Aula 18

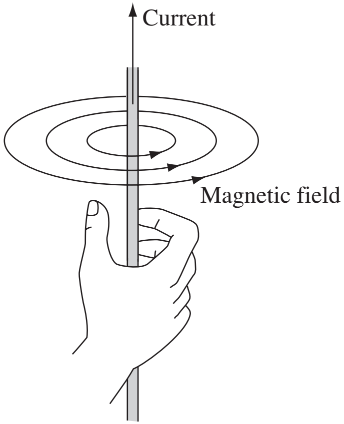
F 502 – Eletromagnetismo I

2º semestre de 2020

17/11/2020

Aula passada

Correntes geram campos magnéticos



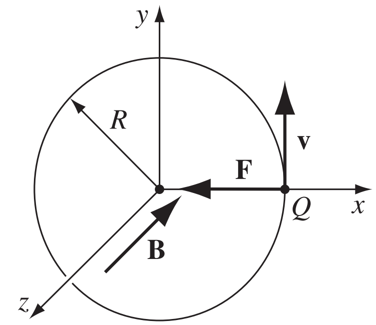
Regra da mão direita

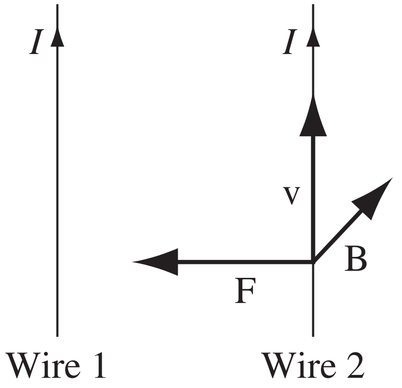
Aula passada

Campos magnéticos atuam sobre correntes/cargas em movimento

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

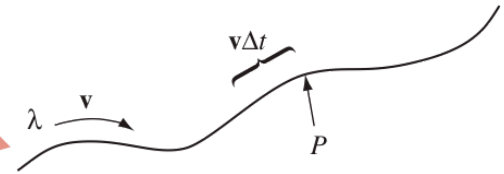
$$d\mathbf{F} = d\mathbf{I} \times \mathbf{B}$$

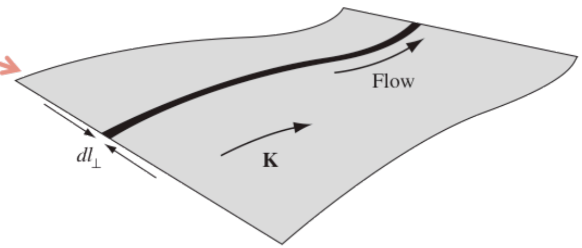


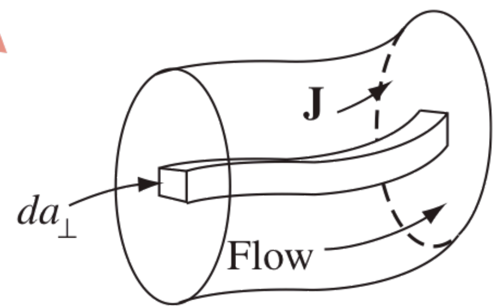


Aula passada

$$
\begin{aligned}
d\mathbf{I} &= \mathbf{I}dl = I d\mathbf{l} \\
d\mathbf{I} &= \mathbf{K}dS \\
d\mathbf{I} &= \mathbf{J}dV
\end{aligned}
$$







$$\boxed{d\mathbf{F} = d\mathbf{I} \times \mathbf{B}}$$

Aula passada

Lei de conservação (local) da carga

$$\boldsymbol{\nabla} \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\oint_{S(V)} \mathbf{J} \cdot d\mathbf{S} = -\frac{dQ\,(V)}{dt}$$

