PROB. 5 DA LISTA 4 (CAP. 10)

$|\varepsilon_1, \varepsilon_2, \varepsilon_3\rangle$

$\vec{S}_{12} = \vec{S}_1 + \vec{S}_2$ E $\vec{S} = \vec{S}_{12} + \vec{S}_3 = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$

$\Rightarrow$ C.C.O.C. $\{S_1^2, S_2^2, S_3^2, S_{12}^2, S^2, S_z\}$

(a) $|s_1 = 1/2, s_2 = 1/2, s_3 = 1/2, s_{12}, s, m\rangle$

$$s_{12} = \begin{cases} 0 \longrightarrow s = 1/2 \longrightarrow m = \pm\frac{1}{2} \\ 1 \begin{cases} \longrightarrow s = 1/2 \longrightarrow m = \pm 1/2 \\ \longrightarrow s = 3/2 \longrightarrow m = \pm 1/2, \pm 3/2 \end{cases} \end{cases}$$

NOTAR QUE, PARA IDENTIFICAR COMPLETAMENTE CADA AUTO-VETOR, PRECISAMOS ESPECIFICAR CADA UM DE: s_{12}, s, m

(b) NO CASO GERAL:

$$\vec{J} = \vec{J}_1 + \vec{J}_2 \Rightarrow \vec{J}^2 = (\vec{J}_1 + \vec{J}_2)\cdot(\vec{J}_1 + \vec{J}_2)$$

$$= \vec{J}_1^2 + \vec{J}_2^2 + \vec{J}_1\cdot\vec{J}_2 + \vec{J}_2\cdot\vec{J}_1$$

(SE $[\vec{J}_1, \vec{J}_2] = 0$) $= \vec{J}_1^2 + \vec{J}_2^2 + 2\vec{J}_1\cdot\vec{J}_2$

$$\Rightarrow \vec{J}_1\cdot\vec{J}_2 = \frac{\vec{J}^2 - \vec{J}_1^2 - \vec{J}_2^2}{2} \longrightarrow \frac{\hbar^2}{2}\left[j(j+1) - j_1(j_1+1) - j_2(j_2+1)\right]$$

ONDE: $j = |j_1 - j_2|, \ldots\ldots, j_1 + j_2$

c) $H = A\,\vec{S}_1\cdot\vec{S}_2 + B(\vec{S}_1+\vec{S}_2)\cdot\vec{S}_3$

USANDO A LETRA (b)

$$H = \frac{A}{2}\left[\vec{S}_{12}^{\,2} - \vec{S}_1^{\,2} - \vec{S}_2^{\,2}\right] + B\,\vec{S}_{12}\cdot\vec{S}_3$$

$$\vec{S} = \vec{S}_{12} + \vec{S}_3 \Rightarrow \vec{S}_{12}\cdot\vec{S}_3 = \frac{1}{2}\left[\vec{S}^{\,2} - \vec{S}_{12}^{\,2} - \vec{S}_3^{\,2}\right]$$

$$H = \frac{(A-B)}{2}\vec{S}_{12}^{\,2} - \frac{A}{2}(\vec{S}_1^{\,2}+\vec{S}_2^{\,2}) - \frac{B}{2}\vec{S}_3^{\,2} + \frac{B}{2}\vec{S}^{\,2}$$

AUTO-VALORES E AUTO-VETORES DE H

AUTO-VETORES: $|s_1 = 1/2, s_2 = 1/2, s_3 = 1/2, s_{12}, s, m\rangle$

AUTO-VALORES:

$$E(s_{12}, s) = -\frac{3A\hbar^2}{4} - \frac{3}{8}B\hbar^2 + \frac{(A-B)}{2}s_{12}(s_{12}+1)\hbar^2 + \frac{B}{2}s(s+1)\hbar^2$$

