FI 193 – Teoria Quântica de Sistemas de Muitos Corpos

2º Semestre de 2023

19/09/2023

Aula 13

Aulas passadas

Função de Green de um corpo ordenada temporalmente a T=0:

$$\begin{aligned} iG_{\alpha\beta}\left(\mathbf{r}t;\mathbf{r}'t'\right) &= \frac{\left\langle \Psi_{0H}\right| T\left[\psi_{H\alpha}\left(\mathbf{r},t\right)\psi^{\dagger}_{H\beta}\left(\mathbf{r}',t'\right)\right]\left|\Psi_{0H}\right\rangle}{\left\langle \Psi_{0H}\middle|\Psi_{0H}\right\rangle} \\ &= \theta\left(t-t'\right)\frac{\left\langle \Psi_{0H}\right| \psi_{H\alpha}\left(\mathbf{r},t\right)\psi^{\dagger}_{H\beta}\left(\mathbf{r}',t'\right)\left|\Psi_{0H}\right\rangle}{\left\langle \Psi_{0H}\middle|\Psi_{0H}\right\rangle} \\ &\quad +\zeta\theta\left(t'-t\right)\frac{\left\langle \Psi_{0H}\right| \psi^{\dagger}_{H\beta}\left(\mathbf{r}',t'\right)\psi_{H\alpha}\left(\mathbf{r},t\right)\left|\Psi_{0H}\right\rangle}{\left\langle \Psi_{0H}\middle|\Psi_{0H}\right\rangle} \end{aligned}$$

$$\boxed{\zeta = \begin{cases} +1, & \text{bósons} \\ -1, & \text{férmions} \end{cases}}$$

Aulas passadas

Propriedades:

(a) Sistema isolado:

$$G_{\alpha\beta}\left(\mathbf{r}t;\mathbf{r}'t'\right)=G_{\alpha\beta}\left(\mathbf{r},\mathbf{r}',t-t'\right)$$

(b) Invariância translacional:

$$G_{\alpha\beta}\left(\mathbf{r}t;\mathbf{r}'t'\right)=G_{\alpha\beta}\left(\mathbf{r}-\mathbf{r}',t-t'\right)\equiv G_{\alpha\beta}\left(\mathbf{R},T\right)$$

(c) Reversão temporal (ausência de campo magnético ou de magnetização):

$$G_{\alpha\beta}\left(\mathbf{r}t;\mathbf{r}'t'\right)=\delta_{\alpha\beta}G\left(\mathbf{r}-\mathbf{r}',t-t'\right)\equiv\delta_{\alpha\beta}G\left(\mathbf{R},T\right)$$

Aula passada

Valores esperados no estado fundamental de operadores de um corpo:

$$\widehat{U} = \sum_{\alpha\beta} \int d^3 r \psi_\alpha^\dagger(\mathbf{r}) U_{\alpha\beta}(\mathbf{r}) \psi_\beta(\mathbf{r})$$

$$\boxed{\langle U \rangle = \zeta i \sum_{\alpha\beta} \int d^3 r \lim_{\eta \to 0^+} \lim_{\mathbf{r} \to \mathbf{r}'} \left[U_{\alpha\beta}(\mathbf{r}) G_{\beta\alpha}(\mathbf{r}t; \mathbf{r}'t + \eta) \right]}$$

$$\boxed{\langle U \rangle = \zeta i V \sum_{\alpha\beta} \int \frac{d^3 k d\omega}{(2\pi)^4} e^{i\omega\eta} \left[U_{\alpha\beta}(\mathbf{k}) G_{\beta\alpha}(\mathbf{k}, \omega) \right]}$$

Aula passada

Valores esperados no estado fundamental de operadores de um corpo: exemplos.

ENERGIA CINÉTICA

$$\langle H_0 \rangle = \zeta i V \sum_{\alpha} \int \frac{d^3k d\omega}{(2\pi)^4} e^{i\omega\eta} \left(\frac{k^2}{2m} \right) G_{\alpha\alpha}(\mathbf{k}, \omega)$$

$$\langle N \rangle = \zeta i V \sum_{\alpha} \int \frac{d^3k d\omega}{(2\pi)^4} e^{i\omega\eta} G_{\alpha\alpha}(\mathbf{k}, \omega)$$

$$\langle \mathbf{S} \rangle = \zeta i V \sum_{\alpha\beta} \int \frac{d^3k d\omega}{(2\pi)^4} e^{i\omega\eta} \left(\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} \right) G_{\beta\alpha}(\mathbf{k}, \omega)$$

Aula passada

Valores esperados de operadores de dois corpos: sistema homogêneo com interação de pares.

$$H = \sum_{\alpha} \int d^3 r \psi_{\alpha}^{\dagger}(\mathbf{r}) \left(-\frac{\hbar^2 \nabla^2}{2m}\right) \psi_{\alpha}(\mathbf{r}) + \frac{1}{2} \sum_{\alpha\beta} \int d^3 r d^3 r' \psi_{\alpha}^{\dagger}(\mathbf{r}) \psi_{\beta}^{\dagger}(\mathbf{r}') V(\mathbf{r}-\mathbf{r}') \psi_{\beta}(\mathbf{r}') \psi_{\alpha}(\mathbf{r})$$

$$\boxed{\begin{aligned} \langle V \rangle &= \frac{i\zeta}{2} \sum_{\alpha} \int d^3 r \lim_{t' \to t^+} \lim_{\mathbf{r} \to \mathbf{r}'} \left[\left(i\partial_t + \frac{\hbar^2 \nabla^2}{2m} \right) G_{\alpha\alpha}(\mathbf{r}t; \mathbf{r}'t') \right] \\ \langle H \rangle &= \frac{i\zeta}{2} \sum_{\alpha} \int d^3 r \lim_{t' \to t^+} \lim_{\mathbf{r} \to \mathbf{r}'} \left[\left(i\partial_t - \frac{\hbar^2 \nabla^2}{2m} \right) G_{\alpha\alpha}(\mathbf{r}t; \mathbf{r}'t') \right] \end{aligned}}$$

Aula passada

Valores esperados de operadores de dois corpos: sistema homogêneo com interação de pares.

