FI 193 – Teoria Quântica de Sistemas de Muitos Corpos

2º Semestre de 2023

30/11/2023

Aula 30

Teoria BCS – Aula passada

$$H_{BCS} = \sum_{\mathbf{p},\sigma} \epsilon\left(\mathbf{p}\right) a^{\dagger}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} - \frac{\lambda}{V} \sum_{\mathbf{p},\mathbf{p}'} a^{\dagger}_{\mathbf{p}',\uparrow} a^{\dagger}_{-\mathbf{p}',\downarrow} a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} F\left(\mathbf{p},\mathbf{p}'\right) \equiv H_0 + H_1$$

$$F\left(\mathbf{p},\mathbf{p}'\right) = \Theta\left(\omega_D - \left|\epsilon\left(\mathbf{p}\right)\right|\right) \Theta\left(\omega_D - \left|\epsilon\left(\mathbf{p}'\right)\right|\right)$$

“Casquinha” de “espessura” ω_D em torno da energia de Fermi.

Teoria de campo médio:

$$a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} \rightarrow \left\langle a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} \right\rangle + \text{flut.} \equiv A_{\mathbf{p}} + \delta_{\mathbf{p}}$$

$$a^{\dagger}_{\mathbf{p}',\uparrow} a^{\dagger}_{-\mathbf{p}',\downarrow} \rightarrow \left\langle a^{\dagger}_{\mathbf{p}',\uparrow} a^{\dagger}_{-\mathbf{p}',\downarrow} \right\rangle + \text{flut.} \equiv A^{*}_{\mathbf{p}'} + \delta^{*}_{\mathbf{p}'}$$

Desprezam-se termos quadráticos em $\delta_{\mathbf{p}}$.

Teoria BCS – Aula passada

$$H_{BCS}^{MF} = \sum_{\mathbf{p},\sigma} \epsilon\left(\mathbf{p}\right) a^{\dagger}_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma} + \sum_{\mathbf{p}} \left(\Delta^{*} a_{-\mathbf{p},\downarrow} a_{\mathbf{p},\uparrow} + \Delta a^{\dagger}_{\mathbf{p},\uparrow} a^{\dagger}_{-\mathbf{p},\downarrow}\right) + \frac{V\left|\Delta\right|^{2}}{\lambda}$$

$$\Delta \equiv -\frac{\lambda}{V} \sum_{\mathbf{p}} A_{\mathbf{p}}$$

Nambu-Gorkov:

$$\mathbf{\Psi}_{\mathbf{p}} = \begin{pmatrix} a_{\mathbf{p},\uparrow} \\ a^{\dagger}_{-\mathbf{p},\downarrow} \end{pmatrix}$$

$$H_{BCS}^{MF} = \sum_{\mathbf{p}} \mathbf{\Psi}^{\dagger}_{\mathbf{p}} \mathbf{H} \mathbf{\Psi}_{\mathbf{p}} + \text{const.}$$

$$\mathbf{H} = \begin{pmatrix} \epsilon\left(\mathbf{p}\right) & \Delta \\ \Delta & -\epsilon\left(\mathbf{p}\right) \end{pmatrix}$$

$$= \epsilon(\vec{p})\,\tau_3 + \Delta\,\tau_1$$

Transformação de Bogoliubov

$$\mathbf{B}_{\mathbf{p}} \equiv \begin{pmatrix} b_{\mathbf{p},\uparrow} \\ b^{\dagger}_{-\mathbf{p},\downarrow} \end{pmatrix} = \mathbf{S} \begin{pmatrix} a_{\mathbf{p},\uparrow} \\ a^{\dagger}_{-\mathbf{p},\downarrow} \end{pmatrix} \equiv \mathbf{S}\mathbf{\Psi}_{\mathbf{p}}$$

$$\mathbf{\Psi}_{\mathbf{p}} = \mathbf{S}^{-1}\mathbf{B}_{\mathbf{p}} = \mathbf{S}^{\dagger}\mathbf{B}_{\mathbf{p}}$$

