CAP. 13

PROBLEMA 1 DO COHEN

$$W(t) = \begin{cases} -qEX & t \in [0, \tau] \\ 0 & t<0 \text{ E } t > \tau \end{cases}$$

$$P_{0n} = |\langle \varphi_n | \psi(t) \rangle|^2 \qquad |\psi(t=0)\rangle = |\varphi_0\rangle$$

$$\text{a) } P_{01} = \frac{1}{\hbar^2} \left| \int_0^t e^{i\omega_{10}t'} W_{10}(t') dt' \right|^2$$

$$\omega_{10} = \frac{E_1 - E_0}{\hbar} = \frac{3/2 \hbar\omega_0 - 1/2 \hbar\omega_0}{\hbar} = \omega_0$$

$$W_{10}(t) = \langle \varphi_1 | -qEX | \varphi_0 \rangle \qquad t \in [0, \tau]$$

$$= -qE \langle \varphi_1 | X | \varphi_0 \rangle = -qE \langle \varphi_1 | \sqrt{\frac{\hbar}{2m\omega_0}} (a + a^\dagger) | \varphi_0 \rangle$$

$$= -qE \sqrt{\frac{\hbar}{2m\omega_0}}$$

$$\int_0^t e^{i\omega_0 t'} \overbrace{\left[ -qE \sqrt{\frac{\hbar}{2m\omega_0}} \right]}^{\overline{W}} \theta(\tau - t') \, dt'$$

$$t < \tau : \overline{W} \int_0^t e^{i\omega_0 t'} dt' = \frac{\overline{W}}{i\omega_0} \left[ e^{i\omega_0 t} - 1 \right]$$

$$= \frac{\overline{W}}{i\omega_0} e^{i\omega_0 t/2} \left[ e^{i\omega_0 t/2} - e^{-i\omega_0 t/2} \right]$$

$$= \left( 2\overline{W}/\omega_0 \right) e^{i\omega_0 t/2} \sin\left( \frac{\omega_0 t}{2} \right)$$

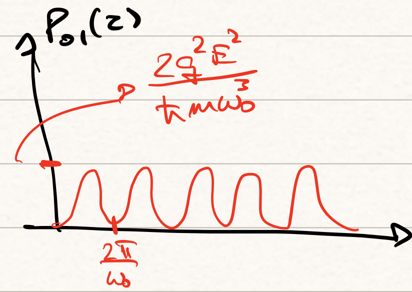
$$P_{01}(t) = 4\frac{\overline{W}^2}{\hbar^2\omega_0^2}\sin^2\left(\frac{\omega_0 t}{2}\right) \qquad t<\tau$$

$$= \frac{4}{\hbar^2\omega_0^2}\, q^2E^2\left(\frac{\hbar}{2m\omega_0}\right)\sin^2\left(\frac{\omega_0 t}{2}\right)$$

$$= \frac{2q^2E^2}{\hbar m\omega_0^3}\sin^2\left(\frac{\omega_0 t}{2}\right)$$

SE $t>\tau$:

$$P_{01}(\tau) = \frac{2q^2E^2}{\hbar m\omega_0^3}\sin^2\left(\frac{\omega_0 \tau}{2}\right)$$




(b) $b_2^{(1)}(t) \propto \langle\varphi_2|(a+a^\dagger)|\varphi_0\rangle = 0$

$b_2^{(2)}(t) = \;?$

DE (B-14): $$i\hbar\,\dot b_n^{(1)}(t) = \sum_k e^{i\omega_{nk}t}\,\hat W_{nk}\, b_k^{(0)}(t)$$

$$i\hbar\,\dot b_n^{(2)}(t) = \sum_k e^{i\omega_{nk}t}\,\hat W_{nk}\, b_k^{(1)}(t)$$

$$b_0^{(0)}(t) = b_0^{(0)}(0) = 1 \qquad b_n^{(0)}(t) = 0 \quad (n\neq 0)$$

$$b_1^{(1)}(t) = \frac{1}{i\hbar}\int_0^t e^{i\omega_{10}t'}\,\hat W_{10}(t')\,dt'$$