$$H=\sum_{\alpha}\int d^3r\psi_{\alpha}^{\dagger}\left(\mathbf{r}\right)\left(-\frac{\hbar^2\nabla^2}{2m}\right)\psi_{\alpha}\left(\mathbf{r}\right)+\frac{1}{2}\sum_{\alpha\beta}\int d^3rd^3r'\psi_{\alpha}^{\dagger}\left(\mathbf{r}\right)\psi_{\beta}^{\dagger}\left(\mathbf{r}'\right)V\left(\mathbf{r}-\mathbf{r}'\right)\psi_{\beta}\left(\mathbf{r}'\right)\psi_{\alpha}\left(\mathbf{r}\right)$$

$$\boxed{\begin{aligned}\langle V\rangle &= \frac{i\zeta V}{2}\sum_{\alpha}\int\frac{d^3kd\omega}{\left(2\pi\right)^4}e^{i\omega\eta}\left(\omega-\frac{k^2}{2m}\right)G_{\alpha\alpha}\left(\mathbf{k},\omega\right)\\ \langle H\rangle &= \frac{i\zeta V}{2}\sum_{\alpha}\int\frac{d^3kd\omega}{\left(2\pi\right)^4}e^{i\omega\eta}\left(\omega+\frac{k^2}{2m}\right)G_{\alpha\alpha}\left(\mathbf{k},\omega\right)\end{aligned}}$$

Aula passada

Função de Green não interagente (férmions):

$$\boxed{iG^{(0)}_{\alpha\beta}\left(\mathbf{r}t;\mathbf{r}'t'\right)=\frac{\delta_{\alpha\beta}}{V}\sum_{\mathbf{k}}e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}e^{-i\epsilon(\mathbf{k})(t-t')}\left[\theta\left(t-t'\right)\theta\left(k-k_F\right)-\theta\left(t'-t\right)\theta\left(k_F-k\right)\right]}$$

$$G_{\alpha\beta}\left(\mathbf{k},\omega\right) = \int d^3R dT e^{-i\mathbf{k}\cdot\mathbf{R}}e^{i\omega T}G_{\alpha\beta}\left(\mathbf{R},T\right)$$

$$G_{\alpha\beta}\left(\mathbf{R},T\right) = \int \frac{d^3k d\omega}{\left(2\pi\right)^4}e^{i\mathbf{k}\cdot\mathbf{R}}e^{-i\omega T}G_{\alpha\beta}\left(\mathbf{k},\omega\right)$$

$$G^{(0)}_{\alpha\beta}\left(\mathbf{k},\omega\right)=\delta_{\alpha\beta}G^{(0)}\left(\mathbf{k},\omega\right)$$

$$\boxed{G^{(0)}\left(\mathbf{k},\omega\right) = \frac{1}{\omega-\epsilon\left(\mathbf{k}\right)+i\eta\,\mathrm{sgn}\left(k-k_F\right)}} \qquad \eta \to 0^+$$

A representação espectral de Lehmann

$\langle \Phi_{0H} | \Phi_{0H} \rangle = 1$ $\quad x = \vec{r}, t \quad x' = \vec{r}', t'$

$$i G_{\alpha\beta}(x, x') = \theta(t - t') \langle \Phi_{0H} | \psi_{\alpha H}(\vec{r}, t)\, \psi^{+}_{\beta H}(\vec{r}', t') | \Phi_{0H} \rangle$$

$$- \theta(t' - t) \langle \Phi_{0H} | \psi^{+}_{\beta H}(\vec{r}', t')\, \psi_{\alpha H}(\vec{r}, t) | \Phi_{0H} \rangle$$

INSERIR O FECHAMENTO NO "MEIO" DOS "SANDUÍCHES"

$$\sum_n |\Phi_n\rangle\langle\Phi_n| = \mathbb{1}$$

$$i G_{\alpha\beta}(x, x') = \sum_n \Big[\theta(t - t') \underbrace{\langle \Phi_{0H} | \psi_{\alpha H}(\vec{r}, t) | \Phi_n \rangle}_{1} \underbrace{\langle \Phi_n | \psi^{+}_{\beta H}(\vec{r}', t') | \Phi_{0H} \rangle}_{2}$$

$$- \theta(t' - t) \langle \Phi_{0H} | \psi^{+}_{\beta H}(\vec{r}', t') | \Phi_n \rangle \langle \Phi_n | \psi_{\alpha H}(\vec{r}, t) | \Phi_{0H} \rangle \Big]$$

$$1 = \langle \Phi_{0H} | e^{iHt} \psi_\alpha(\vec{r}) e^{-iHt} | \Phi_n \rangle = e^{iE_0 t} e^{-iE_n t} \langle \Phi_0 | \psi_\alpha(\vec{r}) | \Phi_n \rangle$$

$$2 = e^{iE_n t'} e^{-iE_0 t'} \langle \Phi_n | \psi^{+}_\beta(\vec{r}') | \Phi_0 \rangle$$

$$iG_{\alpha\beta}(x,x') = \sum_n \Big[\theta(t-t')\, e^{-i(E_n-E_0)(t-t')} \langle \Phi_0|\psi_\alpha(\vec{r})|\Phi_n\rangle\langle\Phi_n|\psi^\dagger_\beta(\vec{r}\,')|\Phi_0\rangle$$

$$-\theta(t'-t)\, e^{i(E_n-E_0)(t-t')} \langle \Phi_0|\psi^\dagger_\beta(\vec{r}\,')|\Phi_n\rangle\langle\Phi_n|\psi_\alpha(\vec{r})|\Phi_0\rangle\Big]$$

