FI 193 – Teoria Quântica de Sistemas de Muitos Corpos

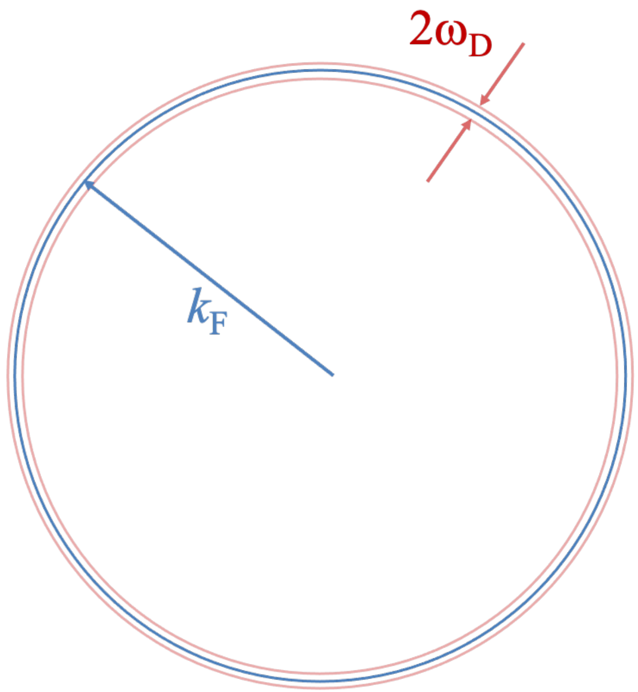
2º Semestre de 2023
28/11/2023
Aula 29

Aula passada

$$H_{BCS} = \sum_{\mathbf{p},\sigma} \epsilon\left(\mathbf{p}\right) a^{\dagger}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \frac{\lambda}{V} \sum_{\mathbf{p},\mathbf{p}'} a^{\dagger}_{\mathbf{p}',\uparrow} a^{\dagger}_{-\mathbf{p}',\downarrow} a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} F\left(\mathbf{p},\mathbf{p}'\right) \equiv H_0 + H_1$$

$$F\left(\mathbf{p},\mathbf{p}'\right) = \Theta\left(\omega_D - \left|\epsilon\left(\mathbf{p}\right)\right|\right) \Theta\left(\omega_D - \left|\epsilon\left(\mathbf{p}'\right)\right|\right)$$

"Casquinha" de "espessura" ω_D em torno da energia de Fermi.



Fenomenologia da supercondutividade

i) ABAIXO DE UMA TEMPERATURA CRÍTICA T_c:

$T < T_c$: $\sigma \to \infty$, $\rho \to 0$ $\Rightarrow$ $\vec{J} = \sigma \vec{E}$

PODEMOS TER: $J \neq 0$ E $\vec{E} = 0$

$$\Rightarrow \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \Rightarrow \quad \frac{\partial \vec{B}}{\partial t} = 0 \Rightarrow \vec{B} = \text{CONST.}$$

SE $\vec{B} = 0$ E RESFRIAMOS A $T < T_c$, E ENTÃO LIGAMOS $\vec{B}$, CONTINUAMOS A TER $\vec{B} = 0$ NO SC.

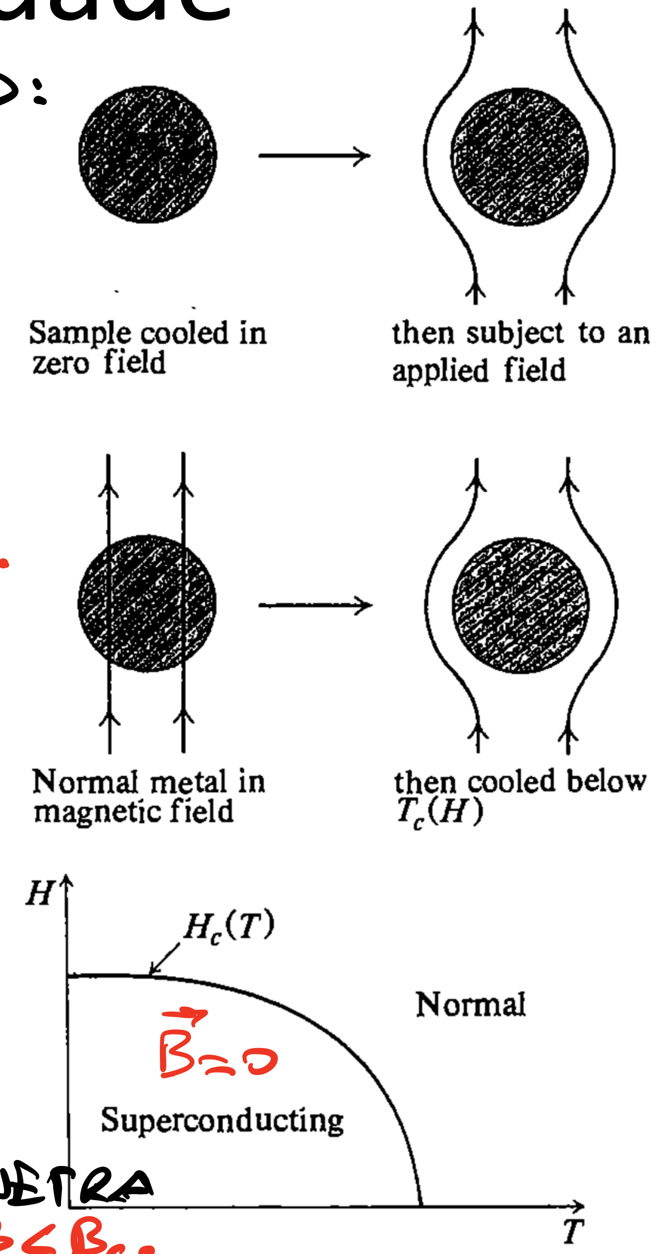
Fenomenologia da supercondutividade

ii) EFEITO MEISSNER-OCHSENFELD:

SE $\vec{B}\neq 0$ A $T>T_c$ E ENTÃO
RESFRIAMOS O MATERIAL,
SE $B<B_{c1}$, O MATERIAL
EXPULSA O CAMPO MAGNÉTICO.

