COHEN (2) $H(t) = a(t)\vec{S}_1 \cdot \vec{S}_2$

(a) $t \to -\infty$ $|\psi_i\rangle = |+,-\rangle$

$P(|+,-\rangle \to |-+\rangle)$ $t \to \infty$

$$H(t) = a(t)\hbar^2\left[\frac{S(S+1)}{2} - \frac{3}{4}\right]$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2 \Rightarrow \vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}\left[S(S+1) - 2\times\frac{3}{4}\right]\hbar^2$$

$$= \left[\frac{S(S+1)}{2} - \frac{3}{4}\right]\hbar^2$$

$|S, M\rangle$

$$|\psi\rangle = c(t)|S=0, M=0\rangle + \sum_{M=-1}^{1} b_M(t)|S=1, M\rangle$$

$$i\hbar \frac{\partial}{\partial t}|\psi\rangle = H(t)|\psi\rangle$$

$$\cancel{\hbar}\dot{c}(t) = \frac{3i\hbar^{\cancel{2}}}{4} a(t)\, c(t)$$

$$\dot{b}_M(t) = -\frac{i}{4}\hbar\, a(t)\, b_M(t)$$

$$c(t) = c(-\infty)\, e^{+i\frac{3}{4}\hbar\int_{-\infty}^{t} a(t')\,dt'}$$

$$b_M(t) = b_M(-\infty)\, e^{-\frac{i}{4}\hbar\int_{-\infty}^{t} a(t')\,dt'}$$

$$|\psi(t)\rangle = c(t)|s=0, m=0\rangle + \sum_{m=-1}^{1} b_m(t)|s=1, m\rangle$$

$$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = (i\hbar)\left[\dot{c}(t)|s=0, m=0\rangle + \sum_{m=-1}^{1} \dot{b}_m(t)|s=1, m\rangle\right]$$

$$H(t)|\psi(t)\rangle = a(t)\hbar^2\left[\frac{s(s+1)}{2} - \frac{3}{4}\right]|\psi(t)\rangle$$

$$= a(t)\hbar^2\left[-\frac{3}{4}c(t)|s=0, m=0\rangle + \frac{1}{4}\sum_{m=-1}^{1} b_m(t)|s=1, m\rangle\right]$$

IGUALANDO OS COEFICIENTES:

$$i\cancel{\hbar}\,\dot{c}(t) = -\frac{3}{4}a(t)\hbar^{\cancel{2}}c(t) \quad (1)$$

$$i\cancel{\hbar}\,\dot{b}_m(t) = \frac{1}{4}a(t)\hbar^{\cancel{2}}b_m(t) \quad (2)$$

$$(1): \frac{\dot{c}}{c} = \frac{3i\hbar}{4}a(t) \Rightarrow \frac{d}{dt}[\ln c(t)] = \frac{3i\hbar}{4}a(t)$$

$$\Rightarrow \ln[c(t)] = \frac{3i\hbar}{4}\int_{-\infty}^{t} a(t')\,dt' + K$$

$$c(t) = c(-\infty)\exp\left[\frac{3i\hbar}{4}\int_{-\infty}^{t} a(t')\,dt'\right]$$

$$b_m(t) = b_m(-\infty)\exp\left[-\frac{i\hbar}{4}\int_{-\infty}^{t} a(t')\,dt'\right]$$

$$|S=0, M=0\rangle = \frac{1}{\sqrt{2}}\left[|+-\rangle - |-+\rangle\right]$$

$$|S=1, M=0\rangle = \frac{1}{\sqrt{2}}\left[|+-\rangle + |-+\rangle\right]$$

$$\Rightarrow |+-\rangle = \frac{1}{\sqrt{2}}\left[|S=0, M=0\rangle + |S=1, M=0\rangle\right]$$

$$c(-\infty) = \frac{1}{\sqrt{2}} \qquad b_0(-\infty) = \frac{1}{\sqrt{2}} \qquad b_{\pm 1}(\infty) = 0$$

$$c(t) = \frac{1}{\sqrt{2}} e^{\frac{3i}{4}\hbar f(t)} \qquad b_{\pm 1}(t) = 0$$

$$b_0(t) = \frac{1}{\sqrt{2}} e^{-\frac{i}{4}\hbar f(t)} \qquad f(t) = \int_{-\infty}^{t} a(t')\,dt'$$

$$P = \lim_{t\to\infty} |\langle -+|\psi(t)\rangle|^2$$

$$\lim_{t\to\infty} \left|\frac{1}{\sqrt{2}}\left[\langle S=1, M=0| - \langle S=0, M=0|\right]|\psi(t)\rangle\right|^2$$

