FI 193 – Teoria Quântica de Sistemas de Muitos Corpos

1º Semestre de 2025

01/07/2025

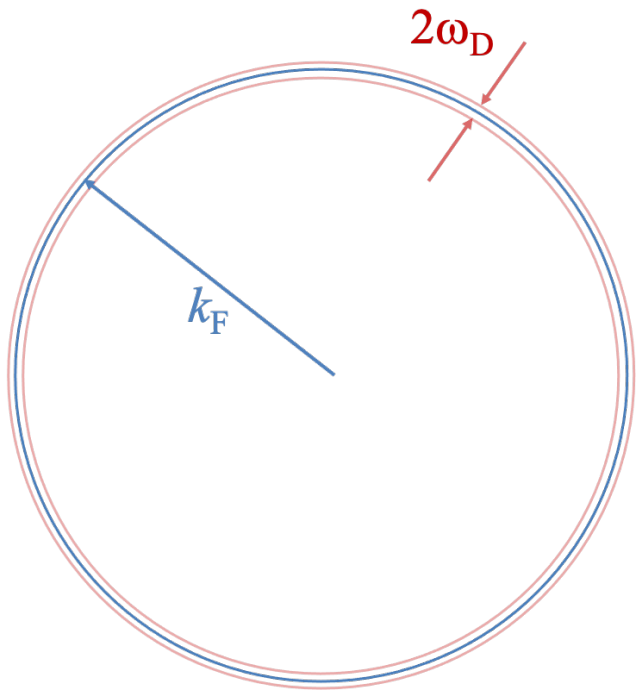
Aula 29

Aula passada

$$H_{BCS} = \sum_{\mathbf{p},\sigma} \epsilon\left(\mathbf{p}\right) a^{\dagger}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \frac{\lambda}{V} \sum_{\mathbf{p},\mathbf{p}'} a^{\dagger}_{\mathbf{p}',\uparrow} a^{\dagger}_{-\mathbf{p}',\downarrow} a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} F\left(\mathbf{p},\mathbf{p}'\right) \equiv H_0 + H_1$$

$$F\left(\mathbf{p},\mathbf{p}'\right) = \Theta\left(\omega_D - \left|\epsilon\left(\mathbf{p}\right)\right|\right) \Theta\left(\omega_D - \left|\epsilon\left(\mathbf{p}'\right)\right|\right)$$

"Casquinha" de "espessura" ω_D em torno da energia de Fermi.



Aula passada

$$H_{BCS} = \sum_{\mathbf{p},\sigma} \epsilon\left(\mathbf{p}\right) a^{\dagger}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \frac{\lambda}{V} \sum_{\mathbf{p},\mathbf{p}'} a^{\dagger}_{\mathbf{p}',\uparrow} a^{\dagger}_{-\mathbf{p}',\downarrow} a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} F\left(\mathbf{p},\mathbf{p}'\right) \equiv H_0 + H_1$$

$$F\left(\mathbf{p},\mathbf{p}'\right) = \Theta\left(\omega_D - \left|\epsilon\left(\mathbf{p}\right)\right|\right) \Theta\left(\omega_D - \left|\epsilon\left(\mathbf{p}'\right)\right|\right)$$

"Casquinha" de "espessura" ω_D em torno da energia de Fermi.

Teoria de campo médio:

$$a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} \rightarrow \left\langle a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} \right\rangle + \text{flut.} \equiv A_{\mathbf{p}} + \delta_{\mathbf{p}}$$

$$a^{\dagger}_{\mathbf{p}',\uparrow} a^{\dagger}_{-\mathbf{p}',\downarrow} \rightarrow \left\langle a^{\dagger}_{\mathbf{p}',\uparrow} a^{\dagger}_{-\mathbf{p}',\downarrow} \right\rangle + \text{flut.} \equiv A^{*}_{\mathbf{p}'} + \delta^{*}_{\mathbf{p}'}$$

Desprezam-se termos quadráticos em $\delta_{\mathbf{p}}$.

Aula passada

$$H_{BCS}^{MF} = \sum_{\mathbf{p},\sigma} \epsilon(\mathbf{p})\, a_{\mathbf{p},\sigma}^{\dagger} a_{\mathbf{p},\sigma} + \sum_{\mathbf{p}} \left(\Delta^* a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} + \Delta a_{\mathbf{p},\uparrow}^{\dagger} a_{-\mathbf{p},\downarrow}^{\dagger} \right) + \frac{V\,|\Delta|^2}{\lambda}$$

$$\Delta \equiv -\frac{\lambda}{V} \sum_{\mathbf{p}} A_{\mathbf{p}} \qquad \Rightarrow \Delta \in \mathbb{R}$$

Nambu-Gorkov:

$$\mathbf{\Psi}_{\mathbf{p}} = \begin{pmatrix} a_{\mathbf{p},\uparrow} \\ a_{-\mathbf{p},\downarrow}^{\dagger} \end{pmatrix}$$

$$H_{BCS}^{MF} = \sum_{\mathbf{p}} \mathbf{\Psi}_{\mathbf{p}}^{\dagger} \mathbf{H} \mathbf{\Psi}_{\mathbf{p}} + \text{const.}$$

$$\mathbf{H} = \begin{pmatrix} \epsilon(\mathbf{p}) & \Delta \\ \Delta & -\epsilon(\mathbf{p}) \end{pmatrix} = \epsilon(\mathbf{p})\tau_3 + \Delta\tau_1$$

