3) DO COHEN

$$\psi(x) = \begin{cases} A e^{ikx} + B e^{-ikx} & (x<0) \\ C e^{ikx} & (x>0) \end{cases}$$

$$K\psi(x) = \begin{cases} KA e^{ikx} + KB e^{-ikx} & (x<0) \\ CK e^{ikx} & (x>0) \end{cases}$$

$$K = \frac{1}{A}$$

$$\psi(x) = \begin{cases} e^{ikx} + \overbrace{\frac{B}{A}}^{B'} e^{-ikx} & (x<0) \\ \underbrace{\frac{C}{A}}_{C'} e^{ikx} & (x>0) \end{cases}$$

$$R = \left|\frac{B}{A}\right|^2 \qquad T = \left|\frac{C}{A}\right|^2$$

$$R = |B'|^2 \qquad T = |C'|^2$$

2) b)

$$\begin{pmatrix} A_2 \\ A_2' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ A_1' \end{pmatrix}$$

$$\Rightarrow A_2 = M_{11} A_1 + M_{12} A_1'$$

$$A_2' = M_{21} A_1 + M_{22} A_1'$$

4) $-\dfrac{\hbar^2}{2m} \psi''(x) - \alpha\, \delta(x)\, \psi(x) = E \psi(x)$

MULTIPLIQUE A EQ. POR $e^{-ipx/\hbar}$ E INTEGRE EM X DE $(-\infty, +\infty)$:

$$E \int_{-\infty}^{+\infty} e^{-ipx/\hbar} \psi(x)\, dx = E \sqrt{2\pi\hbar}\, \Phi(p)$$

$$-\alpha \int_{-\infty}^{+\infty} e^{-ipx/\hbar} \delta(x) \psi(x)\, dx = -\alpha\, \psi(0)$$

$$\underbrace{-\frac{\hbar^2}{2m} \int_{-\infty}^{+\infty} e^{-ipx/\hbar} \psi''(x)\, dx}_{\mathrm{I}} = \frac{p^2}{2m} \sqrt{2\pi\hbar}\, \Phi(p)$$

$$I = \int_{-\infty}^{+\infty} e^{-ipx/\hbar}\, \psi''(x)\, dx$$

INTEGRO POR PARTES

$$I = \cancelto{0}{e^{-ipx/\hbar}\, \psi'(x) \Big|_{x\to-\infty}^{x\to+\infty}} - \int_{-\infty}^{+\infty} \psi'(x) \left(-\frac{ip}{\hbar}\right) e^{-ipx/\hbar}\, dx$$

PORQUE $\psi(x)$ E $\psi'(x) \to 0$ (EXPONENCIALMENTE)
QUANDO $x \to \pm\infty$

$$I = +\frac{ip}{\hbar} \int_{-\infty}^{+\infty} \psi'(x)\, e^{-ipx/\hbar}\, dx$$

$$= \frac{ip}{\hbar} \left[ \cancelto{0}{\psi(x)\, e^{-ipx/\hbar} \Big|_{x\to-\infty}^{x\to+\infty}} + \frac{ip}{\hbar} \underbrace{\int_{-\infty}^{+\infty} \psi(x)\, e^{-ipx/\hbar}\, dx}_{\sqrt{2\pi\hbar}\,\Phi(p)} \right]$$

$$= -\frac{p^2}{\hbar^2} \sqrt{2\pi\hbar}\, \Phi(p)$$

—//—

$$\frac{p^2}{2m} \sqrt{2\pi\hbar}\, \Phi(p) - \alpha\, \psi(0) = E \sqrt{2\pi\hbar}\, \Phi(p)$$

$$\sqrt{2\pi\hbar} \left[ \frac{p^2}{2m} - E \right] \Phi(p) = \alpha\, \psi(0)$$

$$\tilde{\psi}(p) = \frac{\alpha\,\psi(0)/\sqrt{2\pi\hbar}}{\frac{p^2}{2m} - E} \equiv \frac{A}{\frac{p^2}{2m} - E}$$

$$A = \frac{\alpha\,\psi(0)}{\sqrt{2\pi\hbar}}$$

$$\psi(x) = \int_{-\infty}^{+\infty} \frac{dp}{\sqrt{2\pi\hbar}}\, e^{+ipx/\hbar}\, \underbrace{\frac{A}{\frac{p^2}{2m} - E}}_{\tilde{\psi}(p)} \qquad (1)$$

