

# FI 193 – Teoria Quântica de Sistemas de Muitos Corpos

2º Semestre de 2023

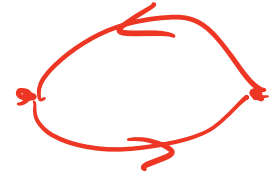
31/10/2023

Aula 22

# Aula passada

$$D_{nn}(\mathbf{k}, \omega) = \tilde{\pi}(\mathbf{k}, \omega) = \frac{\pi(\mathbf{k}, \omega)}{1 - V(\mathbf{k})\pi(\mathbf{k}, \omega)}$$

$$D_{nn}(\mathbf{k}, \omega) \approx \frac{\pi^{(0)}(\mathbf{k}, \omega)}{1 - V(\mathbf{k})\pi^{(0)}(\mathbf{k}, \omega)} \text{ se } r_s \ll 1$$



Da representação de Lehmann:

$$D_{nn}(\mathbf{k}, \omega) = V \sum_n \left[ \frac{\langle \Psi_0 | \hat{n}(-\mathbf{k}) | n\mathbf{k} \rangle \langle n\mathbf{k} | \hat{n}(\mathbf{k}) | \Psi_0 \rangle}{\omega - (E_n - E_0) + i\eta} - \frac{\langle \Psi_0 | \hat{n}(\mathbf{k}) | n\mathbf{k} \rangle \langle n\mathbf{k} | \hat{n}(-\mathbf{k}) | \Psi_0 \rangle}{\omega + (E_n - E_0) - i\eta} \right]$$

$$D_{nn}^R(\mathbf{k}, \omega) = V \sum_n \left[ \frac{\langle \Psi_0 | \hat{n}(-\mathbf{k}) | n\mathbf{k} \rangle \langle n\mathbf{k} | \hat{n}(\mathbf{k}) | \Psi_0 \rangle}{\omega - (E_n - E_0) + i\eta} - \frac{\langle \Psi_0 | \hat{n}(\mathbf{k}) | n\mathbf{k} \rangle \langle n\mathbf{k} | \hat{n}(-\mathbf{k}) | \Psi_0 \rangle}{\omega + (E_n - E_0) + i\eta} \right]$$

Os polos dão as **excitações de carga/densidade** do sistema.

# Aula passada

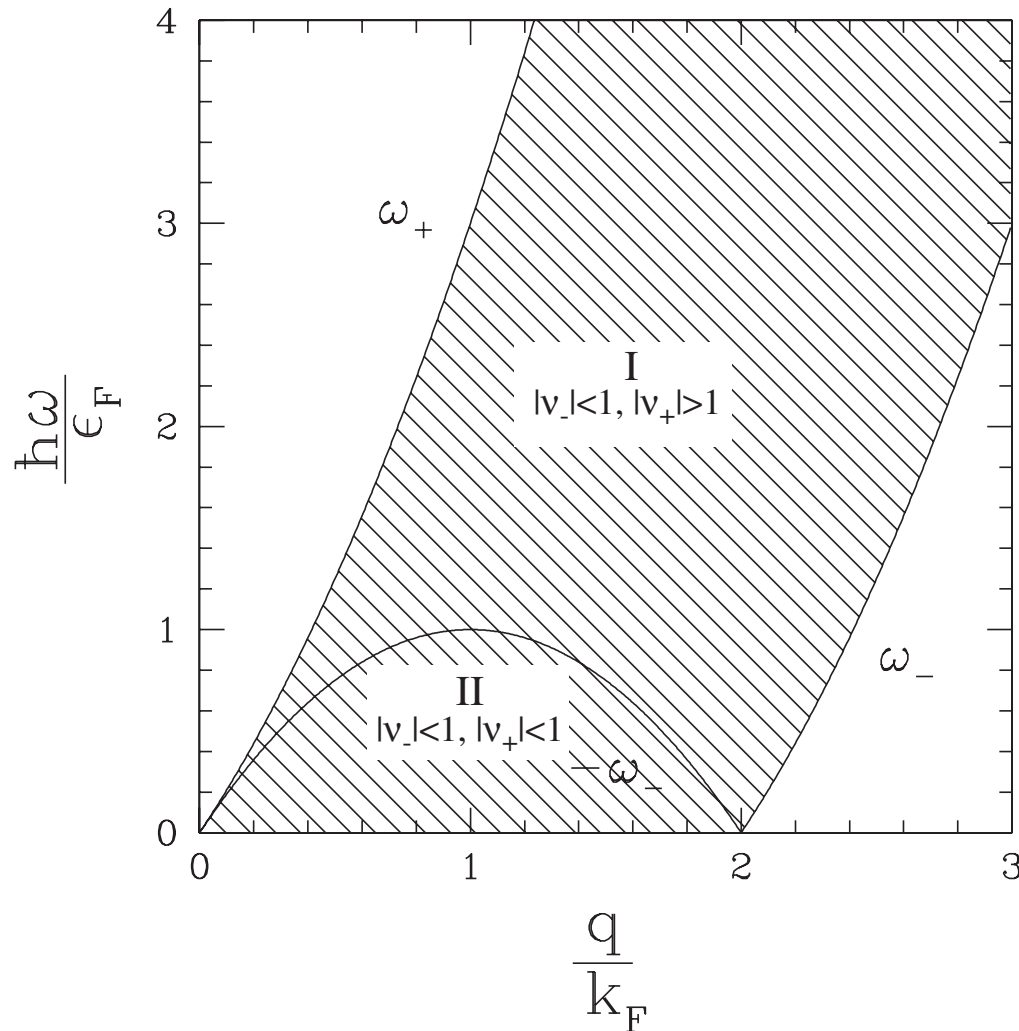
$$\pi^{(0)R}(q) = 2 \int \frac{d^3k}{(2\pi)^3} \left[ \frac{\theta(k_F - k) \theta(|\mathbf{k} + \mathbf{q}| - k_F)}{\nu + \epsilon(\mathbf{k}) - \epsilon(\mathbf{k} + \mathbf{q}) + i\eta} - \frac{\theta(k - k_F) \theta(k_F - |\mathbf{k} + \mathbf{q}|)}{\nu + \epsilon(\mathbf{k}) - \epsilon(\mathbf{k} + \mathbf{q}) + i\eta} \right]$$

$$\nu = \epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}} = \frac{q^2}{2m} + \frac{\mathbf{k} \cdot \mathbf{q}}{m}; \quad \left\{ \begin{array}{ll} k < k_F & \text{e } |\mathbf{k} + \mathbf{q}| > k_F \\ \text{buraco} & \text{ou } \text{partícula} \\ k > k_F & \text{e } |\mathbf{k} + \mathbf{q}| < k_F \\ \text{partícula} & \text{buraco} \end{array} \right.$$

Os polos de  $\pi^{(0)}(\mathbf{q}, \nu)$  são as **excitações partícula-buraco**.