OU SEJA, PARA $B<B_{c1}$,
A CONDIÇÃO $\vec{B}=0$ É UMA
PROPRIEDADE DE EQUILÍBRIO.

ESSA É A FENOMENOLOGIA DOS
SC's DO TIPO I. EXISTEM SCs
DO TIPO II, NOS QUAIS O CAMPO PENETRA
PARCIALMENTE, SE $B_{c1}<B<B_{c2}$



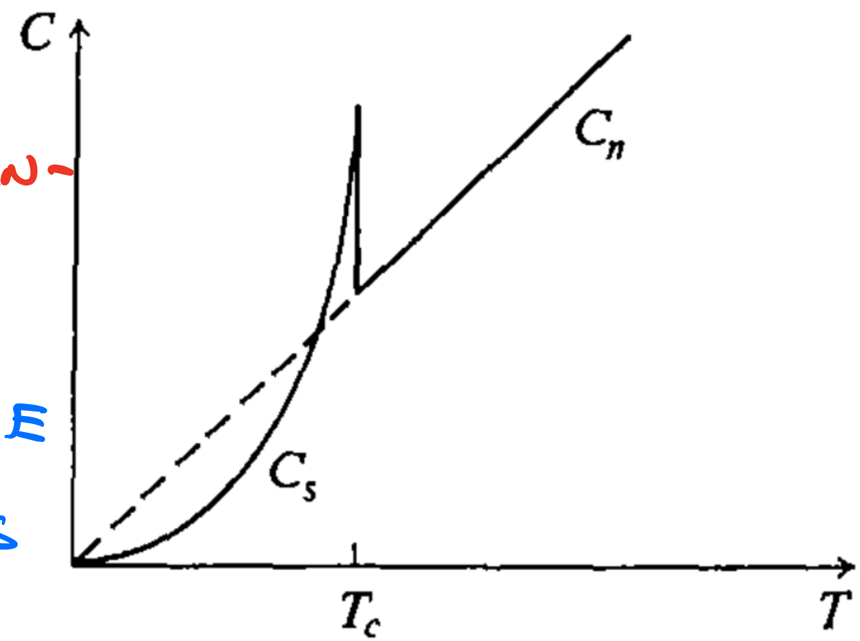
Fenomenologia da supercondutividade

iii) CALOR ESPECÍFICO COMO FUNÇÃO DE T:

a) SALTO EM T_c

b) COMPORTAMENTO EXPONENCIAL $T << T_c$

SUGERE A PRESENÇA DE UM GAP DE EXCITAÇÕES



Fenomenologia da supercondutividade

(iv) EFEITO ISOTÓPICO:

$$T_c \sim \frac{1}{\sqrt{M}} \sim \frac{1}{M^{\beta}} \quad \beta = \frac{1}{2}$$

M: MASSA DOS ÍONS

TABLE II

"Best" Experimental Values for β Obtained in Fitting the Relation $T_c \propto M^{-\beta}$ Compared with the Theoretical Value of BCS (*1*), Swihart (*22,23*), Morel and Anderson (*9*), and Garland (*26*)

Element	β_{exptl}	β_{BCS}	β_{SWI}	β_{MA}	β_{GAR}	$\beta_{\text{GAR(latest)}}$
Zn	$\begin{cases}0.45 \pm 0.01 \\ 0.30\ (27)\end{cases}$	0.5	0.2	0.35	0.40	0.415 ± 0.015
Cd	0.50 ± 0.10	0.5	0.2	0.34	0.37	0.385 ± 0.025
Hg	0.50 ± 0.03	0.5	0.4	0.46	0.465	0.48 ± 0.005
Al		0.5	0.3	0.34	0.35	0.37 ± 0.025
Tl	0.50 ± 0.10	0.5	0.3	0.43	0.45	0.48 ± 0.02
Sn	0.47 ± 0.02	0.5	0.3	0.42	0.44	0.455 ± 0.01
Pb	0.48 ± 0.01	0.5	0.3	0.47	0.47	0.485 ± 0.005
Ti		0.5		0.25	0.2	0.145 ± 0.17
Zr	0.0	0.5		0.30	0.35	0.15 ± 0.17
V		0.5		0.41	0.15	0.25 ± 0.125
Ta		0.5		0.42	0.35	0.35 ± 0.075
Mo	0.33 ± 0.05	0.5	0.15	0.3	0.35	0.35 ± 0.075
Ru	0.0 ± 0.10	0.5	0.0	0.35	0.0	0.065 ± 0.15
Os	0.20 ± 0.05	0.5	0.1	0.25	0.1	0.225 ± 0.10
Ir		0.5		0.3	-0.2	-0.015 ± 0.17
Hf		0.5		0.5	0.3	0.1 ± 0.2
Re	0.39 ± 0.01	0.5		0.41	0.3	0.355 ± 0.05
U(α)	-2.2 ± 0.2	0.5	[(*32a*); see also (*198a*)].			

[a] In some cases the values for β_{SWI} and β_{MA} have been calculated by Garland using their models, respectively. (See Garland for references and (*27*) for Zn and (*32* for Zr, Mo, Re, Ru, and Os.)

Teoria BCS

$$H_{BCS} = \sum_{\mathbf{p},\sigma} \epsilon(\mathbf{p})\, a^\dagger_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \frac{\lambda}{V} \sum_{\mathbf{p},\mathbf{p}'} a^\dagger_{\mathbf{p}',\uparrow} a^\dagger_{-\mathbf{p}',\downarrow} a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} F(\mathbf{p},\mathbf{p}')$$

TENDÊNCIA À FORMAÇÃO DE PARES ENTRE ELÉTRONS COM $(\vec{p},\uparrow)$ E $(-\vec{p},\downarrow)$.

$$a_{-\vec{p}\downarrow} a_{\vec{p},\uparrow} = \underbrace{\langle a_{-\vec{p}\downarrow} a_{\vec{p}\uparrow} \rangle}_{A_{\vec{p}}} + \delta_{\vec{p}} \, ; \qquad a^\dagger_{\vec{p}\uparrow} a^\dagger_{-\vec{p}\downarrow} = A^*_{\vec{p}} + \delta^*_{\vec{p}}$$

