Problem 1.64 In case you're not persuaded that $\nabla^2(1/r) = -4\pi\delta^3(\mathbf{r})$ (Eq. 1.102 with $\mathbf{r}' = \mathbf{0}$ for simplicity), try replacing r by $\sqrt{r^2+\epsilon^2}$, and watching what happens as $\epsilon \to 0$.[16] Specifically, let

$$D(r, \epsilon) \equiv -\frac{1}{4\pi}\nabla^2 \frac{1}{\sqrt{r^2+\epsilon^2}}.$$

To demonstrate that this goes to $\delta^3(\mathbf{r})$ as $\epsilon \to 0$:

(a) Show that $D(r, \epsilon) = (3\epsilon^2/4\pi)(r^2+\epsilon^2)^{-5/2}$.

(b) Check that $D(0, \epsilon) \to \infty$, as $\epsilon \to 0$.

(c) Check that $D(r, \epsilon) \to 0$, as $\epsilon \to 0$, for all $r \neq 0$.

(d) Check that the integral of $D(r, \epsilon)$ over all space is 1.