NOTEM QUE ISSO PROVA QUE $G_{\alpha\beta}$ SÓ DEPENDE DE $(t-t')$:

$$G_{\alpha\beta}(\vec{r},t;\vec{r}\,',t') \equiv G_{\alpha\beta}(\vec{r},\vec{r}\,';\,t-t')$$

$$G_{\alpha\beta}(\vec{r},\vec{r}\,',\omega) = \int_{-\infty}^{+\infty} d\underbrace{(t-t')}_{T}\, e^{i\omega\underbrace{(t-t')}_{T}}\, G_{\alpha\beta}(\vec{r},\vec{r}\,',t-t')$$

$$\int_{-\infty}^{\infty} dT\, \theta(\pm T)\, e^{\mp i\Delta E T}\, e^{i\omega T} = \int_0^\infty dT\, e^{-i\Delta E T}\, e^{\pm i\omega T} =$$

$$\int_0^\infty dT\, e^{i[\pm\omega + i\eta - \Delta E]T} = \left.\frac{e^{i[\pm\omega+i\eta-\Delta E]T}}{i[\pm\omega+i\eta-\Delta E]}\right|_0^\infty =$$

$$= \frac{i}{\pm\omega + i\eta - (E_n-E_0)} = \frac{\pm i}{\omega \pm i\eta \mp (E_n-E_0)}$$

$$G_{\alpha\beta}(\vec{r},\vec{r}',\omega)=\sum_n\left[\frac{\langle\Psi_0|\psi_\alpha(\vec{r})|\Psi_n\rangle\langle\Psi_n|\psi_\beta^\dagger(\vec{r}')|\Psi_0\rangle}{\omega-(E_n-E_0)+i\eta}+\right.$$

$$\left.+\frac{\langle\Psi_0|\psi_\beta^\dagger(\vec{r}')|\Psi_n\rangle\langle\Psi_n|\psi_\alpha(\vec{r})|\Psi_0\rangle}{\omega+(E_n-E_0)-i\eta}\right]$$

- NO 1º (2º) TERMO, $|\Psi_n\rangle$ TEM $N+1$ $(N-1)$ PARTÍCULAS SE $|\Psi_0\rangle$ TEM $\underline{N}$ PARTÍCULAS.

$E_0 \to E_0(N)$ $\quad E_n \longrightarrow \begin{cases} 1º \text{ TERMO: } E_n(N+1) \\ 2º \text{ TERMO: } E_n(N-1) \end{cases}$

$$-\begin{cases} 1º \text{ TERMO: } E_n(N+1)-E_0(N)=[E_n(N+1)-E_0(N+1)]-[E_0(N)-E_0(N+1)] \\ 2º \text{ TERMO: } E_n(N-1)-E_0(N)=[E_n(N-1)-E_0(N-1)]-[E_0(N)-E_0(N-1)] \end{cases}$$

$$E_0(N)-E_0(N+1)\sim-\frac{\partial E_0}{\partial N}=-\mu$$

$$E_0(N)-E_0(N-1)\sim\frac{\partial E_0}{\partial N}=\mu$$

$$E_n(N\pm1)-E_0(N\pm1)\equiv\epsilon_n(N\pm1)\geq0$$

ENERGIA DE EXCITAÇÃO DO NÍVEL $\underline{n}$

$$G_{\alpha\beta}(\bar{r},\bar{r}',\omega) = \sum_n \left[\frac{\langle \Phi_0 | \psi_\alpha(\bar{r}) | \Phi_n \rangle \langle \Phi_n | \psi_\beta^\dagger(\bar{r}') | \Phi_0 \rangle}{\omega - \epsilon_n(N+1) - \mu + i\eta} + \right.$$

$$\left. + \frac{\langle \Phi_0 | \psi_\beta^\dagger(\bar{r}') | \Phi_n \rangle \langle \Phi_n | \psi_\alpha(\bar{r}) | \Phi_0 \rangle}{\omega + \epsilon_n(N-1) - \mu - i\eta} \right]$$

- ESTRUTURA ANALÍTICA DE $G_{\alpha\beta}(\bar{r},\bar{r}',\omega)$ NO PLANO COMPLEXO DE $\underline{\omega}$:
  - PÓLOS SIMPLES EM:

    Ⓐ $\cdot\ \epsilon_n(N+1)+\mu - i\eta$: SEMI-PLANO INFERIOR

    Ⓑ $\cdot\ -\epsilon_n(N-1)+\mu + i\eta$: " " SUPERIOR

Ⓐ PARTE REAL $\geq \mu$

Ⓑ PARTE REAL $\leq \mu$

SE OS ϵ_n FORMAM UM CONJUNTO DENSO, OS POLOS PODEM FORMAR UM "CORTE DE RAMOS"

SUPONDO AGORA QUE O SISTEMA TENHA INVARIÂNCIA TRANSLACIONAL : $[H, \vec{P}]=0$ $\vec{P}$ = MOMENTO LINEAR TOTAL

NOTE PRIMEIRO QUE:

$$\psi_\alpha(\vec{r}) = e^{-i\vec{P}\cdot\vec{r}}\, \psi_\alpha(0)\, e^{i\vec{P}\cdot\vec{r}}$$

PROVA: $\vec{P} = \sum_{\vec{k}\alpha} \vec{k}\, c^\dagger_{\vec{k}\alpha} c_{\vec{k}\alpha} = \sum_\alpha \int d^3r\, \psi^\dagger_\alpha(\vec{r}) \left(\frac{\nabla}{i}\right) \psi_\alpha(\vec{r})$

USANDO: $\psi_\alpha(\vec{r}) = \sum_{\vec{k}} e^{-i\vec{k}\cdot\vec{r}}\, c_{\vec{k}\alpha}$ (TENTEM PROVAR)

$$\langle \Psi_n | \psi_\alpha(\vec{r}) | \Psi_0 \rangle = \langle \Psi_n | e^{-i\vec{P}\cdot\vec{r}}\, \psi_\alpha(\vec{0})\, e^{i\vec{P}\cdot\vec{r}} | \Psi_0 \rangle$$

NA IMENSA MAIORIA DOS CASOS $\vec{P}|\Psi_0\rangle = 0$

JÁ PARA OS ESTADOS EXCITADOS:

