10) $\langle x|p\rangle = \dfrac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}$ $\qquad X|x\rangle = x|x\rangle$

$\langle x|X = \langle x|x$

a) $\langle x|XP|\psi\rangle = x\langle x|P|\psi\rangle$

b) $\langle x|PX|\psi\rangle = \int dp \, \langle x|P|p\rangle\langle p|X|\psi\rangle$

$$= \int dp \, p \, \langle x|p\rangle\langle p|X|\psi\rangle$$

$$= \int dp \, p \, \frac{e^{ipx/\hbar}}{\sqrt{2\pi\hbar}} \langle p|X|\psi\rangle$$

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\frac{e^{ipx/\hbar}}{\sqrt{2\pi\hbar}} = \frac{\hbar}{i}\frac{\partial}{\partial x}\left(\langle x|p\rangle\right)$$

$$= \frac{\hbar}{i}\frac{\partial}{\partial x}\int dp \, \langle x|p\rangle\langle p|X|\psi\rangle$$

$$= \frac{\hbar}{i}\frac{\partial}{\partial x}\left[\langle x|X|\psi\rangle\right] = \frac{\hbar}{i}\frac{\partial}{\partial x}\left[x \underbrace{\langle x|\psi\rangle}_{\psi(x)}\right]$$

$$= \frac{\hbar}{i}\frac{\partial}{\partial x}\left[x\psi(x)\right]$$

g)

a) $\langle \varphi_n | [A, H] | \varphi_n \rangle = 0$

b) $H = \dfrac{P^2}{2m} + V(X)$

α. $[H, P] = \left[\dfrac{P^2}{2m} + V(X), P\right] = [V(X), P]$

$= V(X)P - PV(X)$

$[H, X] = \left[\dfrac{P^2}{2m}, X\right] = \dfrac{1}{2m}[P^2, X] = -\dfrac{i\hbar}{m} P$

$[X, P^2] = \underbrace{[X, P]}_{i\hbar} P + P \underbrace{[X, P]}_{i\hbar} = 2i\hbar P$

$[A, BC] = [A, B]C + B[A, C]$

$[H, XP] = \dfrac{1}{2m}[P^2, XP] + [V(X), XP]$

$= \dfrac{1}{2m}\left\{[P^2, X]P\right\} + X[V(X), P]$

$= \dfrac{1}{2m}\left\{-2i\hbar P^2\right\} + X\left[V(X)P - PV(X)\right] \quad (1)$

$\beta.$ $\langle \varphi_n | P | \varphi_n \rangle = 0$

$$\langle \varphi_n | P | \varphi_n \rangle = \frac{im}{\hbar} \langle \varphi_n | [H, X] | \varphi_n \rangle = 0$$

PELO ITEM (a).

$\gamma.$ $E_K = \langle \varphi_n | \frac{P^2}{2m} | \varphi_n \rangle$

LISTA 2a:

$$[f(x), P] = i\hbar f'(x)$$

$$\langle \varphi_n | X \frac{dV}{dx} | \varphi_n \rangle$$

$$[V(x), P] = i\hbar V'(x)$$

$$[V(x), P] = V(x)P - PV(x) = C$$

ATUANDO SOBRE $\psi(x)$ GENÉRICA:

$$C\psi(x) = [V(x)P - PV(x)]\psi(x)$$

$$= V(x)\frac{\hbar}{i}\frac{\partial \psi}{\partial x} - \frac{\hbar}{i}\frac{\partial}{\partial x}[V(x)\psi(x)]$$

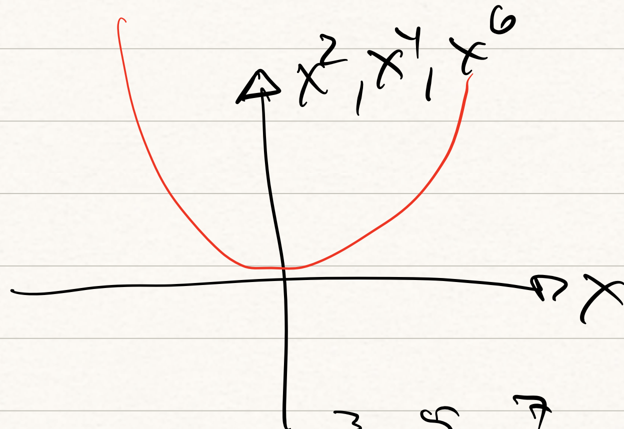
$$= +i\hbar V'(x)\psi(x) \Rightarrow C = i\hbar V'(x)$$

