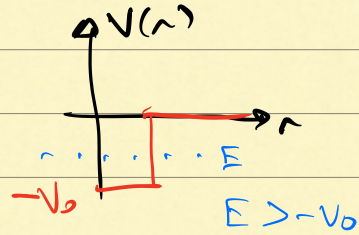
6)

$$V(r) = \begin{cases} -V_0 & 0 < r < a \\ 0 & r > a \end{cases}$$



a) EQ. RADIAL ($l=0$) ESTADOS LIGADOS ($E<0$)

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + V(r)\right] u_{k,0}(r) = E\, u_{k,0}(r)$$

COND. CONTORNO (COMO EM 1D):

$u_{k,0}(r)$ CONTÍNUA EM $r=a$

$u'_{k,0}(r)$ " " "

$r<a$: $V(r) = -V_0$

$$+\frac{\hbar^2}{2\mu} u''_{k,0}(r) + \overbrace{(V_0+E)}^{>0}\, u_{k,0}(r) = 0$$

$$\boxed{u_{k,0}(r) = A \sin kr + \cancel{B} \cos kr} \quad (r<a)$$

$$u''_{k,0}(r) = -k^2\left[A \sin kr + B \cos kr\right]$$

$$-\frac{\hbar^2 k^2}{2\mu} = -(V_0+E) \Rightarrow k = \frac{\sqrt{2(V_0+E)\mu}}{\hbar}$$

$$u_{k,0}(r) \xrightarrow[r\to 0]{} r^{(l+1)} = r \Rightarrow B=0$$

$r > a \qquad V(r) = 0$

$$+\frac{\hbar^2}{2\mu} u''_{k,0}(r) + E\, u_{k,0}(r) = 0 \qquad E < 0$$

$$u_{k,0}(r) = C e^{-Kr} + \cancel{D} e^{+Kr} \qquad K = \frac{\sqrt{-2\mu E}}{\hbar}$$

COND. CONTORNO $(r \to \infty)$

$$u_{k,0}(r) \xrightarrow[r \to \infty]{} 0 \Rightarrow D = 0$$

$$\boxed{u_{k,0}(r) = C e^{-Kr}} \qquad (r > a)$$

b) CASAR AS SOLUÇÕES EM $r = a$:

$u_{k,0}(r)$ CONTÍNUA EM $r = a$:

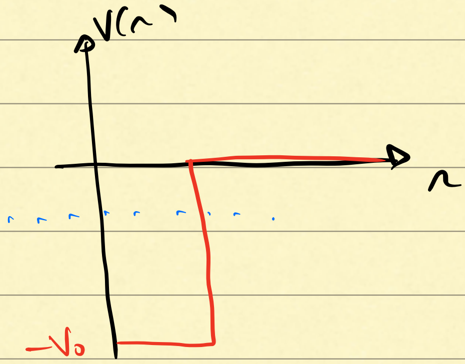
$$A \sin(ka) = C e^{-Ka} \qquad (1)$$

$u'_{k,0}(r)$ CONTÍNUA EM $r = a$:

$$+kA\cos(ka) = -CKe^{-Ka} \qquad (2)$$

$$(1) \div (2) \Rightarrow \frac{\tan(ka)}{k} = -\frac{1}{K} \Rightarrow \boxed{\tan(ka) = -\frac{k}{K}}$$

c) MÍNIMO DE V_0 PARA QUE HAJA UM ESTADO LIGADO



SE V_0 FOR DIMINUINDO CHEGARÁ UM MOMENTO EM QUE O ÚLTIMO ESTADO LIGADO DESAPARECERÁ

CONDIÇÃO PARA O $V_{0c} \Rightarrow E = 0$

$$\tan(ka) = -\frac{k}{K} \qquad K = \frac{\sqrt{2\mu|E|}}{\hbar} \xrightarrow[V_0 \to V_{0c}]{} 0$$

$$\frac{k}{K} \xrightarrow[V_0 \to V_{0c}]{} \infty \qquad k = \frac{\sqrt{2\mu(E+V_0)}}{\hbar} \xrightarrow[V_0 \to V_{0c}]{} \frac{\sqrt{2\mu V_{0c}}}{\hbar}$$

$$ka \to (2n+1)\frac{\pi}{2}$$

$$\frac{\sqrt{2\mu V_{0c}}}{\hbar} a \to (2n+1)\frac{\pi}{2}$$

MENOR $V_{0c} \Rightarrow n = 0 \Rightarrow \sqrt{2\mu V_{0c}} = \frac{\pi\hbar}{2a}$

$$\Rightarrow \boxed{V_{0c} = \frac{\pi^2 \hbar^2}{8\mu a^2}}$$

d) DÊUTERON : ESTADO LIGADO DE UM p E UM n

$$\mu = \frac{m_p m_n}{m_p + m_n} \cong \frac{m_p}{2}$$

$$V_{0c} = \frac{\pi^2 \hbar^2}{8 \mu a^2} = \frac{\pi^2 \hbar^2}{4 m_p a^2} \sim 26 \text{ MeV}$$

$$m_p c^2 \sim 980 \text{ MeV}$$

5) $V(\rho)$ SIMETRIA CILÍNDRICA

$\Rightarrow$ COORD. CILÍNDRICAS (ρ, ϕ, z)

a) $H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(\rho)$

$$\nabla^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}$$

$$H \longrightarrow \left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\right) + V(\rho)\right]$$

