^2}\right] . \tag{19.3}$$

In Section 21 of the same work he introduced another constant by the same letter a, emphasizing that its value was different from the constant a introduced in Section 19. He then expressed his force law in terms of this new constant a as follows:[467]

$$\frac{ee'}{r^2}\left[1-\frac{a^2}{16}\left(\frac{dr}{dt}\right)^2+\frac{a^2}{8}r\frac{d^2r}{dt^2}\right] . \tag{19.4}$$

In 1848 he presented a potential energy from which he could deduce his force.[468]

In 1852 he replaced the constant $a^2/16$ of Equation (19.4) by a constant $1/c^2$ and expressed his force as follows:[469]

$$\frac{ee'}{r^2}\left[1-\frac{1}{c^2}\cdot\left(\frac{dr}{dt}\right)^2+\frac{2r}{c^2}\frac{d^2r}{dt^2}\right] . \tag{19.5}$$

This was the first time in which Weber introduced this constant $c = 4/a$. This constant c had the dimension of a velocity, that is, distance per time. In 1852 Weber had no idea of its numerical value (it might be, for instance, $10^5\ m/s$ or $10^{11}\ m/s$). Moreover, at that time no one had the slightest idea that it might have any relation whatsoever with light velocity in vacuum. At that time it was a purely electrodynamic constant.

The reason why he chose the letter c for this new constant was probably alphabetical. After all, he had already utilized the letters a and b in the expression of his fundamental force law.

He showed, in particular, that when two particles were moving relative to one another with a constant relative velocity $dr/dt = \pm c$, such that $d^2r/dt^2 = 0$, they would exert no net force on one another according to his law. This was the meaning he gave to this constant, namely:[470]

> The meaning of c is that constant value of the relative velocity dr/dt, in which two electric masses have no effect on each other.

He expressed the same meaning of his constant c in later works.[471] For instance:

> The meaning of the constant c is that of a well-defined velocity, and indeed the velocity with which two electric masses must approach or separate from each other if neither attraction nor repulsion is to exist between them.

In 1869 he also expressed his potential energy in terms of this constant c:[472]

[467] [Web46, Section 21, pp. 152 and 157 of Weber's *Werke*] and [Web07, Section 21, pp. 94 and 98].
[468] [Web48a, p. 245 of Weber's *Werke*] with English translation in [Web52e], [Web66d] and [Web19].
[469] [Web52c, p. 366 of Weber's *Werke*] with English translation in [Web21b].
[470] [Web52c, p. 367 of Weber's *Werke*] with English translation in [Web21b].
[471] [Web55b, p. 594 of Weber's *Werke*], [WK56, p. 20 of the 1856 paper and p. 605 of Weber's *Werke*], [WK03, p. 294], [WK08, p. 99] and [KW57, pp. 617 and 651].
[472] [Web69, p. 243 of Weber's *Werke*] with English translation in [Web21g].

$$\frac{ee'}{r}\left[1-\frac{1}{c^2}\cdot\left(\frac{dr}{dt}\right)^2\right] . \tag{19.6}$$

Weber was also involved in determining current intensities and their dimensions through the effects produced by these currents. The electrodynamic measure of current intensity was related to Ampère's force between two current elements and the analogous force between two current-carrying wires.[473] The electromagnetic measure of current intensity was related to the torque exerted by a current carrying wire on a magnet. The electrolytic measure of current intensity was related to the amount of water that is decomposed when a current flows through it. He also determined current intensity and its dimension through the origin of a current, that is, by the amount of charge which flows through the cross-section of a circuit at a given amount of time. In this case the amount of charge was determined mechanically in electrostatics by means of the force with which the electricities act on each other at a distance, that is, utilizing Coulomb's force.[474] Weber showed that the ratio of these measures of current intensity was related with his constant c.

Weber was then interested to measure the numerical value of c for two main reasons. The first one was to complete the determination of his fundamental force law. In this way he would be able to compare quantitatively the electrostatic component of his force (which depends only on the distance r) with the dynamic components (which depend on dr/dt and d^2r/dt^2). The second reason was to be able to compare completely with one another the intensities and dimensions of the electric currents obtained through the origin of the current and its different effects (electrodynamic, electromagnetic and electrolytic).

The first experiment to measure the constant c was performed by Weber and Rudolf Kohlrausch (1809-1858) in 1854-1855. Their result was presented in three works of 1855, 1856 and 1857.[475] The final value of Weber's constant after taking into account all corrections was given by:[476]

$$c = 4.39450 \times 10^8 \ m/s . \tag{19.7}$$

19.2 The Relation of Weber's Constant c with Light Velocity

In 1857 Weber and Gustav Kirchhoff (1824-1887) were the first to derive theoretically the complete telegraph equation. Utilizing the modern concepts and usual terminology of circuit theory, it is possible to say that they were the first to take into account not only the capacitance and resistance of the wire, but also its self-inductance. They deduced the telegraph equation utilizing Weber's 1846 force law between electrified particles. Kirchhoff's papers were published in 1857.[477] Weber's simultaneous and independent work was delayed in pub-

[473] See footnote 44 on page 57.

[474] See footnote 43 on page 56.

[475] [Web55b], [WK56] and [KW57], with English translations in [Web21c], [WK03] and [KW21], and with a Portuguese translation in [WK08]. See Chapters 5, 6 and 7.

[476] [WK56, p. 605 of Weber's *Werke*], [WK03, p. 294], [WK08, p. 100], [KW57, p. 652 of Weber's *Werke*] and [KW21, p. 52]. See also page 179 on Section 7.17.

[477] [Kir57b] and [Kir57c] with English translations in [Kir57a] and [GA94], respectively. See Chapters 8, 9 and 12.

lication due to the death of R. Kohlrausch in 1858, with whom Weber was performing some experiments related to electric oscillations, being published only in 1864.[478]

Kirchhoff's papers of 1857 were published in the *Annalen der Physik und Chemie*, now known as *Annalen der Physik*. In this same year of 1857 Johann Christian Poggendorff (1796-1877), the editor of this Journal, published a paper in the same Volume of the *Annalen*, after Kirchhoff's first work.[479] He related that after seeing Kirchhoff's paper, he had occasion to meet Weber in Berlin. Weber showed him the paper he intended to publish, with essentially the same results as Kirchhoff's. But Weber had not yet sent it to print, as he was waiting for the results of experiments on this topic which were being performed together with R. Kohlrausch. These experiments dealt with the propagation of electromagnetic signals along wires, taking into consideration variable currents and the effects of all surface charges upon the current. In his published work, Weber compared his theoretical results with those of Kirchhoff and mentioned Poggendorff's paper.[480]

Kirchhoff and Weber showed, in particular, that when the conductor had negligible resistance, the velocity of propagation of an electric wave is very nearly equal to the velocity of light in vacuum. As Kirchhoff pointed out in his paper (referring to Weber's constant c):[481]

> The velocity of propagation of an electric wave is here found to be $= c/\sqrt{2}$, hence it is independent of the cross section, of the conductivity of the wire, also, finally, of the density of the electricity: its value is 41950 German miles in a second, hence very nearly equal to the velocity of light *in vacuo*.