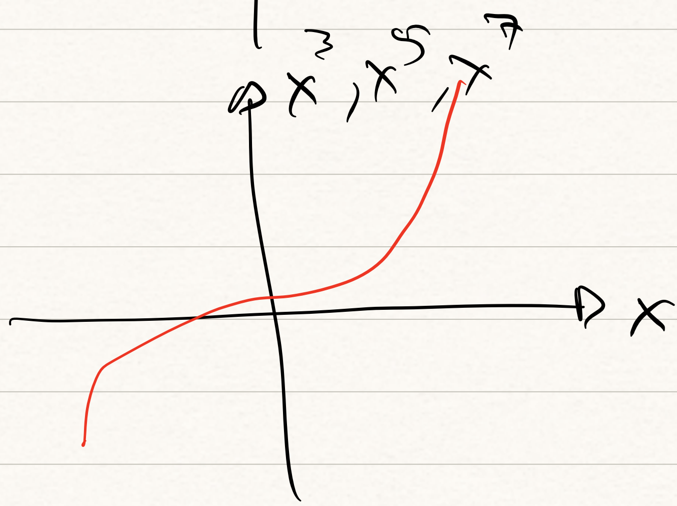
TOMANDO $\langle \varphi_n | (1) | \varphi_n \rangle$

$$0 = -2\cancel{i\hbar} \underbrace{\langle \varphi_n | \frac{P^2}{2m} | \varphi_n \rangle}_{E_k} + \cancel{i\hbar} \langle \varphi_n | X V'(x) | \varphi_n \rangle$$

$$\Rightarrow \langle \varphi_n | X V'(x) | \varphi_n \rangle = 2 E_k$$

SE $V(x) = V_0 X^s$ $\quad \underbrace{s = 2, 4, 6, \ldots}_{\text{PARA HAVER ESTADOS LIGADOS}}$





$$x V'(x) = x \, V_0 s \, x^{(s-1)} = s V_0 x^{(s)} = s V(x)$$

$$\Rightarrow \boxed{s \langle \varphi_n | V(x) | \varphi_n \rangle = 2 E_K}$$

C.C.O.C.

BASE $\{|1\rangle, |2\rangle, |3\rangle\}$

$$A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & -3i \\ 0 & 3i & 0 \end{pmatrix}$$

A e B SÃO HERMITIANOS

$[A, B] = 0$ ✓

$$AB = \begin{pmatrix} 12 & 0 & 0 \\ 0 & 0 & 12i \\ 0 & -12i & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} 12 & 0 & 0 \\ 0 & 0 & 12i \\ 0 & -12i & 0 \end{pmatrix}$$

$\Rightarrow$ DIAGONALIZE $\underline{B}$

$$B|1\rangle = 3|1\rangle \longrightarrow |\lambda_0\rangle = |1\rangle$$

$$\lambda_+ = 3 \Rightarrow |\lambda_+\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle + i|3\rangle\right)$$

$$\hookrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ i \end{pmatrix}$$

$$\lambda_- = -3 \Rightarrow |\lambda_-\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle - i|3\rangle\right)$$

NA BASE $\{|\lambda_0\rangle, |\lambda_+\rangle, |\lambda_-\rangle\}$

$$\Rightarrow B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{pmatrix} \qquad A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$

SIM, $\{A, B\}$ É UM CCOC

$$|a_i, b_j\rangle \Rightarrow \begin{cases} |4,3\rangle = |1\rangle = |\lambda_0\rangle \\ |-4,3\rangle = |\lambda_+\rangle \\ |-4,-3\rangle = |\lambda_-\rangle \end{cases}$$