Transformação de Bogoliubov

$$\mathbf{B}_{\mathbf{p}} \equiv \begin{pmatrix} b_{\mathbf{p},\uparrow} \\ b^{\dagger}_{-\mathbf{p},\downarrow} \end{pmatrix} = \mathbf{S} \begin{pmatrix} a_{\mathbf{p},\uparrow} \\ a^{\dagger}_{-\mathbf{p},\downarrow} \end{pmatrix} \equiv \mathbf{S}\mathbf{\Psi}_{\mathbf{p}}$$

$$\mathbf{\Psi}_{\mathbf{p}} = \mathbf{S}^{-1}\mathbf{B}_{\mathbf{p}} = \mathbf{S}^{\dagger}\mathbf{B}_{\mathbf{p}}$$

$$\mathbf{\Psi}^{\dagger}_{\mathbf{p}} = \mathbf{B}^{\dagger}_{\mathbf{p}}\mathbf{S}$$

$$b_{\vec{p}\uparrow} = u_p a_{\vec{p}\uparrow} + v_p a^{\dagger}_{-\vec{p}\downarrow}$$

$$b^{\dagger}_{-\vec{p}\downarrow} = -v_p a_{\vec{p}\uparrow} + u_p a^{\dagger}_{-\vec{p}\downarrow}$$

$$\left\{ b_{\mathbf{p},\sigma}, b^{\dagger}_{\mathbf{p}',\sigma'} \right\} = \delta_{\mathbf{p},\mathbf{p}'}\delta_{\sigma,\sigma'}$$

$$\boxed{\begin{aligned} u^2_{\mathbf{p}} &= \frac{1}{2}\left[1 + \frac{\epsilon(\mathbf{p})}{E(\mathbf{p})}\right] \\ v^2_{\mathbf{p}} &= \frac{1}{2}\left[1 - \frac{\epsilon(\mathbf{p})}{E(\mathbf{p})}\right] \\ u_{\mathbf{p}}, v_{\mathbf{p}} &> 0 \\ u^2_{\mathbf{p}} + v^2_{\mathbf{p}} &= 1 \\ E(\mathbf{p}) &= \sqrt{\epsilon^2(\mathbf{p}) + \Delta^2} \end{aligned}}$$

$$\mathbf{S} = \begin{pmatrix} u_{\mathbf{p}} & v_{\mathbf{p}} \\ -v_{\mathbf{p}} & u_{\mathbf{p}} \end{pmatrix}, \quad \mathbf{S}^{-1} = \begin{pmatrix} u_{\mathbf{p}} & -v_{\mathbf{p}} \\ v_{\mathbf{p}} & u_{\mathbf{p}} \end{pmatrix} = \mathbf{S}^{\dagger}$$

Transformação de Bogoliubov

$$H_{BCS}^{MF} = \sum_{\mathbf{p}} \mathbf{\Psi}_{\mathbf{p}}^{\dagger} \mathbf{H} \mathbf{\Psi}_{\mathbf{p}} + \text{const.}$$

$$\mathbf{\Psi}_{\mathbf{p}} = \mathbf{S}^{-1}\mathbf{B}_{\mathbf{p}} = \mathbf{S}^{\dagger}\mathbf{B}_{\mathbf{p}}$$

$$\mathbf{H} = \begin{pmatrix} \epsilon(\mathbf{p}) & \Delta \\ \Delta & -\epsilon(\mathbf{p}) \end{pmatrix}$$

$$\mathbf{SHS}^{\dagger} = \begin{pmatrix} E(\mathbf{p}) & 0 \\ 0 & -E(\mathbf{p}) \end{pmatrix} \equiv \mathbf{h}$$

$$\begin{aligned}
H_{BCS}^{MF} &= \sum_{\mathbf{p}} \mathbf{B}_{\mathbf{p}}^{\dagger} \mathbf{SHS}^{\dagger} \mathbf{B}_{\mathbf{p}} + \text{const.} \\
&= \sum_{\mathbf{p}} \mathbf{B}_{\mathbf{p}}^{\dagger} \mathbf{h} \mathbf{B}_{\mathbf{p}} + \text{const.} \\
&= \sum_{\mathbf{p}} E(\mathbf{p}) \left[b_{\mathbf{p},\uparrow}^{\dagger} b_{\mathbf{p},\uparrow} - b_{-\mathbf{p},\downarrow} b_{-\mathbf{p},\downarrow}^{\dagger} \right] + \text{const.} \\
&= \sum_{\mathbf{p}} E(\mathbf{p}) \left[b_{\mathbf{p},\uparrow}^{\dagger} b_{\mathbf{p},\uparrow} + b_{-\mathbf{p},\downarrow}^{\dagger} b_{-\mathbf{p},\downarrow} - 1 \right] + \text{const.} \\
&= \sum_{\mathbf{p},\sigma} E(\mathbf{p}) b_{\mathbf{p},\sigma}^{\dagger} b_{\mathbf{p},\sigma} \underbrace{+ \text{const.}}_{E_0}
\end{aligned}$$

$$E_0 = \sum_{\vec{k}} \left[\epsilon(\vec{k}) - E(\vec{k}) \right] + \frac{V\Delta^2}{\lambda}$$

O estado fundamental BCS e as excitações de quasi-partículas

EST. FUND.: $b_{\vec{p}\sigma}|\Phi_0\rangle = 0$

$$\Rightarrow \quad u_p a_{\vec{p}\uparrow} + v_p a^{\dagger}_{-\vec{p}\downarrow}|\Phi_0\rangle = 0$$

$$u_p a_{\vec{p}\uparrow}|\Phi_0\rangle = -v_p a^{\dagger}_{-\vec{p}\downarrow}|\Phi_0\rangle$$