MOSTRE: $[H, L_z] = [H, P_z] = 0$

$L_z = \frac{\hbar}{i}\frac{\partial}{\partial \phi}$ $\qquad$ $P_z = \frac{\hbar}{i}\frac{\partial}{\partial z}$

COMO V SÓ DEPENDE DE ρ $\therefore$ $[L_z, V(\rho)] = [P_z, V(\rho)] = 0$

$$[L_z, T] = \left[\frac{\hbar}{i}\frac{\partial}{\partial \phi}, -\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\right)\right]$$

$$\left[\frac{\partial}{\partial \phi}, \frac{\partial^2}{\partial \rho^2}\right] = 0 = \left[\frac{\partial}{\partial \phi}, \frac{1}{\rho}\frac{\partial}{\partial \rho}\right] = \left[\frac{\partial}{\partial \phi}, \frac{\partial^2}{\partial z^2}\right]$$

$$\left[\frac{\partial}{\partial \phi}, \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2}\right] = \frac{1}{\rho}\left[\frac{\partial}{\partial \phi}, \frac{\partial^2}{\partial \phi^2}\right] = 0 \Rightarrow [L_z, H] = 0$$

ANALOGAMENTE: $[H, P_z] = 0$

$$\varphi_{n,m,k}(\rho,\phi,z) = f_{n,m}(\rho)\, e^{im\phi}\, e^{ikz}$$

(b)

$\{H, L_z, P_z\}$ FORMAM UM C.O.C.

$$H\varphi = E\varphi$$
$$L_z\varphi = m\hbar\varphi$$
$$P_z\varphi = \hbar k\varphi$$

$\frac{\hbar}{i}\frac{\partial}{\partial z}$ $\qquad$ $\frac{\hbar}{i}\frac{\partial}{\partial \phi}$

$$P_z\, e^{ikz} = \hbar k\, e^{ikz} \qquad L_z\, e^{im\phi} = m\hbar\, e^{im\phi}$$

É RAZOÁVEL CHUTAR A FORMA SEPARÁVEL:

$$\varphi_{n,m,k}(\rho,\phi,z) = f_{n,m}(\rho)\, e^{im\phi}\, e^{ikz}$$

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}\right) + V(\rho)\right]\varphi_{n,m,k} =$$

$$= \left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - \frac{m^2}{\rho^2} - k^2\right) + V(\rho)\right] f\, e^{im\phi} e^{ikz}$$

$$= E\, f\, e^{im\phi} e^{ikz}$$

$$\Rightarrow \left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - \frac{m^2}{\rho^2} - k^2\right) + V(\rho)\right] f_{n,m}(\rho) = E\, f_{n,m}(\rho)$$

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - \frac{m^2}{\rho^2}\right) + V(\rho)\right] f_{n,m}(\rho) = \overbrace{\left(E - \frac{\hbar^2 k^2}{2\mu}\right)}^{E'} f_{n,m}(\rho)$$

4)

$$\psi(\vec{r},0) = -A\,(x+iy)\,e^{-r/2a_0}$$

$$\psi_{n,\ell,m}(r,\theta,\phi) = R_{n,\ell}(r)\,Y_{\ell,m}(\theta,\phi)$$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$

$$B = -\sqrt{\frac{8\pi}{3}}$$

$$x+iy = r\sin\theta(\cos\phi + i\sin\phi) = r\underbrace{\sin\theta\, e^{i\phi}}_{B\,Y_{1,1}(\theta,\phi)}$$

$$\psi(\vec{r},0) = -A \times B\, r\, Y_{1,1}(\theta,\phi)\,e^{-r/2a_0}$$

$$= \sqrt{\frac{8\pi}{3}}\,A\,\underbrace{\left(r\,e^{-r/2a_0}\right)}_{\propto R_{n=2,\ell=1}(r)}Y_{1,1}(\theta,\phi)$$

DE (C-39c) DO LIVRO: $R_{n=2,\ell=1}(r) = \frac{1}{(2a_0)^{3/2}}\frac{1}{\sqrt{3}}\frac{r}{a_0}e^{-r/2a_0}$

$$\Rightarrow \psi(\vec{r},0) = K\,R_{n=2,\ell=1}(r)\,Y_{1,1}(\theta,\phi)$$

$L^2 \xrightarrow{\ell(\ell+1)\hbar^2}$ É AUTO-FUNÇÃO COM AUTO-VALOR $2\hbar^2$

$L_z \rightarrow$ É " " " $\hbar$

NORMALIZAÇÃO: $\int |\psi|^2 d^3r = 1$

$$\Rightarrow \underbrace{\int_0^\infty r^2 dr\,|R_{n,\ell}(r)|^2}_{1}\;\underbrace{\int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi\,|Y_{\ell m}(\theta,\phi)|^2}_{1} = 1$$