Correntes estacionárias

1. CORRENTES QUE NÃO VARIAM NO TEMPO:

$$\vec{J}(x,y,z,\cancel{t}) \longrightarrow \vec{J}(x,y,z)$$

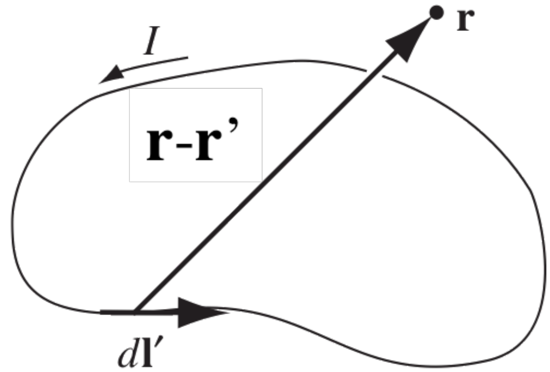
2. A CORRENTE FLUI NO ESPAÇO SEM QUE HAJA ACÚMULO DE CARGA EM QUALQUER PONTO NO ESPAÇO

$$\Rightarrow \frac{\partial \rho(x,y,z,t)}{\partial t} = 0 \Rightarrow \vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t} = 0 \Rightarrow \boxed{\vec{\nabla}\cdot\vec{J} = 0}$$

FISICAMENTE, A CORRENTE APENAS CIRCULA NO ESPAÇO.

Lei de Biot-Savart

CAMPO MAGNÉTICO $d\vec{B}(\vec{r})$ CRIADO POR UM ELEMENTO DE CORRENTE $d\vec{I}(\vec{r}\,')$



$$d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{d\vec{I}(\vec{r}\,') \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

COMPARE COM:

$$d\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} dq(\vec{r}\,') \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

PARTICULARIZANDO PARA OS VÁRIOS ELEMENTOS DE CORRENTES:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int I'\, d\vec{l}\,' \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

OU

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{K}(\vec{r}\,') \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \, ds'$$

E:

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{J}(\vec{r}\,') \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \, dV'}$$

$(\vec{\nabla} \cdot \vec{J} = 0)$

COMPARE COM:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \rho(\vec{r}\,') \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \, dV'$$

O divergente de **B**

$$\vec{\nabla}\cdot\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\,\vec{\nabla}\cdot\left[\int \vec{J}(\vec{r}\,')\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,dv'\right]$$

$$=\frac{\mu_0}{4\pi}\int \vec{\nabla}\cdot\left[\underbrace{\vec{J}(\vec{r}\,')}_{\vec{A}}\times\underbrace{\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}}_{\vec{B}}\right]dv'$$

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\vec{\nabla}\times\vec{A})-\vec{A}\cdot(\vec{\nabla}\times\vec{B})$$

$$\hookrightarrow\ \vec{\nabla}\times\vec{J}(\vec{r}\,')=0$$

$$\vec{\nabla}\cdot\vec{B}(\vec{r})=\frac{\mu_0}{4\pi}\int(-1)\,\vec{J}(\vec{r}\,')\cdot\left[\vec{\nabla}\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right]dv'=0\ \Rightarrow\ \boxed{\vec{\nabla}\cdot\vec{B}=0}$$

$$\vec{\nabla}\times\left(\frac{\vec{r}}{|\vec{r}|^3}\right)=\vec{\nabla}\times\left(\frac{\hat{r}}{r^2}\right)=0\ \Rightarrow\ \vec{\nabla}\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}=0$$

$$\frac{\partial}{\partial x}f(x-x')=f'(x-x')$$

O rotacional de **B**

$$\vec{\nabla}\times\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{\nabla}\times\left[\vec{J}(\vec{r}\,')\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right]dV'$$

$$\vec{\nabla}\times(\vec{A}\times\vec{B}) = \underbrace{\cancel{(\vec{B}\cdot\vec{\nabla})\vec{A}}}_{0} - (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla}\cdot\vec{B}) - \underbrace{\cancel{\vec{B}(\vec{\nabla}\cdot\vec{A})}}_{0}$$

AMBOS TÊM $\vec{\nabla}$ ATUANDO EM $\vec{J}(\vec{r}\,')$

$$\vec{\nabla}\times\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\,\vec{\nabla}\cdot\left[\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right]dV' - \underbrace{\frac{\mu_0}{4\pi}\int\left[\vec{J}(\vec{r}\,')\cdot\vec{\nabla}\right]\left[\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right]dV'}_{\vec{I}}$$

$$\vec{\nabla}\cdot\left[\frac{\vec{r}}{r^3}\right] = \vec{\nabla}\cdot\left[\frac{\hat{r}}{r^2}\right] = 4\pi\,\delta^{(3)}(\vec{r})$$