Representing light velocity in vacuum by v_L and assuming that $c/\sqrt{2}$ is equal to this velocity yields:

$$\frac{c}{\sqrt{2}} = v_L \qquad \text{or} \qquad c = \sqrt{2} \cdot v_L \ . \tag{19.8}$$

Weber's force and potential energy given by Equations (19.5) and (19.6) can be expressed in terms of light velocity in vacuum as, respectively:

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{c^2}\frac{d^2r}{dt^2}\right] = \frac{ee'}{r^2}\left[1 - \frac{1}{2v_L^2}\left(\frac{dr}{dt}\right)^2 + \frac{r}{v_L^2}\frac{d^2r}{dt^2}\right] \ , \tag{19.9}$$

and

$$\frac{ee'}{r}\left[1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2\right] = \frac{ee'}{r}\left[1 - \frac{1}{2v_L^2}\left(\frac{dr}{dt}\right)^2\right] \ . \tag{19.10}$$

Utilizing Weber and Kohlrausch's experimental value of Weber's constant c, Equation (19.7), the velocity of propagation of an electric wave obtained by Weber and Kirchhoff was given by

$$\frac{c}{\sqrt{2}} = \frac{4.39450 \times 10^8 \ m/s}{\sqrt{2}} = 3.10738 \times 10^8 \ m/s \ . \tag{19.11}$$

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[478] [Web64] with English translation in [Web21d]. See Chapter 18.

[479] [Pog57] with English translation in [Pog21]. See Chapters 10 and 11.

[480] [Web64, Section 6, pp. 130-132 of Weber's *Werke*] with English translation in [Web21d, Section 6]. See page 289 of Section 18.6.

[481] [Kir57b, pp. 209-210] and [Kir57a, p. 406]. See page 214 on Chapter 8.

This value was really very close to the known velocity of light as given by astronomical observations and optical terrestrial experiments.

Several scientists were impressed by this result. In 1858 Georg Friedrich Bernhard Riemann (1826-1866) wrote a paper (first published posthumously in 1867) developing a theory of a retarded potential propagating at this velocity of $c/\sqrt{2}$.[482] Ludvig Valentin Lorenz (1829-1891) developed an independent theory of electromagnetic waves propagating at a velocity of $c/\sqrt{2}$ utilizing retarded potentials.[483] Carl Gottfried Neumann (1832-1925) also developed an independent theory on the propagation of electrodynamic potentials which generated a debate with Rudolf Julius Emanuel Clausius (1822-1888).[484]

Maxwell was aware of Kirchhoff's 1857 papers. The first paper was published in the *Annalen der Physik und Chemie* and an English translation appeared in the same year in the *Philosophical Magazine* with the title "On the motion of electricity in wires".[485] In note 26 of Schaffer's paper we find the following important remark regarding Maxwell's knowledge of Kirchhoff's first paper of 1857:[486]

> In the early 1870s Maxwell made detailed notes on Kirchhoff's paper on electricity in wires: see Cambridge University Library MSS ADD 7655 Vn/1, p. 44 ff.

Kirchhoff's second paper of 1857 was also published in the *Annalen der Physik und Chemie*.[487] It was quoted explicitly by Maxwell in the Note included in Article [805] of the second Volume of his *Treatise on Electricity and Magnetism* published in 1873.[488] However, this specific quotation did not appear in the final Index of the *Treatise* under Kirchhoff's name.[489] For this reason some researches discussing Maxwell's works may have missed this important information. Ludvig Lorenz' 1867 work is also discussed by Maxwell in this Note included in Article [805] of the *Treatise*.

Two of the main inspirations for Maxwell's electromagnetic theory of light developed in 1861-1864 were related to these two results, namely, (a) the experimental measurement of c by Weber and Kohlrausch, and (b) the result obtained by Kirchhoff and Weber on the propagation of electromagnetic signals at the velocity $c/\sqrt{2} = 3.1 \times 10^8\ m/s$, that is, essentially with the same value as the known velocity of light obtained by astronomical observations and by terrestrial experiments as those conducted by Fizeau and Foucault. In the next Section, I discuss some works by Maxwell related to this topic.

Weber and Kohlrausch's measurement of c have been discussed by many authors.[490]

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[482] [Rie67b] with English translation in [Rie67a] and [Rie77].

[483] [Lor67b] with English translation in [Lor67a].

[484] [Neu68a] with English translation in [Neu20a], [Neu68b], [Neu69] with English translation in [Neu20b], [Cla68] with English translation in [Cla69], [Arc86].

[485] [Kir57b] with English translation in [Kir57a]. See Chapter 8.

[486] [Sch95].

[487] [Kir57c], see Chapter 12.

[488] [Max73a, Article 805, p. 398] and [Max54, Article 805, p. 450].

[489] [Max73a, Index on p. 441] and [Max54, Index on p. 498].

[490] [Duh02] with English translation in [Duh15], [Kir56] with English translation in [Kir57], [Ros56], [Wie60], [Woo62], [O'R65, p. 534], [Wie67], [Woo68], [Whi73, p. 232], [Woo81], [Wis81], [Ros81], [D'A81], [Har82], [JM86, Vol. 1, pp. 144-146 and 296-297], [Wie92], [Wie93c], [Wie93b], [Wie93a], [Wie94], [Hec96b], [Hec96a], [Hec97], [Wie97], [Gib97], [Dar00], [ARW02], [ARW04], [Wie04], [Men06], [Men15], [Tom20] and [Hun21].

19.3 Maxwell's Constant c and the Confusion Introduced by Many Scientists up to Einstein

I will now trace the origin of Maxwell's constant c.[491] This constant appears in most textbooks representing light velocity in vacuum and also the ratio of electromagnetic and electrostatic units of charge. Its value is given as a number close to 3×10^8 m/s. The origin of Maxwell's constant c is closely related with Weber's constant c. The symbol of Maxwell's constant c is the same as the symbol for Weber's constant c. Despite this fact, the meaning and numerical value of Maxwell's constant c are different from the meaning and numerical value of Weber's constant c. The value of Maxwell's constant c is given by Weber's constant c divided by the square root of two. The goal of this Section is to clarify this confusion in which the same letter represents two different magnitudes.

In his paper on physical lines of force published in 1861-1862, Maxwell utilized the upper case letter V to represent the velocity of light (or the velocity of propagation of electromagnetic disturbances in a non-conducting medium according to his electromagnetic theory of light) and continued to utilize the symbol V with this meaning in his other publications.[492] In his paper on a dynamical theory of the electromagnetic field published in 1865, Maxwell utilized the lower case letter v to represent the number of electrostatic units of electricity contained in one electromagnetic unit of electricity (or the number of units of statical electricity which are transmitted by the unit electric current — estimated in electromagnetic units — in the unit of time) and continued to utilize the symbol v with this meaning in his other papers.[493] He also utilized this small case letter v to represent this ratio in his *Treatise on Electricity and Magnetism* first published in 1873.[494] Maxwell utilized the letters V and v for these two magnitudes because they had the dimension of a velocity.