$$= \lim_{t\to\infty} \left|\frac{1}{\sqrt{2}}\left[b_0(t) - c(t)\right]\right|^2$$

$$= \lim_{t\to\infty} \frac{1}{4}\left|e^{-i\hbar/4 f(t)} - e^{\frac{3i\hbar}{4} f(t)}\right|^2 \qquad f(\infty) = F$$

$$= \frac{1}{4}\left|e^{-\frac{i\hbar}{4}F} - e^{\frac{3i\hbar}{4}F}\right|^2 = \frac{1}{4}\left[1 + 1 - 2\cos(\hbar F)\right] =$$

$$= \frac{1}{2}\left[1 - \cos(\hbar F)\right] = \sin^2\left(\frac{\hbar F}{2}\right)$$

$$= \sin^2\left[\frac{\hbar}{2}\int_{-\infty}^{\infty} a(t')\,dt'\right]$$

(b) $$b_i^{(0)}(-\infty) = \text{const.} = \begin{cases} i=0 \; |S=0, M=0\rangle : \frac{1}{\sqrt{2}} \\ i=1 \; |S=1, M=1\rangle : 0 \\ i=2 \; |S=1, M=0\rangle : \frac{1}{\sqrt{2}} \\ i=3 \; |S=1, M=-1\rangle : 0 \end{cases}$$

$$\dot{b}_m^{(1)}(t) = \frac{1}{i\hbar}\sum_k \hat{W}_{mk}(t)\, b_k^{(0)}(t)$$

$$b_m^{(1)}(t) = \frac{1}{i\hbar}\int_{-\infty}^{t}\left[\hat{W}_{m,0}(t')\frac{1}{\sqrt{2}} + \hat{W}_{m,2}(t')\frac{1}{\sqrt{2}}\right]dt'$$

$$\hat{W}_{m,k}(t) = \langle m|\, a(t)\, \vec{S}_1 \cdot \vec{S}_2 \,|k\rangle$$

$$= a(t)\hbar^2 \langle m|\left[\frac{S(S+1)}{2} - \frac{3}{4}\right]|k\rangle$$

$$\hat{W}_{m,0}(t) = a(t)\hbar^2\left(-\frac{3}{4}\right)\delta_{m,0} \; ; \; \hat{W}_{m,2}(t) = \frac{\hbar^2}{4} a(t)\,\delta_{m,2}$$

$$b_0^{(1)}(t) = -\frac{3}{4}\frac{\hbar^2}{i\hbar}\int_{-\infty}^{t} a(t')\frac{dt'}{\sqrt{2}} = i\frac{3\hbar}{4\sqrt{2}} f(t)$$

$$b_2^{(1)}(t) = \frac{1}{4}\frac{\hbar^2}{i\hbar}\int_{-\infty}^{t} a(t')\frac{dt'}{\sqrt{2}} = -i\frac{\hbar}{4\sqrt{2}} f(t)$$

$$P(+- \to -+) = |\langle -+|\psi(t)\rangle|^2$$

$$= \left|\frac{1}{\sqrt{2}}\left(\langle S=1, M=0| - \langle S=0, M=0|\right)\left(b_0^{(1)}(t)|S=0, M=0\rangle + b_2^{(1)}(t) \times \right.\right.$$

$$\left.\left. \times |S=1, M=0\rangle\right)\right|^2$$

$$P(+- \to -+) = \left|\frac{1}{\sqrt{2}}\left[b_2^{(1)}(+\infty) - b_0^{(1)}(+\infty)\right]\right|^2$$

$$= \frac{1}{4}\left|\frac{\hbar}{4}F + \frac{3}{4}\hbar F\right|^2 = \frac{1}{4}\left|\hbar F\right|^2 = \frac{\hbar^2 F^2}{4}$$

PROBLEMA 4 DA L5:

$$|\varphi_0\rangle = |\varphi_{1,0,0}\rangle$$

$$\langle\varphi_n|x,y,z|\varphi_0\rangle = ? \quad \text{PARA} \quad |\varphi_n\rangle = |\varphi_{2,0,0}\rangle, |\varphi_{2,1,1}\rangle, |\varphi_{2,1,0}\rangle$$
$$|\varphi_{2,1,-1}\rangle$$

$$\langle\varphi_{2,0,0}|x,y,z|\varphi_{1,0,0}\rangle = 0 \qquad \Delta l = 0$$

$$\hookrightarrow \int r^2 dr\, d\Omega\, R_{2,0}(r)\underbrace{Y_{0,0}}_{\frac{1}{\sqrt{4\pi}}}(x,y,z)R_{1,0}(r)\underbrace{Y_{0,0}(\Omega)}_{\frac{1}{\sqrt{4\pi}}}$$

$$= \int_0^\infty r^2 dr\, \frac{R_{2,0}(r)R_{1,0}(r)}{4\pi}\int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi \begin{bmatrix} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{bmatrix}$$