NOTEM QUE $\langle a_{-\vec{p}\downarrow} a_{\vec{p}\uparrow} \rangle \neq 0 = \langle N-2 | a_{-\vec{p}\downarrow} a_{\vec{p}\uparrow} | N \rangle$

LEVO NO HAMILTONIANO:

$$-\frac{\lambda}{V}\left[\left(\sum_{\vec{p}} A_{\vec{p}}\right) \sum_{\vec{p}'} a^\dagger_{\vec{p}'\uparrow} a^\dagger_{-\vec{p}'\downarrow} + \left(\sum_{\vec{p}'} A^*_{\vec{p}'}\right) \sum_{\vec{p}} a_{-\vec{p}\downarrow} a_{\vec{p}\uparrow} - \left(\sum_{\vec{p}'} A^*_{\vec{p}'}\right)\left(\sum_{\vec{p}} A_{\vec{p}}\right) \right]$$

DEFINIMOS: $\Delta = -\frac{\lambda}{V} \sum_{\vec{p}} A_{\vec{p}}$

$$H = \sum_{\vec{p}\sigma} \epsilon(\vec{p})\, a^\dagger_{\vec{p}\sigma} a_{\vec{p}\sigma} + \sum_{\vec{p}} \left(\Delta^* a_{-\vec{p}\downarrow} a_{\vec{p}\uparrow} + h.c. \right) + V \frac{\Delta^2}{\lambda}$$

DEFINO: $\Psi_{\vec{p}} = \begin{pmatrix} a_{\vec{p}\uparrow} \\ a^\dagger_{-\vec{p}\downarrow} \end{pmatrix}$ $\qquad \Psi^\dagger_{\vec{p}} = \left(a^\dagger_{\vec{p}\uparrow} \;\; a_{-\vec{p}\downarrow} \right)$

DEFINO MATRIZES NESSE ESPAÇO (NAMBU-GORKOV):

$$\tau_x = \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};\quad \tau_y = \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix};\quad \tau_z = \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\tau_0 = \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$H_0 = \sum_{\vec{p}} \epsilon(\vec{p}) \left[a^\dagger_{\vec{p}\uparrow} a_{\vec{p}\uparrow} + a^\dagger_{\vec{p}\downarrow} a_{\vec{p}\downarrow} \right]$$

$$= \sum_{\vec{p}} \left[\epsilon(\vec{p})\, a^\dagger_{\vec{p}\uparrow} a_{\vec{p}\uparrow} + \epsilon(-\vec{p})\, \underbrace{a^\dagger_{-\vec{p}\downarrow} a_{-\vec{p}\downarrow}}_{-a_{-\vec{p}\downarrow} a^\dagger_{-\vec{p}\downarrow} + 1} \right] \qquad \left[\epsilon(-\vec{p}) = \epsilon(\vec{p}) \right]$$

$$= \sum_{\vec{p}} \left[\epsilon(\vec{p})\, a^\dagger_{\vec{p}\uparrow} a_{\vec{p}\uparrow} - \epsilon(\vec{p})\, a_{-\vec{p}\downarrow} a^\dagger_{-\vec{p}\downarrow} \right] + \sum_{\vec{p}} \epsilon(\vec{p})$$

MAS:
$$\begin{pmatrix} a^{\dagger}_{\vec{p}\uparrow} & a_{-\vec{p}\downarrow} \end{pmatrix} \begin{pmatrix} \epsilon(\vec{p}) & 0 \\ 0 & -\epsilon(\vec{p}) \end{pmatrix} \begin{pmatrix} a_{\vec{p}\uparrow} \\ a^{\dagger}_{-\vec{p}\downarrow} \end{pmatrix} =$$

$$= \epsilon(\vec{p})\, a^{\dagger}_{\vec{p}\uparrow} a_{\vec{p}\uparrow} - \epsilon(\vec{p})\, a_{-\vec{p}\downarrow} a^{\dagger}_{-\vec{p}\downarrow}$$

$$\Rightarrow H_0 = \sum_{\vec{p}} \Psi^{\dagger}_{\vec{p}} \left[\epsilon(\vec{p})\, \tau_3\right] \Psi_{\vec{p}} + \sum_{\vec{p}} \epsilon(\vec{p})$$

VAMOS VER QUE, PARA UM ÚNICO SC, SEMPRE PODEMOS ESCOLHER $\Delta \in \mathbb{R}$

$$H_1 = \sum_{\vec{p}} \Delta \left(a_{-\vec{p}\downarrow} a_{\vec{p}\uparrow} + a^{\dagger}_{\vec{p}\uparrow} a^{\dagger}_{-\vec{p}\downarrow} \right) = \sum_{\vec{p}} \Psi^{\dagger}_{\vec{p}} (\Delta \tau_1) \Psi_{\vec{p}}$$

SE TOMARMOS $\Delta \in \mathbb{C}$: $\Delta = \Delta' + i\Delta''$

$$H_1 = \sum_{\vec{p}} \Psi^{\dagger}_{\vec{p}} (\Delta' \tau_1 - \Delta'' \tau_2) \Psi_{\vec{p}} = \frac{1}{2} \sum_{\vec{p}} \Psi^{\dagger}_{\vec{p}} \left[\Delta \tau^{+} + \Delta^{*} \tau^{-} \right] \Psi_{\vec{p}}$$

$$\tau^{\pm} = \tau_x \pm i \tau_y$$

$$H_{MF} = \sum_{\vec p} \Psi^{+}_{\vec p} \Big[\underbrace{\epsilon(\vec p)\tau_3 + \Delta\tau_1}_{\begin{pmatrix} \epsilon(\vec p) & \Delta \\ \Delta & -\epsilon(\vec p) \end{pmatrix} = H_{\vec p}} \Big] \Psi_{\vec p} + \underbrace{\sum_{\vec p} \epsilon(\vec p) + V\frac{\Delta^2}{\lambda}}_{\text{CONST.}}$$

COMO UM CAMPO MAGNÉTICO: $\begin{pmatrix} B_z & B_x \\ B_x & -B_z \end{pmatrix} = \vec{B}\cdot\vec{\sigma}$

PARA DIAGONALIZAR H_{MF} PRECISAMOS REALIZAR UMA "ROTAÇÃO" NO ESPAÇO DE NAMBU-GORKOV.