Maxwell mentioned in all his papers quoted above that the first numerical determination of the ratio of the electromagnetic to the electrostatic unit of electricity, v, was made by Weber and Kohlrausch, quoting their papers of 1856 and 1857. Maxwell expressed this measure as:[495]

$$v = 3.10740 \times 10^8 \ m/s \ . \tag{19.12}$$

Maxwell himself made an experiment to determine this magnitude in 1868.[496] He obtained the following value:

$$v = 2.8798 \times 10^8 \ m/s \ . \tag{19.13}$$

I will now discuss these numerical values obtained by Weber and Maxwell in their experiments. The value of Weber's $c/\sqrt{2}$ obtained by Weber and Kohlrausch in 1854-1855, as presented in 1856-1857, was 3.10738×10^8 m/s, see Equation (19.11). Maxwell's measurement of 1868 gave 2.8798×10^8 m/s, see Equation (19.13). I will compare these numbers with the modern value of light velocity in vacuum as presented in modern textbooks, namely,

[491] [Gib97], [Men06] and [Ano06].

[492] [Max62, p. 22], [Max65, p. 497], [Max68, p. 655], [Max54, Vol. 2, Article 784, p. 434] and [Hop59].

[493] [Max65, p. 491] and [Max68, p. 645].

[494] [Max73b, Vol. 2, Article 628, p. 243, and Articles 768-780, pp. 368-382] and [Max54, Vol. 2, Article 628, p. 268, and Articles 768-780, pp. 413-430].

[495] [Max65, p. 499], [Max68, p. 651] and [Max54, Vol. 2, Article 771, p. 417].

[496] [Max68, p. 651], [Max70, p. 438], [Max54, Vol. 2, Article 787, p. 436] and [Hop59].

$2.99792 \times 10^8\ m/s$. The relative changes Δ (or percentage differences Δ) of Weber and Maxwell's numbers when compared with the modern value of light velocity are given by, respectively:

$$\Delta_{Weber} = \frac{|3.10738 - 2.99792|}{2.99792} \times 100\% = 3.65\% \ , \tag{19.14}$$

$$\Delta_{Maxwell} = \frac{|2.8798 - 2.99792|}{2.99792} \times 100\% = 3.94\% \ . \tag{19.15}$$

Before performing this experiment, Weber and Kohlrausch had no idea of the order of magnitude of c. It might be, for instance, $10^5\ m/s$ or $10^{11}\ m/s$. Moreover, in 1854-1856 nobody had any idea that Weber's constant c might have any relation whatsoever with light velocity in vacuum. That is, it had not been predicted that $c/\sqrt{2}$ might be v_L. More than ten years later Maxwell measured the same magnitude. He knew precisely its order of magnitude as obtained by Weber's previous experiment. Moreover, Maxwell expected its value to have exactly the same value as light velocity in vacuum. Despite these 3 facts, Maxwell's measured value for this magnitude is farther from the modern value of light velocity in vacuum than Weber's measured value of $c/\sqrt{2}$, as can be seen by comparing Equations (19.14) and (19.15).

Assuming that Weber's constant c should be exactly light velocity times the square root of 2 and the modern value of light velocity in vacuum as $2.99792 \times 10^8\ m/s$, then the precise value of Weber's constant should be $4.23970 \times 10^8\ m/s$. Weber and Kohlraush obtained $4.39450 \times 10^8\ m/s$.

It is worth while quoting here Kirchner's words related to this experiment:[497]

> [...] if we are to use the words of Weber and Kohlrausch, we have to formulate the obtained results in the following way.
>
> The ratio between the mechanical and the magnetic measurement of current intensity is as $1 : 3.1074 \times 10^{11}$ in the mm-mg-sec system or as $1 : 3.1074 \times 10^{10}$ in the cgs system.
>
> Considering that this ratio was then not even known as to its order of magnitude, that we deal therefore with a real pioneering effort, and if one realizes furthermore the primitive equipment they had to work with, one has to admire the work by Weber and Kohlrausch as a masterpiece in the art of experimentation, very few of which exist in the history of our science.

Maxwell compared the measured values of these two magnitudes, V and v, in several places.[498] As regards light velocity, V, he quoted the measures of Fizeau, Foucault and those based on aberration and Sun's parallax. As regards the ratio of electromagnetic and electrostatic units of charge, v, he quoted the measures of Weber and Kohlrausch, his own experiment and that of Thomson of 1869.[499] In his electromagnetic theory of light it was expected that $V = v$.

Other authors continued to utilize this meaning of the constant v.[500]

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[497] [Kir56, p. 531] with English translation in [Kir57, p. 625].

[498] [Max65, p. 499], [Max68, pp. 651-652] and [Max54, Vol. 2, Article 771, p. 417, together with Articles 786-787, pp. 435-436].

[499] [Max70] and [Kin70].

[500] [Kin70], [Tho83], [Ros89] and [TS90].

Up to now everything was fine. The confusion began by Maxwell in a postcard he sent to Tait of 1871,[501] and mainly in the last Chapter of his *Treatise of Electricity and Magnetism* published in 1873.[502] In his papers of 1855 (published in 1858) and 1864 (published in 1865), he had praised Weber's electrodynamics but saw only one problem in it, namely, he believed that Weber's law was incompatible with the principle of the conservation of energy.[503] In 1869 and 1871 Weber proved in more detail that his force was compatible with the principle of the conservation of energy utilizing his potential energy which he had introduced in 1848.[504] Maxwell changed his mind only in 1871, after Weber's proof. Harman reproduced a postcard from Maxwell to Peter Guthrie Tait (1831-1901), from 7 November 1871. In this postcard Maxwell informed Tait that Weber had reason. Weber's force has a potential. Hence in any cyclic operation no work is spent or gained. There is then conservation of energy in Weber's electrodynamics.[505] In this postcard Maxwell wrote Weber's potential energy ψ and Weber's force as given by, respectively:

$$\psi = \frac{ee'}{r}\left[1 - \frac{1}{2c^2}\left(\frac{\partial r}{\partial t}\right)^2\right] , \tag{19.16}$$

and

$$m\frac{\partial^2 r}{\partial t^2} = \frac{ee'}{r^2}\left[1 - \frac{1}{2c^2}\left(\frac{\partial r}{\partial t}\right)^2 + \frac{r}{c^2}\frac{\partial^2 r}{\partial t}^2\right] . \tag{19.17}$$

In his *Treatise of Electricity and Magnetism* Maxwell discussed Weber's deduction of Ampère's formula for the force between two current elements of lengths ds and ds'. Maxwell considered two current elements carrying electric currents of intensities i and i' estimated in electromagnetic units. He also considered two electric particles e and e_1 (with the magnitude of their charges estimated in electrostatic units) moving with velocities v and v_1 along the first current element. He then stated that:[506]

> the total transfer of electricity, reckoned algebraically, along the first circuit, is represented by
>
> $$ve + v_1 e_1 = cids , \tag{19.18}$$
>
> where c is the number of units of statical electricity which are transmitted by the unit electric current in the unit of time.