5) DA L5

(a) $w = \frac{\pi}{2\hbar} \left| \langle \Omega_f, E_f = \hbar\omega - E_I | W | \varphi_{1,0,0} \rangle \right|^2 \rho(\Omega_f, \hbar\omega - E_I)$

PRIMEIRAMENTE: $\rho(\Omega_f, E) = m\sqrt{2mE}$

ALÉM DISSO: $W = \frac{q E_0}{m\omega} P_z \rightarrow i q \frac{\overbrace{(E_f - E_i)}^{\hbar\omega}}{\hbar\omega} E_0 z$

$= i q E_0 z$

O ELEMENTO DE MATRIZ FICA:

$$\langle \vec{p}_f | q E_0 z | \varphi_{100} \rangle = q E_0 \int \frac{e^{-i\vec{p}_f \cdot \vec{r}/\hbar}}{(2\pi\hbar)^{3/2}} z \frac{e^{-r/a_0}}{\sqrt{\pi}\, a_0^{3/2}} d^3r$$

$$= \frac{q E_0}{(2\pi\hbar)^{3/2} \sqrt{\pi}\, a_0^{3/2}} (+i) \frac{\partial}{\partial k_z} \underbrace{\left[\int e^{-i\vec{k}\cdot\vec{r}} e^{-r/a_0} d^3r \right]}_{I} \Bigg|_{\vec{k} = \vec{p}_f/\hbar}$$

$$I = \int e^{-ikr\cos\theta} e^{-r/a_0} r^2 dr\, 2\pi \sin\theta\, d\theta$$

$$= 2\pi \int_0^\infty r^2 dr\, e^{-r/a_0} \int_{-1}^{+1} d\mu\, e^{-ik\mu r}$$

$$= 2\pi \int_0^\infty r^{2} dr\, e^{-r/a_0} \left(\frac{e^{-ikr} - e^{ikr}}{-ikr} \right) = \frac{2\pi i}{k} \int_0^\infty r dr e^{-r/a_0} \left(e^{-ikr} - e^{ikr} \right)$$

$$\int_0^\infty r e^{-r(1/a_0 \pm ik)} = \underbrace{\frac{r e^{-r()}}{-(\frac{1}{a_0} \pm ik)} \Bigg|_0^\infty}_{0} + \frac{1}{1/a_0 \pm ik} \int_0^\infty e^{-r(1/a_0 \pm ik)} dr =$$

$$= \frac{1}{(1/a_0 \pm ik)^2}$$

$$I = \frac{2\pi i}{k}\left[\frac{1}{(1/a_0+ik)^2} - \frac{1}{(1/a_0-ik)^2}\right] = \frac{2\pi i}{k}\left[\frac{1}{z^2} - \frac{1}{\bar{z}^2}\right]$$

$$[\;] = \frac{\bar{z}^2 - z^2}{|z|^4} = \frac{\rho^2 e^{-2i\phi} - \rho^2 e^{2i\phi}}{\rho^4} \qquad (z = \rho e^{i\phi})$$

$$= \frac{-2i \sin 2\phi}{\rho^2} = -\frac{4i}{\rho^2} \sin\phi \cos\phi$$

$$= -\frac{4i}{\rho^4} \rho^2 \sin\phi \cos\phi = -\frac{4i}{\rho^4} \mathrm{Re}(z)\,\mathrm{Im}(z)$$

$$I = \frac{8\pi}{\cancel{k}} \frac{\cancel{k}/a_0}{\left(\frac{1}{a_0^2}+k^2\right)^2} = \frac{8\pi a_0^3}{[1+(ka_0)^2]^2}$$

$$\frac{\partial}{\partial k_z}\left[\frac{8\pi a_0^3}{[1+(ka_0)^2]^2}\right] = 8\pi a_0^3 \frac{(-2)}{[1+(ka_0)^2]^3} a_0^2 (2k_z)$$

$$= \frac{-32\pi a_0^5}{[1+(ka_0)^2]^3} k_z$$

$$|\;|^2 = \frac{q^2 E_0^2}{(2\pi\hbar)^3 \cancel{\pi} \cancel{a_0^3}} \frac{(32)^2 \pi^{\cancel{2}} a_0^{\cancel{10}\,7}}{[1+(ka_0)^2]^6} k^2 \cos^2\theta$$

$$= \frac{128}{\pi^2 \hbar^3} \frac{q^2 E_0^2 a_0^7}{[1+(ka_0)^2]^6} k^2 \cos^2\theta$$

$$W = \frac{64}{\pi \hbar^4} \frac{q^2 E_0^2 a_0^7}{[1+(ka_0)^2]^6} k^2 \cos^2\theta \, m \sqrt{2mE_f}$$