# Aula passada

O contínuo partícula-buraco:



$$\max(0, \omega_-(q)) \leq |\omega| \leq \omega_+(q)$$

$$\omega_{\pm}(q) = \frac{\hbar q^2}{2m} \pm v_F q$$

QUANTUM THEORY OF THE ELECTRON LIQUID  
Gabriele Giuliani, Giovanni Vignale

$$\bar{q}_\sigma \equiv \frac{q}{k_{F\sigma}},$$

$$v_{\pm\sigma} \equiv \frac{\omega}{qv_{F\sigma}} \pm \frac{q}{2k_{F\sigma}},$$

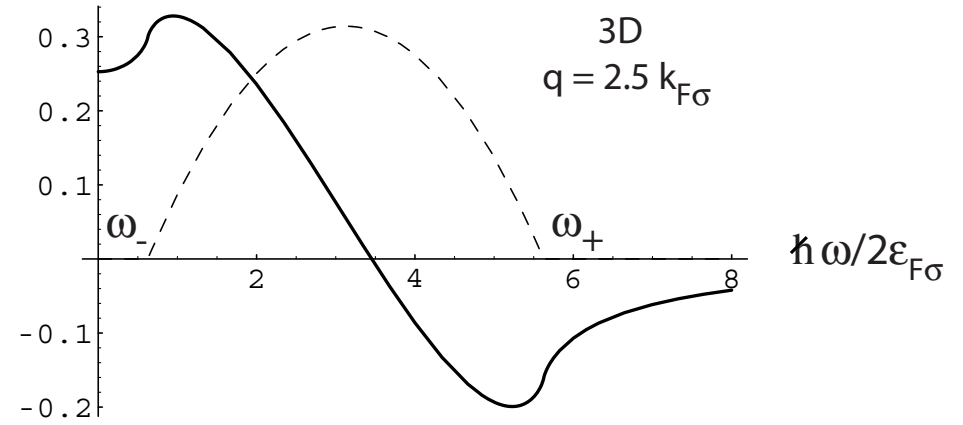
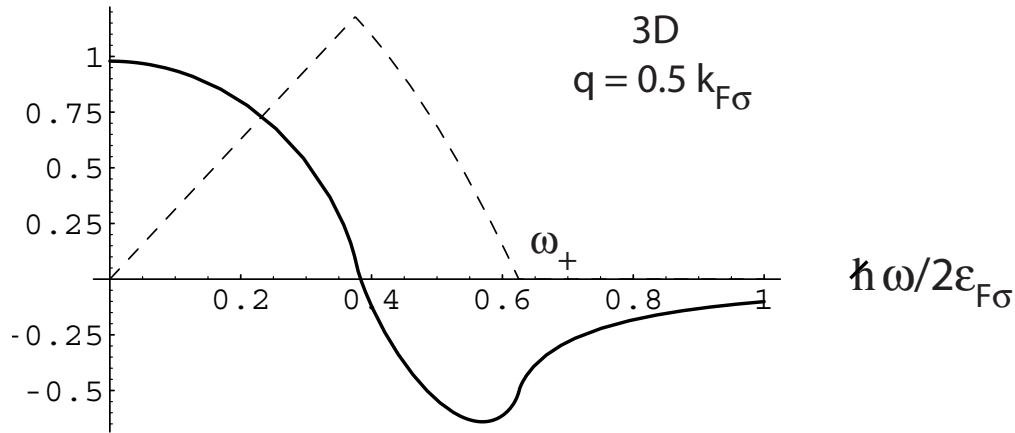
$$\frac{\Re\chi_{0\sigma}(q, \omega)}{N_\sigma(0)} = - \begin{cases} \frac{1}{2} - \frac{1-v_{-\sigma}^2}{4\bar{q}_\sigma} \ln \left| \frac{v_{-\sigma}+1}{v_{-\sigma}-1} \right| + \frac{1-v_{+\sigma}^2}{4\bar{q}_\sigma} \ln \left| \frac{v_{+\sigma}+1}{v_{+\sigma}-1} \right|, & 3D \\ 1 + \frac{1}{\bar{q}_\sigma} \left[ \text{sign}(v_{-\sigma})\Theta(v_{-\sigma}^2 - 1)\sqrt{v_{-\sigma}^2 - 1} \right. \\ \left. - \text{sign}(v_{+\sigma})\Theta(v_{+\sigma}^2 - 1)\sqrt{v_{+\sigma}^2 - 1} \right], & 2D \\ \frac{1}{2\bar{q}_\sigma} \ln \left| \frac{v_{-\sigma}-1}{v_{-\sigma}+1} \right| - \frac{1}{2\bar{q}_\sigma} \ln \left| \frac{v_{+\sigma}-1}{v_{+\sigma}+1} \right|, & 1D \end{cases}$$

$$\chi_0(\mathbf{k}, \omega) \equiv \pi^{(0)}(\mathbf{k}, \omega)$$

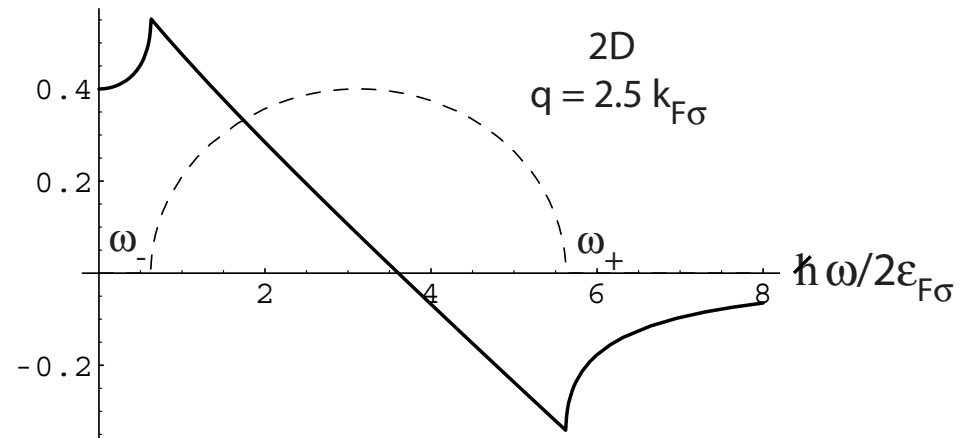
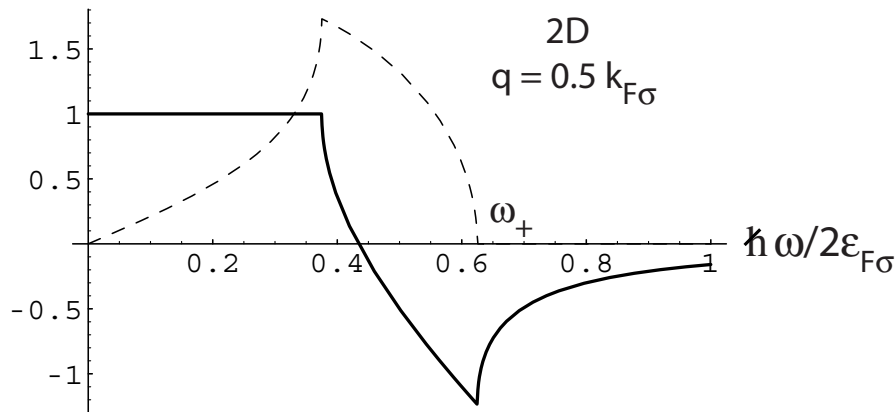
$$\frac{\Im\chi_{0\sigma}(q, \omega)}{N_\sigma(0)} = - \begin{cases} \frac{\pi}{4\bar{q}_\sigma} \left[ \Theta(1 - v_{-\sigma}^2)(1 - v_{-\sigma}^2) - \Theta(1 - v_{+\sigma}^2)(1 - v_{+\sigma}^2) \right], & 3D \\ \frac{1}{\bar{q}_\sigma} \left[ \Theta(1 - v_{-\sigma}^2)\sqrt{1 - v_{-\sigma}^2} - \Theta(1 - v_{+\sigma}^2)\sqrt{1 - v_{+\sigma}^2} \right], & 2D \\ \frac{\pi}{2\bar{q}_\sigma} \left[ \Theta(1 - v_{-\sigma}^2) - \Theta(1 - v_{+\sigma}^2) \right]. & 1D \end{cases}$$