(VER AULAS INICIAIS)

$$\Rightarrow \vec{\nabla}\cdot\left[\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right] = 4\pi\,\delta^{(3)}(\vec{r}-\vec{r}\,')$$

1º TERMO: $\dfrac{\mu_0}{\cancel{4\pi}}\displaystyle\int \vec{J}(\vec{r}\,')(\cancel{4\pi})\,\delta^{(3)}(\vec{r}-\vec{r}\,')\,dV' = \mu_0\vec{J}(\vec{r})$

PARA O CÁLCULO DO 2º TERMO EU PRECISO DE:

$$\left[\vec{J}(\vec{r}\,')\cdot\vec{\nabla}\right]\left[\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right] = -\left[\vec{J}(\vec{r}\,')\cdot\vec{\nabla}'\right]\left[\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right]$$

SE TIVERMOS $f(x-x')$ ENTÃO:

$$\underbrace{\frac{d}{dx}f(x-x')}_{f'(x-x')} = -\underbrace{\frac{d}{dx'}f(x-x')}_{f'(x-x')(-1)}$$

TENHO ALGO COMO $(\vec{A}\cdot\vec{\nabla}')f$. USO A IDENTIDADE:

$$\vec{\nabla}'\left[f\vec{A}\right] = \vec{A}\cdot\left(\vec{\nabla}' f\right) + f\left(\vec{\nabla}'\cdot\vec{A}\right)$$

$$\Rightarrow \vec{I} = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\cdot\vec{\nabla}'\left[\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\right]dV'$$

$$I_x = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\cdot\vec{\nabla}'\left[\frac{(x-x')}{|\vec{r}-\vec{r}\,'|^3}\right]dV'$$

$$I_x = \frac{\mu_0}{4\pi}\int \vec{\nabla}' \cdot \left[\frac{(x-x')}{|\vec{r}-\vec{r}'|^3}\vec{J}(\vec{r}')\right] dV' - \frac{\mu_0}{4\pi}\int \frac{(x-x')}{|\vec{r}-\vec{r}'|^3}\underbrace{\vec{\nabla}'\cdot\vec{J}(\vec{r}')}_{=0} dV'$$

CORRENTE ESTACIONÁRIA

$$\int_{S_\infty} \frac{(x-x')}{|\vec{r}-\vec{r}'|^3}\vec{J}(\vec{r}')\cdot d\vec{S}'$$

SE $\vec{J}(\vec{r}')$ FOR LOCALIZADA, O INTEGRANDO É NULO NO INFINITO E A INTEGRAL É ZERO

ANALOGAMENTE: $I_y = 0$ $\quad I_z = 0$

FINALMENTE:

$$\boxed{\vec{\nabla}\times\vec{B}(\vec{r}) = \mu_0\vec{J}} \qquad (\vec{\nabla}\cdot\vec{J}=0)$$

LEI DE AMPÈRE

Leis da magnetostática

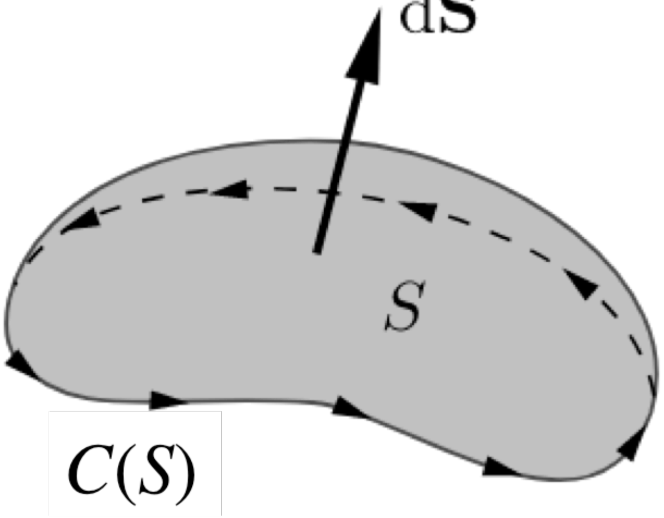
$$\boxed{\begin{aligned} \boldsymbol{\nabla} \cdot \mathbf{B} &= 0 \\ \boldsymbol{\nabla} \times \mathbf{B} &= \mu_0 \mathbf{J} \end{aligned}}$$

