9.4 The transitional charge density for the radiative transition from the $m=0$ $2p$ state in hydrogen to the $1s$ ground state is, in the notation of (9.1),

$$\rho(r, \theta, \phi) = \frac{2e}{\sqrt{6}} a_0^{-4} \cdot r e^{-3r/2a_0} Y_{00} Y_{10} e^{-i\omega_0 t}$$

where $a_0 = \hbar^2/me^2$ is the Bohr radius and $\omega_0 = 3e^2/8\hbar a_0$ is the frequency difference of the levels.

(*a*) Evaluate all the radiation multipoles for this charge density in the long wavelength limit.

(*b*) In the electric dipole approximation calculate the total time-averaged power radiated. Express your answer in units of $(\hbar\omega_0) \cdot (\alpha^4 c/a_0)$, where $\alpha = e^2/\hbar c$ is the fine-structure constant.

(*c*) Interpreting the classically calculated power as the photon energy $(\hbar\omega_0)$ times the transition probability, evaluate numerically the transition probability in units of $\sec^{-1}$.

(*d*) If, instead of the semiclassical charge density used above, the electron in the $2p$ state was described by a *circular* Bohr orbit of radius $2a_0$, rotating with the transitional frequency ω_0, what would the radiated power be? Express your answer in the same units as in (*b*) and evaluate the ratio of the two powers numerically.