Fl 193 – Teoria Quântica de Sistemas de Muitos Corpos

1° Semestre de 2025 10/04/2025 Aula 13

Função de Green de um corpo ordenada temporalmente a *T*=0:

$$iG_{\alpha\beta}\left(\mathbf{r}t;\mathbf{r}'t'\right) = \frac{\left\langle\Psi_{0H}\right|T\left[\psi_{H\alpha}\left(\mathbf{r},t\right)\psi_{H\beta}^{\dagger}\left(\mathbf{r}',t'\right)\right]\left|\Psi_{0H}\right\rangle}{\left\langle\Psi_{0H}\right|\Psi_{0H}\right\rangle}$$
$$= \theta\left(t-t'\right)\frac{\left\langle\Psi_{0H}\right|\psi_{H\alpha}\left(\mathbf{r},t\right)\psi_{H\beta}^{\dagger}\left(\mathbf{r}',t'\right)\left|\Psi_{0H}\right\rangle}{\left\langle\Psi_{0H}\right|\Psi_{0H}\right\rangle}$$
$$+\zeta\theta\left(t'-t\right)\frac{\left\langle\Psi_{0H}\right|\psi_{H\beta}^{\dagger}\left(\mathbf{r}',t'\right)\psi_{H\alpha}\left(\mathbf{r},t\right)\left|\Psi_{0H}\right\rangle}{\left\langle\Psi_{0H}\right|\Psi_{0H}\right\rangle}$$
$$\left\{\zeta = \begin{cases} +1, \text{ bósons}\\ -1, \text{ férmions} \end{cases}$$

Propriedades:

(a) Sistema isolado:

$$G_{\alpha\beta}\left(\mathbf{r}t;\mathbf{r}'t'\right) = G_{\alpha\beta}\left(\mathbf{r},\mathbf{r}',t-t'\right)$$

(b) Sistema isolado e invariância translacional:

$$G_{\alpha\beta}\left(\mathbf{r}t;\mathbf{r}'t'\right) = G_{\alpha\beta}\left(\mathbf{r}-\mathbf{r}',t-t'\right) \equiv G_{\alpha\beta}\left(\mathbf{R},T\right)$$

 (c) Sistema isolado, invariância translacional e simetria de reversão temporal (ausência de campo magnético ou de magnetização):

$$G_{\alpha\beta}\left(\mathbf{r}t;\mathbf{r}'t'\right) = \delta_{\alpha\beta}G\left(\mathbf{r}-\mathbf{r}',t-t'\right) \equiv \delta_{\alpha\beta}G\left(\mathbf{R},T\right)$$

Valores esperados no estado fundamental de operadores de um corpo:

$$\widehat{U} = \sum_{\alpha\beta} \int d^3 r \psi_{\alpha}^{\dagger} \left(\mathbf{r} \right) U_{\alpha\beta} \left(\mathbf{r} \right) \psi_{\beta} \left(\mathbf{r} \right)$$

$$\langle U \rangle = \zeta i \sum_{\alpha\beta} \int d^3 r \lim_{\eta \to 0^+} \lim_{\mathbf{r} \to \mathbf{r}'} \left[U_{\alpha\beta} \left(\mathbf{r} \right) G_{\beta\alpha} \left(\mathbf{r} t; \mathbf{r}' t + \eta \right) \right]$$

Função de Green não interagente (férmions):

$$iG_{\alpha\beta}^{(0)}\left(\mathbf{r}t;\mathbf{r}'t'\right) = \frac{\delta_{\alpha\beta}}{V}\sum_{\mathbf{k}}e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}e^{-i\epsilon(\mathbf{k})(t-t')}\left[\theta\left(t-t'\right)\theta\left(k-k_F\right) - \theta\left(t'-t\right)\theta\left(k_F-k\right)\right]$$