That is, the constant c introduced here by Maxwell is the same constant which he had always represented by v, namely, the number of electrostatic units of electricity contained in one electromagnetic unit of electricity.

501[Har82, pp. 96-97] and [Max95, pp. 686-688].

502See Chapter XXIII, Theories of Action at a Distance, in volume 2 of Maxwell's *Treatise*, [Max73b] and [Max54]. German translation in [Max83a]. Portuguese translation in [Ass92].

503[Max58, pp. 207-208 of Niven's book] and [Max65, pp. 526-527 of Niven's book].

504[Web69] with English translation in [Web21g]; [Web71] with English translation in [Web72].

505[Har82, pp. 96-97] and [Max95, pp. 686-688].

506[Max73b, Vol. 2, Article 849, p. 428] and [Max54, Vol. 2, Article 849, p. 482].

In Articles 850 and 853 of his *Treatise*, Maxwell considered two electrical particles e and e' separated by a distance r. He then expressed Weber's force and potential energy ψ in the same way as he had written them in his 1871 postacard to Tait, namely:[507]

$$\frac{ee'}{r^2}\left[1+\frac{1}{c^2}\left(r\frac{\partial^2 r}{\partial t^2}-\frac{1}{2}\left(\frac{\partial r}{\partial t}\right)^2\right)\right] , \tag{19.19}$$

and

$$\psi=\frac{ee'}{r}\left[1-\frac{1}{2c^2}\left(\frac{\partial r}{\partial t}\right)^2\right] . \tag{19.20}$$

Maxwell utilized here the symbols $\partial r/\partial t$ and $\partial^2 r/\partial t^2$ to indicate Weber's relative velocity dr/dt and relative acceleration d^2r/dt^2, respectively. What Maxwell wrote as $\partial r/\partial t$ would be written today as dr/dt, as is evident from what he wrote in Article [847] of the *Treatise*. Although Weber and Maxwell utilized different symbols to express these magnitudes, they had the same meanings for both authors.

The main difference between Equations (19.5), (19.17) and (19.19) — or between Equations (19.6), (19.16) and (19.20) — is that Maxwell's constant c is not the same as Weber's constant c. Let Weber's constant c be represented by c_{Weber}, while Maxwell's constant c is represented by $c_{Maxwell}$. A comparison of Equations (19.5) and (19.19) — or a comparison between Equations (19.6) and (19.20) — yields $1/c^2_{Weber} = 1/(2c^2_{Maxwell})$, that is, $c_{Maxwell} = c_{Weber}/\sqrt{2}$. This comparison shows that Maxwell's constant c is Weber's c divided by the square root of two. Moreover, combining this result with Equation (19.8) yields

$$c_{Maxwell}=\frac{c_{Weber}}{\sqrt{2}}=v_L . \tag{19.21}$$

I do not know why Maxwell wrote Weber's force and potential energy as Equations (19.16) and (19.17). Maybe he had a memory lapse and had not available Weber's original works with him when he sent the postcard to Tait in 1871. In any event, I also do not know why did he continue to express Weber's force and potential energy in this way two years later in the *Treatise*, as can be seen in Equation (19.19) and (19.20).

In the *Treatise* he may have utilized the letter c instead of his usual letter v to represent the number of electrostatic units of electricity which are contained in one electromagnetic unit of electricity in order to avoid confusion with the letter v which he was utilizing now in this last Chapter of the *Treatise* to represent the velocity of the electric particle e relative to the metal wire. However, Maxwell did not warn his readers of this change of letters, nor that his present c in the Article [849] of the last Chapter of the *Treatise* was the same magnitude as his earlier v in Articles [628] and [771] of the same *Treatise*, namely, the number of units of statical electricity which are transmitted by the unit electric current in the unit of time.

Moreover, Maxwell did not emphasize in the *Treatise* that his present constant c included in Equations (19.19) and (19.20) was numerically different from Weber's constant c appearing in Equations (19.5) and (19.6). Weber's constant c had been widely utilized in physics. It is amazing that without a single warning, Maxwell wrote Weber's force and potential energy

[507][Max73b, Vol. 2, Articles 850 and 853, pp. 428-429] and [Max54, Vol. 2, Articles 850 and 853, pp. 483-484].

utilizing a letter c which had a different numerical value from the letter c which Weber himself had introduced exactly in the expression of this force!

The importance acquired by Maxwell's *Treatise* created a confusion related to the meaning and numerical value of c which lasts up to the present time.

Paul K. L. Drude (1863-1906) was a German physicist who wrote papers and books integrating optics with Maxwell's theory of electromagnetism. Like Maxwell, he utilized the capital letter V to represent the velocity of light propagating in the ether.[508] In 1894 he followed Maxwell and called $c = 3 \times 10^8\ m/s$ the number of electrostatic units of electricity contained in one electromagnetic unit. He also claimed that Kirchhoff had shown in 1864 that electric waves propagate along metal wires with this velocity c:[509]

> Kirchhoff (G. Kirchhoff, Pogg. Ann. 121, 1864) has proven, that electric waves must propagate along a metal wire, located in air, with the speed c, where c is the ratio of an amount of electricity measured according to the electrostatic unit by an amount measured in electromagnetic unit.

Two corrections must be made here. Kirchhoff had proven the result pointed out by Drude in 1857 and not in this quoted paper of 1864.[510] Moreover, in his 1857 paper Kirchhoff utilized Weber's constant c with the same meaning as that given by Weber, with its numerical value as measured by Weber and Kohlrausch, namely, $c = 4.39 \times 10^8\ m/s$. As pointed out before, Kirchhoff showed that electric waves propagate along the wire with the velocity $c/\sqrt{2} = 3.1 \times 10^8\ m/s$.

In his books of 1894, *Physik des Aethers auf elektromagnetischer Grundlage*, and 1900 (translated into English in 1902, *The Theory of Optics*), Drude continued to utilize c with the numerical value given by Maxwell, $c = 3 \times 10^8\ m/s$, and with its meaning as the number of electrostatic units of electricity contained in one electromagnetic unit.[511] Moreover, he followed his paper of 1894 and also represented by c the speed of electrodynamic waves propagating along wires and through air.