$$\mathbf{\Psi}^{\dagger}_{\mathbf{p}} = \mathbf{B}^{\dagger}_{\mathbf{p}}\mathbf{S}$$

$$\boxed{\begin{aligned} u^2_{\mathbf{p}} &= \frac{1}{2}\left[1 + \frac{\epsilon(\mathbf{p})}{E(\mathbf{p})}\right] \\ v^2_{\mathbf{p}} &= \frac{1}{2}\left[1 - \frac{\epsilon(\mathbf{p})}{E(\mathbf{p})}\right] \\ u_{\mathbf{p}}, v_{\mathbf{p}} &> 0 \\ u^2_{\mathbf{p}} + v^2_{\mathbf{p}} &= 1 \\ E(\mathbf{p}) &= \sqrt{\epsilon^2(\mathbf{p}) + \Delta^2} \end{aligned}}$$

$$\left\{ b_{\mathbf{p},\sigma}, b^{\dagger}_{\mathbf{p}',\sigma'} \right\} = \delta_{\mathbf{p},\mathbf{p}'}\delta_{\sigma,\sigma'}$$

$$\mathbf{S} = \begin{pmatrix} u_{\mathbf{p}} & v_{\mathbf{p}} \\ -v_{\mathbf{p}} & u_{\mathbf{p}} \end{pmatrix}, \quad \mathbf{S}^{-1} = \begin{pmatrix} u_{\mathbf{p}} & -v_{\mathbf{p}} \\ v_{\mathbf{p}} & u_{\mathbf{p}} \end{pmatrix} = \mathbf{S}^{\dagger}$$

Transformação de Bogoliubov

$$H_{BCS}^{MF} = \sum_{\mathbf{p}} \mathbf{\Psi}_{\mathbf{p}}^{\dagger} \mathbf{H} \mathbf{\Psi}_{\mathbf{p}} + \mathrm{const.}$$

$$\mathbf{\Psi}_{\mathbf{p}} \;=\; \mathbf{S}^{-1}\mathbf{B}_{\mathbf{p}} = \mathbf{S}^{\dagger}\mathbf{B}_{\mathbf{p}}$$

$$\mathbf{H} = \begin{pmatrix} \epsilon\left(\mathbf{p}\right) & \Delta \\ \Delta & -\epsilon\left(\mathbf{p}\right) \end{pmatrix}$$

$$\mathbf{S}\mathbf{H}\mathbf{S}^{\dagger} = \begin{pmatrix} E\left(\mathbf{p}\right) & 0 \\ 0 & -E\left(\mathbf{p}\right) \end{pmatrix} \equiv \mathbf{h}$$

$$\begin{aligned} H_{BCS}^{MF} &= \sum_{\mathbf{p}} \mathbf{B}_{\mathbf{p}}^{\dagger} \mathbf{S}\mathbf{H}\mathbf{S}^{\dagger} \mathbf{B}_{\mathbf{p}} + \mathrm{const.} \\ &= \sum_{\mathbf{p}} \mathbf{B}_{\mathbf{p}}^{\dagger} \mathbf{h} \mathbf{B}_{\mathbf{p}} + \mathrm{const.} \\ &= \sum_{\mathbf{p}} E\left(\mathbf{p}\right) \left[b_{\mathbf{p},\uparrow}^{\dagger} b_{\mathbf{p},\uparrow} - b_{-\mathbf{p},\downarrow} b_{-\mathbf{p},\downarrow}^{\dagger} \right] + \mathrm{const.} \\ &= \sum_{\mathbf{p}} E\left(\mathbf{p}\right) \left[b_{\mathbf{p},\uparrow}^{\dagger} b_{\mathbf{p},\uparrow} + b_{-\mathbf{p},\downarrow}^{\dagger} b_{-\mathbf{p},\downarrow} - 1 \right] + \mathrm{const.} \\ &= \sum_{\mathbf{p},\sigma} E\left(\mathbf{p}\right) b_{\mathbf{p},\sigma}^{\dagger} b_{\mathbf{p},\sigma} + \mathrm{const.} \end{aligned}$$