$$G_{\alpha\beta}(\mathbf{k},\omega) = \int d^{3}R dT e^{-i\mathbf{k}\cdot\mathbf{R}} e^{i\omega T} G_{\alpha\beta}(\mathbf{R},T)$$
$$G_{\alpha\beta}(\mathbf{R},T) = \int \frac{d^{3}k d\omega}{(2\pi)^{4}} e^{i\mathbf{k}\cdot\mathbf{R}} e^{-i\omega T} G_{\alpha\beta}(\mathbf{k},\omega)$$

$$G_{\alpha\beta}^{(0)}\left(\mathbf{k},\omega\right) = \delta_{\alpha\beta}G^{(0)}\left(\mathbf{k},\omega\right)$$

$$G^{(0)}(\mathbf{k},\omega) = \frac{1}{\omega - \epsilon(\mathbf{k}) + i\eta \operatorname{sgn}(k - k_F)}, \quad \eta \to 0^+$$

Valor esperado de um operador de dois corpos a partir da função de Green

Sistema homogêneo com interação de pares:

$$H = \sum_{\alpha} \int d^3 r \psi_{\alpha}^{\dagger} \left(\mathbf{r} \right) \left(-\frac{\hbar^2 \nabla^2}{2m} \right) \psi_{\alpha} \left(\mathbf{r} \right) + \frac{1}{2} \sum_{\alpha\beta} \int d^3 r d^3 r' \psi_{\alpha}^{\dagger} \left(\mathbf{r} \right) \psi_{\beta}^{\dagger} \left(\mathbf{r}' \right) U \left(\mathbf{r} - \mathbf{r}' \right) \psi_{\beta} \left(\mathbf{r}' \right) \psi_{\alpha} \left(\mathbf{r} \right)$$

$$\left\langle U \right\rangle = \frac{i\zeta}{2} \sum_{\alpha} \int d^3r \lim_{t' \to t^+} \lim_{\mathbf{r} \to \mathbf{r}'} \left[\left(i\partial_t + \frac{\hbar^2 \nabla^2}{2m} \right) G_{\alpha\alpha} \left(\mathbf{r}t; \mathbf{r}'t' \right) \right]$$

$$\left\langle H \right\rangle = \frac{i\zeta}{2} \sum_{\alpha} \int d^3r \lim_{t' \to t^+} \lim_{\mathbf{r} \to \mathbf{r}'} \left[\left(i\partial_t - \frac{\hbar^2 \nabla^2}{2m} \right) G_{\alpha\alpha} \left(\mathbf{r}t; \mathbf{r}'t' \right) \right]$$

Transformadas de Fourier

$$G_{\alpha\beta}(\mathbf{k},\omega) = \int d^{3}R dT e^{-i\mathbf{k}\cdot\mathbf{R}} e^{i\omega T} G_{\alpha\beta}(\mathbf{R},T)$$

$$G_{\alpha\beta}(\mathbf{R},T) = \int \frac{d^{3}k d\omega}{(2\pi)^{4}} e^{i\mathbf{k}\cdot\mathbf{R}} e^{-i\omega T} G_{\alpha\beta}(\mathbf{k},\omega)$$

$$\swarrow$$

$$C^{i\omega(t-t')} = e^{i\omega(t-t-u)} = e^{i\omega t}$$

$$C^{i\omega(t-t')} = e^{i\omega(t-t-u)}$$

Operadores de um corpo

Valores esperados no estado fundamental de operadores de um corpo: exemplos.

$$\langle H_0 \rangle = \zeta i V \sum_{\alpha} \int \frac{d^3 k d\omega}{(2\pi)^4} e^{i\omega\eta} \left(\frac{k^2}{2m}\right) G_{\alpha\alpha} \left(\mathbf{k}, \omega\right)$$
$$\langle N \rangle = \zeta i V \sum_{\alpha} \int \frac{d^3 k d\omega}{(2\pi)^4} e^{i\omega\eta} G_{\alpha\alpha} \left(\mathbf{k}, \omega\right)$$
$$\langle \mathbf{S} \rangle = \zeta i V \sum_{\alpha\beta} \int \frac{d^3 k d\omega}{(2\pi)^4} e^{i\omega\eta} \left(\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2}\right) G_{\beta\alpha} \left(\mathbf{k}, \omega\right)$$