A lei de Ampère na forma integral

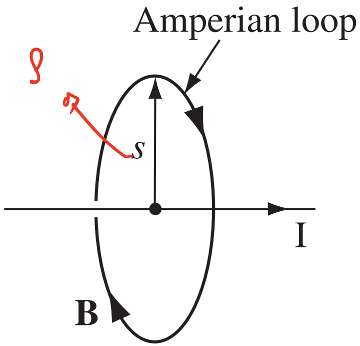
$$\int_S \vec{\nabla} \times \vec{B} \cdot d\vec{S} = \oint_{C(S)} \vec{B} \cdot d\vec{\ell}$$

$$\int_S \mu_0 \vec{J} \cdot d\vec{S} = \mu_0 I(S)$$

$$\Rightarrow \boxed{\oint_{C(S)} \vec{B} \cdot d\vec{\ell} = \mu_0 I(S)}$$



Campo de um fio reto infinito



SIMETRIA DO PROBLEMA:

- $\vec{B}(\vec{r}) = B_\phi(\vec{r})\,\hat{\phi}$
- $\vec{B}(\vec{r}) = B_\phi(\rho)\,\hat{\phi}$

APLICANDO A LEI DE AMPÈRE À AMPERIANA AO LADO.

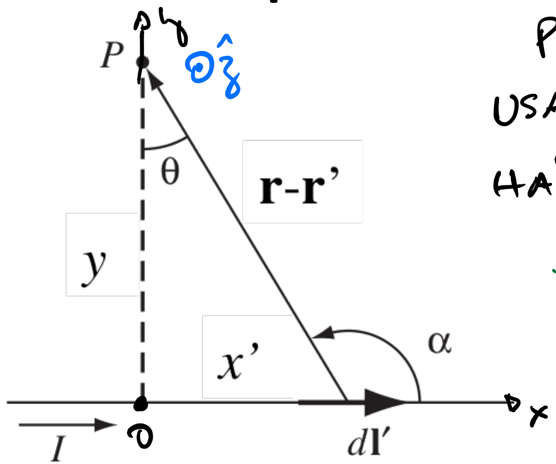
$$\oint_{C(s)} \vec{B}\cdot d\vec{\ell} = \oint_{C(s)} \vec{B}\cdot d\ell\,\hat{\phi} = \oint_{C(s)} B_\phi(\rho)\,\rho\, d\phi = 2\pi\rho\, B_\phi(\rho)$$

$$I(s) = I$$

$$\Rightarrow 2\pi\rho\, B_\phi(\rho) = \mu_0 I \Rightarrow B_\phi(\rho) = \frac{\mu_0 I}{2\pi\rho}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi\rho}\,\hat{\phi}}$$

Exemplo 5.5: campo de um fio **finito**



PARA O FIO FINITO NÃO DÁ PARA USAR LEI DE AMPÈRE PORQUE NÃO HÁ SIMETRIA SUFICIENTE ⇒ BIOT-SAVART

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int I\, d\vec{l}' \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{r} = y\,\hat{y} \qquad \vec{r}' = x'\,\hat{x} \qquad d\vec{l}' = dx'\,\hat{x}$$

$$d\vec{l}' \times (\vec{r}-\vec{r}') = dx'\,\hat{x} \times (y\,\hat{y} - x'\,\hat{x})$$

$$= y\, dx'\,\hat{z}$$

$$|\vec{r}-\vec{r}'| = |y\,\hat{y} - x'\,\hat{x}| = \sqrt{(x')^2 + y^2}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I y}{4\pi}\hat{z}\int_{x_1}^{x_2} \frac{dx'}{\left[(x')^2+y^2\right]^{3/2}}$$