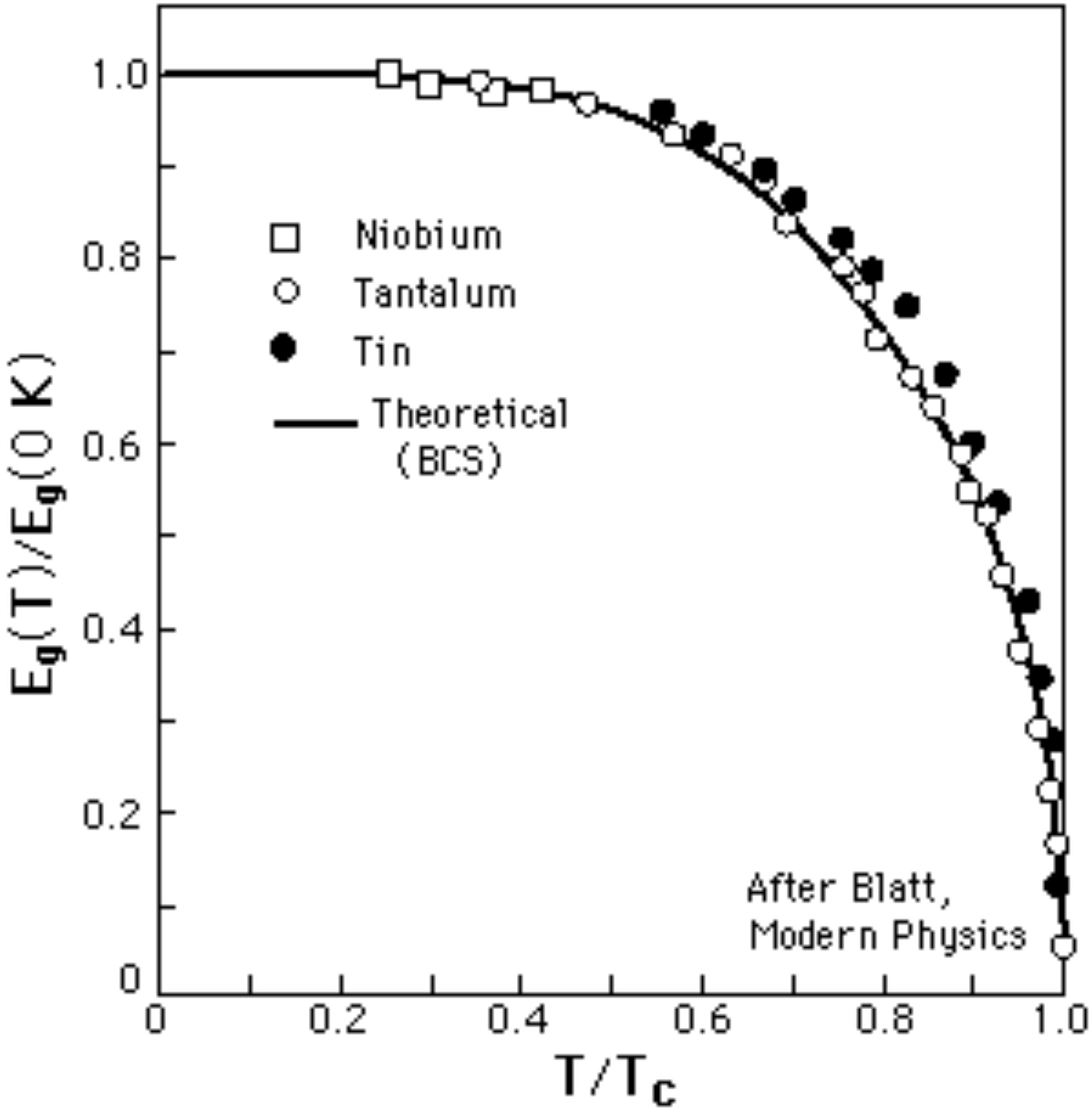
O gap superconductor e a temperatura crítica