Operadores de dois corpos

Valores esperados de operadores de dois corpos: sistema homogêneo com interação de pares.

$$H = \sum_{\alpha} \int d^3 r \psi_{\alpha}^{\dagger} \left(\mathbf{r} \right) \left(-\frac{\hbar^2 \nabla^2}{2m} \right) \psi_{\alpha} \left(\mathbf{r} \right) + \frac{1}{2} \sum_{\alpha\beta} \int d^3 r d^3 r' \psi_{\alpha}^{\dagger} \left(\mathbf{r} \right) \psi_{\beta}^{\dagger} \left(\mathbf{r}' \right) V \left(\mathbf{r} - \mathbf{r}' \right) \psi_{\beta} \left(\mathbf{r}' \right) \psi_{\alpha} \left(\mathbf{r} \right)$$

$$\langle V \rangle = \frac{i\zeta V}{2} \sum_{\alpha} \int \frac{d^3 k d\omega}{(2\pi)^4} e^{i\omega\eta} \left(\omega - \frac{k^2}{2m}\right) G_{\alpha\alpha} \left(\mathbf{k}, \omega\right)$$
$$\langle H \rangle = \frac{i\zeta V}{2} \sum_{\alpha} \int \frac{d^3 k d\omega}{(2\pi)^4} e^{i\omega\eta} \left(\omega + \frac{k^2}{2m}\right) G_{\alpha\alpha} \left(\mathbf{k}, \omega\right)$$

A representação espectral de Lehmann · FORMULA EXATA PARA GAR (Ew) , POUCO UTIL PARA CALCULOS DE GAB (Re, 2) . MUITO UTIL PARA ANALISARMOS PROPRIEDADES DE GAR(KW) GENERALIZAVEL PARA AS OUTRAS FUNÇÕES DE GREEN. $t_{ah}(x) = e^{iHt} + f_{x}e^{iMt}$ < for (For) = 1 $i G_{\alpha\beta}(x_1x') = O(t-t') \leq \Phi_0 | t_{\alpha\mu}(x) + t_{\beta\mu}(x') | t_{\beta\gamma} = e^{i\mu t_1'} + t_{\beta}(x') e^{i\mu t_1'}$ $-\partial(t'-t) \leq \mathcal{L}_{I}\psi_{\mu+}(x')\psi_{\mu+}(x$ iGr (x,x')= 2 [o(t-t') < foldan(x) 1/2 × 1/4 × 1 -0 (t-t) < Iol 1 = (x) 1 = 2 < 1 1 + (x) 1 =]

$$\begin{split} & (G_{\mu}(x_{i},x_{i})) = \sum_{n} \left[\Theta(t-t') < \frac{1}{2} O(t_{\alpha H}(x) | t_{n} \times t_{n}| t_{\beta H}(x_{i}) | \frac{1}{2} O(t_{n} \times t_{n}) \right] \\ &= \sum_{n} \left[\Theta(t-t') e^{i(E_{n}-E_{n})(t-t')} < \frac{1}{2} O(t_{n}(x) | \frac{1}{2}_{n}) > (\frac{1}{2}_{n} \times t_{n}) \right] \\ &= O(t'-t) e^{i(E_{n}-E_{n})(t-t')} < \frac{1}{2} O(t_{n}(x) | \frac{1}{2}_{n}) > (\frac{1}{2}_{n} \times t_{n}) \right] \\ &= O(t'-t) e^{i(E_{n}-E_{n})(t-t')} < \frac{1}{2} O(t_{n}) \left[\frac{1}{2}_{n} \times t_{n} \right] \\ &= O(t'-t) e^{i(E_{n}-E_{n})(t-t')} < \frac{1}{2} O(t_{n}) \left[\frac{1}{2}_{n} \times t_{n} \right] \\ &= O(t'-t) e^{i(E_{n}-E_{n})(t-t')} < \frac{1}{2} O(t_{n}) \left[\frac{1}{2}_{n} \times t_{n} \right] \\ &= O(t'-t) e^{i(E_{n}-E_{n})(t-t')} \\ &= O(t'-t$$