EN. EST. FUND. $\Rightarrow E_0$

EM $T \neq 0$:

$$\langle b^{\dagger}_{\vec{k}\sigma} b_{\vec{k}\sigma}\rangle = \frac{1}{e^{\beta E(\vec{k})}+1} = f(E(\vec{k}))$$

$$E(\vec{k}) = \sqrt{[\epsilon(\vec{k})-\mu]^2 + \Delta(T)}$$

$$\min(E(\vec{k})) = \Delta(T) = \text{GAP}$$

EXCITAÇÕES CRIADAS POR $b^{\dagger}_{\vec{k}\sigma}$ SÃO AS QUASI-PARTÍCULAS

Auto-consistência e a equação do gap

$$\Delta \equiv -\frac{\lambda}{V}\sum_{\mathbf{p}} A_{\mathbf{p}}$$

ONDE $A_{\vec{p}} = \langle a_{-\vec{p}\downarrow} a_{\vec{p}\uparrow} \rangle$

LEVANDO: $\Phi_{\vec{k}} = S^{-1} B_{\vec{k}}$ ACIMA

$$A_{\vec{k}} = u_k v_{\vec{k}} \langle b^{\dagger}_{\vec{k}\uparrow} b_{\vec{k}\uparrow} + b^{\dagger}_{-\vec{k}\downarrow} b_{-\vec{k}\downarrow} \rangle = u_k v_{\vec{k}}$$

JÁ QUE $\langle b^{\dagger}_{\vec{k}\uparrow} b^{\dagger}_{-\vec{k}\downarrow} \rangle = 0 = \langle b_{-\vec{k}\downarrow} b_{\vec{k}\uparrow} \rangle$

$$\Rightarrow \Delta = -\frac{\lambda}{V}\sum_{\vec{k}} u_k v_k \underbrace{\left[\sum_{\sigma} \langle b^{\dagger}_{\vec{k}\sigma} b_{\vec{k}\sigma} \rangle - 1\right]}_{2f(E_{\vec{k}})-1} = \frac{\lambda}{V}\sum_{\vec{k}} u_k v_k \tanh\left(\frac{\beta E(\vec{k})}{2}\right)$$

$$2f(x) - 1 = \frac{2}{e^{\beta x}+1} - 1 = \frac{-e^{\beta x}+1}{e^{\beta x}+1} = \frac{e^{-\frac{\beta x}{2}} - e^{\frac{\beta x}{2}}}{e^{\frac{\beta x}{2}} + e^{-\frac{\beta x}{2}}} = -\tanh\left(\frac{\beta x}{2}\right)$$

$$u_k v_k = \frac{\Delta}{2E(\vec{k})}$$

$$\Rightarrow \cancel{\Delta} = \frac{\lambda}{V} \sum_{\vec{k}} \frac{\cancel{\Delta}}{2E(\vec{k})} \tanh\left(\frac{\beta E(\vec{k})}{2}\right) \qquad \text{SE } \Delta \neq 0$$

$$\sum_{\vec{k}} = V \int \frac{d^3k}{(2\pi)^3} \qquad E(\vec{k}) = \sqrt{\epsilon_{\vec{k}}^2 + \Delta^2}$$

$$\Rightarrow \frac{2}{\lambda} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{E(\vec{k})} \tanh\left(\frac{E(\vec{k})}{2T}\right) \theta(\omega_D - |\epsilon_{\vec{k}}|)$$

$$\int \frac{d^3k}{(2\pi)^3} B(\epsilon(\vec{k})) = \int d\epsilon \int \frac{d^3k}{(2\pi)^3} B(\epsilon)\, \delta(\epsilon - \epsilon(\vec{k})) = (*)$$

MAS: $\displaystyle\int \frac{d^3k}{(2\pi)^3} \delta(\epsilon - \epsilon(\vec{k})) \equiv \rho_0(\epsilon) =$ DENS. DE ESTADOS POR SPIN

$$(*) = \int d\epsilon\, \rho_0(\epsilon)\, B(\epsilon)$$

$$\Rightarrow \frac{1}{\lambda} = \int_0^{\omega_D} d\epsilon \, \frac{\rho_0(\epsilon)}{\sqrt{\epsilon^2+\Delta^2}} \tanh\left[\frac{\sqrt{\epsilon^2+\Delta^2}}{2T}\right]$$

EQUAÇÃO DO GAP $\Rightarrow \Delta(T)$

DOIS LIMITES IMPORTANTES:

(i) $T=0$

$$\Rightarrow \tanh\left(\frac{x}{T}\right) \xrightarrow[T\to 0]{} 2\theta(x)-1$$

$$\Rightarrow \frac{1}{\lambda} = \int_0^{\omega_D} d\epsilon \, \frac{\rho_0(\epsilon)}{\sqrt{\epsilon^2+\Delta_0^2}}$$

$$\frac{1}{\lambda} \cong \rho_0 \int_0^{\omega_D} \frac{d\epsilon}{\sqrt{\epsilon^2+\Delta_0^2}} = \rho_0 \sinh^{-1}\left(\frac{\omega_D}{\Delta_0}\right)$$

$$\Rightarrow \Delta_0 = \frac{\omega_D}{\sinh\left(\frac{1}{\rho_0\lambda}\right)} \xrightarrow[\rho_0\lambda \ll 1]{} \boxed{\omega_D e^{-\frac{1}{\rho_0\lambda}} = \Delta_0}$$