TROCANDO VARIÁVEIS:

$$\sin\theta = \frac{x'}{\sqrt{(x')^2+y^2}} \qquad \cos\theta = \frac{y}{\sqrt{(x')^2+y^2}}$$

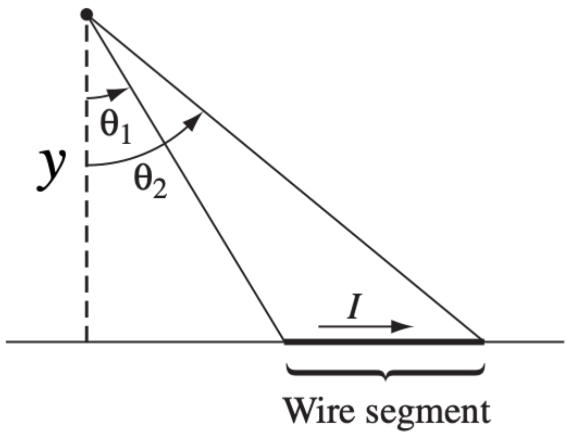
$$\tan\theta = \frac{x'}{y}\ ; \quad dx' = y\sec^2\theta\, d\theta$$

$$\int \frac{dx'}{\left[(x')^2+y^2\right]^{3/2}} = \int \cancel{y}\, \cancel{\sec^2}\theta\, d\theta\, \frac{\cancel{\cos^3}\theta}{y^{\cancel{3}\,2}} = \frac{1}{y^2}\int \cos\theta\, d\theta = \frac{\sin\theta}{y^2}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi y}\,\hat{z}\,(\sin\theta)\Big|_{\theta_1}^{\theta_2} = \frac{\mu_0 I}{4\pi y}\,\hat{z}\,(\sin\theta_2 - \sin\theta_1)$$

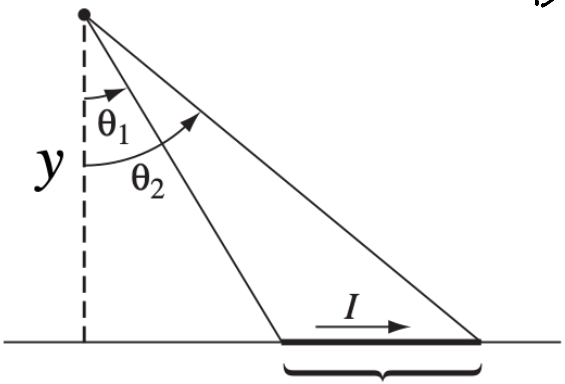
$$\Rightarrow \vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi s}\,\hat{\phi}\,(\sin\theta_2 - \sin\theta_1)$$

Campo de um fio finito



$$B = \frac{\mu_0 I}{4\pi y}\left(\sin\theta_2 - \sin\theta_1\right)$$

Recuperando o campo do fio infinito



$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi s}\hat{\phi}\,(\sin\theta_2 - \sin\theta_1)$$

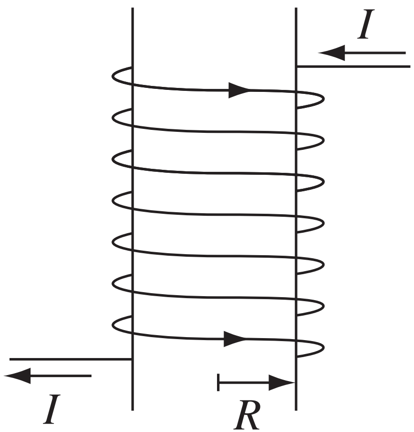
FIO INFINITO: $\theta_2 \to \frac{\pi}{2}$