$\vec{P}\,|\Psi_n\rangle = \vec{p}_n\,|\Psi_n\rangle$

ONDE ASSUMIMOS TRABALHAR NUMA BASE $|\psi_n, \vec{p}_n\rangle$ DE AUTO-ESTADOS DE H E $\vec{P}$

$$\langle \Psi_n | e^{-i\vec{P}\cdot\vec{x}} \psi_\alpha(\vec{0})\, e^{i\vec{P}\cdot\vec{x}} |\Psi_0\rangle = e^{-i\vec{P}_n\cdot\vec{x}} \langle \Psi_n | \psi_\alpha(0) | \Psi_0\rangle$$

$$\langle \Psi_0 | \psi^\dagger_\beta(\vec{x}') | \Psi_n\rangle \langle \Psi_n | \psi_\alpha(\vec{x}) | \Psi_0\rangle = e^{-i\vec{P}_n(\vec{x}-\vec{x}')} \langle \Psi_0 | \psi^\dagger_\beta(0) | \Psi_n\rangle \times$$

$$\times \langle \Psi_n | \psi_\alpha(0) | \Psi_0\rangle$$

$$|\Psi_n\rangle \Rightarrow |n, \vec{P}_n\rangle$$

$$\langle \Psi_0 | \psi_\alpha(\vec{x}) | \Psi_n\rangle \langle \Psi_n | \psi^\dagger_\beta(\vec{x}') | \Psi_0\rangle = e^{+i\vec{P}_n\cdot(\vec{x}-\vec{x}')} \langle \Psi_0 | \psi_\alpha(0) | n, \vec{P}_n\rangle \times \langle n, \vec{P}_n | \psi^\dagger_\beta(0) | \Psi_0\rangle$$

$$G_{\alpha\beta}(\vec{x},\vec{x}';\omega) = \sum_{n,\vec{P}_n} \Bigg[e^{i\vec{P}_n\cdot(\vec{x}-\vec{x}')} \frac{\langle \Psi_0 | \psi_\alpha(0) | n, \vec{P}_n\rangle \langle n, \vec{P}_n | \psi^\dagger_\beta(0) | \Psi_0\rangle}{\omega - \epsilon_{n,\vec{P}_n}(N+1) - \mu + i\eta}$$

$$+ e^{-i\vec{P}_n\cdot(\vec{x}-\vec{x}')} \frac{\langle \Psi_0 | \psi^\dagger_\beta(0) | n, \vec{P}_n\rangle \langle n, \vec{P}_n | \psi_\alpha(0) | \Psi_0\rangle}{\omega + \epsilon_{n,\vec{P}_n}(N-1) - \mu - i\eta}$$

ISSO PROVA QUE $G_{\alpha\beta}(\vec{r},\vec{r}',\omega) \equiv G_{\alpha\beta}(\vec{r}-\vec{r}',\omega)$

TOMAMOS FOURIER:

$$G_{\alpha\beta}(\vec{k},\omega) = \int d^3(\vec{r}-\vec{r}')\, e^{-i\vec{k}\cdot(\vec{r}-\vec{r}')}\, G_{\alpha\beta}(\vec{r}-\vec{r}';\omega)$$

$$G_{\alpha\beta}(\vec{k},\omega) = V\sum_{n,\vec{p}_n}\Bigg[\delta_{\vec{k},\vec{p}_n}\frac{\langle\Psi_0|\psi_\alpha(0)|n,\vec{p}_n\rangle\langle n,\vec{p}_n|\psi^\dagger_\beta(0)|\Psi_0\rangle}{\omega-\epsilon_{n,\vec{p}_n}(N+1)-\mu+i\eta}$$

$$+\,\delta_{\vec{k},-\vec{p}_n}\frac{\langle\Psi_0|\psi^\dagger_\beta(0)|n,\vec{p}_n\rangle\langle n,\vec{p}_n|\psi_\alpha(0)|\Psi_0\rangle}{\omega+\epsilon_{n,\vec{p}_n}(N-1)-\mu-i\eta}\Bigg]$$

$$= V\sum_n\Bigg[\frac{\langle\Psi_0|\psi_\alpha(0)|n,\vec{k}\rangle\langle n,\vec{k}|\psi^\dagger_\beta(0)|\Psi_0\rangle}{\omega-\epsilon_{n,\vec{k}}(N+1)-\mu+i\eta}+$$

$$+\frac{\langle\Psi_0|\psi^\dagger_\beta(0)|n,-\vec{k}\rangle\langle n,-\vec{k}|\psi_\alpha(0)|\Psi_0\rangle}{\omega+\epsilon_{n,-\vec{k}}(N-1)-\mu-i\eta}\Bigg]$$

SE $|\Psi_0\rangle$ TEM $S_z=0$, É FÁCIL VER QUE $G_{\alpha\beta}=\delta_{\alpha\beta}G$

NESSE CASO: $G_{\alpha\beta}(\vec{k},\omega) = \delta_{\alpha\beta}\, G(\vec{k},\omega)$