$$= \frac{1}{i\hbar}\,\frac{\overline{W}}{i\omega_0}\left(e^{i\omega_0 t}-1\right)\Theta(\tau-t)$$

$$b_n^{(1)}(t) = 0 \qquad n\neq 0,1$$

$$\dot b_n^{(2)}(t) = \frac{1}{i\hbar}e^{i\omega_{n1}t}\,\hat W_{n1}\, b_1^{(1)}(t) = -\frac{1}{\hbar^2}\frac{\overline{W}}{i\omega_0}e^{i\omega_{n1}t}\left(e^{i\omega_0 t}-1\right)\Theta(\tau-t)\,\hat W_{n1}$$

$$\dot b_n^{(2)}(t) = \frac{1}{i\hbar} e^{i\omega_{n1} t}\, \hat W_{n1}\, b_1^{(1)}(t) = -\frac{1}{\hbar^2}\frac{\overline W}{i\omega_0} e^{i\omega_{n1} t}(e^{i\omega_0 t}-1)\theta(\tau-t)\hat W_{n1}$$

$n=2$:

$$\lambda^2 \dot b_2^{(2)}(t) = \frac{i(\overline W)^2}{\hbar^2\omega_0} e^{i\omega_0 t}(e^{i\omega_0 t}-1)\,\theta(t-\tau)$$

$$\hat W_{21} = -qE\sqrt{\frac{\hbar}{2m\omega_0}}\,\overbrace{\langle\varphi_2|(a+a^\dagger)|\varphi_1\rangle}^{\sqrt{2}} = \sqrt{2}\,\overline W$$

$$\omega_{21} = \frac{E_2-E_1}{\hbar} = \omega_0$$

$$\lambda^2 b_2^{(2)}(t) = \frac{i(\overline W)^2}{\hbar^2\omega_0}\int_0^\tau \left(e^{2i\omega_0 t} - e^{i\omega_0 t}\right)dt$$

$$= \frac{(\overline W)^2}{\hbar^2\omega_0}\left[\frac{1}{2\omega_0}(e^{2i\omega_0\tau}-1) - \frac{1}{\omega_0}(e^{i\omega_0\tau}-1)\right]$$

$$= \frac{(\overline W)^2}{\hbar^2\omega_0^2}\left[\frac{1}{2} + \frac{1}{2}e^{2i\omega_0\tau} - e^{i\omega_0\tau}\right]$$

$$P_{20}^{(2)}(\tau) = |\lambda^2 b_2^{(2)}(\tau)|^2$$

$$W_{nk}(t) = \overline{W} \langle \varphi_n | (a + a^{\dagger}) | \varphi_k \rangle$$

$$= \sqrt{k} \langle \varphi_n | \varphi_{k-1} \rangle + \sqrt{k+1} \langle \varphi_n | \varphi_{k+1} \rangle \overline{W}$$

$$= \Big[ \sqrt{k} \underbrace{\delta_{n,k-1}}_{k=n+1} + \sqrt{k+1} \underbrace{\delta_{n,k+1}}_{k=n-1} \Big] \overline{W}$$

$$i\hbar \dot{b}_n^{(n)}(t) = \overline{W} \Big[ e^{i\omega_{n,n-1} t} \sqrt{n}\, b_{n-1}^{(n-1)}(t) + e^{i\omega_{n,n+1} t} \sqrt{n+1}\, b_{n+1}^{(n-1)}(t) \Big]$$

COND. INICIAIS: $b_0(0) = 1$ $\quad b_{n \neq 0}(0) = 0$

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$b_0^{(0)}(0) = 1$ $\quad b_{n \neq 0}^{(0)}(0) = 0$

$b_0^{(n \neq 0)}(0) = 0$ $\quad b_{n \neq 0}^{(n \neq 0)}(0) = 0$