MAS $E_f = \dfrac{P_f^2}{2m} = \dfrac{\hbar^2 k^2}{2m}$

$$W = \frac{64 m q^2 E_0^2 a_0^7}{\pi \hbar^3 [1+(ka_0)^2]^6} k^3 \cos^2\theta$$

(b) INTEGRANDO W SOBRE $d\Omega$ $(\mu = \cos\theta)$

$$\int \cos^2\theta \, d\Omega = \int_{-1}^{+1} 2\pi \mu^2 d\mu = \frac{2\pi}{3} \times 2 = \frac{4\pi}{3}$$

$$W_T = \frac{256}{3\hbar^3} \frac{m q^2 E_0^2 a_0^7}{[1+(ka_0)^2]^6} k^3$$

$$\sigma(\omega) = \frac{\hbar\omega W_T}{\frac{\varepsilon_0 c}{2} E_0^2} = \frac{512}{3\hbar^3 \varepsilon_0 c} \frac{m q^2 a_0^7 k^3}{[1+(ka_0)^2]^6} \hbar\omega$$

MAS $\hbar\omega = E_f + E_I = \dfrac{\hbar^2 k^2}{2m} + \dfrac{\hbar^2}{2ma_0^2} = \dfrac{\hbar^2}{2ma_0^2}[1 + k^2 a_0^2]$

$$\sigma(k) = \frac{256}{3\hbar\epsilon_0 c} \frac{q^2 a_0^5 k^3}{[1+(ka_0)^2]^5}$$

USANDO AGORA : $a_0 = \frac{\hbar^2}{me^2} \Rightarrow q^2 a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m}$

$$\sigma(k) = \frac{1024\pi}{3} \frac{\hbar}{mc} \frac{k^3 a_0^4}{[1+(ka_0)^2]^5}$$

NOTE QUE :

$\frac{\hbar}{mc} = \frac{\lambda_{COMPTON}}{2\pi} = \frac{2.43 \times 10^{-2} \text{ Å}}{2\pi}$ $\quad a_0 = 0.53 \text{ Å}$

$$\sigma(k) = \frac{512}{3} \times 1.29 \times 10^{-2} \text{ Å}^2 \frac{k^3 a_0^3}{[1+(ka_0)^2]^5}$$

$$\sigma(k) \simeq 2.2 \text{ Å}^2 \frac{k^3 a_0^3}{[1+(ka_0)^2]^5}$$

PARA $\lambda = 200 \text{ Å} \Rightarrow \frac{hc}{\lambda} = \frac{\hbar^2}{2ma_0^2}[1+(ka)^2]$

$$\Rightarrow 1+(ka)^2 = \frac{hc/\lambda}{13.6 \text{ eV}} \simeq \frac{62}{13.6} = 4.56$$

$(ka)^2 = 3.56 \qquad (ka)^3 = 6.71$

$$\Rightarrow \frac{(ka)^3}{[1+(ka_0)^2]^5} = 3.4 \times 10^{-3} \Rightarrow \boxed{\sigma = 7.5 \times 10^{-3} \text{ Å}^2}$$

PROBLEMA 6 DA L5:

$\vec{S} \cdot \vec{I}$ → $|F=1, M\rangle$ (EST. EXC.)
→ $|F=0, M=0\rangle$ (EST. FUND.)

$S = 1/2 \quad I = 1/2$

$$\langle \varphi_M | S_x, S_y, S_z | \varphi_0 \rangle = ?$$

$$|\varphi_0\rangle = |F=0, M=0\rangle = \frac{1}{\sqrt{2}} \left[|+-\rangle - |-+\rangle \right]$$

$$|\varphi_1\rangle = \begin{cases} |++\rangle = |S=1, M=1\rangle \\ |--\rangle = |S=1, M=-1\rangle \\ \frac{1}{\sqrt{2}}\left(|+-\rangle + |-+\rangle\right) = |S=1, M=0\rangle \end{cases}$$