--- Im  $\Pi^0$   
 — Re  $\Pi^0$

$$\chi_0(\mathbf{k}, \omega) \equiv \pi^{(0)}(\mathbf{k}, \omega)$$

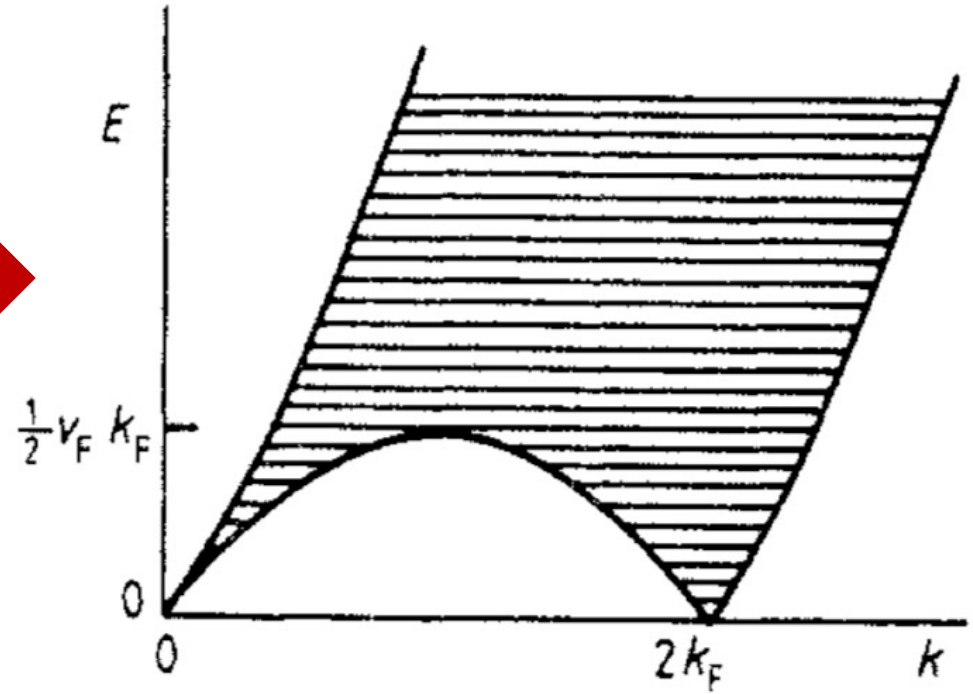
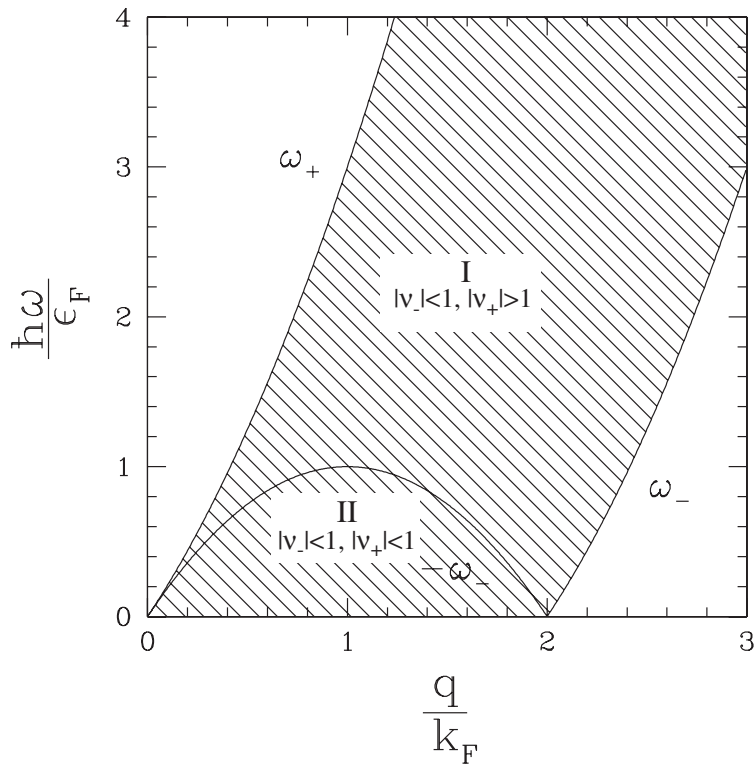


$$\Im m \chi_{0\sigma}(q, \omega) = -\frac{\pi}{2} N_{\sigma}(0) \frac{\omega}{q v_{F\sigma}}, \quad 3D \rightarrow \text{LANDAU DAMPING}$$

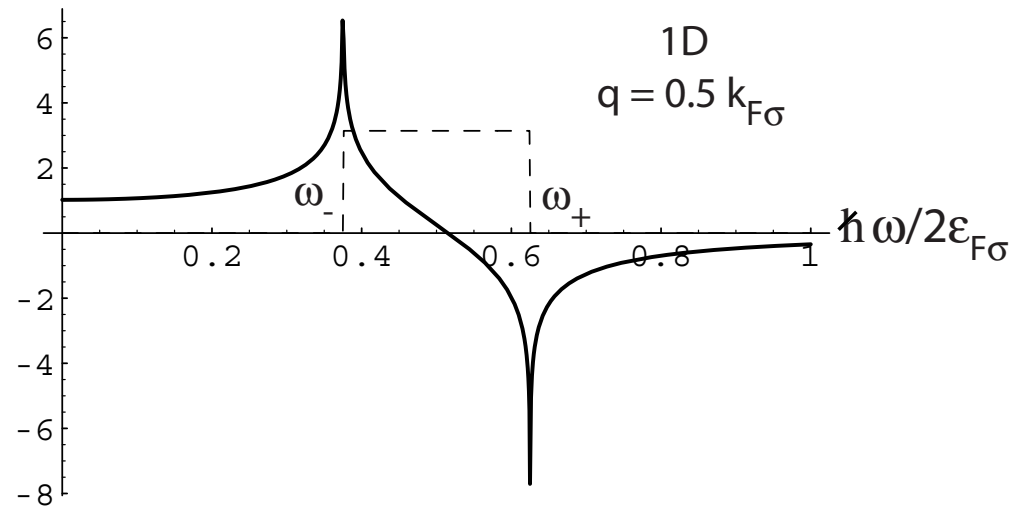


$$\Im m \chi_{0\sigma}(q, \omega) \simeq -\frac{N_{\sigma}(0)}{\sqrt{1 - \left(\frac{q}{2k_{F\sigma}}\right)^2}} \frac{\omega}{q v_{F\sigma}}, \quad 2D$$