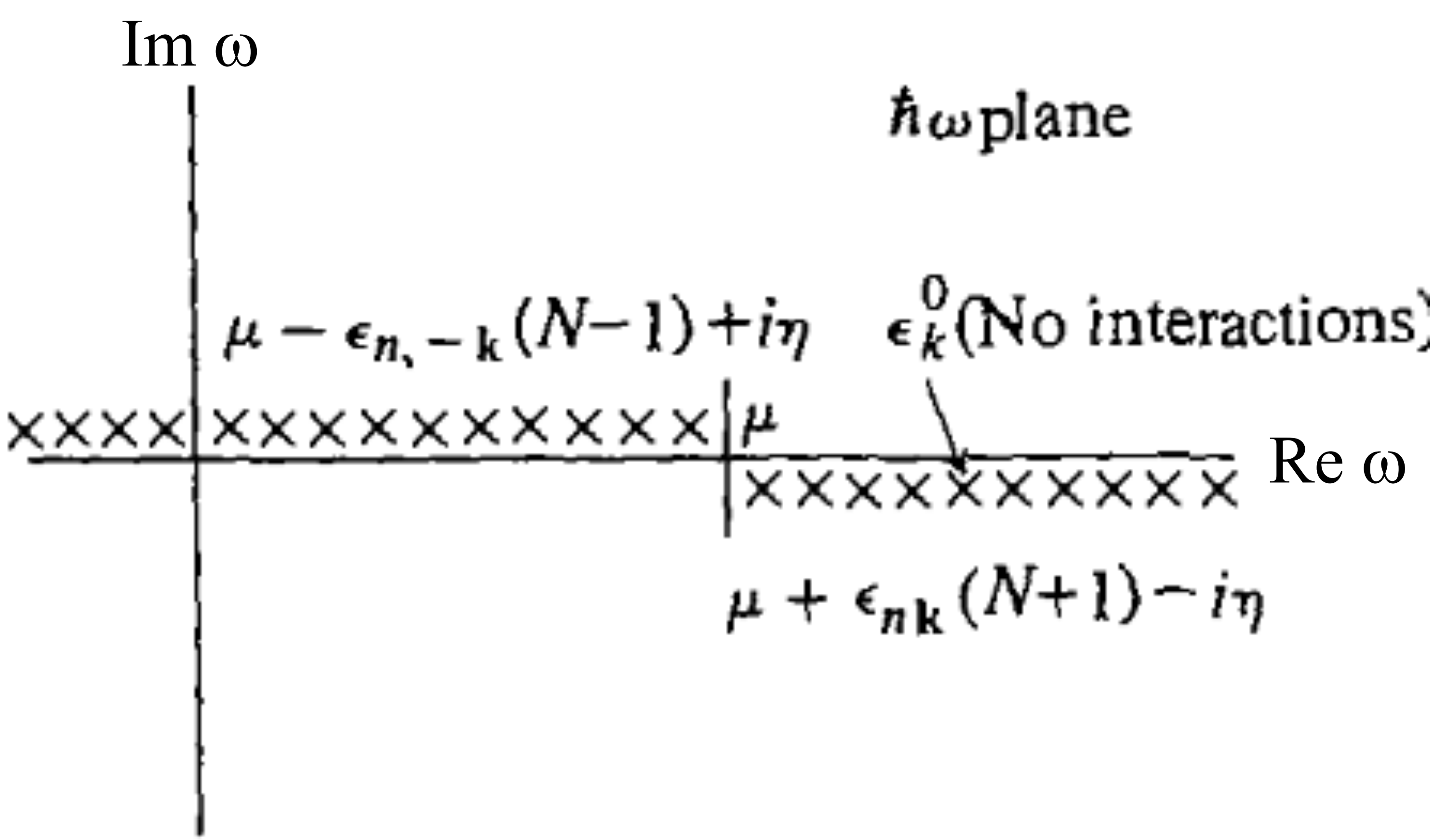
$$G(\vec{k},\omega) = V \sum_{n} \left[\frac{\langle \Phi_0 | \psi_\alpha(0) | n,\vec{k} \rangle \langle n,\vec{k} | \psi_\alpha^\dagger(0) | \Phi_0 \rangle}{\omega - \epsilon_{n,\vec{k}}(N+1) - \mu + i\eta} + \frac{\langle \Phi_0 | \psi_\alpha^\dagger(0) | n,-\vec{k} \rangle \langle n,-\vec{k} | \psi_\alpha(0) | \Phi_0 \rangle}{\omega + \epsilon_{n,-\vec{k}}(N-1) - \mu - i\eta} \right]$$

$$G(\vec{k},\omega) = V \sum_{n} \left[\frac{\overbrace{|\langle n,\vec{k} | \psi_\alpha^\dagger(0) | \Phi_0 \rangle|^2}^{A_{n,\vec{k}}}}{\omega - \epsilon_{n,\vec{k}}(N+1) - \mu + i\eta} + \frac{\overbrace{|\langle n,-\vec{k} | \psi_\alpha(0) | \Phi_0 \rangle|^2}^{B_{n,\vec{k}}}}{\omega + \epsilon_{n,-\vec{k}}(N-1) - \mu - i\eta} \right]$$

$$A_{n,\vec{k}} = |\langle n,\vec{k} | \psi_\alpha^\dagger(0) | \Phi_0 \rangle|^2 \geq 0$$

$$B_{n,\vec{k}} = |\langle n,-\vec{k} | \psi_\alpha(0) | \Phi_0 \rangle|^2 \geq 0$$

Estrutura analítica da função de Green no plano ω complexo



USANDO: $$\frac{1}{x \pm i\eta} = P\left(\frac{1}{x}\right) \mp i\pi\delta(x)$$

$$\frac{1}{(x \pm i\eta)}\frac{(x \mp i\eta)}{(x \mp i\eta)} = \frac{x \mp i\eta}{x^2+\eta^2} =$$

$$\frac{x}{x^2+\eta^2} \mp i\pi \underbrace{\frac{(\eta/\pi)}{x^2+\eta^2}}_{\eta \to 0^+:\ \delta(x)}$$

$$\mathrm{Im}\, G(\vec{k},\omega) = \pi V \sum_{n} \Big[-A_{n,\vec{k}}\,\delta[\omega-\mu-\epsilon_{n,\vec{k}}(N+1)] + B_{n,\vec{k}}\,\delta[\omega-\mu+\epsilon_{n,-\vec{k}}(N-1)] \Big]$$

$$\mathrm{Re}\, G(\vec{k},\omega) = V \sum_{n} \left[P\left(\frac{A_{n,\vec{k}}}{\omega-\mu-\epsilon_{n,\vec{k}}(N+1)}\right) + P\left(\frac{B_{n,\vec{k}}}{\omega-\mu+\epsilon_{m,-\vec{k}}(N-1)}\right) \right]$$

$$\mathrm{Im}\,G(\vec{k},\omega) = \begin{cases} <0 & \omega > \mu \\ >0 & \omega < \mu \end{cases}$$

$$\mathrm{sgn}\left[\mathrm{Im}\,G(\vec{k},\omega)\right] = -\,\mathrm{sgn}(\omega-\mu)$$