Hamiltoniano BCS em notação de Nambu-Gorkov

$$\mathbf{\Psi}_{\mathbf{p}} = \begin{pmatrix} a_{\mathbf{p},\uparrow} \\ a^{\dagger}_{-\mathbf{p},\downarrow} \end{pmatrix}$$

$$H^{MF}_{BCS} = \sum_{\mathbf{p}} \mathbf{\Psi}^{\dagger}_{\mathbf{p}} \mathbf{H} \mathbf{\Psi}_{\mathbf{p}} + \mathrm{const.}$$

$$\mathbf{H} = \begin{pmatrix} \epsilon\left(\mathbf{p}\right) & \Delta \\ \Delta & -\epsilon\left(\mathbf{p}\right) \end{pmatrix}$$

Transformação de Bogoliubov

$$\mathbf{B}_{\mathbf{p}} \equiv \begin{pmatrix} b_{\mathbf{p},\uparrow} \\ b^{\dagger}_{-\mathbf{p},\downarrow} \end{pmatrix} = \mathbf{S} \begin{pmatrix} a_{\mathbf{p},\uparrow} \\ a^{\dagger}_{-\mathbf{p},\downarrow} \end{pmatrix} \equiv \mathbf{S}\mathbf{\Psi}_{\mathbf{p}}$$

$$\mathbf{\Psi}_{\mathbf{p}} = \mathbf{S}^{-1}\mathbf{B}_{\mathbf{p}} = \mathbf{S}^{\dagger}\mathbf{B}_{\mathbf{p}}$$

$$\mathbf{\Psi}^{\dagger}_{\mathbf{p}} = \mathbf{B}^{\dagger}_{\mathbf{p}}\mathbf{S}$$

$$\boxed{\begin{aligned} u^2_{\mathbf{p}} &= \frac{1}{2}\left[1 + \frac{\epsilon(\mathbf{p})}{E(\mathbf{p})}\right] \\ v^2_{\mathbf{p}} &= \frac{1}{2}\left[1 - \frac{\epsilon(\mathbf{p})}{E(\mathbf{p})}\right] \\ u_{\mathbf{p}}, v_{\mathbf{p}} &> 0 \\ u^2_{\mathbf{p}} + v^2_{\mathbf{p}} &= 1 \\ E(\mathbf{p}) &= \sqrt{\epsilon^2(\mathbf{p}) + \Delta^2} \end{aligned}}$$

$a_{p\uparrow}, b_{p\uparrow}$ SÃO FÉRMIONS

$u_p, v_p \in \mathbb{R}$

$$\left\{ b_{\mathbf{p},\sigma}, b^{\dagger}_{\mathbf{p}',\sigma'} \right\} = \delta_{\mathbf{p},\mathbf{p}'}\delta_{\sigma,\sigma'}$$

$$\mathbf{S} = \begin{pmatrix} u_{\mathbf{p}} & v_{\mathbf{p}} \\ -v_{\mathbf{p}} & u_{\mathbf{p}} \end{pmatrix}, \quad \mathbf{S}^{-1} = \begin{pmatrix} u_{\mathbf{p}} & -v_{\mathbf{p}} \\ v_{\mathbf{p}} & u_{\mathbf{p}} \end{pmatrix} = \mathbf{S}^{\dagger}$$

$$S = \begin{pmatrix} \cos\theta_p & \sin\theta_p \\ -\sin\theta_p & \cos\theta_p \end{pmatrix}$$

$$\begin{cases} b_{\vec{p}\uparrow} = u_{\vec{p}}\, c_{\vec{p}\uparrow} + v_{\vec{p}}\, a^{+}_{-\vec{p}\downarrow} \\ b^{+}_{-\vec{p}\downarrow} = -v_{\vec{p}}\, c_{\vec{p}\uparrow} + u_{\vec{p}}\, a^{+}_{-\vec{p}\downarrow} \end{cases}$$

$$a_{-\vec{p}\downarrow} = v_{\vec{p}}\, b^{+}_{\vec{p}\uparrow} + u_{\vec{p}}\, b_{-\vec{p}\downarrow}$$

$$a_{\vec{p}\uparrow} = u_{\vec{p}}\, b_{\vec{p}\uparrow} - v_{\vec{p}}\, b^{+}_{-\vec{p}\downarrow}$$

Transformação de Bogoliubov

$$H_{BCS}^{MF} = \sum_{\mathbf{p}} \mathbf{\Psi}_{\mathbf{p}}^{\dagger} \mathbf{H} \mathbf{\Psi}_{\mathbf{p}} + \text{const.}$$

$$\mathbf{\Psi}_{\mathbf{p}} = \mathbf{S}^{-1}\mathbf{B}_{\mathbf{p}} = \mathbf{S}^{\dagger}\mathbf{B}_{\mathbf{p}}$$

$$\mathbf{\Psi}_{\mathbf{p}}^{\dagger} = \mathbf{B}_{\mathbf{p}}^{\dagger}\mathbf{S}$$

$$\mathbf{H} = \begin{pmatrix} \epsilon(\mathbf{p}) & \Delta \\ \Delta & -\epsilon(\mathbf{p}) \end{pmatrix}$$

$$\mathbf{SHS}^{\dagger} = \begin{pmatrix} E(\mathbf{p}) & 0 \\ 0 & -E(\mathbf{p}) \end{pmatrix} \equiv \mathbf{h}$$

$$\begin{aligned} H_{BCS}^{MF} &= \sum_{\mathbf{p}} \mathbf{B}_{\mathbf{p}}^{\dagger}\mathbf{SHS}^{\dagger}\mathbf{B}_{\mathbf{p}} + \text{const.} \\ &= \sum_{\mathbf{p}} \mathbf{B}_{\mathbf{p}}^{\dagger}\mathbf{h}\mathbf{B}_{\mathbf{p}} + \text{const.} \\ &= \sum_{\mathbf{p}} E(\mathbf{p}) \left[b_{\mathbf{p},\uparrow}^{\dagger} b_{\mathbf{p},\uparrow} - b_{-\mathbf{p},\downarrow} b_{-\mathbf{p},\downarrow}^{\dagger} \right] + \text{const.} \\ &= \sum_{\mathbf{p}} E(\mathbf{p}) \left[b_{\mathbf{p},\uparrow}^{\dagger} b_{\mathbf{p},\uparrow} + b_{-\mathbf{p},\downarrow}^{\dagger} b_{-\mathbf{p},\downarrow} - 1 \right] + \text{const.} \\ &= \sum_{\mathbf{p},\sigma} E(\mathbf{p})\, b_{\mathbf{p},\sigma}^{\dagger} b_{\mathbf{p},\sigma} + \text{const!} \end{aligned}$$