$$S_{\alpha\rho}(\pi,\pi',\omega) = \sum_{n} \left[\frac{\langle \underline{A}_{o}|\underline{L}_{\alpha}(\pi) |\underline{A}_{n} \rangle \langle \underline{A}_{n}|\underline{L}_{\mu}(\pi')|\underline{L}_{p} \rangle}{\omega - (\underline{E}_{n} - \underline{E}_{o}) + iN} + \frac{\langle \underline{A}_{o}|\underline{L}_{\mu}(\pi')|\underline{L}_{n} \rangle \langle \underline{A}_{n}|\underline{L}_{\alpha}(\pi)|\underline{L}_{p} \rangle}{\omega + (\underline{E}_{n} - \underline{E}_{o}) - iN} \right]$$

$$NO \quad I^{2} (2^{2}) TERNO_{j} \quad (\underline{A}_{n}) TEN \quad N+1 (N-1) PARTICULAS$$

$$SE \quad V_{o}^{2} TEN \quad N \quad PARTICULAS.$$

$$E_{n} - \underline{E}_{o} = E_{n} (N \pm 1) - E_{o} (N) = E_{n} (N \pm 1) - E_{o} (N \pm 1) - \frac{1}{2} (N$$

 $G_{\alpha\rho}(\pi,\pi',\omega) = \sum_{n} \left[\frac{\langle \Phi_{0}| t_{\alpha}(\pi) | \Phi_{n} \rangle \langle \Phi_{n}| \langle \psi_{\beta}(\pi')| F_{0} \rangle}{\omega - \varepsilon_{n}(\omega + i) - \mu + i N} + \frac{\langle \Phi_{0}| \psi_{\beta}(\pi') | F_{0} \rangle}{\omega - \varepsilon_{n}(\omega + i) - \mu + i N} \right]$ $(\underline{A}) (\underline{A}) [\underline{A}) (\underline{A})$ $\omega + \epsilon_{\mu}(N-1) - \mu - in$ 31 SE HA' INVARIANCIA TRANSLACIONAL: O HOMENTO LINEAR TOTAL E CONSERVADO: $\vec{P} = \int \vec{d}_{n} \vec{z} + \vec{z} (\vec{x}) \left[\vec{\nabla} \right] \vec{+}_{a} (\vec{x}) = \sum_{k} \vec{k} \cdot \vec{c}_{k} \cdot \vec{c}_{k$ やれい ~~ たいと -n [B, H]=0 PODE-SE PROVAR: $\psi_{\alpha}(\vec{x}) = e^{\frac{1}{2}\cdot\vec{x}} + \frac{1}{2}(\vec{x}) = e^{-\frac{1}{2}\cdot\vec{x}} + \frac{1}{2}(\vec{x}) =$ LEVANDO NA EKPRESSÃO PARA GARA (A, ,,) E SUPONDO QUE PIPOZO (MOMENTO LINEAR TOTAL DO EST. FUND. É ZERO)