ESCALAS DE ENERGIA

D: LARGURA DE BANDA

$\omega_D \ll D$

$\Delta_0 \ll \omega_D$

$T_c \ll \omega_D$

(ii) $T \to T_c^-$ $\quad \Delta(T \to T_c^-) \to 0$

$$\frac{1}{\rho_0 \lambda} = \int_0^{\omega_D} \frac{d\epsilon}{\epsilon} \tanh\left(\frac{\epsilon}{2T_c}\right) = I$$ EQUAÇÃO P/ T_c

$$x = \frac{\epsilon}{T_c} \Rightarrow I = \int_0^{\omega_D/T_c} \frac{dx}{x} \tanh\left(\frac{x}{2}\right)$$

$$\int_0^{\Lambda} \frac{dx}{x} \tanh\left(\frac{x}{2}\right) = \ln(x) \tanh\left(\frac{x}{2}\right)\Big|_0^{\Lambda} - \underbrace{\frac{1}{2}\int_0^{\Lambda \to \infty} \frac{dx \ln x}{\cosh^2\left(\frac{x}{2}\right)}}_{\ln\left(\frac{\pi}{2}\right) - \gamma}$$

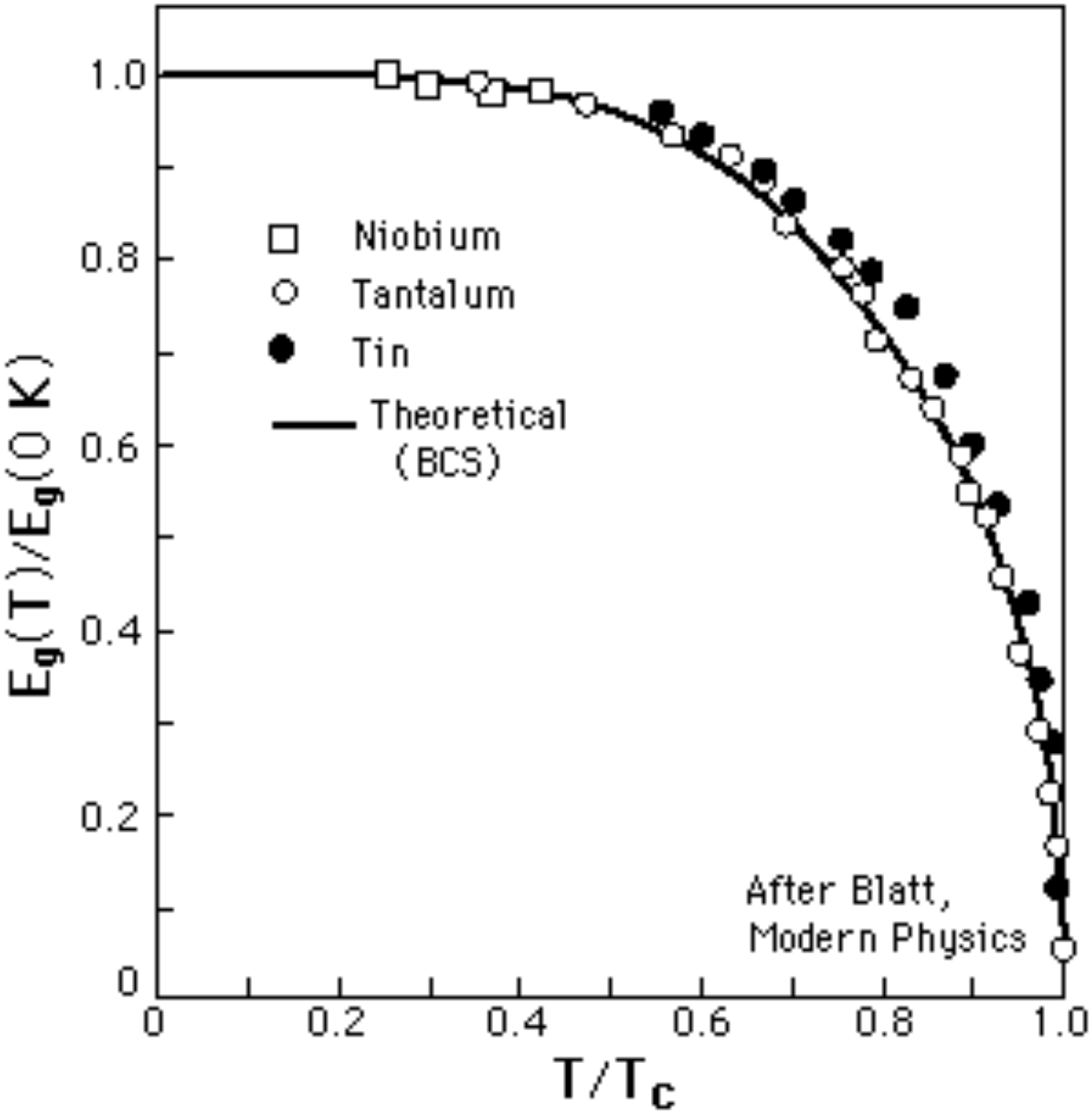
$$= \ln \Lambda - \ln\left(\frac{\pi}{2}\right) + \gamma$$

ONDE : $\gamma \cong 0.577$ = CONST. EULER-MASCHERONI

$$\Rightarrow \frac{1}{\rho_0 \lambda} = \ln\left(\frac{\omega_D}{T_c}\right) + \ln\left(\frac{2e^{\gamma}}{\pi}\right) = \ln\left[\frac{2e^{\gamma}\omega_D}{\pi T_c}\right]$$

$$\Rightarrow T_c = \left(\frac{2e^{\gamma}}{\pi}\right) \underbrace{\omega_D e^{-\frac{1}{\rho_0 \lambda}}}_{\Delta(0)} \cong 0.57\, \Delta(0) \Rightarrow \boxed{\frac{2\Delta(0)}{T_c} = \frac{2}{0.57} \cong 3.53}$$

