Force on a dielectric slab: Fringing field approach

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Force on a dielectric slab: Fringing field approach

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The calculation of the force on a dielectric material partially inserted between two parallel charged conducting plates, typically done in undergraduate classrooms using energy considerations, is performed by explicitly considering the effect of the fringing electric field. © 2004 *American Association of Physics Teachers.*

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Most contemporary undergraduate electromagnetism textbooks include, either as part of the expository text or as a problem, the calculation of the force exerted on a slab of dielectric material partially inserted between two parallel charged conducting plates (see Fig. 1). This problem has enjoyed a resurgence of interest as the basis for a simple laboratory demonstration with a homemade apparatus.[1] The calculation is ubiquitous because it showcases the power and efficiency of the energy method in such situations and is an adumbration of later discussions of Lagrangian and Hamiltonian mechanics.

In the energy method solution,[2] the virtual work done by an external force along the y axis is equated to the change in the electrostatic energy of assembly, δU_E, of the charges in the system if no change in kinetic energy occurs during the virtual displacement, that is, if the applied force is equal and opposite to the electric force $\mathbf{F}_D$ on the dielectric. The expression for U_E as a function of the y position of the air–dielectric interface is then used to obtain F_{Dy}: $F_{Dy} = -(\partial U_E/\partial y)$. Students are told that it is actually the fringing field which is responsible for the effect, an observation that generates a bit of skepticism initially, because the emphasis of the calculation of δU_E during the virtual displacement seems to be on the interface between the air and the dielectric. Sometimes this derivation is accompanied by a remark about how, by virtue of the energy method, they were spared the nastiness of having to deal with the complexity of the fringing field itself.

In this paper I argue that a simple extension of this discussion, suitable for upper-division courses, that attributes the force *directly* to the fringing field is not only pedagogically useful, but is even simpler. The field lines are shown schematically in Fig. 1 for a system comprising the charged plates with separation d and a partially inserted slab of linear, homogeneous, isotropic dielectric material of relative permittivity κ. The dielectric slab has a width W along the z direction (out of the page) which is much smaller than the corresponding dimension of the plates, so that the z component of the fringing electric field is nearly zero in the material. The insertion is symmetric with respect to the $x-y$ plane and the material extends to infinity along $-y$ (or at least to where the electric field is small enough to be ignored). The air–dielectric interface at $y=0$ is far enough away from the plate edges so that the electric field $\mathbf{E}$ here has only an x component [denoted by $E_x(0)$] with negligible partial derivatives, and thus the potential difference between the plates is $V = E_x(0)d$. The force on a single electric dipole $\mathbf{p}$ within the dielectric is given[2] as $(\mathbf{p}\cdot\nabla)\mathbf{E}$, so at a point where the polarization is $\mathbf{P}$, the force on a volume element $d\tau$ of material is $d\mathbf{F}_D = (\mathbf{P}\cdot\nabla)\mathbf{E}\,d\tau$. By using the constitutive relationship $\mathbf{P} = \varepsilon_0(\kappa-1)\mathbf{E}$ for the dielectric, the y component of the total force on the slab is

$$F_{Dy} = \varepsilon_0(\kappa-1)\int\int\int\left[E_x\frac{\partial E_y}{\partial x} + E_y\frac{\partial E_y}{\partial y} + E_z\frac{\partial E_y}{\partial z}\right] \times dy\,dx\,dz, \tag{1}$$

where the integration is over the volume of the slab. Because we have assumed that the plates are wide enough compared to W so that E_z in the slab is negligible, the third term in the integrand does not contribute substantially to the force. The middle term can be rewritten as

$$\frac{1}{2}\frac{\partial}{\partial y}(E_y^2)$$

so that the innermost integral becomes

$$\int_{-\infty}^{0}\frac{1}{2}\frac{\partial}{\partial y}(E_y^2)dy = \frac{1}{2}[E_y^2(0) - E_y^2(-\infty)] = 0. \tag{2}$$

The force is thus derived in its entirety from the first term in the integrand of Eq. (1), as can be understood physically by noting that the existence of a gradient of E_y along x has the effect of producing an unbalanced force along $+y$; this