# Uma dimensão é diferente!



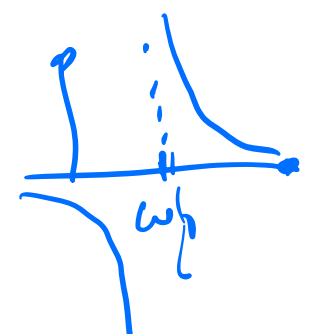
A região II está ausente em uma dimensão!

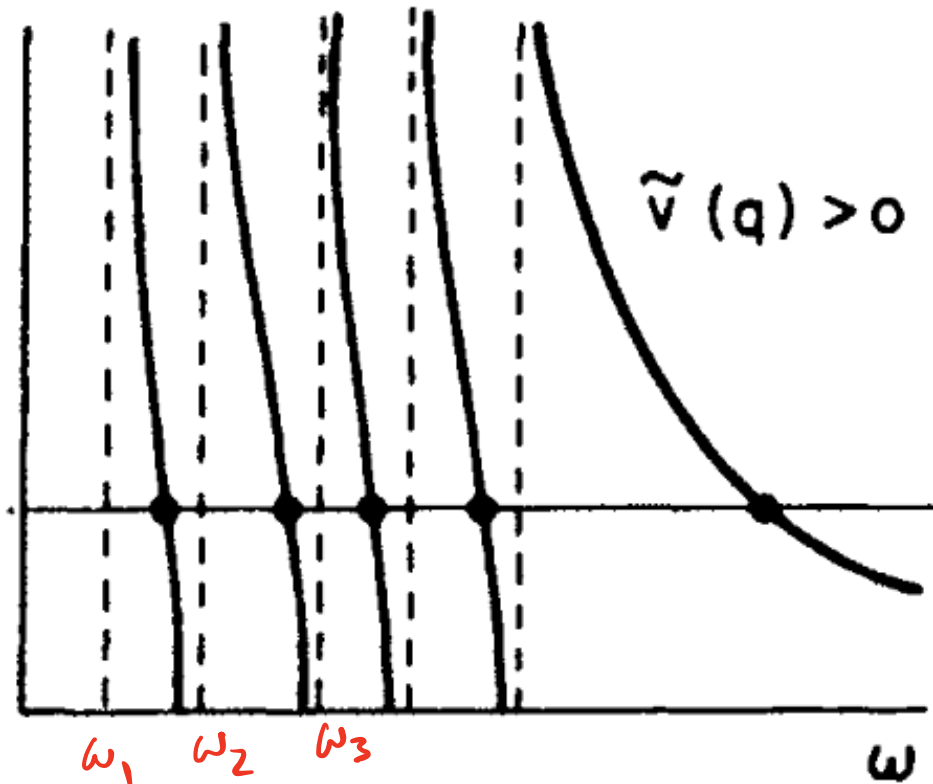


$$\pi^{(0)R}(q) = 2 \int \frac{d^3k}{(2\pi)^3} \left[ \frac{\theta(k_F - k) \theta(|\mathbf{k} + \mathbf{q}| - k_F)}{\nu + \epsilon(\mathbf{k}) - \epsilon(\mathbf{k} + \mathbf{q}) + i\eta} - \frac{\theta(k - k_F) \theta(k_F - |\mathbf{k} + \mathbf{q}|)}{\nu + \epsilon(\mathbf{k}) - \epsilon(\mathbf{k} + \mathbf{q}) + i\eta} \right]$$

$$\nu = \epsilon_{\mathbf{k}+\mathbf{q}} - \epsilon_{\mathbf{k}} = \frac{q^2}{2m} + \frac{\mathbf{k} \cdot \mathbf{q}}{m}; \quad \left\{ \begin{array}{l} k < k_F \text{ e } |\mathbf{k} + \mathbf{q}| > k_F \\ \text{ou} \\ k > k_F \text{ e } |\mathbf{k} + \mathbf{q}| < k_F \end{array} \right.$$

LIMITE DISCRETO:

$$\frac{A_1}{\omega - \omega_1} + \frac{A_2}{\omega - \omega_2} + \dots + \frac{A_N}{\omega - \omega_N}$$






# O plasmon

$$D_{nn}(\mathbf{k}, \omega) \approx \frac{\pi^{(0)}(\mathbf{k}, \omega)}{1 - V(\mathbf{k}) \pi^{(0)}(\mathbf{k}, \omega)} \text{ se } r_s \ll 1$$

PÓLOS ESTÃO EM:  $V(\vec{k}) \pi^{(0)}(\vec{k}, \omega) = 1 \Rightarrow \pi^{(0)}(\vec{k}, \omega) = \frac{1}{V(\vec{k})} = \frac{k^3}{4\pi e^2}$

NO LIMITE DE  $|\vec{k}| \ll k_F$ :

$$\pi^{(0)}(\vec{k}, \omega) = \frac{k_F^3}{3\pi^2} \frac{\cancel{k^2}}{m\omega^2} \left[ 1 + \frac{3}{5} \left( \frac{k_F k}{m\omega} \right)^2 + \dots \right] = \frac{\cancel{k^2}}{4\pi e^2}$$

EM ORDEM DOMINANTE EM  $k/k_F$ :