Gap como função da temperatura



Previsão de BCS

$$\frac{2\Delta(0)}{k_B T_c} = 3.53$$

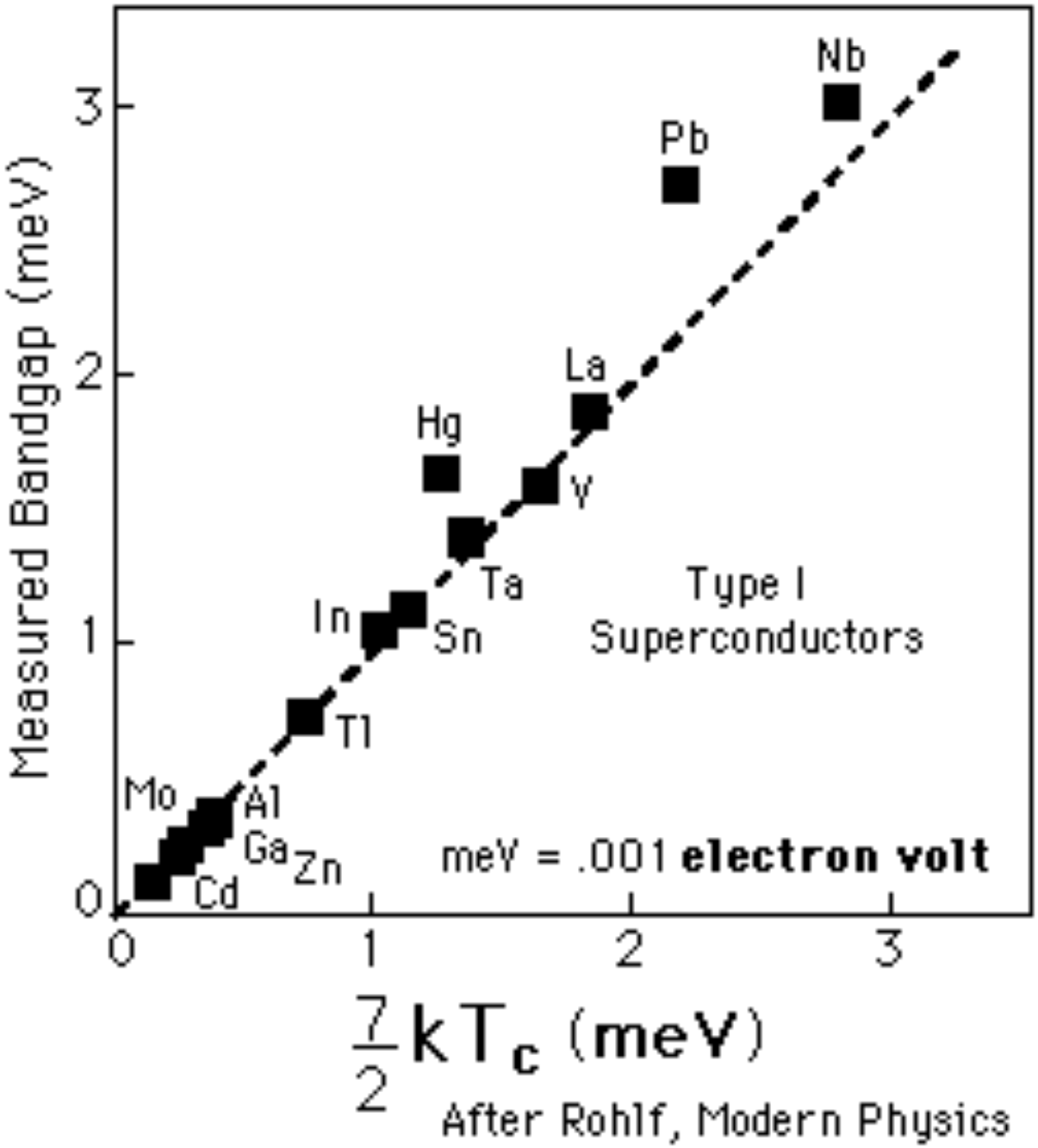


TABLE III

Measured Values of $2\Delta(0)/kT_c$
(BCS) theoretical value = 3.53)

Superconductor	Tunneling measurements	Ref.	Thermodynamic measurements[a]
Al	4.2 ± 0.6	(*49*)	3.53
	2.5 ± 0.3	(*50*)	
	$2.8 - 3.6$	(*68*)	
	3.37 ± 0.1	(*65*)	
Cd	3.2 ± 0.1	(*65a*)	3.44
Ga			3.52, 3.50, 3.48
Hg(α)	4.6 ± 0.1	(*84*)	3.95
In	3.63 ± 0.1	(*49*)	3.65
	3.45 ± 0.07	(*65*)	
	3.61	(*94*)	
La	$1.65 - 3.0$ (fcc)[b]	(*100*)	3.72 (fcc) (*d*-hep)
	3.2	(*100*)	
Nb	3.84 ± 0.06	(*85*)	3.65
	3.6	(*95*)	
	3.6	(*96*)	
Pb[c]	4.29 ± 0.04	(*69*)	3.95
	4.38 ± 0.01[d]	(*74*)	
Sn	3.46 ± 0.1	(*49*)	3.61, 3.57
	3.10 ± 0.05	(*50*)	
	3.51 ± 0.18	(*85*)	
	$2.8 - 4.06$	(*65*)	
	$3.1 - 4.3$	(*87*)	
Ta	3.60 ± 0.1	(*85*)	3.63
	3.5	(*95*)	
	3.65 ± 0.1	(*97*)	
Tl	3.57 ± 0.05	(*98*)	3.63
	3.9	(*94*)	
V	3.4	(*95*)	3.50
Zn	3.2 ± 0.1	(*99*)	3.44

[a] The values given here were calculated from values of $\gamma T_c^2/V_M H_c^2$ (0) assuming the equation $[2\pi V_M H_c^2(0)/3\gamma T_c^2]^{1/2} = 2\Delta(0)/kT_c$.