$$\mathbf{B}_{\mathbf{p}} \equiv \begin{pmatrix} b_{\mathbf{p},\uparrow} \\ b_{-\mathbf{p},\downarrow}^{\dagger} \end{pmatrix}$$

$$E(\vec{p}) = \sqrt{\epsilon^2(p) + \Delta^2}$$
$$= \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + \Delta^2}$$
$$= \sqrt{\left(\frac{p^2}{2m} - \frac{p_F^2}{2m}\right)^2 + \Delta^2}$$

$$\text{const.} \rightarrow \frac{V}{\lambda}\Delta^2 + \sum_{\vec{p}} \left(\epsilon(\vec{p}) - E(\vec{p})\right)$$

$$H = \sum_{\vec{p}\sigma} E(\vec{p})\, b^\dagger_{\vec{p}\sigma} b_{\vec{p}\sigma} + \underbrace{\text{CONST!}}_{E_0}$$

ESTADO FUNDAMENTAL : $b^\dagger_{\vec{p}\sigma} b_{\vec{p}\sigma} = n^b_{\vec{p}\sigma} = 0$

EXCITAÇÕES (QUASI-PARTÍCULAS) TÊM DISPERSÃO:

$$E(\vec{p}) = \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + \Delta^2}$$

MENOR VALOR DE $E(\vec{p})$ OCORRE EM $\vec{p} = k_F \hat{p}$

$$\Rightarrow E(p_F) = \Delta > 0$$

$\Rightarrow$ GAP PARA EXCITAÇÕES

MAS: $A_{\vec{p}} = \langle a_{-\vec{p}\downarrow} a_{\vec{p}\uparrow} \rangle$ NO ESTADO DE EQUILÍBRIO A TEMP. T

$A_{\vec{p}} = u_{\vec{p}} v_{\vec{p}} \langle b^\dagger_{\vec{p}\uparrow} b_{\vec{p}\uparrow} + b^\dagger_{-\vec{p}\downarrow} b_{-\vec{p}\downarrow} \rangle - u_{\vec{p}} v_{\vec{p}}$ ONDE JÁ FIZEMOS $\langle b^\dagger_{\vec{p}\uparrow} b^\dagger_{-\vec{p}\downarrow} \rangle = \langle b_{-\vec{p}\downarrow} b_{\vec{p}\uparrow} \rangle = 0$

À TEMPERATURA T:

$$\langle b^\dagger_{\vec{p}\sigma} b_{\vec{p}\sigma} \rangle = f(E_{\vec{p}}) = \frac{1}{e^{\beta E_{\vec{p}}}+1}$$

FINALMENTE:

$$\Delta = -\frac{\lambda}{V}\sum_{\vec{p}} A_{\vec{p}} = -\frac{\lambda}{V}\sum_{\vec{p}} u_{\vec{p}} v_{\vec{p}} \underbrace{\left[\sum_{\sigma} \underbrace{\langle b^\dagger_{\vec{p}\sigma} b_{\vec{p}\sigma} \rangle}_{f(E_{\vec{p}})} - 1\right]}_{2f(E_{\vec{p}})-1 = -\tanh\left(\frac{\beta E_{\vec{p}}}{2}\right)}$$

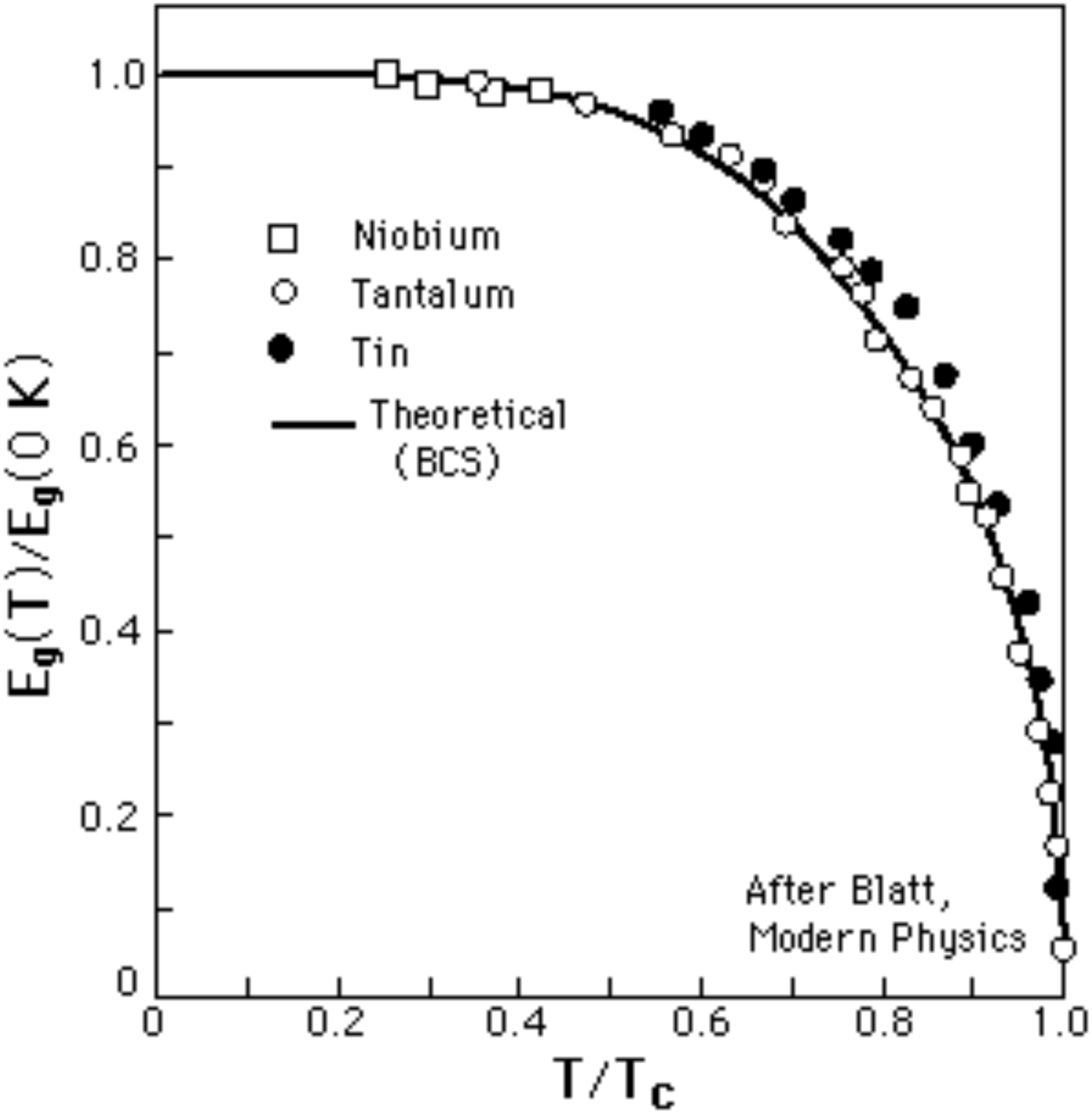
$$2f(x)-1 = \frac{2}{e^{\beta x}+1} - 1 = \frac{1-e^{\beta x}}{1+e^{\beta x}} = \frac{e^{-\frac{\beta x}{2}} - e^{\frac{\beta x}{2}}}{e^{-\frac{\beta x}{2}} + e^{\frac{\beta x}{2}}} = -\tanh\left(\frac{\beta x}{2}\right)$$