In 1892 and 1895 Hendrik Antoon Lorentz (1853-1928) utilized the upper case V to represent light velocity relative to the ether and also the ratio of electromagnetic and electrostatic units of electricity.[512] However, by 1903 he began utilizing the lower case c for these magnitudes.[513]

In 1900 Max Planck (1858-1947) utilized c as the symbol for light velocity in vacuum, being followed in 1902 by Max Abraham.[514]

Albert Einstein (1879-1955) utilized the symbol V to represent the velocity of light from 1905 to 1907.[515] For instance, in his paper creating the special theory of relativity, "On the electrodynamics of moving bodies", he said the following:[516]

> Wir wollen diese Vermutung (deren Inhalt im folgenden "Prinzip der Relativität" genannt werden wird) zur Voraussetzung erheben und auβerdem die mit ihm nur

[508] [Dru94b], [Dru94a, pp. 343-391 and 483] and [Dru02, pp. 114-119].
[509] [Dru94b, p. 191].
[510] [Kir57b] with English translation in [Kir57a], see Chapter 8; and [Kir64].
[511] [Dru94a, pp. 447-455] and [Dru02, pp. 115-123, 263-265 and 276].
[512] [Lor92, p. 46], [Lor95, pp. 17 and 139] and [Lor06, pp. 17 and 139].
[513] [Lor22, pp. 81-82], [Lor04, p. 809] and [Gib97].
[514] [Pla00], [Abr02a] and [Abr02b, pp. 114 and 116].
[515] [Ein05b, pp. 891-2 and 894], [Ein05a, p. 639] and [Ein07b, p. 371].
[516] [Ein05b, pp. 891-2].

scheinbar unverträgliche Vorausetzung einführen, daβ sich das Licht im leeren Raume stets mit einer bestimmten, vom Bewegungszustande des emittierenden Körpers unabhängigen Geschwindigkeit V fortplanze.

The English translations of these papers by Princeton University Press maintained the capital letter V as the symbol for light velocity.[517] Curiously, when these papers were translated into English in the famous Dover edition, the capital letter V was replaced by the lower case c without warning to the readers.[518] For instance, the quoted sentence of this paper on the special theory of relativity has been translated as follows:[519]

> We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body.

It was in 1907 that Einstein changed his notation and utilized the lower case c to represent light velocity in vacuum:[520]

> Lorentz bewies allerdings bekanntlich in jener Arbeit,[521] daß nach seinen Grundannahmen eine Beeinflussung des Strahlensganges bei optischen Versuchen durch jene Relativbewegung nicht zu erwarten sei, sofern man sich bei der Rechnung auf die Glieder beschränkt, in denen das Verhältnis v/c jener Relativgeschwindigkeit im Vakuum in der ersten Potenz auftritt. [...] Wer nehmen nun an, die Uhren können so gerichtet werden, daß die Fortpflanzungsgeschwindigkeit eines jeden Licht-strahles im Vakuum — mit Hilfe dieser Uhren gemessen — allenthalben gleich einer universellen Kostante c wird, vorausgesetzt, daß das Koordinatensystem nicht beschleunigt wird.

English translation:[522]

> It is true that in the study cited[523] Lorentz proved that in optical experiments, as a consequence of his basic assumptions, an effect of that relative motion on the ray path is not to be expected as long as the calculation is limited to terms in which the ratio v/c of the relative velocity to the velocity of light in vacuum appears in the first power. [...] We now assume *that the clocks can be adjusted in such a way that the propagation velocity of every light ray in vacuum—measured by means of these clocks—becomes everywhere equal to a universal constant* c, provided that the coordinate system is not accelerated.

[517] [Ein89b, pp. 140 and 143], [Ein89a, p. 172] and [Ein89c, p. 238].
[518] [Ein52b, pp. 38 and 40] and [Ein52a, pp. 69-70].
[519] [Ein52b, p. 38].
[520] [Ein07a, pp. 412 and 415].
[521] [Note by Einstein:] H. A. Lorentz, Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern. Leiden 1895. Neudruck Leipzig, 1906.
[522] [Ein89d, pp. 252-253 and 256].
[523] [Note appearing in this English translation:] H. A. Lorentz, *Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern.* [Attempt at a theorie of electric and optical phenomena in moving bodies] Leiden 1895. Reprinted Leipzig, 1906.

As discussed before, in his work of 1895 (reprinted in 1906) quoted by Einstein, Lorentz utilized V to represent light velocity relative to the ether and also the ratio of electromagnetic and electrostatic units of electricity. Einstein utilized now in 1907 the letter c to represent light velocity in vacuum.

It was also in this paper of 1907 that Einstein introduced his most famous equation, $E = mc^2$.

Einstein was certainly influenced by the works of Drude, Planck and Lorentz (from 1903 onwards) in this utilization of the letter c to represent light velocity.[524] He was only indirectly influenced by Maxwell, but not directly. As pointed out by Miller,[525] in his works he spoke a lot about Maxwell's equations and his electromagnetic theory of light, but he never read Maxwell's *Treatise*, although a German translation had been published in 1883.[526]

From Einstein's 1907 paper onwards, the lower case c with the value of the order of 3×10^8 m/s has been widely utilized to represent light velocity in vacuum not only in relativity papers, but in physics textbooks in general and also in popular media. Its original meaning and numerical value, as first obtained by Wilhelm Weber, were essentially forgotten after 1907. This situation continued during the whole of the XXth century.

In my own papers and books, I utilized the symbol c to represent light velocity in vacuum with its value of the order of 3×10^8 m/s. The reason for this choice was that I was addressing myself to modern scientists and physics students who had learned this meaning of the letter c from their textbooks. In any event, whenever I was discussing Weber's original works, I was careful to call attention that Weber's c was different from light velocity in vacuum.[527]

In this book with the English translation of Weber's main works on electrodynamics, I am maintaining the definition, meaning and numerical values of c as those given by Weber himself.

I hope that in the XXIst century, after Weber's electrodynamics reaches once more the dominant role it deserves in physics, the definition and meaning of the constant c will be those given by Weber, with a numerical value given by $c = \sqrt{2} \cdot v_L$ as first measured by Weber himself. In this case my suggestion is that light velocity in vacuum should be represented by v_L, with its numerical value of the order of 3×10^8 m/s. That is, light velocity in vacuum should no longer be represented by c. This letter should be reserved for Weber's constant in order to avoid confusion, to respect his original work and in honour of this great scientist.

[524] [Gib97].

[525] [Mil81, pp. 138-139, note 7].

[526] [Max83b] and [Max83a].

[527] [Ass94, Section 3.1], [Ass95, Section 2.1], [AW03, p. 55], [ARW04, p. 25], [AWW11, Section 1.6], [AWW14, Section 1.6], [Ass15a, Section 2.1] and [AWW18, Section 1.6].

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Available in PDF format at www.ifi.unicamp.br/~assis

- Page 13, the 2nd line of the 2nd paragraph should be replaced by:

discuss more carefully the consideration which led to the conjecture of a *diamagnetic induction of*

- Page 14, the 6th line of Section 2.1 should be replaced by:

iron and a bar of bismuth, the iron exerts magnetic forces at a distance, compared to which the

- Page 15, the 2nd line below Figure 1 should be replaced by:

trodiamagnet consisted of two spiraling copper wires. Each of these spirals had a length of 190

- Page 18, the 4th to 6th lines should be replaced by:

important to achieve this *without changing the strength of their diamagnetism and without inducing through this movement a current in the conductor bismuth*. here the advantage of a *electrodiamagnet* compared to a *usual* one became manifest. In fact, a *usual* diamagnetic material

- Page 20, the line of item 8 of the Table should be replaced by:

8.	above	489.7	487.3	± 7.0

- Page 21, the title of Section 2 should be replaced by:

Experiments with One Little Bar of Iron

- Page 21, the 3rd line of Section 2 should be replaced by:

spiral up and down and then in the second one. The little iron bar had the same length

- Page 23, the 18th line of Section 2.4 should be replaced by:

$= \log\frac{3}{2}$ and therefore it suffices to divide the value of the nth oscillation arc by $\left(1-\left(\frac{2}{3}\right)^n\right)$ or

- Page 24, the 12th line should be replaced by:

rest states of the needle were obtained alternately for the *upper* and *lower* position:

- Page 28, the last line should be replaced by:

opposite direction, as with the bismuth bar.

