

7.1)

(a) $S_0 = 3$ $S_1 = -1$ $S_2 = 2$ $S_3 = -2$

$$\begin{cases} a_1^2 + a_2^2 = 3 \\ a_1^2 - a_2^2 = -1 \end{cases} \Rightarrow \begin{cases} a_1^2 = 1 \Rightarrow a_1 = 1 \\ a_2^2 = 2 \Rightarrow a_2 = \sqrt{2} \end{cases}$$

$$-2 = 2a_1 a_2 \sin(\delta_2 - \delta_1) \Rightarrow \sin(\delta_2 - \delta_1) = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \delta_2 - \delta_1 = -\frac{\pi}{4} \text{ , Tomando } \delta_1 = 0$$

$$\boxed{E_1 = 1} ; \boxed{E_2 = \sqrt{2} e^{-i\pi/4} = \sqrt{2} \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right) = 1 - i}$$

$$E_{\pm} = \frac{1}{\sqrt{2}} (E_1 \mp i E_2) = \frac{1}{\sqrt{2}} (1 \mp i \mp 1) \Rightarrow \begin{cases} E_+ = -\frac{i}{\sqrt{2}} = \frac{1}{\sqrt{2}} e^{-i\pi/2} \\ E_- = \frac{2+i}{\sqrt{2}} = \frac{\sqrt{5}}{\sqrt{2}} e^{i\delta} \end{cases}$$

$$r e^{i\alpha} = \frac{E_1 + i E_2}{E_1 - i E_2} = \frac{E_-}{E_+} = \frac{2+i}{-i} = -1 + 2i$$

$$\delta = \arctan\left(\frac{1}{2}\right)$$

$$r = \sqrt{5} \quad \tan \alpha = -2 \Rightarrow \alpha = -\arctan 2 \approx -63^\circ$$

A razão entre semi-eixos é $\left| \frac{1+\sqrt{5}}{1-\sqrt{5}} \right| = \frac{6+2\sqrt{5}}{4} = \frac{3}{2} + \frac{\sqrt{5}}{2}$

e a elipse é inclinada de α em relação a \hat{E}_1
 $\alpha \approx -32^\circ$

(b) $S_0 = 25$ $S_1 = 0$ $S_2 = 24$ $S_3 = 7$

$$\begin{cases} a_1^2 + a_2^2 = 25 \\ a_1^2 - a_2^2 = 0 \end{cases} \Rightarrow a_1 = a_2 = \frac{5\sqrt{2}}{2}$$

$$7 = 2a_1^2 \sin(\delta_2 - \delta_1) \Rightarrow \sin(\delta_2 - \delta_1) = \frac{7}{25} \Rightarrow \cos(\delta_2 - \delta_1) = \frac{24}{25}$$

Envolvendo $\delta_1 = 0$

$$\boxed{E_1 = \frac{5\sqrt{2}}{2}} \quad \text{e} \quad E_2 = \frac{5\sqrt{2}}{2} \left(\frac{24+i7}{25} \right) = \frac{\sqrt{2}}{10} (24+7i) = E_2$$

$$E_+ = \frac{1}{\sqrt{2}} \frac{5\sqrt{2}}{2} \left[1 + \frac{24i}{25} + \frac{7}{25} \right]$$

$$E_+ = \frac{5}{2} \left(\frac{32-24i}{25} \right) = \frac{4}{5} (4-3i) = E_+ \quad a_+ = 4$$
$$E_- = \frac{5}{2} \left(\frac{18+24i}{25} \right) = \frac{3}{5} (3+4i) = E_- \quad a_- = 3$$

$\arctan \delta_+ = -\frac{3}{4}$
 $\arctan \delta_- = \frac{4}{3}$

$$\lambda e^{i\alpha} = \frac{3}{4} \frac{(3+4i)}{4-3i} = \frac{3}{4} \frac{12-12+(16+9)i}{25} = \frac{3}{4} i$$

$$\Rightarrow \lambda = \frac{3}{4} \quad \alpha = \frac{\pi}{2}$$

$$\frac{a}{b} = \frac{|1+\lambda|}{|1-\lambda|} = \frac{7/4}{1/4} = 7$$

A razão entre semi-eixos é 7 e a elipse está inclinada de $\frac{\pi}{4} = 45^\circ$ em relação a \hat{E}_1 .