$$u_p v_p = \frac{\Delta}{2E(\vec{p})}$$

EQUAÇÃO DO GAP $\Delta(T)$

$$\cancel{\Delta} = +\frac{\lambda \cancel{\Delta}}{2V}\sum_{\vec{p}} \frac{\tanh\left(\frac{\beta E(\vec{p})}{2}\right)}{E(\vec{p})} \Rightarrow \boxed{\frac{2}{\lambda} = \int \frac{d^3p}{(2\pi)^3} \frac{\tanh\left(\frac{\beta E(\vec{p})}{2}\right)}{E(\vec{p})} \theta(\omega_D - |\epsilon_p|)}$$

Gap como função da temperatura



A EQUAÇÃO PODE SER RESOLVIDA ANALITICAMENTE NOS DOIS LIMITES: $T=0$ E $T \simeq T_c$

$T=0$: $\Delta(0) \cong 2\omega_D \, e^{-1/\rho_0 \lambda}$ $\rho_0 = \rho(E_F)$

↓

NÃO ANALÍTICA NA INTERAÇÃO λ $\omega_D \sim \frac{1}{\sqrt{M}}$

$T \simeq T_c$: $\Delta^2(T) \simeq \frac{8\pi^2 T_c^2}{7\zeta(3)} \left(1 - \frac{T}{T_c}\right)$

$\Delta(T) \sim \sqrt{T_c - T}$

A temperatura crítica

PRÓXIMO DE $T = T_c$: $\Delta \sim 0$

$$\frac{2}{\lambda} = \int \frac{d^3p}{(2\pi)^3} \frac{\tanh\left(\frac{|\epsilon(\bar{p})|}{2T_c}\right)}{|\epsilon(\bar{p})|} \theta(\omega_D - |\epsilon(\bar{p})|)$$

EQUAÇÃO PARA T_c:

$$T_c = \frac{2e^{\gamma}}{\pi} \omega_D \, e^{-\frac{1}{\rho_0 \lambda}} = \frac{e^{\gamma}}{\pi} \Delta(0) = 0{,}57\, \Delta(0)$$