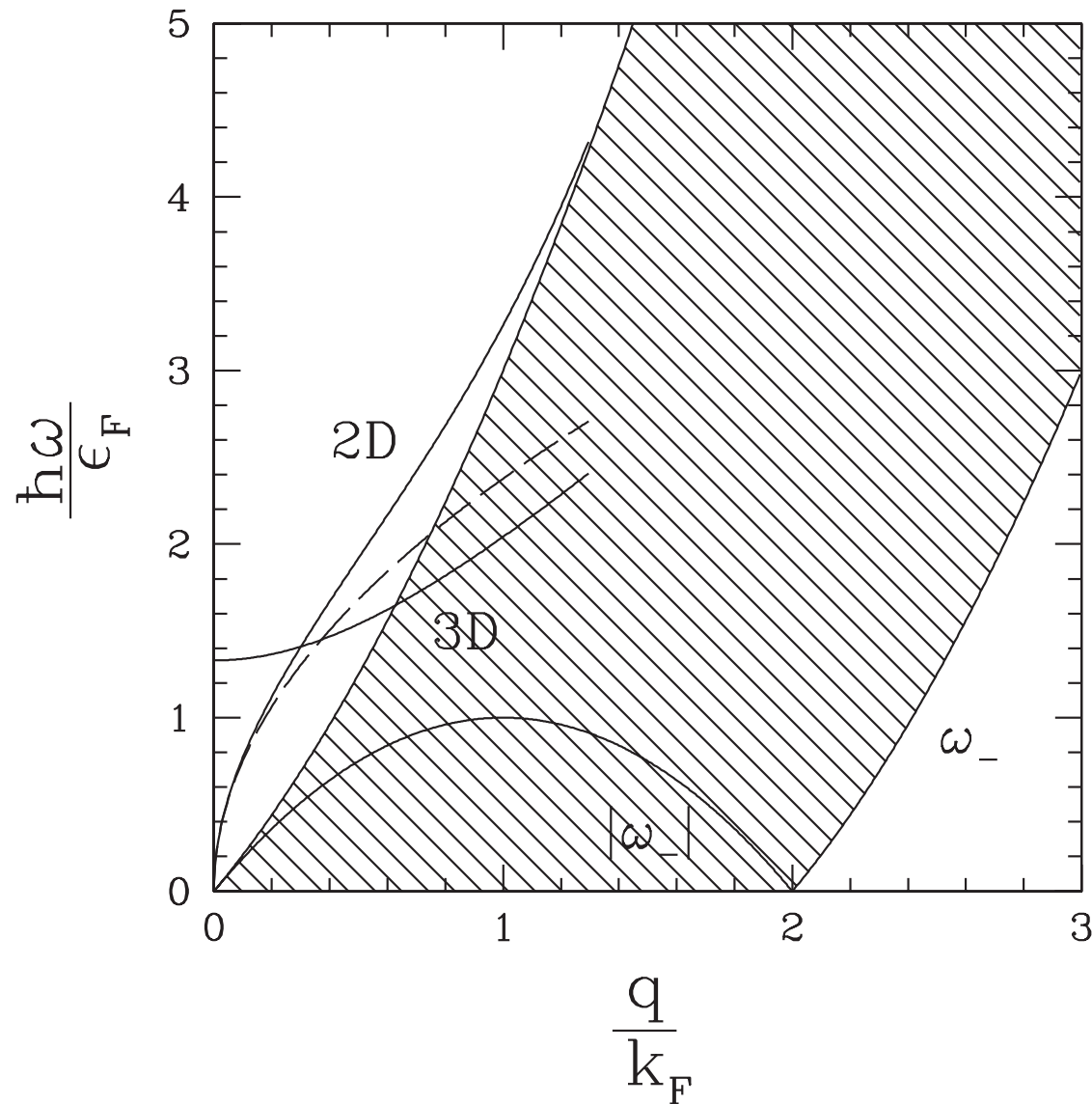
$$\Rightarrow m\omega_p^2 = 4\pi e^2 n \Rightarrow \omega_p^2 = \frac{4\pi n e^2}{m} = \frac{4\pi e^2}{m} \left( \frac{k_F^3}{3\pi^2} \right) \Rightarrow \text{"PLASMON"}$$

AS CORREÇÕES DÃO:

$$\omega_k = \omega_p \left[ 1 + \frac{3}{10} \left( \frac{k}{k_F} \right)^2 + \dots \right]$$

DISPERSÃO  
DO  
PLASMON

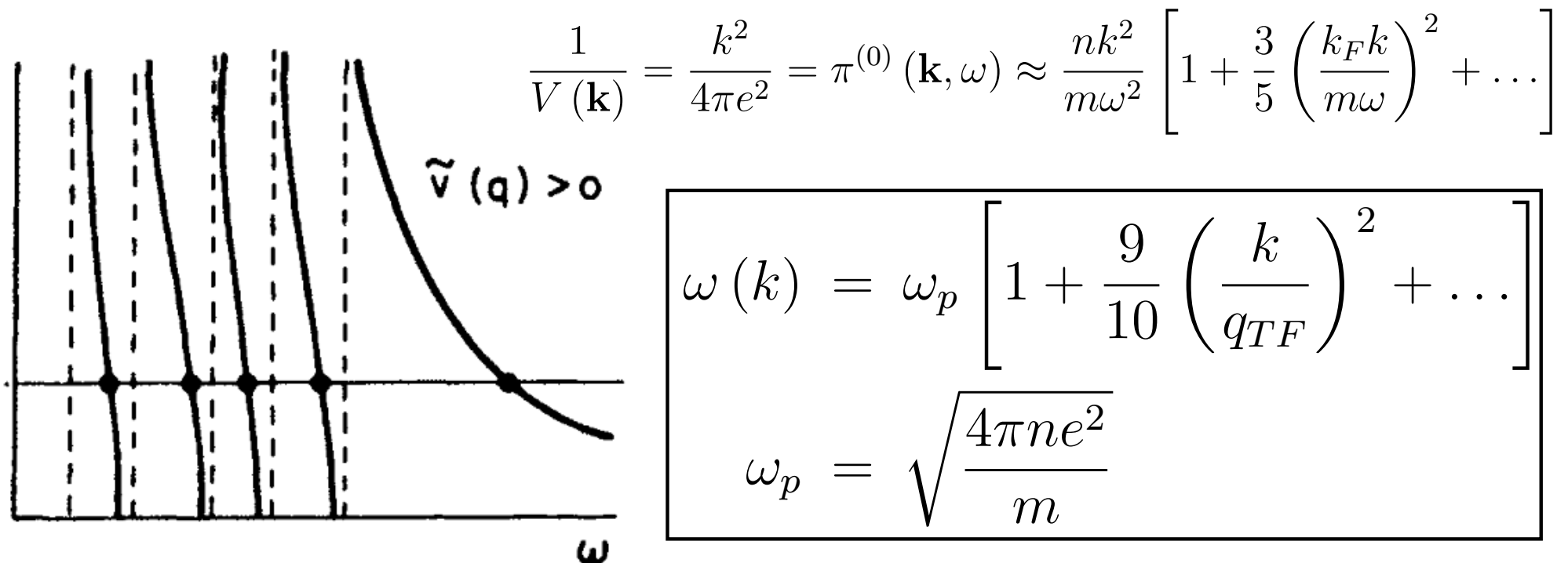
# Dispersão do plasmon e contínuo de elétron-buraco



# O plasmon

$$D_{nn}(\mathbf{k}, \omega) \approx \frac{\pi^{(0)}(\mathbf{k}, \omega)}{1 - V(\mathbf{k}) \pi^{(0)}(\mathbf{k}, \omega)} \text{ se } r_s \ll 1$$

$$\pi^{(0)R}(q) = 2 \int \frac{d^3k}{(2\pi)^3} \left[ \frac{\theta(k_F - k) \theta(|\mathbf{k} + \mathbf{q}| - k_F)}{\nu + \epsilon(\mathbf{k}) - \epsilon(\mathbf{k} + \mathbf{q}) + i\eta} - \frac{\theta(k - k_F) \theta(k_F - |\mathbf{k} + \mathbf{q}|)}{\nu + \epsilon(\mathbf{k}) - \epsilon(\mathbf{k} + \mathbf{q}) + i\eta} \right]$$