NO LIMITE EM OS NÍVEIS DE ENERGIA SE TORNAM DENSOS $(V \to \infty)$, DEFINIMOS:

$$A(\vec{k},\epsilon)\,d\epsilon = V \sum_{n} A_{n,\vec{k}} > 0 \qquad \epsilon < \epsilon_{n\vec{k}}(N+1) < \epsilon + d\epsilon$$

$$B(\vec{k},\epsilon)\,d\epsilon = V \sum_{n} B_{n,\vec{k}} > 0 \qquad \epsilon < \epsilon_{n,\vec{k}}(N-1) < \epsilon + d\epsilon$$

$$\Rightarrow G(\vec{k},\omega) = \int_0^\infty d\epsilon \left[\frac{A(\vec{k},\epsilon)}{\omega-\mu-\epsilon+i\eta} + \frac{B(\vec{k},\epsilon)}{\omega-\mu+\epsilon-i\eta} \right]$$

$$\mathrm{Im}\,G(\vec{k},\omega) = \begin{cases} -\pi A(\vec{k},\omega-\mu) & \omega > \mu \\ \pi B(\vec{k},\omega-\mu) & \omega < \mu \end{cases}$$

$$\mathrm{Re}\, G(\vec{k},\omega) = P\int_{-\infty}^{\infty} \frac{dx}{\pi} \frac{\mathrm{Im}\, G(\vec{k},x)\,\mathrm{sgn}(x-\mu)}{x-\omega}$$

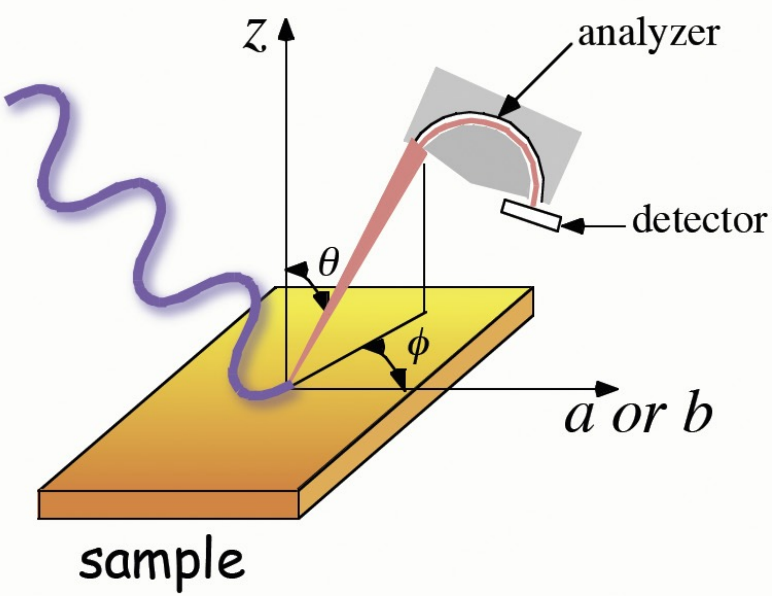
SE $A(\vec{k},\epsilon)$ E $B(\vec{k},\epsilon)$ SÓ TÊM SUPORTE COMPACTO NO EIXO REAL OU SE CAÍREM SUFICIENTEMENTE RÁPIDO COM ϵ:

$$G(\vec{k},\omega) \xrightarrow[\omega\to\infty]{} \frac{1}{\omega}\underbrace{\left[\int_0^{\infty} \left(A(\vec{k},\epsilon)+B(\vec{k},\epsilon)\right)d\epsilon\right]}_{\text{PODE-SE PROVAR QUE } = 1}$$

$$G(\vec{k},\omega) \xrightarrow[\omega\to\infty]{} \frac{1}{\omega}$$

Angle-resolved photoemission spectroscopy (ARPES)

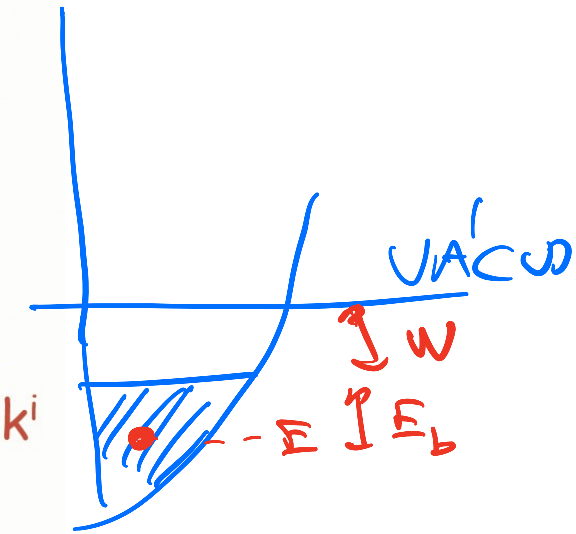
ARPES experiment



We need:

binding energy - E_b

initial momentum - k^i



$$E_b = E - h\nu + W$$

Função trabalho

$$k^i_{\parallel} = k^f_{\parallel} = \sqrt{2mE/\hbar^2}\,\sin\theta$$

$$k^i_{\perp} = k^f_{\perp} - G = \sqrt{2mE/\hbar^2}\,\cos\theta - G$$

Perda na superfície

Angle-resolved photoemission spectroscopy (ARPES)

$$H_I = -\int d^3x \, \vec{j}(x) \cdot \vec{A}(x)$$

$$\langle \zeta, \mathbf{k}+\mathbf{q} | -\vec{j} \cdot \vec{A} | \lambda, \mathbf{q} \rangle \sim \Lambda(\mathbf{q}, \hat{e}_\lambda) \langle \zeta | c_{\mathbf{k}\sigma} | \lambda \rangle$$

Aproximação súbida ("sudden"): Dependência de Λ com momento e energia pode ser ignorada.