Equação do gap:

$$\frac{1}{\rho_F \lambda} = \int_0^{\omega_D} \frac{\tanh\left[\sqrt{\epsilon^2 + \Delta\left(T\right)}/\left(2T\right)\right]}{\sqrt{\epsilon^2 + \Delta\left(T\right)}} d\epsilon$$

$$\begin{aligned} \Delta\left(0\right) &= 2\omega_D e^{-1/\rho_F \lambda}, \\ T_c &= \left(\frac{e^\gamma}{\pi}\right) 2\omega_D e^{-1/\rho_F \lambda} = \left(\frac{e^\gamma}{\pi}\right) \Delta\left(0\right), \\ \frac{2\Delta\left(0\right)}{T_c} &= \frac{2\pi}{e^\gamma} \approx 3.53. \end{aligned}$$

Gap como função da temperatura



Previsão de BCS

$$\frac{2\Delta(0)}{k_B T_c} = 3.53$$

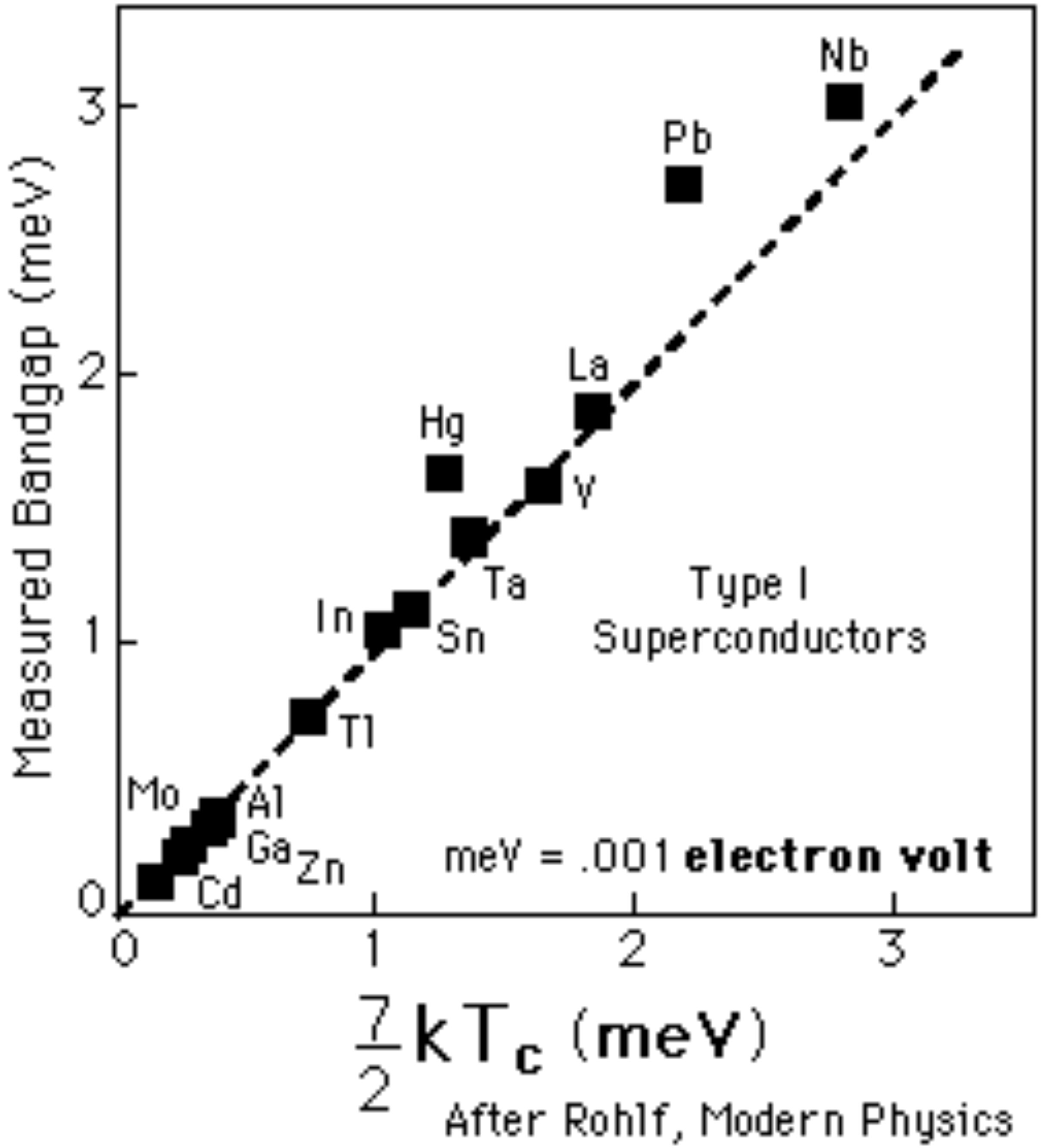


TABLE III

Measured Values of $2\Delta(0)/kT_c$
(BCS) theoretical value = 3.53)

Superconductor	Tunneling measurements	Ref.	Thermodynamic measurements[a]
Al	4.2 ± 0.6	*(49)*	3.53
	2.5 ± 0.3	*(50)*	
	$2.8 - 3.6$	*(68)*	
	3.37 ± 0.1	*(65)*	
Cd	3.2 ± 0.1	*(65a)*	3.44
Ga			3.52, 3.50, 3.48
Hg(α)	4.6 ± 0.1	*(84)*	3.95
In	3.63 ± 0.1	*(49)*	3.65
	3.45 ± 0.07	*(65)*	
	3.61	*(94)*	
La	$1.65 - 3.0$ (fcc)[b]	*(100)*	3.72 (fcc) (*d*-hep)
	3.2	*(100)*	
Nb	3.84 ± 0.06	*(85)*	3.65
	3.6	*(95)*	
	3.6	*(96)*	
Pb[c]	4.29 ± 0.04	*(69)*	3.95
	4.38 ± 0.01[d]	*(74)*	
Sn	3.46 ± 0.1	*(49)*	3.61, 3.57
	3.10 ± 0.05	*(50)*	
	3.51 ± 0.18	*(85)*	