# O plasmon como oscilação clássica

EM EQUILÍBRIO:  $n_e(\vec{x}, t) = n_0$ ,  $\vec{v}(\vec{x}, t) = 0$ ,  $\vec{E}(\vec{x}, t) = 0$

FORA DO EQUILÍBRIO:  $\delta n_e(\vec{x}, t) = n_e(\vec{x}, t) - n_0$

TEORIA HIDRODINÂMICA LINEARIZADA EM  $\delta n, v, E$ :

$\delta \rho(\vec{x}, t) = -e \delta n_e(\vec{x}, t)$  DENSIDADE DE CARGA

DA LEI DE GAUSS:  $\vec{\nabla} \cdot \vec{E} = 4\pi \delta \rho = -4\pi e \delta n_e$  (1)

EQ. DE EULER DA HIDRODINÂMICA (2ª LEI DE NEWTON):

$$m \frac{d[m_e \vec{v}]}{dt} = -e m_e \vec{E} = -e n_0 \vec{E} + O(\delta n \vec{E})$$

$$\hookrightarrow \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} = \frac{\partial}{\partial t} + v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}$$

$$\Rightarrow m \left[ \frac{\partial (m_e \vec{v})}{\partial t} + \underbrace{\vec{v} \cdot \vec{\nabla} (m_e \vec{v})}_{O(v^2)} \right] = m n_0 \frac{\partial \vec{v}}{\partial t} + O(\delta n v, v^2)$$
$$\Rightarrow \boxed{m \frac{\partial \vec{v}}{\partial t} = -e \vec{E}} \quad (2)$$

CONSERVAÇÃO DE MASSA:

$$\frac{\partial n_e}{\partial t} + \vec{\nabla} \cdot (n_e \vec{v}) = 0 \Rightarrow \boxed{\frac{\partial \delta n_e}{\partial t} + n_0 \vec{\nabla} \cdot \vec{v} = 0} \quad (3)$$

$$\frac{\partial}{\partial t} (3): \quad \frac{\partial^2 (\delta n_e)}{\partial t^2} = -n_0 \vec{\nabla} \cdot \left[ \frac{\partial \vec{v}}{\partial t} \right] = \frac{n_0 e}{m} \underbrace{\vec{\nabla} \cdot \vec{E}}_{(1) = -4\pi e \delta n_e} = -\frac{4\pi n_0 e^2}{m} \delta n_e$$

(2)  $\Rightarrow -\frac{e}{m} \vec{\nabla} \cdot \vec{E}$

ASSUMINDO UMA DEPENDÊNCIA HARMÔNICA:

$$\delta n_e(\vec{x}, t) = e^{-i\omega t} \delta n_e(\vec{x})$$

$$\Rightarrow -\omega_p^2 \delta n_e = -\frac{4\pi n_0 e^2}{m} \delta n_e$$

$$\Rightarrow \boxed{\omega_p^2 = \frac{4\pi e^2 n_0}{m}}$$

QUE É A FREQUÊNCIA DE PLASMA DE RPA.

# Férmions com interação de curto alcance: o som zero

$$D_{nn}(\mathbf{k}, \omega) = \tilde{\pi}(\mathbf{k}, \omega) = \frac{\pi(\mathbf{k}, \omega)}{1 - V(\mathbf{k})\pi(\mathbf{k}, \omega)}$$

$V(\vec{k}) \xrightarrow{k \rightarrow 0} V_0$  MAS COMO ENCONTRAR  $\pi(\vec{k}, \omega)$ ?

NO LIMITE DILUÍDO ( $k_F a \ll 1$ ) O CÁLCULO DIAGRAMÁTICO NOS DÁ:

$$\pi(\vec{k}, \omega) = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots$$

$$\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots \Rightarrow \text{diagram 4}$$

$$\pi(\vec{k}, \omega) = \text{diagram 1} + \text{diagram 5}$$

SE O POTENCIAL É FRACO, POSSO MANTER APENAS A "BOLHA":

$$\pi(\vec{k}, \omega) \approx \pi^{(0)}(\vec{k}, \omega)$$

NESSE CASO, OS POLOS PARA  $|\vec{k}| \ll k_F$ :

$$\pi^{(0)}(\vec{k}, \omega) = \frac{1}{v_0}$$

MAS O LIMITE DE  $\pi^{(0)}(\vec{k}, \omega) \xrightarrow{k \rightarrow 0} ?$  DEPENDE DE  $\omega$ !

ANTECIPANDO O RESULTADO, O POLO É TAL QUE:

$$\omega \sim c k$$

PRECISAMOS, PORTANTO, DO LIMITE DE  $\pi^{(0)}(\vec{k}, \omega)$

QUANDO  $k \rightarrow 0$ ,  $\omega \rightarrow 0$ , MAS  $\frac{\omega}{k} = \text{FINITO} = v_F$

ONDE  $\underline{S}$  É ADIMENSIONAL:

$$\pi^{(0)R}(\vec{k}, \omega) = \frac{mk_F}{\pi^2} \left[ \frac{s}{2} \ln \left| \frac{1+s}{1-s} \right| - 1 - i \frac{\pi}{2} s \theta(1-|s|) \right]$$

$$\pi^{(0)R}(\vec{k}, \omega) = \frac{1}{v_0} \quad \text{DEFINIO: } \lambda = v_0 \frac{mk_F}{\pi^2} = s_F v_0 \text{ (ADIMENSIONAL)}$$