DIVERGE, PARA $E>0$, QUANDO

$$p = \pm\sqrt{2mE}$$

PARA $E<0$, NÃO HÁ DIVERGÊNCIA:

$$E = -|E_L| \qquad E_L < 0$$

$$\tilde{\psi}(p) = \frac{A}{\frac{p^2}{2m} + |E_L|} = \frac{2mA}{p^2 + 2m|E_L|} = \frac{A'}{p^2 + p_0^2}$$

$$A' = 2mA \qquad \frac{p_0^2}{2m} = |E_L|$$

NORMALIZAÇÃO DE $\Phi(p)$

$$\int_{-\infty}^{+\infty} |\Phi(p)|^2 dp = 1$$

$$\Rightarrow \int_{-\infty}^{+\infty} \frac{A'^2}{(p^2+p_0^2)^2} dp = 1 \Rightarrow A' = \sqrt{\frac{2p_0^3}{\pi}}$$

$$A' = 2mA = \frac{2m\alpha\,\psi(0)}{\sqrt{2\pi\hbar}}$$

DE (1) EM $x=0$:

$$\cancel{\psi(0)} = \int_{-\infty}^{+\infty} \frac{dp}{\sqrt{2\pi\hbar}} \frac{A'}{p^2+p_0^2} = \frac{\cancel{2}m\alpha\,\cancel{\psi(0)}}{\cancel{2}\pi\hbar} \underbrace{\int_{-\infty}^{+\infty} \frac{dp}{p^2+p_0^2}}_{\frac{\pi}{p_0}}$$

FAZENDO A INTEGRAL:

$$\frac{\cancel{\pi}\hbar}{m\alpha} = \frac{\cancel{\pi}}{p_0} \Rightarrow p_0 = \frac{m\alpha}{\hbar}$$

$$E_L = -\frac{p_0^2}{2m} = -\frac{m^{\cancel{2}}\alpha^2}{\hbar^2}\frac{1}{2\cancel{m}} = -\frac{m\alpha^2}{2\hbar^2}$$

$$\Phi(p) = \int \frac{dx}{\sqrt{2\pi\hbar}} e^{-ipx/\hbar} \psi(x)$$

$$\psi(x) = \int \frac{dp}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} \Phi(p)$$

$$\psi(x) = \int \frac{dp}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} \left[ \int \frac{dx'}{\sqrt{2\pi\hbar}} e^{-ipx'/\hbar} \psi(x') \right]$$

$$= \int \frac{dx'}{2\pi\hbar} \underbrace{\int dp\, e^{ip(x-x')/\hbar}}_{I'(x-x')} \psi(x')$$

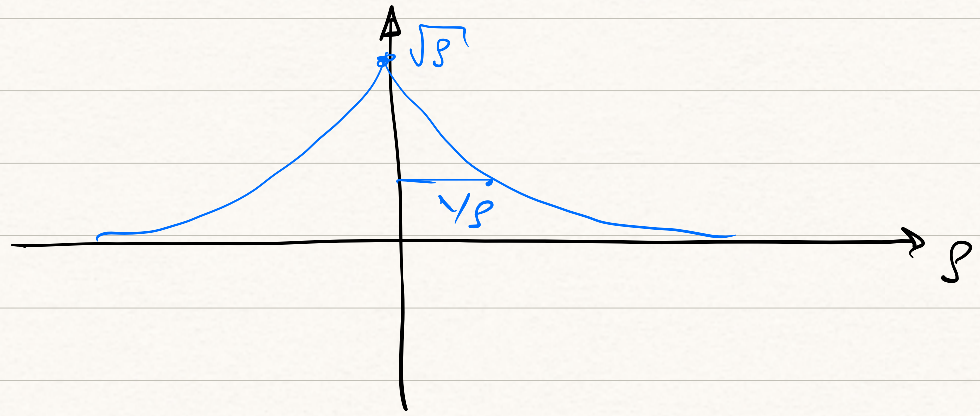
$$I'(y) = \int_{-\infty}^{+\infty} dp\, e^{ipy/\hbar} = \hbar \int_{-\infty}^{+\infty} \frac{dp}{\hbar} e^{ipy/\hbar}$$

$$\frac{p}{\hbar} = q$$

$$I'(y) = \hbar \underbrace{\int_{-\infty}^{+\infty} dq\, e^{iqy}}_{2\pi\delta(y)} = 2\pi\hbar\,\delta(y)$$

$$\psi(x) = \int \frac{dx'}{\cancel{2\pi\hbar}} \cancel{2\pi\hbar}\, \delta(x-x')\psi(x') = \psi(x)$$

2c) $\varphi(x) = \sqrt{\rho}\, e^{-\rho|x|}$




$\Delta x = ? = \frac{1}{\rho}$

RC $\quad I(t) \sim e^{-t/\tau}$

$\tau = RC$

$e^{-\frac{x^2}{a^2}}$ $\qquad \Delta x \sim a$

$\sin(kx)$ $\qquad$ ESCALA DE OSCILAÇÃO EM $x$

$\frac{1}{k}$

$$\alpha = E L = \frac{\hbar^2}{m L^{\cancel{2}}} \cancel{L}$$

$$E = \frac{\hbar^2}{m L^2}$$

$$\boxed{L = \frac{m\alpha}{\hbar^2}}$$

$$\delta \cancel{\alpha} \frac{\hbar^2}{m\alpha}$$