γ = EULER-MASCHERONI

Previsão de BCS

$$\frac{2\Delta(0)}{k_B T_c} = 3.53$$

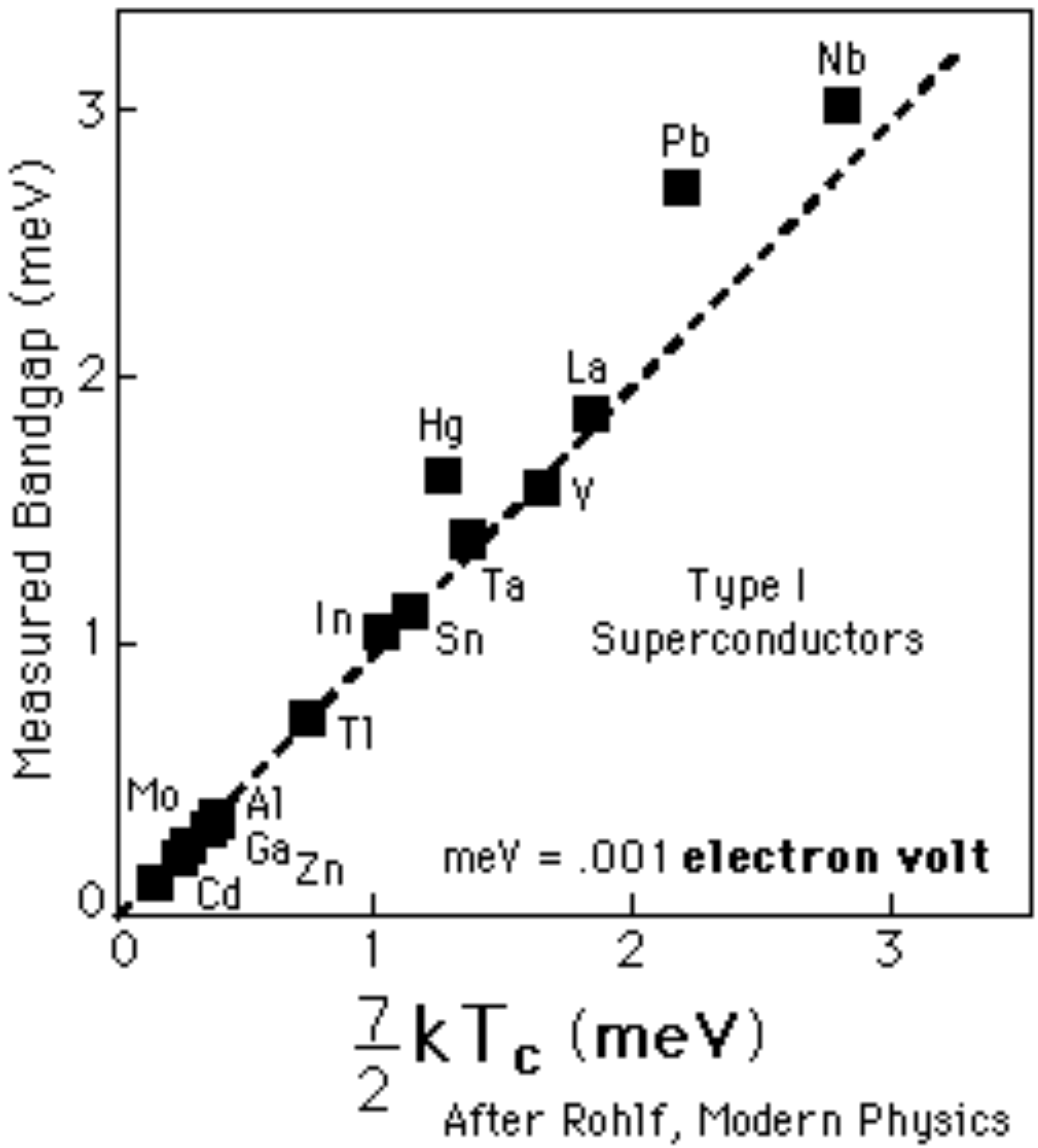


TABLE III

Measured Values of $2\Delta(0)/kT_c$
(BCS) theoretical value = 3.53)

Superconductor	Tunneling measurements	Ref.	Thermodynamic measurements[a]
Al	4.2 ±0.6	*(49)*	3.53
	2.5 ±0.3	*(50)*	
	2.8 − 3.6	*(68)*	
	3.37 ±0.1	*(65)*	
Cd	3.2 ±0.1	*(65a)*	3.44
Ga			3.52, 3.50, 3.48
Hg(α)	4.6 ±0.1	*(84)*	3.95
In	3.63 ±0.1	*(49)*	3.65
	3.45 ±0.07	*(65)*	
	3.61	*(94)*	
La	1.65 − 3.0 (fcc)[b]	*(100)*	3.72 (fcc) (d-hcp)
	3.2	*(100)*	
Nb	3.84 ±0.06	*(85)*	3.65
	3.6	*(95)*	
	3.6	*(96)*	
Pb[c]	4.29 ±0.04	*(69)*	3.95
	4.38 ±0.01[d]	*(74)*	
Sn	3.46 ±0.1	*(49)*	3.61, 3.57
	3.10 ±0.05	*(50)*	
	3.51 ±0.18	*(85)*	
	2.8 − 4.06	*(65)*	
	3.1 − 4.3	*(87)*	
Ta	3.60 ±0.1	*(85)*	3.63
	3.5	*(95)*	
	3.65 ±0.1	*(97)*	
Tl	3.57 ±0.05	*(98)*	3.63
	3.9	*(94)*	
V	3.4	*(95)*	3.50
Zn	3.2 ±0.1	*(99)*	3.44

[a] The values given here were calculated from values of $\gamma T_c^2/V_M H_c^2(0)$ assuming the equation $[2\pi V_M H_c^2(0)/3\gamma T_c^2]^{1/2} = 2\Delta(0)/kT_c$.

[b] The measured tunneling results of Edelstein and Toxen *(100)* in La are very low and widely scattered and perhaps reflect the great structure sensitivity of La. Hauser's *(100a)* later measurements are higher and less scattered.

[c] Other older measurements are collected in *(51)*.

[d] T_c assumed to be 7.193 °K.

Efeito isotópico

TABLE II

"Best" Experimental Values for β Obtained in Fitting the Relation $T_c \propto M^{-\beta}$ Compared with the Theoretical Value of BCS (*1*), Swihart (*22,23*), Morel and Anderson (*9*), and Garland (*26*)

Element	β_{exptl}	β_{BCS}	β_{SWI}	β_{MA}	β_{GAR}	$\beta_{\mathrm{GAR(latest)}}$
Zn	$\left\{\begin{matrix} 0.45 \pm 0.01 \\ 0.30\ (27) \end{matrix}\right\}$	0.5	0.2	0.35	0.40	0.415 ± 0.015
Cd	0.50 ± 0.10	0.5	0.2	0.34	0.37	0.385 ± 0.025
Hg	0.50 ± 0.03	0.5	0.4	0.46	0.465	0.48 ± 0.005
Al		0.5	0.3	0.34	0.35	0.37 ± 0.025
Tl	0.50 ± 0.10	0.5	0.3	0.43	0.45	0.48 ± 0.02
Sn	0.47 ± 0.02	0.5	0.3	0.42	0.44	0.455 ± 0.01
Pb	0.48 ± 0.01	0.5	0.3	0.47	0.47	0.485 ± 0.005
Ti		0.5		0.25	0.2	0.145 ± 0.17
Zr	0.0	0.5		0.30	0.35	0.15 ± 0.17
V		0.5		0.41	0.15	0.25 ± 0.125
Ta		0.5		0.42	0.35	0.35 ± 0.075
Mo	0.33 ± 0.05	0.5	0.15	0.3	0.35	0.35 ± 0.075
Ru	0.0 ± 0.10	0.5	0.0	0.35	0.0	0.065 ± 0.15
Os	0.20 ± 0.05	0.5	0.1	0.25	0.1	0.225 ± 0.10
Ir		0.5		0.3	-0.2	-0.015 ± 0.17
Hf		0.5		0.5	0.3	0.1 ± 0.2
Re	0.39 ± 0.01	0.5		0.41	0.3	0.355 ± 0.05
U(α)	-2.2 ± 0.2	0.5	[(*32a*); see also (*198a*)].			