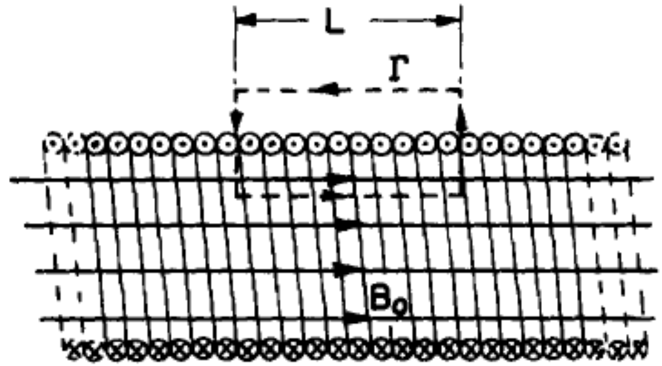
$\theta_1 \to -\frac{\pi}{2}$

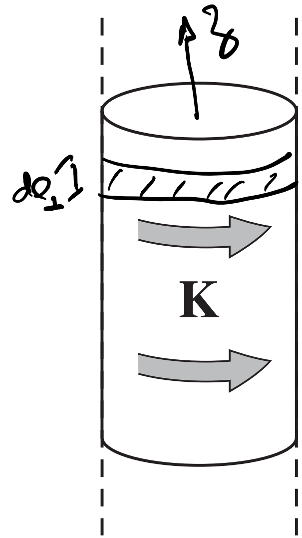
$$\sin\theta_2 - \sin\theta_1 \to 2$$

$$\Rightarrow \boxed{\vec{B}(\vec{r}) = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

Exemplo 5.9: solenóide infinito







n = NÚMERO DE ESPIRAS POR UNIDADE DE COMPRIMENTO

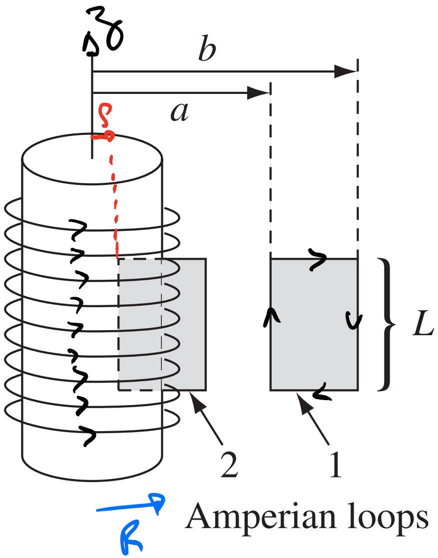
$$K = \frac{dI}{d\ell_\perp} = \frac{(n\, d\ell_\perp)\, I}{d\ell_\perp} = nI \Rightarrow \vec{K} = (nI)\,\hat{\phi}$$

SIMETRIA (USANDO COORDENADAS CILÍNDRICAS)

$\cdot\ \vec{B} = B_z \hat{z}$

$\cdot\ B_z(\rho)$

$$\vec{B} = B_z(\rho)\hat{z}$$



VAMOS APLICAR LEI DE AMPÈRE AOS DOIS CIRCUITOS AO LADO:

1. $\oint_C \vec{B} \cdot d\vec{\ell} = B_z(\rho=a)\cancel{L} - B_z(\rho=b)\cancel{L} = 0$

NÃO HÁ CORRENTE ATRAVESSANDO A SUPERFÍCIE 1

$\Rightarrow B_z(a) = B_z(b)$

$\Rightarrow B_z(\rho) =$ CONST. PARA $\rho > R$

MAS $B_z(\rho) \xrightarrow[\rho \to \infty]{} 0 \Rightarrow \boxed{B_z(\rho) = 0 \quad \text{PARA } \rho > R}$

VER ARGUMENTO RIGOROSO NO PAPER LINKADO NA PÁGINA DO CURSO

USANDO AGORA A SUPERFÍCIE 2:

$$\oint_{C(S)} \vec{B}\cdot d\vec{\ell} = B_z(\rho)L - \cancel{B_z(\rho)}L = \mu_0 I(S) = \mu_0(n I)L$$

0

FORA DO SOLENOIDE

$$\Rightarrow \boxed{B_z(\rho) = \mu_0 n I} \qquad (\rho < R)$$

EM TERMOS DE $\vec{K}$:

$$B_z(\rho) = \begin{cases} \mu_0 K & \rho < R \\ 0 & \rho > R \end{cases}$$