PROB. (2) DO CAP. 11

H_0 : POÇO QUADRADO INFINITO 2D DE LADO a

$$\langle \vec{r} | n_x, n_y \rangle = \left(\frac{2}{a}\right) \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{a}\right) = \varphi_{n_x n_y}(\vec{r})$$

$$E(n_x, n_y) = \frac{\pi^2 \hbar^2 (n_x^2 + n_y^2)}{2ma^2}$$

PERTURBAÇÃO $\underline{W}$

a) EST. FUNDAMENTAL $|1,1\rangle$ NÃO DEGENERADO

$$E^0(1,1) = \frac{\pi^2 \hbar^2}{ma^2}$$

$$E^1(1,1) = \langle 1,1 | W | 1,1 \rangle$$

$$= \int \varphi_{11}^*(\vec{r})\, W(\vec{r})\, \varphi_{11}(\vec{r})\, d^2 r$$

$$= \int_0^{a/2} dx \int_0^{a/2} dy\; W_0 \sin^2\left(\frac{\pi x}{a}\right) \sin^2\left(\frac{\pi y}{a}\right) \left(\frac{4}{a^2}\right)$$

$$= \frac{4 W_0}{\cancel{a^2}} \left(\frac{\cancel{a}}{4}\right)^2 = \frac{W_0}{4}$$

$$E(1,1) \simeq \frac{\pi^2 \hbar^2}{ma^2} + \frac{W_0}{4} + O(W_0^2)$$

b) 1º ESTADO EXCITADO

$$E^0(1,2) = E^0(2,1) = \frac{5\pi^2\hbar^2}{2ma^2}$$ DUPLAMENTE DEGENERADO.

SUB-ESPAÇO $\{|1,2\rangle, |2,1\rangle\}$ → MATRIZ DE W

$\langle 1,2|W|1,2\rangle$, $\langle 1,2|W|2,1\rangle$, $\langle 2,1|W|2,1\rangle$

$$\Rightarrow W = w_0 \begin{pmatrix} 1/4 & \frac{16}{9}\pi^2 \\ \frac{16\pi^2}{9} & 1/4 \end{pmatrix} = \frac{w_0}{4}\mathbb{1} + \frac{16\pi^2}{9}\sigma_x$$

AUTO-VALORES: $e_{+,-} = w_0\left[\frac{1}{4} \pm \frac{16\pi^2}{9}\right]$

$$|\psi(\pm)\rangle = \frac{1}{\sqrt{2}}\left[|1,2\rangle \pm |2,1\rangle\right]$$

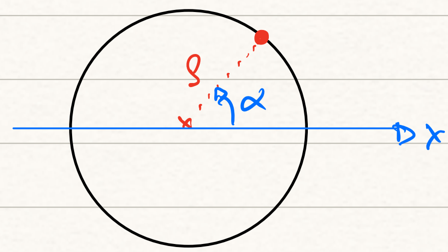
$$E(\pm) = \frac{5\pi^2\hbar^2}{2ma^2} + \frac{w_0}{4} \pm \frac{16\pi^2}{9} w_0 + O(w_0^2)$$

DE MANEIRA GERAL, NO CASO 2x2

$$W = a\mathbb{1} + b\sigma_x + c\sigma_y + d\sigma_z$$

AUTO-VALORES: $a \pm \sqrt{b^2 + c^2 + d^2}$

PROB. 4 DO CAP. 11 DO COHEN



$\psi(\alpha)$

$\Rightarrow \psi(\alpha + 2\pi) = \psi(\alpha)$

$$\int_0^{2\pi} |\psi(\alpha)|^2 \, d\alpha = 1$$

a) $M = \dfrac{\hbar}{i} \dfrac{d}{d\alpha}$

$$\left(\langle \varphi | M | \psi \rangle\right)^* \stackrel{?}{=} \langle \psi | M | \varphi \rangle$$

$$\left[\int_0^{2\pi} \varphi^*(\alpha) \frac{\hbar}{i} \frac{d}{d\alpha} \psi(\alpha) \, d\alpha\right]^* \stackrel{?}{=} \int_0^{2\pi} \psi^*(\alpha) \frac{\hbar}{i} \frac{d}{d\alpha} \varphi(\alpha) \, d\alpha$$