- Page 29, the 1st line of Section 2.6 should be replaced by:

The experiments about *diamagneto-electric induction* are obviously more difficult than the previous

- Page 29, the 1st line of Section 2.7 should be replaced by:

Here I describe a different diamagnetic inductor than the one with the help of which I

- Page 29, the 5th line of Section 2.7 should be replaced by:

in the *Philos. Transact. 1850, P. I.*[20] However, Faraday did not succeed to detect diamagnetic

- Page 29, the 1st line of item 1. of Section 2.7 should be replaced by:

1. Instead of a usual electromagnet, an *electrodiamagnet* is used for the induction, whose moment due

- Page 32, the 3rd line of the Section The Induction Spiral should be replaced by:

belonging to the electrodiamagnet through which the current of the galvanic pile flows and has

- Page 32, the 9th line of the Section The Induction Spiral should be replaced by:

electrodiamagnet or more precisely the spiral was wound arount it.

- Page 33, the 4th line should be replaced by:

of the needle to the other side, a second commutator *ee* next to the observation telescope in Figure 6 E is

- Page 34, the 14th line should be replaced by:

West *reversed displacement*. Finally the position the *rotating commutator* had during the

- Page 34, the 8th and 9th lines from bottom to top should be replaced by:

induced current created a decrease of the present oscillation arc, which then by a continuous change decreased until zero and then started increasing until it attained its limit. When the

- Page 46, the 8th line after item 2. should be replaced by:

the magnetoelectric effects requires the application of quite different devices than the mag-

- Page 46, the 15th, 16th and 17th lines after item 2. should be replaced by:

been even much larger if the difference between the *masses* of bismuth and iron, which were used for the various electrodiamagnets and electromagnets, had not already been taken into account in determining these ratios. By taking into account the inequality of the masses, the coarsest occurring

- Page 48, the two last lines of the footnote 26 should be replaced by:

the thickness of the wire was 2.8 millimeters. Consequently, the strength of *the separating force* exerted by the current of one layer of turns whose radius $= r$ on a point of the iron bar laying at the distance $= a$ from the center

- Page 49, the 7th line from bottom to top of the footnote should be replaced by:

weight $= 7.78$, so that one finds for its thickness $d' = 0.8342$. The value of X' for this little bar is determined

- Page 51, the 18th line should be replaced by:

poles. Some experiments by Poggendorff (*Annalen* 1848, Vol. 73, p. 475)[32] followed immedi-

- Page 53, the 1st line of the 2nd paragraph should be replaced by:

Among the devides which allow an even higher degree of fineness and accuracy than

- Page 53, the 14th line of the 2nd paragraph should be replaced by:
of the experiments, but that it is in any case far preferable to eliminate this influence

- Page 58, the 12th line should be replaced by:
that through *continued* increase of the magnetic separating force, *in the ideal distribution*

- Page 61, the 5th and 6th lines should be replaced by:
This integral value is the *some of the products* of the *intensity* with the *element of time* during which the force with this intensity is acting, according to the absolute measure [of current intensity] determined on page 321 of this Volume [page 321 of Weber's *Werke*].

- Page 62, the 10th line should be replaced by:
Here i denotes the intensity of the *inducing* current according to the same measure.

- Page 66, the 1st line of the last paragraph should be replaced by:
For each of the two main cases a *theory* can be developed and each of the theories can be split

- Page 67, the 9th line of the 2nd paragraph should be replaced by:
already existing rotatable molecules (molecular magnets or molecular currents) and into the

- Page 71, the 1st to 3rd lines of footnote 61 should be replaced by:
[Note by WW:] Namely, according to this assumption, the magnetic state of equilibrium is defined by the fact that on the surface of all molecular conductors there is a distribution of the two magnetic fluids acting on all points in the interior of the molecules in such a way taht the effect of the external separating forces gets cancelled. It follows

- Page 71, the last two lines of the 2nd paragraph should be replaced by:
a given magnetic or electromagnetic separating force) a substitute for the magnetic fluids through electric currents is possible.

- Page 72, the 10th line from bottom to top should be replaced by:
is then just due to the magnetism of iron which can be determined by the same acuteness

- Page 77, the 1st line should be replaced by:
ND. However, this driving force, which is due to the interaction of the molecules, has to increase according

- Page 88, the last line of footnote 80 should be replaced by:
current" in all places.

- Page 88, the 17th line of the 2nd paragraph should be replaced by:
resistance. The *other* currents, which are excited by the same force of separation in larger trajectories, but

- Page 90, the 6th and 7th lines of footnote 81 should be replaced by:
where n signifies the number of coils, r the radius, and a the length of the axis of the spiral. This value holds first of all for the force of separation at the center of the cylinder, but approaches it for every other

- Page 97, the 7th and 8th lines of Section 3.1.7 should be replaced by:

iron are the rotatable bearers of permanent molecular currents, be assumed, from it will follow a different law of the dependence of the variable magnetism on the magnitude of

- Page 99, the 8th line should be replaced by:

moment y to the force X, in the theory of rotatable molecular currents, has the same meaning as the

- Page 101, the 4th line of the 2nd paragraph should be replaced by:

connexion with the results of certain experiments made at the Institute of Physics in Leipzig by him and Prof. Hankel,[104] M.

- Page 101, the 1th and 2nd lines of the 3rd paragraph should be replaced by:

"A current of four elements of Grove[105] was made use of, and the magnet was maintained in its former position by a multiplier placed on the side, which was achieved on 1 to 1.5 parts of the scale. The bismuth was chemically pure,

- Page 102, the last line should be replaced by:

the bar of bismuth in the middle of each oscillation, the following results were obtained:

- Page 103, the 3rd line below the Table should be replaced by:

smaller numbers; the same was observed when the bar of iron was reversed. The stand

- Page 103, the 6th line from bottom to top should be replaced by:

be increased to 57.5 divisions of the scale, and retained at this magnitude, inasmuch as the action

- Page 106, the 11th line should be replaced by:

a particular form of the iron bar and for a definite strength of the magnetizing force acting on the iron, namely

- Page 109, the number in the last column of the Table should be replaced by:

$32^\circ\ 10'$ W

- Page 113, the number in the last column of the Table should be replaced by:

$31^\circ\ 39'$ W

- Page 115, the 2nd, 3rd and 4th lines of the paragraph below the Table should be replaced by:

the inductive shocks[108] (caused by the motion of the bismuth to and fro) over the entire period of oscillation of the needle, it is easy to deduce the limit-value which would correspond to all the inductive shocks during one period of oscillation concentrated on the center of the period of oscillation. The value of the

- Page 116, the last line should be replaced by:

upon the remote magnetometer, a conclusion capable of easy proof. The entire action exhibited

- Page 117, the penultimate line should be replaced by:

the iron M, divided by the mass of the iron expressed in milligrams, $p = 8190$, and thus reduced to the unit of mass,

- Page 127, the 1st sentence of the last paragraph should be replaced by:

However, from the above, when one observes that only half of the *positive* amount of electricity E flows from the Leyden jar to the Earth, because the other half is neutralized by the *negative* electricity that flows from the Earth to the jar in the opposite direction, one will have the quotient $E/\int idt = c\sqrt{2}$, in which c denotes the desired constant.