[a] In some cases the values for β_{SWI} and β_{MA} have been calculated by Garland using their models, respectively. (See Garland for references and (*27*) for Zn and (*32* for Zr, Mo, Re, Ru, and Os.)

CÁLCULO DO CALOR ESPECÍFICO:

$$\Delta C_V(T_c) = \frac{8\pi^2}{7\zeta(3)} \rho_0 T_c$$

$$\frac{\Delta C_V(T_c)}{C_V(T_c^+)} = \frac{12}{7\zeta(3)} = 1.43$$

Calor específico x T

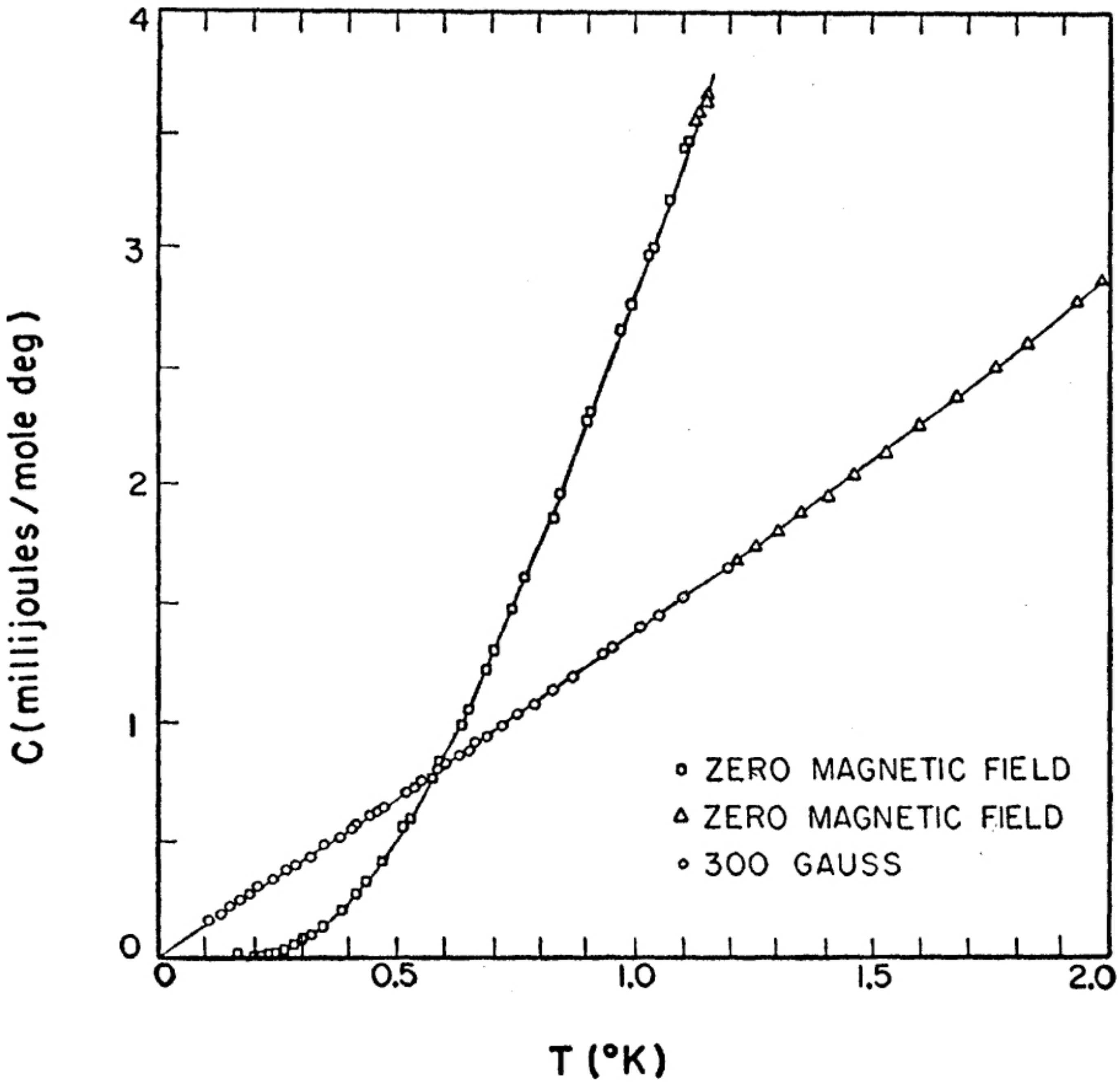


TABLE IV

Measured Values of Two Thermodynamic Functions

	$\left(\frac{C_s - C_n}{C_n}\right)_{T_c}$	Ref.	$\frac{\gamma T_c^2}{V_M H_c^2(0)}$ [a]	Ref.
BCS	1.43[b]	(*1*[b],*4*)	0.168	
		(*166*		
Al	1.29[c]–1.59[c]	(*167*,*168*)	0.171	(*167*)
		(*148*,*170*)		
		(*171*)		
	1.45[c] (av.)		0.170	(*170*[d])
Cd	1.32	(*172*)	0.177	(*172*)
	1.40	(*173*)		
Ga	1.44, 1.41	(*172*,*174*)	0.169, 0.170	(*172*,*174*)
	1.40	(*175*)[e]	0.173	(*175*)
Hg	2.37	(*169*,*176*)	0.134	(*165*)
		(*165*)		
In	1.73	(*156*)	0.157,0.150	(*152*,*165*)
La (hcp)	1.5	(*177*)		
Mo	1.28	(*186*)	0.182	(*148*)
Nb	1.87 (calorimetric)	(*178*)	0.157	(*179*)
	2.0 (magnetic)	(*179*)		
Pb	2.71	(*180*,*176*)	0.134	(*187*[d])
Sn	1.60	(*147*,*181*)	0.161	(*165*)
			0.164	(*152*)
Ta	1.59	(*147*,*181*,*182*)	0.161	(*181*,*182*)
Tl	1.50	(*183*)	0.161	(*181*)
U(β)	1.36, 1.52	(*184*)		
V	1.49	(*162*)	0.170	(*162*)
Zn	1.30	(*174*)	0.177	(*174*)
	1.24	(*185*)		