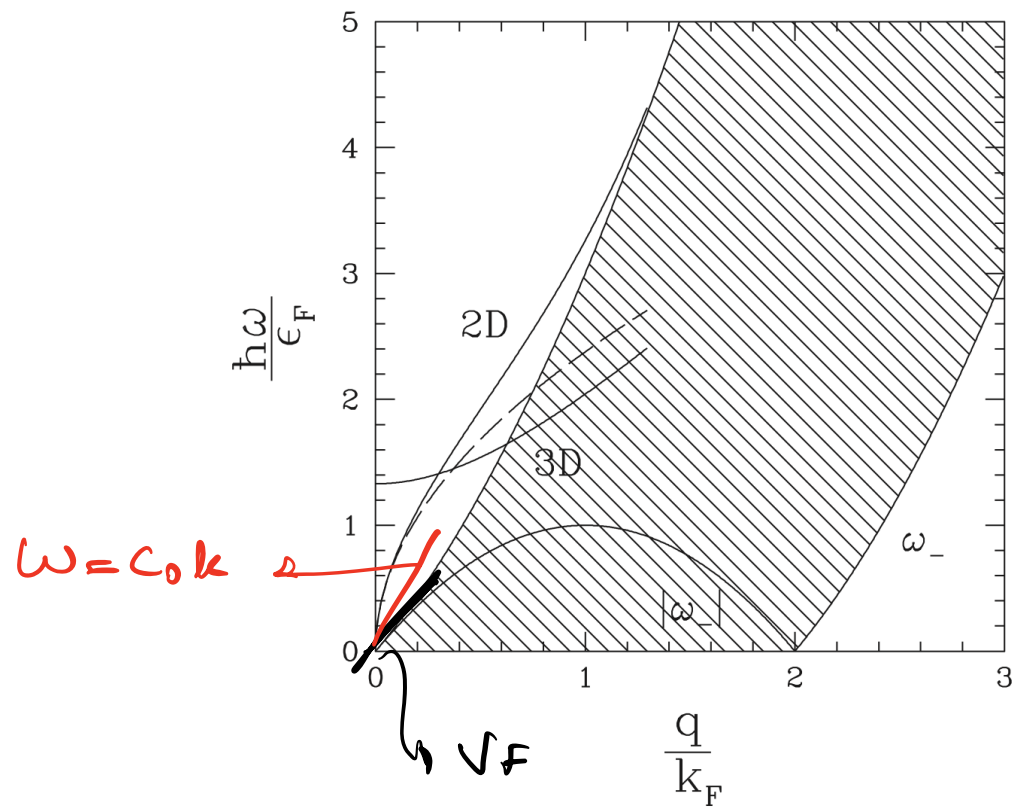
RESOLVENDO PARA  $s$ :  $\omega = c_0 k$  ONDE

$$c_0 = \begin{cases} v_F [1 + 2e^{-\frac{2}{\lambda}-2}] & \lambda \ll 1 \\ v_F \sqrt{\frac{\lambda}{3}} & \lambda \gg 1 \end{cases}$$

EM AMBOS OS CASOS  $c_0 > v_F$ .

ESSA EXCITAÇÃO DE DENSIDADE DO SISTEMA DE FÉRMIONS COM INTERAÇÃO DE CURTO ALCANCE É CARACTERÍSTICA GENEÉRICA DE LÍQUIDOS DE FERMI E É CHAMADA DE "SOM ZERO"





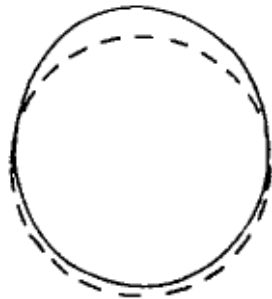
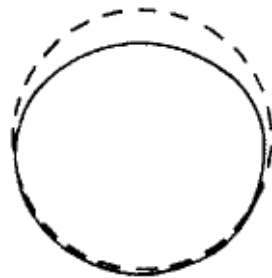
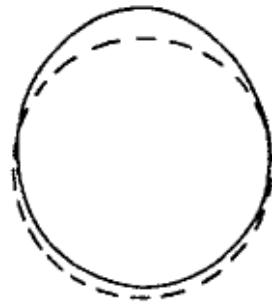
# Comparação entre o som zero e o som hidrodinâmico (primeiro som)

SOM ZERO :

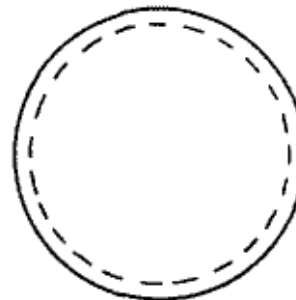
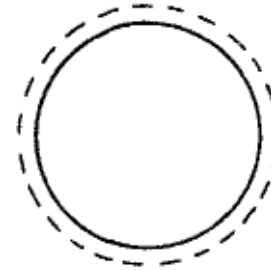
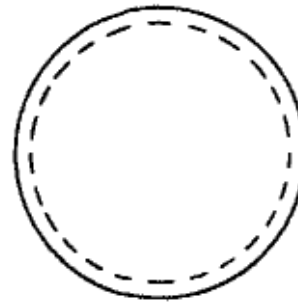
• OCORRE NA AUSÊNCIA DE COLISÕES ENTRE AS PARTÍCULAS

