EXEMPLOS PARA RESOLVER NA AULA

C. C. O. C.

1) SEJAM, NUMA BASE $\{|1\rangle, |2\rangle, |3\rangle\}$ OS SEGUINTES OPERADORES:

$$A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix} ; B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & -3i \\ 0 & 3i & 0 \end{pmatrix}$$

A) CALCULE $[A, B]$

$$\left.\begin{aligned} AB &= \begin{pmatrix} 12 & 0 & 0 \\ 0 & 0 & 12i \\ 0 & -12i & 0 \end{pmatrix} \\ BA &= \begin{pmatrix} 12 & 0 & 0 \\ 0 & 0 & 12i \\ 0 & -12i & 0 \end{pmatrix} \end{aligned}\right\} \Rightarrow [A, B] = 0$$

B) DIAGONALIZE B

B É BLOCO-DIAGONAL. 3 É CLARAMENTE AUTO-VALOR COM AUTO-VETOR $|1\rangle$

MOSTRE A PARTIR DE $|1\rangle \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

PODEMOS DIAGONALIZAR O OUTRO BLOCO

$$\begin{vmatrix} -\lambda & -3i \\ 3i & -\lambda \end{vmatrix} = \lambda^2 - 9 = 0 \Rightarrow \lambda_\pm = \pm 3$$

$\lambda = \lambda_+ = 3$:

$$\begin{pmatrix} -3 & -3i \\ 3i & -3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 0 \qquad \begin{aligned} 3a &= -3ib \\ a &= -ib \end{aligned}$$

$$|\lambda_+\rangle = \frac{1}{\sqrt{2}} \left(|2\rangle + i|3\rangle \right) \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \end{pmatrix}$$

$\lambda = \lambda_- = -3$

$$\begin{pmatrix} 3 & -3i \\ 3i & 3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 0 \Rightarrow a = ib$$

$$|\lambda_-\rangle = \frac{1}{\sqrt{2}} \left(|2\rangle - i|3\rangle \right) \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -i \end{pmatrix}$$

ASSIM, NA NOVA BASE

$$|A\rangle = |1\rangle$$

$$|B\rangle = \frac{1}{\sqrt{2}}(|2\rangle + i|3\rangle)$$

$$|C\rangle = \frac{1}{\sqrt{2}}(|2\rangle - i|3\rangle)$$

$$\Rightarrow B' = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

NA MESMA BASE, O OPERADOR A É O MESMO QUE NA BASE ORIGINAL, POIS $|1\rangle$ JÁ ERA AUTO-VETOR E NO ~~SUB-ESPAÇO~~ SUB-ESPAÇO 2×2 $\{|2\rangle, |3\rangle\}$, A É PROPORCIONAL À IDENTIDADE:

$$\Rightarrow A' = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$

ASSIM, PODEMOS IDENTIFICAR UNIVOCAMENTE OS AUTO-VETORES SIMULTÂNEOS DE A E B POR SEUS AUTO-VALORES $|a, b\rangle$

$$|4,3\rangle = |1\rangle = |A\rangle$$

$$|-4,3\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle + i|3\rangle\right) = |B\rangle$$

$$|-4,-3\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle - i|3\rangle\right) = |C\rangle$$

A E B FORMAM UM C.C.O.C.

CONSIDERE AGORA:

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 7/2 & -1/2 & 0 \\ -1/2 & 7/2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

TEMOS QUE:

$$A \times B = \begin{pmatrix} 11 & -5 & 0 \\ -5 & 11 & 0 \\ 0 & 0 & 6 \end{pmatrix} = BA.$$

DIAGONALIZANDO A:

$$\begin{vmatrix} 3-\lambda & -1 \\ -1 & 3-\lambda \end{vmatrix} = (3-\lambda)^2 - 1 = 0$$

$$\Rightarrow \lambda - 3 = \pm 1 \Rightarrow \begin{cases} \lambda_1 = 4 \\ \lambda_2 = 2 \end{cases}$$

$$\lambda_1 : \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow |\lambda_1\rangle = \frac{1}{\sqrt{2}} (|1\rangle - |2\rangle)$$

$$\lambda_2 : \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = 0 \Rightarrow |\lambda_2\rangle = \frac{1}{\sqrt{2}} (|1\rangle + |2\rangle)$$

ASSIM, SE:

$$S = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

SEGUE QUE:

$$S^{\dagger} A S = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$S^{\dagger} B S = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$\Rightarrow$ $\underline{A}$ E $\underline{B}$ NÃO FORMAM UM

C.C.O.C.