INT. PARTES

$$\left(\frac{\hbar}{i}\right)\left[\cancel{\varphi^*(\alpha) \psi(\alpha)\Big|_0^{2\pi}} - \int_0^{2\pi} \left(\frac{d\varphi^*}{d\alpha}\right) \psi(\alpha) \, d\alpha\right]$$

$$= -\frac{\hbar}{i} \int_0^{2\pi} \psi(\alpha) \frac{d}{d\alpha} \varphi^*(\alpha) \, d\alpha$$

$$[\;]^* = \frac{\hbar}{i} \int_0^{2\pi} \psi^*(\alpha) \frac{d}{d\alpha} \varphi(\alpha) \, d\alpha$$

= ✓ HERMITIANO

$$M\varphi = \lambda \varphi \Rightarrow \frac{\hbar}{i} \frac{d\varphi(\alpha)}{d\alpha} = \lambda \varphi(\alpha)$$

$\varphi(\alpha) = A e^{i\lambda\alpha/\hbar}$ PERIÓDICA $\Rightarrow$ $\varphi(\alpha + 2\pi) = \cancel{A e^{i\lambda\alpha/\hbar}} \, e^{i\lambda 2\pi/\hbar} = \cancel{A e^{i\lambda\alpha/\hbar}}$

$m = (0, \pm 1, \pm 2, \dots) \Rightarrow e^{i\lambda 2\pi/\hbar} = 1 \Rightarrow 2\pi \frac{\lambda}{\hbar} = 2m\pi \Rightarrow \lambda = m\hbar$

AUTO-FUNÇÕES DE M SÃO:

$$\varphi_m(\alpha) = A\, e^{im\alpha} \qquad m = 0, \pm 1, \pm 2, \ldots$$

$$\int_0^{2\pi} |\varphi_m(\alpha)|^2 d\alpha = 1 \Rightarrow |A|^2 \times 2\pi = 1 \Rightarrow |A| = \frac{1}{\sqrt{2\pi}}$$

$$\varphi_m(\alpha) = \frac{e^{im\alpha}}{\sqrt{2\pi}}$$

AUTO-VALORES DE M SÃO: $\lambda = m\hbar$

$M = L_z$ = MOM. ANGULAR PERPENDICULAR AO PLANO

b) $$H_0 = \frac{M^2}{\underbrace{2M\rho^2}_{I}} \longrightarrow \frac{L_z^2}{2I}$$

$$H_0\, \varphi_m(\alpha) = \left(\frac{m^2 \hbar^2}{2M\rho^2}\right) \varphi_m(\alpha)$$

$$E(m) = \frac{\hbar^2 m^2}{2M\rho^2} \qquad m = 0, \pm 1, \pm 2, \ldots$$

$E(0) \rightarrow$ NÃO DEGENERADO

$E(m)$ $(m \neq 0) \longrightarrow$ DUPLAMENTE DEGENERADOS

a) PARA EXPANDIR UMA FUNÇÃO QUALQUER $\psi(\alpha)$ NA BASE $\varphi_m(\alpha)$

$$\psi(\alpha) = \sum_{m=-\infty}^{+\infty} c_m \varphi_m(\alpha)$$

$$c_m = \int_0^{2\pi} \varphi_m^*(\alpha)\psi(\alpha)\, d\alpha$$

$$\cos^2\alpha \longrightarrow \frac{1}{2}(1+\cos 2\alpha) = \underset{\substack{\downarrow \\ \varphi_0(x)}}{\frac{1}{2}} + \frac{1}{4}\Big(\underset{\substack{\downarrow \\ \varphi_1(x)}}{e^{i\alpha}} + \underset{\substack{\downarrow \\ \varphi_{-1}(\alpha)}}{e^{-i\alpha}}\Big)$$