[b] The measured tunneling results of Edelstein and Toxen (*100*) in La are very low and widely scattered and perhaps reflect the great structure sensitivity of La. Hauser's (*100a*) later measurements are higher and less scattered.

[c] Other older measurements are collected in (*51*).

[d] T_c assumed to be 7.193 °K.

Efeito isotópico

TABLE II

"Best" Experimental Values for β Obtained in Fitting the Relation $T_c \propto M^{-\beta}$ Compared with the Theoretical Value of BCS (*1*), Swihart (*22,23*), Morel and Anderson (*9*), and Garland (*26*)

Element	β_{expt1}	β_{BCS}	β_{SWI}	β_{MA}	β_{GAR}	$\beta_{GAR(latest)}$
Zn	$\left\{\begin{matrix}0.45 \pm 0.01 \\ 0.30\ (27)\end{matrix}\right\}$	0.5	0.2	0.35	0.40	0.415 ± 0.015
Cd	0.50 ± 0.10	0.5	0.2	0.34	0.37	0.385 ± 0.025
Hg	0.50 ± 0.03	0.5	0.4	0.46	0.465	0.48 ± 0.005
Al		0.5	0.3	0.34	0.35	0.37 ± 0.025
Tl	0.50 ± 0.10	0.5	0.3	0.43	0.45	0.48 ± 0.02
Sn	0.47 ± 0.02	0.5	0.3	0.42	0.44	0.455 ± 0.01
Pb	0.48 ± 0.01	0.5	0.3	0.47	0.47	0.485 ± 0.005
Ti		0.5		0.25	0.2	0.145 ± 0.17
Zr	0.0	0.5		0.30	0.35	0.15 ± 0.17
V		0.5		0.41	0.15	0.25 ± 0.125
Ta		0.5		0.42	0.35	0.35 ± 0.075
Mo	0.33 ± 0.05	0.5	0.15	0.3	0.35	0.35 ± 0.075
Ru	0.0 ± 0.10	0.5	0.0	0.35	0.0	0.065 ± 0.15
Os	0.20 ± 0.05	0.5	0.1	0.25	0.1	0.225 ± 0.10
Ir		0.5		0.3	-0.2	-0.015 ± 0.17
Hf		0.5		0.5	0.3	0.1 ± 0.2
Re	0.39 ± 0.01	0.5		0.41	0.3	0.355 ± 0.05
U(α)	-2.2 ± 0.2	0.5	[(*32a*); see also (*198a*)].			

[a] In some cases the values for β_{SWI} and β_{MA} have been calculated by Garland using their models, respectively. (See Garland for references and (*27*) for Zn and (*32* for Zr, Mo, Re, Ru, and Os.)

CALOR ESPECÍFICO:

(i) DADO $E(\vec{p}) = \sqrt{\epsilon(\vec{p})^2 + \Delta^2(T)}$ E $H = \sum_{\vec{k}\sigma} E(\vec{k})\, \hat{n}_{b\vec{k}\sigma} + E_0$

$F(T) = -T \ln Z$

PARA CADA $\vec{k}$:

$$Z = \left[\prod_{\vec{k}} \left(1 + e^{-\beta E(\vec{k})}\right)\right]^2 \qquad Z_{\vec{k}} = \left[1 + e^{-\beta E(\vec{k})}\right]^2$$

$$F(T) = -2T \sum_{\vec{k}} \ln\left(1 + e^{-\beta E(\vec{k})}\right) \Rightarrow U(T) \Rightarrow C_V(T) = \frac{\partial U}{\partial T}$$

(ii) CALCULO DIRETAMENTE A ENTROPIA

$$S = \sum_i -P_i \ln P_i$$

P_i = PROBABILIDADE DO ESTADO $\underline{i}$

PARA CADA $\vec{k}$: $\langle b^+_{\vec{k}\sigma} b_{\vec{k}\sigma} \rangle = f(E_{\vec{k}}) = 0 \times P^0_{\vec{k}} + 1 \times P^1_{\vec{k}} = P^1_{\vec{k}}$

$$P^1_{\vec{k}} = f(E_{\vec{k}}) \equiv f_{\vec{k}} \qquad P^0_{\vec{k}} = 1 - P^1_{\vec{k}} = 1 - f(E_{\vec{k}}) = 1 - f_{\vec{k}}$$

$$\vec{S} = -2 \sum_{\vec{k}} \left[P^0_{\vec{k}} \ln P^0_{\vec{k}} + P^1_{\vec{k}} \ln P^1_{\vec{k}}\right] = -2 \sum_{\vec{k}} \left[(1 - f_{\vec{k}}) \ln(1 - f_{\vec{k}}) + f_{\vec{k}} \ln f_{\vec{k}}\right]$$