ONDE: $\boxed{s_{12} = 0,\ s = 1/2}$ OU

$\boxed{s_{12} = 1,\ s = 1/2}$ OU

$\boxed{s_{12} = 1;\ s = 3/2}$

SE QUISERMOS OS AUTO-VETORES

$$|s_1 = 1/2, s_2 = 1/2, s_3 = 1/2, s_{12}, s, m\rangle$$

EM TERMOS DE $|\varepsilon_1, \varepsilon_2, \varepsilon_3\rangle$, PRECISAMOS DOS CLEBSCH-GORDAN:

$$|s_{12}=0, \overset{m_{12}=0}{\checkmark} \varepsilon_2\rangle = \frac{1}{\sqrt{2}}\left[|+,-,\varepsilon_3\rangle - |-,+,\varepsilon_3\rangle\right]$$

$$|s_{1,2}=0, s=1/2, \varepsilon\rangle = \frac{1}{\sqrt{2}}\left[|+,-,\varepsilon\rangle - |-,+,\varepsilon\rangle\right]$$

ONDE $\varepsilon = \pm = 2m$

$$|S_{12}=1, m_{1,2}, \varepsilon_3\rangle = \begin{cases} |S_{12}=1, m_{1,2}=1, \varepsilon_3\rangle = |+,+,\varepsilon_3\rangle \\ |S_{1,2}=1, m_{1,2}=-1, \varepsilon_3\rangle = |-,-,\varepsilon_3\rangle \\ |S_{1,2}=1, m_{1,2}=0, \varepsilon_3\rangle = \end{cases}$$

$$= \frac{1}{\sqrt{2}}\left[|+,-,\varepsilon_3\rangle + |-,+,\varepsilon_3\rangle\right]$$

PARA SOMAS $S_{1,2}=1$ COM $S_3=1/2$ E ACHAR A NOVA BASE, PRECISAMOS DOS CLEBSCH-GORDAN CORRESPONDENTES.

SUPONHA $j_1 = 0, \quad j_2 = 1/2 \Rightarrow j = 1/2$

$$\begin{array}{cc} j_1 = 0 & j_2 = 1/2 \\ \downarrow & \downarrow \\ |m_1 = 0\rangle & |m_2 = \pm 1/2\rangle \end{array}$$

$$|m_1, m_2\rangle = |0, \pm 1/2\rangle$$

BASE SOMADA : $|j, m\rangle = |j = 1/2, m = \pm 1/2\rangle$

$$= |0, \pm 1/2\rangle$$

PROB. 2 DO CAP. IX

$$A = \vec{S} \cdot \vec{P} = S_x P_x + S_y P_y + S_z P_z$$

a) A É HERMITIANO?

$$A^{\dagger} = (S_x P_x)^{\dagger} + (S_y P_y)^{\dagger} + (S_z P_z)^{\dagger}$$

$$= P_x^{\dagger} S_x^{\dagger} + P_y^{\dagger} S_y^{\dagger} + P_z^{\dagger} S_z^{\dagger}$$

$$= P_x S_x + P_y S_y + P_z S_z$$

COMO $[S_i, P_j] = 0$:

$$A^{\dagger} = S_x P_x + S_y P_y + S_z P_z = \vec{S} \cdot \vec{P} = A \Rightarrow \text{HERMITIANO}$$

b) $[P_x, A] = [P_x, \vec{S} \cdot \vec{P}] = [P_x, S_x P_x + S_y P_y + S_z P_z] = 0$

ANALOGAMENTE $[P_y, A] = [P_z, A] = 0$

MATRIZ DE NA BASE $|p_x, p_y, p_z, \pm\rangle$

$$A|p_x, p_y, p_z, \pm\rangle = (p_x S_x + p_y S_y + p_z S_z)|p_x, p_y, p_z, \pm\rangle$$

$$A \rightarrow \frac{\hbar}{2}(p_x \sigma_x + p_y \sigma_y + p_z \sigma_z) = \frac{\hbar}{2}\begin{pmatrix} p_z & p_x - i p_y \\ p_x + i p_y & -p_z \end{pmatrix}$$

c) AUTO-VALORES DE A:

$$\pm \frac{\hbar}{2}\sqrt{p_x^2 + p_y^2 + p_z^2}$$

O AUTO-VALOR DE A FIXA O SINAL E O MÓDULO DE $\vec{p}$. MAS PARA UM MÓDULO DE $\vec{p}$ FIXO HÁ UMA ESFERA DE INFINITOS VALORES DAS COMPONENTES (p_x, p_y, p_z).