$$I_{ARPES}(\mathbf{k}, \omega) \propto f(-\omega) A(\mathbf{k}, -\omega)$$

Parte imaginária da função de Green

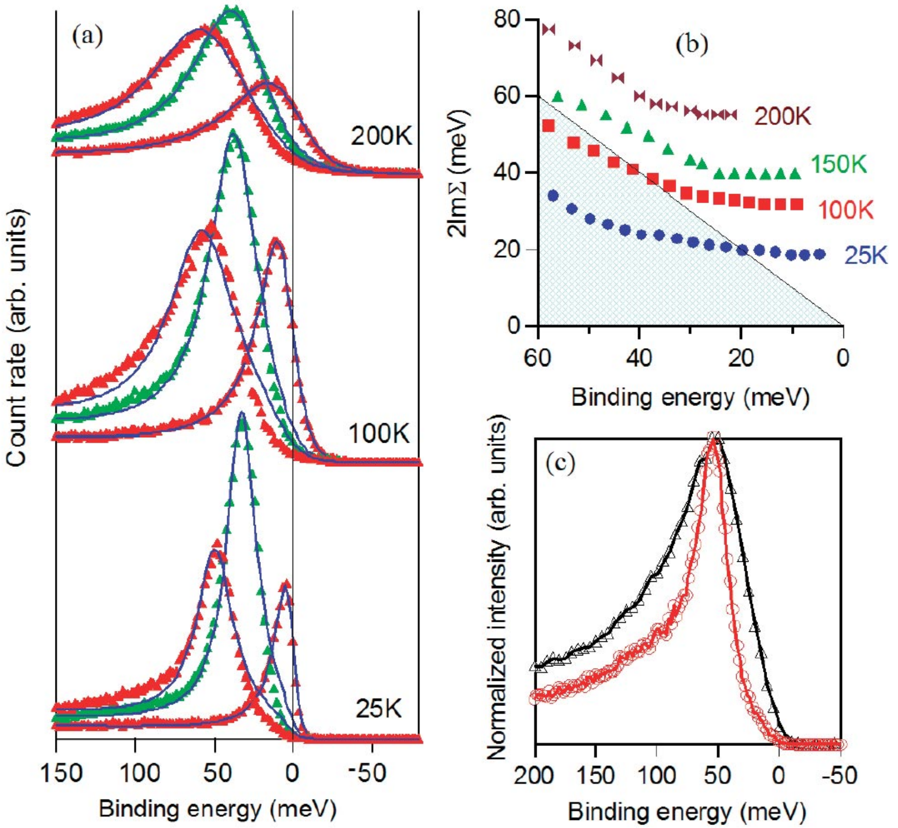


FIG. 3 (color). (a) EDCs (triangles) and Lorentzian fits (blue lines) at different temperatures (offset for clarity) for three emission angles each. (b) Summary of EDC fitting results showing full-width $2\,\mathrm{Im}\Sigma$ versus peak position. The shaded region indicates where peak full widths are sharper than their energy, which should be considered quasiparticle-like. (c) Raw EDCs from the laser (red circles) and 52 eV synchrotron source (black triangles) measured at the same **k** value.

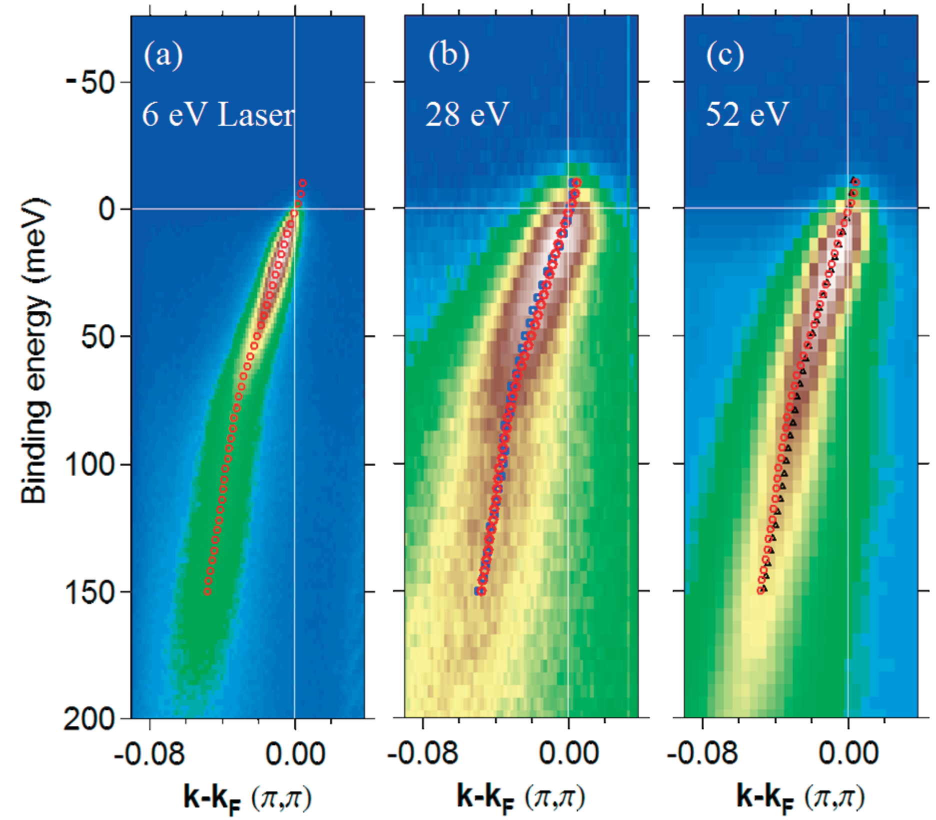


FIG. 1 (color). Comparison of ARPES along the node in near-optimally doped Bi2212 using (a) 6 eV laser photons at $T = 25$ K, (b) 28 eV photons at $T = 26$ K, and (c) 52 eV photons at $T = 16$ K. The images are scaled identically in E and $\mathbf{k}$, and all three contain MDC derived dispersion for the laser data (red circles). Additionally, the dispersions for the data of panels (b) and (c) are shown as blue squares and black triangles, respectively.