LEMBRANDO: $C_V(T) = \frac{đQ}{dT} = T\frac{dS}{dT}\Big|_{V,\mu} = -\beta\frac{\partial S}{\partial \beta}\Big|_{V,\mu}$

$$\Rightarrow C_V = 2\beta\sum_{\vec{k}}\left[-\ln(1-f_k)\cancel{-1} + \ln f_k + \cancel{1}\right]\frac{\partial f_k}{\partial \beta}\Big|_{\mu}$$

$$= 2\beta\sum_{\vec{k}}\ln\left(\frac{f_k}{1-f_k}\right)\frac{\partial f_k}{\partial \beta}\Big|_{\mu} = -2\beta^2\sum_{\vec{k}}E_{\vec{k}}\frac{\partial f_k}{\partial \beta}\Big|_{\mu}$$

$$\frac{f_k}{1-f_k} = \frac{1}{e^{\beta E}+1}\,\frac{1}{1-\frac{1}{e^{\beta E}+1}} = \frac{1}{e^{\beta E}} = e^{-\beta E}$$

$$\frac{\partial f_k}{\partial \beta} = \frac{\partial}{\partial \beta}\left[\frac{1}{e^{\beta E}+1}\right] = -\frac{e^{\beta E}}{(e^{\beta E}+1)^2}\frac{\partial(\beta E)}{\partial \beta} = -\frac{e^{\beta E}}{(e^{\beta E}+1)^2}\left[E+\beta\frac{\partial E}{\partial \beta}\right]$$

$$\boxed{\frac{\partial f(x)}{\partial x} = -\frac{e^{x}}{(e^{x}+1)^2}}$$

$$= -\frac{e^{\beta E}}{(e^{\beta E}+1)^2}\left[E+\frac{\beta\Delta}{E}\frac{\partial \Delta}{\partial \beta}\right]$$

$$C_V = +2\beta^2\sum_{\vec{k}}\frac{E_k e^{\beta E_k}}{(e^{\beta E_k}+1)^2}\left[E_k+\frac{\beta\Delta}{E}\frac{d\Delta}{d\beta}\right]$$

$$\sum_{\vec{k}} = V\int\frac{d^3k}{(2\pi)^3} = V\int g_0(\epsilon)\,d\epsilon$$

$$C_{\mathcal{D}}(T) = 2\beta^2 \rho_0 \int d\epsilon \, \frac{E(\epsilon)\, e^{\beta E(\epsilon)}}{\left[e^{\beta E(\epsilon)}+1\right]^2} \left[E(\epsilon) + \frac{\beta\Delta}{E(\epsilon)} \frac{d\Delta}{d\beta}\right]$$

PODEMOS CALCULAR ANALITICAMENTE O SALTO DE $C_{\mathcal{D}}(T)$ EM $T=T_c$. PARA ISSO, FAZENDO $T \to T_c$ ($\beta \to \beta_c$) ACIMA: ($\Delta(T) \xrightarrow[T\to T_c]{} 0$)

1º TERMO: $C_{\mathcal{D}}(T_c) = 2\beta_c^2 \rho_0 \int d\epsilon \, \frac{\epsilon^2 e^{\beta_c \epsilon}}{(e^{\beta_c \epsilon}+1)^2}$

$x = \beta_c \epsilon$ $\qquad C_{\mathcal{D}}(T_c) = 2\rho_0 T_c \underbrace{\int dx \, \frac{x^2 e^x}{(e^x+1)^2}}_{\pi^2/3} = \frac{2\pi^2 \rho_0 T_c}{3}$

QUE É O CALOR ESPECÍFICO DO ESTADO NORMAL (METAL) EM T_c: $C_{\mathcal{D},n}(T_c)$

LOGO, O 2º TERMO NOS DÁ $\Delta C_{\mathcal{D}}(T_c)$

$$C_v^{(2)}(T_c) = \Delta C_v(T_c) = 2\beta_c^2 \rho_0 \Delta \int d\epsilon \frac{e^{\beta_c \epsilon}}{[e^{\beta_c \epsilon}+1]^2} \beta_c \frac{d\Delta}{d\beta}\Big|_{\beta_c}$$

$$\boxed{\beta \frac{d\Delta}{d\beta} = -T\frac{d\Delta}{dT}}$$

$$= -2\beta_c \rho_0 \Delta \int d\epsilon \frac{e^{\beta_c \epsilon}}{[e^{\beta_c \epsilon}+1]^2} \frac{d\Delta}{dT}\Big|_{T_c}$$

$$= -2\rho_0 \Delta \int dx \frac{e^x}{[e^x+1]^2} \frac{d\Delta}{dT}\Big|_{T_c}$$

$$= -\rho_0 \underbrace{\int dx \frac{e^x}{[e^x+1]^2}}_{=1} \frac{d\Delta^2}{dT}\Big|_{T_c}$$

$$\Rightarrow \Delta C_v(T_c) = -\rho_0 \frac{d\Delta^2}{dT}\Big|_{T_c}$$

DO LANDAU: $-\frac{d\Delta^2}{dT}\Big|_{T_c} = \frac{8\pi^2}{7\zeta(3)} T_c$

$$\Delta C_v(T_c) = \frac{8\pi^2}{7\zeta(3)} \rho_0 T_c \Rightarrow \frac{\Delta C_v(T_c)}{C_{v,n}(T_c)} = \frac{8\pi^2}{7\zeta(3)} \frac{3}{2\pi^2} = \frac{12}{7\zeta(3)} \simeq 1.43$$