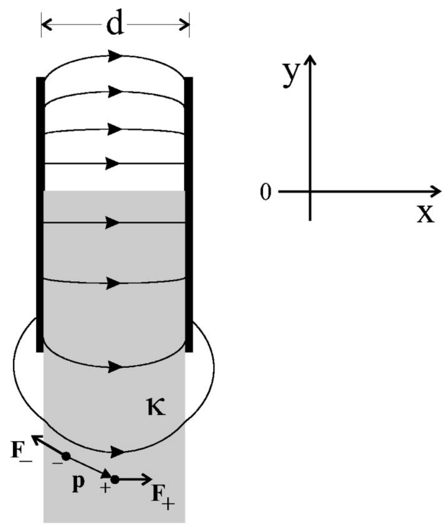

Fig. 1. Linear dielectric slab partially inserted between parallel conducting charged plates. Unbalanced forces $\mathbf{F}_+$ and $\mathbf{F}_-$ are shown acting on a dipole $\mathbf{p}$ in the fringing field region. The $+z$ direction is out of the page.

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situation is depicted qualitatively in Fig. 1 for a physical dipole near the bottom of the slab. Note that the zero-curl condition for the electrostatic field precludes any discontinuity in E_x along the y direction, which implies both that E_x is continuous at the air–dielectric interface between the plates and that there must be fringing at the plate edges, where E_x cannot suddenly vanish. Because $\nabla \times \mathbf{E} = 0$, we have

$$\frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y}$$

and so in Eq. (1) the inner integral along y is

$$\int_{-\infty}^{0} \frac{1}{2} \frac{\partial}{\partial y} E_x^2 dy = \frac{1}{2}[E_x^2(0) - E_x^2(-\infty)]$$

$$= \frac{1}{2} E_x^2(0) = \frac{V^2}{2d^2}. \tag{3}$$

If we substitute Eq. (3) in Eq. (1) and do the remaining integrals along x and z, we obtain the generally accepted result:

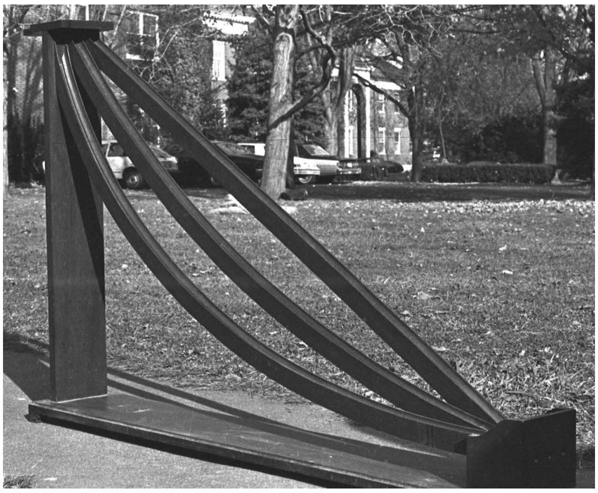
$$F_{Dy} = \frac{\varepsilon_0(\kappa - 1)}{2d} W V^2. \tag{4}$$

The assumed symmetry of the configuration obviates the calculation of the other components of $\mathbf{F}_D$, which vanish. It is interesting to note that the dominant contribution to the integral in Eq. (1) is due to the fringing field region, whereas in the more standard energy approach, the major contribution is apparently from the region of uniform field near $y = 0$.

The present treatment is simple enough so that it should, either by itself or as an extension to the standard application of the energy method, enhance student insight regarding forces on dielectrics. The fact that two apparently very different approaches give the same answer illustrates the marvelous internal consistency of the theory of electrostatics.

[1]Paul Gluck, "Force on the dielectric in a parallel plate capacitor," Phys. Teach. **41**, 521–523 (2003).

[2]See, for example, David J Griffiths, *Introduction to Electrodynamics*, 3rd ed. (Prentice Hall, Upper Saddle River, N.J., 1999), Chap. 4. The calculation of the force on the slab by the energy method starts on p. 194. The force on a dipole is given on p. 165.



Time of Descent Apparatus. A standard problem for mechanics students involves the calculation of the minimum time needed for a body to pass, under the gravitational force, from an upper level to a point on a lower level. This is usually called the Brachistrochone problem, and the path giving the minimum time is a segment of a cycloid. The other paths for the rolling ball are parabolic and straight. This apparatus was at the Lawrenceville School in Lawrenceville, New Jersey when I photographed it in 1980 during a week spent as an exchange teacher at the school. The 1888 James W. Queen catalogue lists a similar device at $18.00. (Photograph and notes by Thomas B. Greenslade, Jr., Kenyon College)