- Page 128, the 4th line of the 3rd paragraph below the Table should be replaced by:

e.g., that a positive amount of electricity of $16\frac{4}{9}$ trillion units of measurement and an equal amount of

- Page 140, the 4th line should be replaced by:

of water in a column 1 millimeter long were linked in a string, and all oxygen particles in another string,

- Page 148, the 4th line of the 3rd paragraph of Section 7.4 should be replaced by:

is ordinarily chosen to be the unit of the strength of all other currents by observing it with

- Page 149, the 4th line of the 5th paragraph should be replaced by:

amount x of positive electricity that flows through the cross-section of the conductor during

- Page 149, the two last lines should be replaced by:

which is a result of whose validity one can easily convince oneself, whatever idea one may have of what happens inside the conductors during the discharge.

- Page 150, the 5th line from bottom to top should be replaced by:

If one then multiplies $\frac{1}{2}E$ by the number that shows how often τ is included in one second,

- Page 151, the 8th line of the 4th paragraph should be replaced by:

electrometer,[186] which will yield the ratio $1 : (n-1)$ of the amount of electricity E remaining in the bottle to the

- Page 151, the penultimate line should be replaced by:

through the Leyden jar, the fixed ball through the large one, and the moving one through the

- Page 153, the last line of the 1st paragraph should be replaced by:

measurements could be performed, especially towards the end of each series of experiments.

- Page 154, the 1st paragraph below the Table should be replaced by:

The last column in this Table, under n, gives the *ratio* of the charge in the jar before contact with the ball to the charge after contact, calculated for the moment of contact from the two observations made immediately before and after, contained in the second and third columns, according to the following rule:

- Page 158, the 6th line of the 3rd paragraph should be replaced by:

observer at the telescope m' watched the elongation of the magnetic needle of the tangent

- Page 160, the 8th line should be replaced by:

That yields for the cited values the desired ratio:

- Page 162, the 4th line from bottom to top should be replaced by:

of the extended line c; i.e., *the repulsive force of the two balls*:

- Page 163, lines 4 to 7 should be replaced by:

Finally, the product of the force of repulsion between the two balls with the perpendicular drawn from the axis of rotation to the direction of this force — i.e., to the line c — gives the value of the *rotational moment* that this force of repulsion exerts upon the torsion balance, which should be equal to 1.

- Page 163, the 15th and 16th lines should be replaced by:

From this follows the *rotational moment* exerted on the torsion balance by the electric force of repulsion between the two balls will be equal to:

- Page 163, the last sentence should be replaced by:

This determination of ε is based on that quantity of electricity as a unit which exerts the unit of repulsive force on an equal quantity of electricity in the unit of distance at relative rest.

- Page 172, the 23rd line should be replaced by:

that is exerted upon the compass will be equal to:

- Page 174, The 7th and 8th lines from bottom to top should be replaced by:

$$T = 1.798\,3 \ ,$$

$$\lambda = 0.070 \ ,$$

- Page 176, the 7th to 11th lines of the 3rd paragraph of Section 7.15 should be replaced by:

of the magnetic unit for current intensity to the mechanical unit, since the amount of electricity that passes through the cross-section in the same time interval will then be:

$$155\,370 \cdot 10^6$$

times greater in the *magnetic* current unit than the amount in the *mechanical* unit of current. As a result, from the cited

- Page 177, footnote 215 should be replaced by:

[Note by HW:] Wilhelm Weber's *Werke*, Vol. III, p. 614.

- Page 179, the 4th line of the last paragraph should be replaced by:

and negative electricity, against which those forces would disappear. Wherever

- Page 180, the 3rd and 4th lines of the penultimate paragraph should be replaced by:

(From Section 7.14), $\frac{1}{2\tau} \cdot E$ then denotes the number of millimetres that both electricities must traverse in the opposite directions in 1 second in order to make:

- Page 182, the 4th line of item (4) should be replaced by:

conductor element of length α' at a distance of r when α makes an angle of ϑ with r and α'

- Page 184, the 1st line of the 2nd paragraph of Section 7.19 should be replaced by:

The above sentence is self-evident if electric masses are so connected to their ponderable carrier that they cannot be moved without it.

- Page 184, the 5th line of the 2nd paragraph of Section 7.19 should be replaced by:

finds a coupling between the electrical masses and the metallic particles that must be dissolved

- Page 184, the 19th line of the 2nd paragraph of Section 7.19 should be replaced by:

immediately vanish as soon as the driving force ceases. — It will then follow from this that,

- Page 185, the 15th line of the 4th paragraph should be replaced by:

a Voltameter is introduced into a circuit, then the electrical separating forces that act in the

- Page 186, the 1st line of the 3rd paragraph should be replaced by:

Now, if the current intensity for this *resistance* is to be $= 106\frac{2}{3}$ in *magnetic* units —

- Page 187, the 2nd line of the 2nd paragraph should be replaced by:

that act in the direction of the current in *each unit* of free positive electricity (in the

- Page 188, the 1st line of the 2nd paragraph should be replaced by:

Should the water be decomposed at a smaller rate under the same conditions — e.g., with a rate

- Page 189, the 12th line of the 3rd paragraph of Section 7.20 should be replaced by:

defined amount of neutral fluid, in addition, and finally, how the negative electricity on

- Page 191, the 1st line should be replaced by:

that is contained in each *element of length* in the conductor is exceptionally large. However, for a given current intensity,

- Page 191, the 7th line of the 2nd paragraph should be replaced by:

units, together with 1/9 milligram of hydrogen, will move in one direction, while an equally-

- Page 191, the 8th line of the 1st paragraph of Section 7.21 should be replaced by:

another mass at a unit distance that would impart in the unit of time a velocity to the latter that would equal

- Page 191, the 2nd line from bottom to top should be replaced by:

each other, by the unit of length, if they are to have no influence on each other according to that law.