$$\begin{split} & G_{xp}(\bar{n},\bar{n}'_{i}\omega) = \sum_{m} \left[\frac{e^{i\vec{p}_{m}\cdot(\bar{n}-\bar{n}')} < \widehat{F}_{0}(\frac{1}{4}\sqrt{e^{i(0)}}) + \frac{1}{4}\sqrt{e^{i(1+\lambda)}} + \frac{1}{4}\sqrt{e^{i(0)}} \right] \\ & \omega - G_{m}(\omega_{+1}) - \mu + \lambda n \\ & t = \frac{e^{i\vec{p}_{m}\cdot(\bar{n}-\bar{n}')} < \widehat{F}_{0}(\frac{1}{4}\sqrt{e^{i(0)}}) + \frac{1}{4}\sqrt{e^{i(0)}} + \frac{1}{4}\sqrt{e^{i(0)}} \right] \\ & \omega + G_{m}(\omega_{-1}) - \mu - \lambda n \\ & 0 \quad 0 \cup E \quad PROVA \quad Q \cup E \quad G_{m} \int \overline{A}, \overline{n}'_{i}\omega > = G_{mp} \left(\overline{A} - \overline{A}', \omega\right) \\ & T. \quad FOURIER \quad NO \quad E \leq PA \leq 0: \quad \overline{R} = \overline{\lambda} - \overline{\lambda} \\ & G_{0}(\overline{L}_{1}\omega) = \int d\overline{A} e^{i\overline{L}\cdot\overline{R}} \quad G_{m}(\overline{R}_{1}\omega) \\ & \int d\overline{A} e^{i\overline{L}\cdot\overline{R}} e^{i\overline{L}\cdot\overline{R}} e^{i\overline{L}\cdot\overline{R}} \quad G_{m}(\overline{R}_{1}\omega) \\ & \int d\overline{A} e^{i\overline{L}\cdot\overline{R}} e^{i\overline{L}\cdot\overline{R}} e^{i\overline{L}\cdot\overline{R}} = \sqrt{S_{m}} \frac{1}{4} \frac{1}{p} e^{i(0)(\frac{1}{2}\sigma)} + \frac{1}{\omega - G_{m}\overline{L}(\omega_{+1}) - \mu + \lambda n} \\ & + \frac{\langle \widehat{F}_{0}(4\frac{1}{4}(\sigma)) - \mu_{1} - \overline{L}) \langle \omega + G_{m} - \overline{L}\rangle \langle \omega - \overline{L}\rangle }{\omega + G_{m} - \overline{L}} \right] \end{split}$$

SE HA' SIMETRIA DE REVERSÃO TEMPORAL,
$$\alpha = \beta$$
:
 $G(\overline{R}_{1}\omega) = \sqrt{2} \left[\begin{array}{c} A_{m,\overline{R}} \\ \hline \\ \omega - \mu - \epsilon_{n,\overline{R}}(\omega + 1) + in \end{array} + \begin{array}{c} B_{m,\overline{R}} \\ \hline \\ \omega - \mu + \epsilon_{m,-\overline{R}}(\omega - 1) - in \end{array} \right]$
ONDE:
 $A_{m,\overline{R}} = \left[\langle M_{1}\overline{R} \right] P_{n}^{\dagger}(o) \left[\frac{4}{3}o \right]^{2} \geqslant 0$
 $B_{m,\overline{R}} = \left[\langle M_{1}\overline{R} \right] P_{n}^{\dagger}(o) \left[\frac{4}{3}o \right]^{2} \geqslant 0$
ESSA E A FERRESENTAÇÃO DE LEHMANN.
Polos: 1º TERNO: $\epsilon_{\mu\overline{R}}(N+1) + \mu - in \Rightarrow \begin{cases} Ro: > \mu \\ \exists m: < 0 \end{cases}$

RESÍDUOS DOS POLOS. AMÍR (1º TERMO) Brit (2º TERMO)

Estrutura analítica da função de Green no plano ω complexo



UTILIZANDO:
$$\frac{1}{\chi \pm iS} = P \perp \mp i\pi S(\chi)$$
 (S=0)

NA REP. DE LEHMANN:

Im $G(\overline{R}, \omega) = \Pi V \sum_{m} \left[-A_{m, \overline{R}} S \left[\omega - \mu - E_{M, \overline{R}}(n + 1) \right] + \right]$ +BM, Te S[W-u+Em, te (N-1)] $Re G(\overline{u}_{i}\omega) = V \stackrel{<}{>} \left[P \frac{A_{n,i}k}{\omega - \mu - \epsilon_{n,i}k} + P \frac{B_{n,i}k}{\omega - \mu + \epsilon_{n,i}k} \right]$ Imb(Ikiw) = (xo SE W>m >0 SE W>m