•  $\omega\tau \gg 1$

Zero sound



Thermodynamic sound



$q$

PRIMEIRO SOM

• OCORRE NO REGIME DE MUITAS COLISÕES ENTRE AS PARTÍCULAS

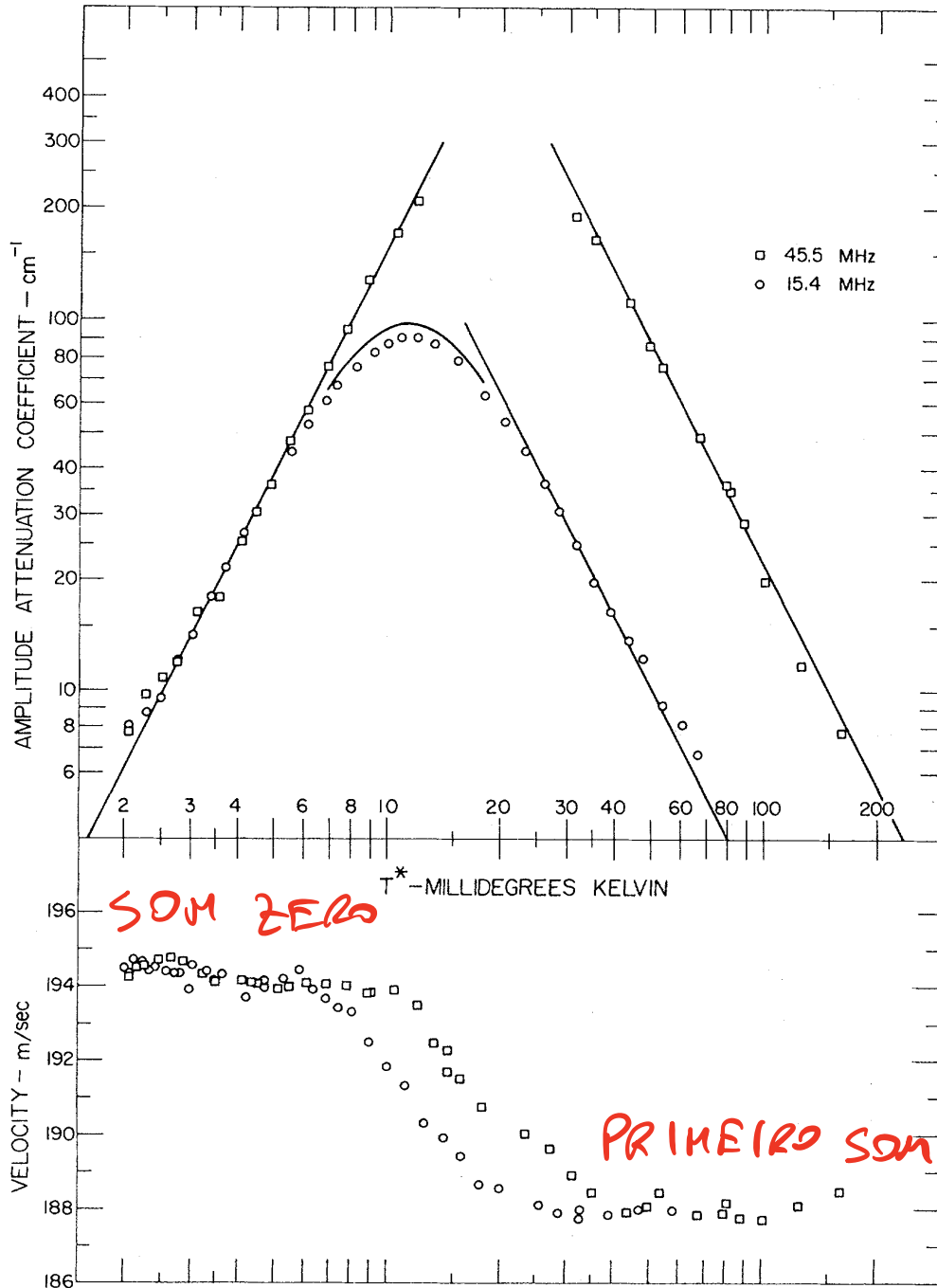
$\omega\tau \ll 1$

$\tau$  = TEMPO DE RELAXAÇÃO  
 $\propto$  TEMPO ENTRE COLISÕES

# Som zero em $^3\text{He}$

Da teoria dos líquidos de Fermi:

This leads to  $[(c_0 - c_1)/c_1]_{0.28 \text{ atm}} = 0.034$ , in remarkable agreement with the measured value at 0.32 atm of  $0.035 \pm 0.003$ .



W. R. Abel, A. C. Anderson, and J. C. Wheatley,  
Phys. Rev. Lett. **17**, 74 (1966).

# Regras de soma

$$D_{nn}^R(\mathbf{k}, \omega) = V \sum_n \left[ \frac{\langle \Psi_0 | \hat{n}(-\mathbf{k}) | n\mathbf{k} \rangle \langle n\mathbf{k} | \hat{n}(\mathbf{k}) | \Psi_0 \rangle}{\omega - (E_n - E_0) + i\eta} - \frac{\langle \Psi_0 | \hat{n}(\mathbf{k}) | n\mathbf{k} \rangle \langle n\mathbf{k} | \hat{n}(-\mathbf{k}) | \Psi_0 \rangle}{\omega + (E_n - E_0) + i\eta} \right]$$

REGRAS DE SOMA SÃO RELAÇÃO EXATAS OBEDECIDAS POR FUNÇÕES DE CORRELAÇÃO. ELAS TÊM IMPORTÂNCIA PARA CHECAR A CONSISTÊNCIA DE CÁLCULOS APROXIMADOS OU NUMÉRICOS.

REGRA DE SOMA f: "f-SUM RULE"

$$\int_{-\infty}^{+\infty} d\omega \omega \operatorname{Im} D_{nn}^R(\vec{q}, \omega) = -\frac{\pi N q^2}{m}$$

$$\operatorname{Im} \left[ \frac{1}{x + i\delta} \right] = -i\pi \delta(x)$$

$$\operatorname{Im} D_{nn}^R(\vec{q}, \omega) = -\pi V \sum_n \left[ \langle \Phi_0 | \hat{n}(-\vec{q}) | \Phi_n \rangle \langle \Phi_n | \hat{n}(\vec{q}) | \Phi_0 \rangle \delta[\omega - (E_n - E_0)] \right. \\ \left. - \langle \Phi_0 | \hat{n}(\vec{q}) | \Phi_n \rangle \langle \Phi_n | \hat{n}(-\vec{q}) | \Phi_0 \rangle \delta[\omega + (E_n - E_0)] \right]$$

$$\begin{aligned} \Rightarrow \int d\omega \omega \text{Im} D^R &= -\pi V \sum_{\vec{n}} \left[ \langle \Phi_0 | \hat{u}(-\vec{q}) | \Phi_n \rangle \langle \Phi_n | \hat{u}(\vec{q}) | \Phi_0 \rangle (E_n - E_0) + \right. \\ &+ \left. \langle \Phi_0 | \hat{u}(\vec{q}) | \Phi_n \rangle \langle \Phi_n | \hat{u}(-\vec{q}) | \Phi_0 \rangle (E_n - E_0) \right] = \\ &= -\pi V \sum_{\vec{n}} \left[ \langle \Phi_0 | \hat{u}(-\vec{q}) | \Phi_n \rangle \langle \Phi_n | [H, \hat{u}(\vec{q})] | \Phi_0 \rangle \right. \\ &\quad \left. - \langle \Phi_0 | [H, \hat{u}(\vec{q})] | \Phi_n \rangle \langle \Phi_n | \hat{u}(-\vec{q}) | \Phi_0 \rangle \right] = \end{aligned}$$

DE  $\sum_{\vec{n}} |\Phi_n\rangle \langle \Phi_n| = 1$

$$\begin{aligned} &= -\pi V \langle \Phi_0 | \left[ \hat{u}(-\vec{q}) [H, \hat{u}(\vec{q})] - [H, \hat{u}(\vec{q})] \hat{u}(-\vec{q}) \right] | \Phi_0 \rangle \\ &= \pi V \langle \Phi_0 | \left[ [H, \hat{u}(\vec{q})], \hat{u}(-\vec{q}) \right] | \Phi_0 \rangle \end{aligned}$$

SE  $\hat{V} = \frac{1}{2} \sum_{\vec{q}} v(\vec{q}) \hat{u}(-\vec{q}) \hat{u}(\vec{q}) \Rightarrow [H, \hat{u}(\vec{q})] = [K, \hat{u}(\vec{q})] =$

$$= \frac{q^2}{2m} \hat{u}(\vec{q}) + \frac{1}{m} \sum_{\vec{p}, \sigma} (\vec{q} \cdot \vec{p}) c_{\vec{p}+\vec{q}, \sigma}^\dagger c_{\vec{p}, \sigma} - c_{\vec{p}, \sigma}^\dagger c_{\vec{p}, \sigma}$$

$$[\hat{H}, \hat{n}(\vec{q})], \hat{n}(-\vec{q})] = -\frac{q^2}{m} N$$

DAQUI, FINALMENTE:

$$\int_{-\infty}^{+\infty} d\omega \omega \operatorname{Im} D_{mn}^R(\vec{q}, \omega) = -\frac{\pi N q^2}{m}$$

REGRA DE  
SOMA "f"

PODE-SE MOSTRAR QUE, SE:

$$D_{mn}^{RPA}(\vec{q}, \omega) = \frac{\pi^{(0)}(\vec{q}, \omega)}{1 - v(\vec{q}) \pi^{(0)}(\vec{q}, \omega)}$$

NO LIMITE  $q \rightarrow 0$ :

$$\int_{-\infty}^{+\infty} d\omega \omega \operatorname{Im} D_{mn}^{RPA}(\vec{q}, \omega) = -\frac{\pi N q^2}{m} !$$

RPA "ESGOTA" A REGRA DE SOMA-f NO LIMITE  $q \rightarrow 0$ .