- Page 192, the 8th line of the 2nd paragraph should be replaced by:

be 439450^2 billion times larger, and the previous acceleration would be equal to:

- Page 193, the 1st sentence of Appendix I (Description of the Torsion Balance) should be replaced by:

In order to avoid as much as possible an unequal reaction of the walls of the torsion balance, electrified by the charged spheres through electrostatic induction, on the movable sphere, the balance is constructed on an unusually large scale.

- Page 195, the 8th and 9th lines should be replaced by:

c a 5 mm protruding threaded spindle, in order to attach either the body, by whose period of oscillation the torsion coefficient should be determined, or the brass wire

- Page 195, the 3rd line of the 3rd paragraph should be replaced by:

at right angles to it, a brass rod rt with a running weight. The tips rested on brass bearings, q in a conical hole, p in a slot. The running weight pushed the

- Page 215, the 10th line of the 2nd paragraph should be replaced by:

If the corresponding expression be formed for i, remembering the equation by which h has

- Page 217, the 6th line of the 2nd paragraph should be replaced by:

which the wire is at each moment divided by this point, the same current intensity exists everywhere

- Page 217, the 1st line of the 3rd paragraph should be replaced by:

The current intensity before the point at which the break occurs, considered without regard to

- Page 219, the 4th line should be replaced by:

This expression shows that the current intensity at the commencement of the wire is 0 up to the

- Page 219, the 1st line of footnote 276 should be replaced by:

[Note by AKTA:] In the *Philosophical Magazine* this equation appeared as

- Page 223, Equation (9.16) should be replaced by:

$$\frac{\partial^2 V}{\partial s^2} - \frac{2}{c^2}\frac{\partial^2 V}{\partial t^2} = \frac{r}{8\gamma l}\frac{\partial V}{\partial t} \ .$$

- Page 241, the 1st line of the 3rd paragraph should be replaced by:

With Jacobi's resistance standard,[319] a copper wire of 7.62 m length, 0.333 mm radius,

- Page 248, the 3rd line should be replaced by:

important that the same determination should be repeated for the two larger needles with

- Page 253, the 7th line of footnote 376 should be replaced by:

reverse the polarity of the needle, [WSH03].

- Page 258, the 1st line should be replaced by:

non-uniform and *rapidly changing* motions; because this law, first formulated by Ohm,[384]

- Page 258, the 5th line of the 2nd paragraph should be replaced by:

rapidly changing currents. Furthermore, the development of the laws, as far as it has been

- Page 271, the 1st line of the 5th paragraph should be replaced by:

Hence, the electromotive force due to a current element of length α with its current

- Page 280, the last sentence should be replaced by:

But now, when α is very small, the latter two parts of this value of U may be considered as vanishing compared to the first part, then one may put

- Page 281, the 11th line should be replaced by:

conducting wire, considered as a cylinder, on any point of the middle cross-section of this piece, is

- Page 287, the 5th line should be replaced by:

under consideration. Hence multiplying this force by the number of positive electric units of

- Page 287, the 1st line of the 2nd paragraph should be replaced by:

In order to take into account all forces which act on the electric particle of the conducting wire under consid-

- Page 289, the 5th line of the 4th paragraph of Section 18.6 should be replaced by:

rather $\lambda = \infty$ when α vanishes, as is easily seen because the number, $= \mathfrak{E}$, of positive electric units

- Page 308, the 7th line of the 2nd paragraph should be replaced by:

locity during the unit time on a mass of one milligram), it follows that r^2 is the force exerted by *one*

- Page 309, the 2nd line of Section 18.15 should be replaced by:

itself in a circular conductor after an arbitrary disturbance of equilibrium turn out to be a series of *wave*

- Page 310, the 1st line of the 3rd paragraph should be replaced by:

The *intensities* of the various wave trains, which are proportional to i^2 according to the rules of wave

- Page 313, the 3rd line should be replaced by:

Already Kirchhoff has found this velocity for the propagation of electric waves and remarked:[434]

- Page 313, the 3rd line from bottom to top should be replaced by:

measures of resistance, we have $W = \pi c^2 a w'/4$ or $a^2 c^2 w'^2/128 = W^2/[8\pi^2 c^2]$, after what[437]

- Page 329, the 2nd line of the 1st paragraph should be replaced by:

have at the same time equal *phase* and oscillation *amplitude* in all parts of a circular conductor, even if the

- Page 330, the 6th line of the 2nd paragraph should be replaced by:

velocities of the small magnet. Choosing the three conducting wires already exemplified in Section 18.16, we get

- Page 337, the 3rd line of the 2nd paragraph should be replaced by:

wires and the solenoid weight carried by them, there is a *static directive* force for the solenoid which can easily be determined and shall be denoted

- Page 339, the penultimate line should be replaced by:

to the first part and the solenoid to the latter part of the closed conductor. Then from this

- Page 345, the 7th line of the 7th paragraph should be replaced by:
wire f^{III} and the multiplier wire f^{IV} of the second dynamometer, but will

- Page 353, the 15th and 16th lines should be replaced by:
After this mutual comparison of the sensitivities of both dynamometers, the observed deflections of one dynamometer alternatively switched onto two different places of the circuit may

- Page 353, the 23rd line should be replaced by:
other dynamometer is valid, and this *deflection calculated for the first position* of the main

- Page 360, the 11th line should be replaced by:

$$625.20, \qquad 625.29, \qquad 621.70 \; ,$$

- Page 380, the 1st line of the 2nd paragraph should be replaced by:
Hence it would be possible to determine *directly* the magnitude $r\mathfrak{E}$ *without consid-*

- Page 393, the first line after Equation (19.19) should be replaced by:
same way as he had written them in his 1871 postcard to Tait, namely:[507]

This is the third of 4 volumes of the book "Wilhelm Weber's Main Works on Electrodynamics Translated into English".

This third Volume contains Weber's main works related to diamagnetism, including his Third major Memoir on Electrodynamic Measurements (1852).

These works are followed by three papers published by Weber and Rudolf Kohlraush in 1855-1857 in which they presented the measurement of Weber's fundamental constant c appearing in his force law. Weber and Kohlrausch's 1857 work is the Fourth major Memoir on Electrodynamic Measurements.

Soon after this measurement, Kirchhoff and Weber succeeded in deducing the complete telegraph equation from Weber's electrodynamics. Their works were published in 1857 and 1864. When the resistance of the wire was negligible, the telegraph equation reduced to the wave equation. The velocity of propagation of an electric wave along the wire was equal to the known light velocity in vacuum. This remarkable result of Weber's electrodynamics indicated for the first time in the history of physics a direct and quantitative connection between electrodynamics and optics. This volume contains the English translations of Kirchhoff's two papers of 1857, together with a paper by J. C. Poggendorff emphasizing the independent researches made by Weber and Kirchhoff on this subject in which both scientists arrived simultaneously at similar results. Weber's 1864 work is his Fifth major Memoir on Electrodynamic Measurements. The translation of this paper is also included in this Volume.

About the Editor: Prof. Andre Koch Torres Assis has been working on Weber's law applied to electromagnetism and gravitation for more than 30 years: https://www.ifi.unicamp.br/~assis

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