NO	LIMITE	TERMODINA	mico, o	ESPECTE		CONTINUO
	VZAnjk	$A(\mathbb{Z}_{e})$	de	E< Empli	p(n+1)	26+8E
	V J Brite	$\rightarrow B(\vec{k},\epsilon)$	96	ELEN	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	$-1) \subset C + \partial C$
6()	$\tilde{n}_{i}(\omega) = \int_{0}^{\infty}$	$de \left[\frac{A(k,e)}{\omega - \mu - \mu} \right]$	$\frac{1}{6 + in} +$	B(R)E) = -in	
I	v 6 (0, w)	$= -\pi A(k_{\mu})$	-u) + TI B	(K, u-w)		
Re	و (آلگ _ا به)	= P [de[-	A(R,E) >-/e-E	-+ -	B(R,E) - set	



DADA A PARTE IMAGINARIA DE 6(R, w), SUA PARTE REAL SEGUE DE MANEIRA UNICA. NO LIMITE [W/ D, SE A(R,E) E B(R,E) TEM SUPORTE EN REGIÕES FINITAS DE G. $G(\vec{k}, \omega) \xrightarrow{n} \int \left\{ \int A(\vec{k}, G) + B(\vec{k}, G) \right\} dG$

POPE-SE PROVAR, A PARTIC DE { 4 (R), 4 (R)]= S (R-I)Sop QUE O TERRO { }=1

 $G(\vec{k},\omega) \xrightarrow{1}{|\omega| \rightarrow \infty} \omega$



Perda na superfície

Angle-resolved photoemission spectroscopy (ARPES)

$$H_I = -\int d^3x \vec{j}(x) \cdot \vec{A}(x)$$

$$\langle \zeta, \mathbf{k} + \mathbf{q} | - \vec{j} \cdot \vec{A} | \lambda, \mathbf{q} \rangle \sim \Lambda(\mathbf{q}, \hat{e}_{\lambda}) \langle \zeta | c_{\mathbf{k}\sigma} | \lambda \rangle$$

Aproximação súbita ("sudden"): Dependência de Λ com momento e energia pode ser ignorada.

Distrib. Fermi-Dirac $I_{ARPES}(\mathbf{k}, \omega) \propto f(-\omega) A(\mathbf{k}, -\omega)$ Parte imaginária da função de Green [$B(\mathbf{k}, \mu-\omega)$]





FIG. 3 (color). (a) EDCs (triangles) and Lorentzian fits (blue lines) at different temperatures (offset for clarity) for three emission angles each. (b) Summary of EDC fitting results showing full-width $2 \text{ Im}\Sigma$ versus peak position. The shaded region indicates where peak full widths are sharper than their energy, which should be considered quasiparticle-like. (c) Raw EDCs from the laser (red circles) and 52 eV synchrotron source (black triangles) measured at the same **k** value.

FIG. 1 (color). Comparison of ARPES along the node in nearoptimally doped Bi2212 using (a) 6 eV laser photons at T =25 K, (b) 28 eV photons at T = 26 K, and (c) 52 eV photons at T = 16 K. The images are scaled identically in *E* and **k**, and all three contain MDC derived dispersion for the laser data (red circles). Additionally, the dispersions for the data of panels (b) and (c) are shown as blue squares and black triangles, respectively.

Laser Based Angle-Resolved Photoemission, the Sudden Approximation, and Quasiparticle-Like Spectral Peaks in Bi2Sr2CaCu2O8, J. D. Koralek et al., PRL **96,** 017005 (2006)

Cones de Dirac no grafeno



FIG. 2 (color online). (a) ARPES measured band structure of an 11-layer C-face graphene film grown on the 6*H* SiC. The ARPES resolution was set at 7 meV at $\hbar \omega = 30$ eV. The sample temperature is 6 K. The scan in k_y is perpendicular to the SiC $\langle 10\bar{1}0 \rangle_{SiC}$ direction at the *K* point (see Fig. 1). Two linear Dirac cones are visible. (b) A MDC at $BE = E_F - 0.675$ eV shows a third faint cone. Heavy solid line is a fit to the sum of six Lorentzians (thin solid lines).



M. Sprinkle *et al.*, Phys. Rev. Lett. **103**, 226803 (2009)