Laser Based Angle-Resolved Photoemission, the Sudden Approximation, and Quasiparticle-Like Spectral Peaks in Bi2Sr2CaCu2O8, J. D. Koralek *et al.*, PRL **96,** 017005 (2006)

Cones de Dirac no grafeno

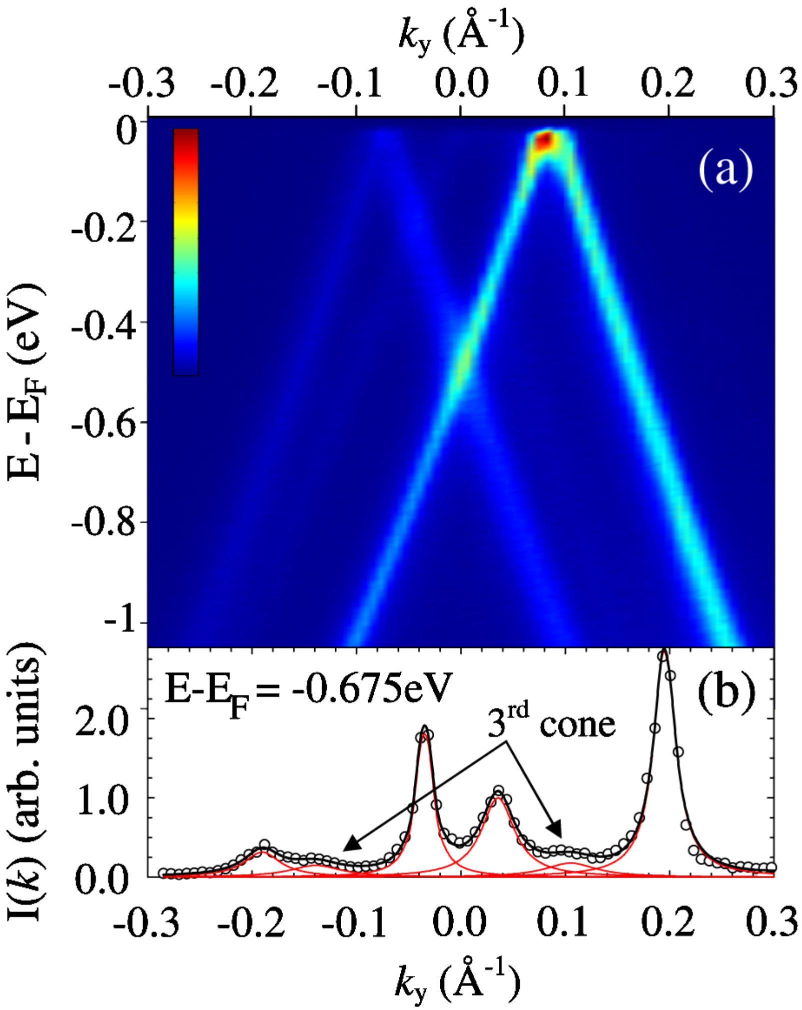
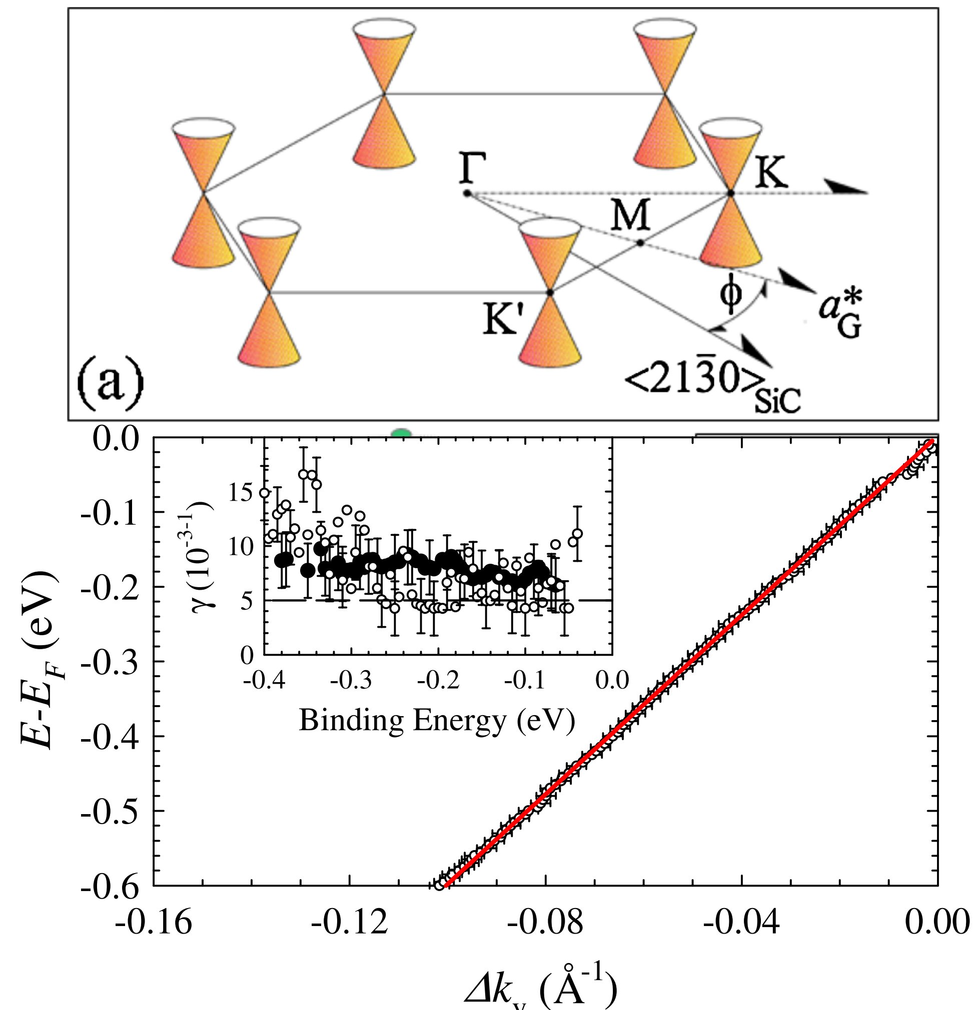


FIG. 2 (color online). (a) ARPES measured band structure of an 11-layer C-face graphene film grown on the 6H SiC. The ARPES resolution was set at 7 meV at $\hbar\omega = 30$ eV. The sample temperature is 6 K. The scan in k_y is perpendicular to the SiC $\langle 10\bar{1}0\rangle_{SiC}$ direction at the K point (see Fig. 1). Two linear Dirac cones are visible. (b) A MDC at $BE = E_F - 0.675$ eV shows a third faint cone. Heavy solid line is a fit to the sum of six Lorentzians (thin solid lines).



M. Sprinkle *et al.*, Phys. Rev. Lett. **103**, 226803 (2009)