Calor específico x T

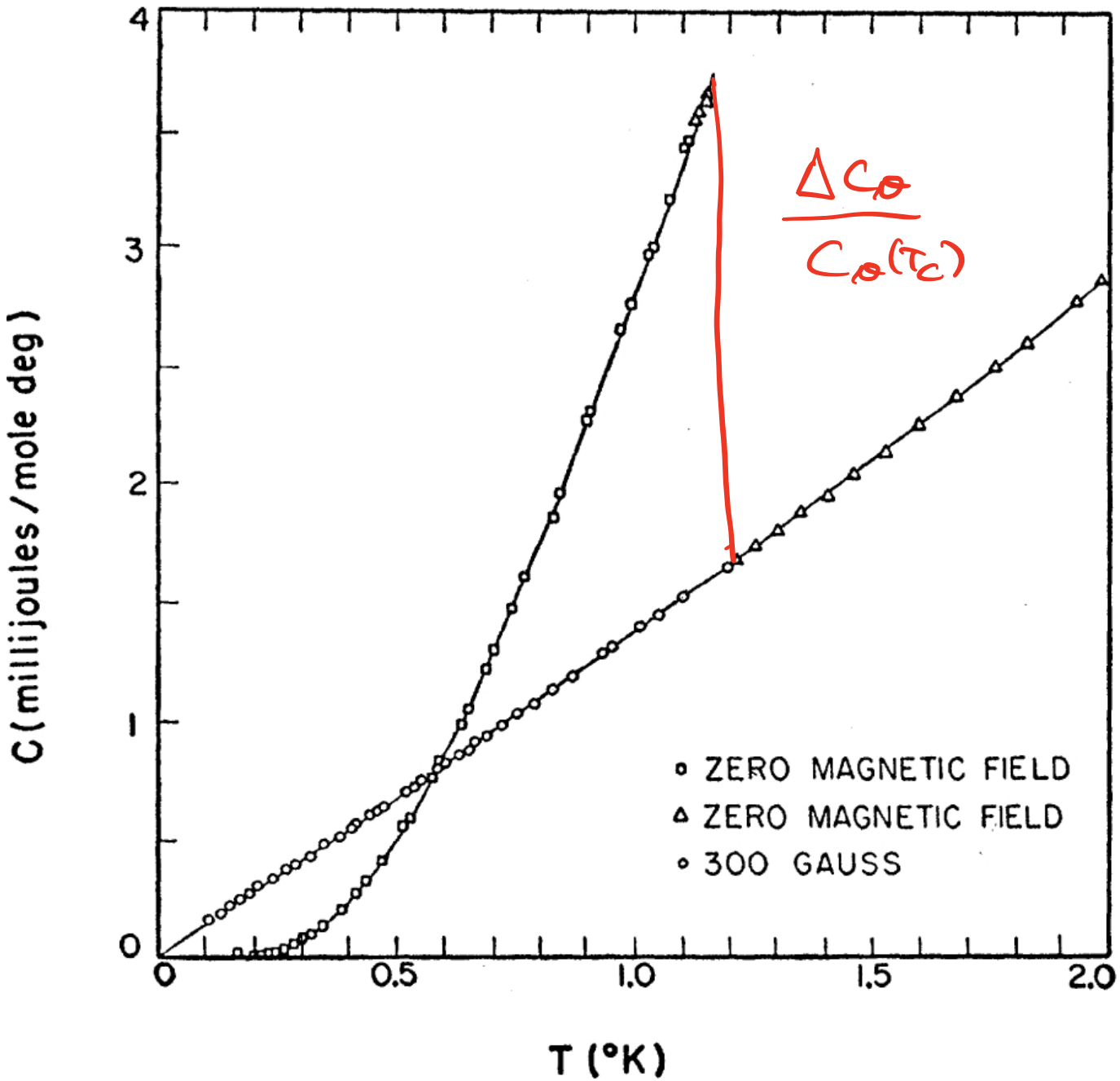


TABLE IV

Measured Values of Two Thermodynamic Functions

	$\left(\frac{C_s - C_n}{C_n}\right)_{T_c}$	Ref.	$\frac{\gamma T_c^2}{V_M H_c^2(0)}$ [a]	Ref.
BCS	1.43[b]	(*1*[b],*4*)	0.168	
		(*166*		
Al	1.29[c]–1.59[c]	(*167,168*)	0.171	(*167*)
		(*148,170*)		
		(*171*)		
	1.45[c] (av.)		0.170	(*170*[d])
Cd	1.32	(*172*)	0.177	(*172*)
	1.40	(*173*)		
Ga	1.44, 1.41	(*172,174*)	0.169, 0.170	(*172,174*)
	1.40	(*175*)[e]	0.173	(*175*)
Hg	2.37	(*169,176*)	0.134	(*165*)
		(*165*)		
In	1.73	(*156*)	0.157,0.150	(*152,165*)
La (hcp)	1.5	(*177*)		
Mo	1.28	(*186*)	0.182	(*148*)
Nb	1.87 (calorimetric)	(*178*)	0.157	(*179*)
	2.0 (magnetic)	(*179*)		
Pb	2.71	(*180,176*)	0.134	(*187*[d])
Sn	1.60	(*147,181*)	0.161	(*165*)
			0.164	(*152*)
Ta	1.59	(*147,181,182*)	0.161	(*181,182*)
Tl	1.50	(*183*)	0.161	(*181*)
U(β)	1.36, 1.52	(*184*)		
V	1.49	(*162*)	0.170	(*162*)
Zn	1.30	(*174*)	0.177	(*174*)
	1.24	(*185*)		