	$2.8 - 4.06$	*(65)*	
	$3.1 - 4.3$	*(87)*	
Ta	3.60 ± 0.1	*(85)*	3.63
	3.5	*(95)*	
	3.65 ± 0.1	*(97)*	
Tl	3.57 ± 0.05	*(98)*	3.63
	3.9	*(94)*	
V	3.4	*(95)*	3.50
Zn	3.2 ± 0.1	*(99)*	3.44

[a] The values given here were calculated from values of $\gamma T_c^2/V_M H_c^2(0)$ assuming the equation $[2\pi V_M H_c^2(0)/3\gamma T_c^2]^{1/2} = 2\Delta(0)/kT_c$.

[b] The measured tunneling results of Edelstein and Toxen *(100)* in La are very low and widely scattered and perhaps reflect the great structure sensitivity of La. Hauser's *(100a)* later measurements are higher and less scattered.

[c] Other older measurements are collected in *(51)*.

[d] T_c assumed to be 7.193 °K.

Efeito isotópico

TABLE II

"Best" Experimental Values for β Obtained in Fitting the Relation $T_c \propto M^{-\beta}$ Compared with the Theoretical Value of BCS (*1*), Swihart (*22,23*), Morel and Anderson (*9*), and Garland (*26*)

Element	β_{exptl}	β_{BCS}	β_{SWI}	β_{MA}	β_{GAR}	$\beta_{\text{GAR(latest)}}$
Zn	$\begin{Bmatrix} 0.45 \pm 0.01 \\ 0.30\ (27) \end{Bmatrix}$	0.5	0.2	0.35	0.40	0.415 ± 0.015
Cd	0.50 ± 0.10	0.5	0.2	0.34	0.37	0.385 ± 0.025
Hg	0.50 ± 0.03	0.5	0.4	0.46	0.465	0.48 ± 0.005
Al		0.5	0.3	0.34	0.35	0.37 ± 0.025
Tl	0.50 ± 0.10	0.5	0.3	0.43	0.45	0.48 ± 0.02
Sn	0.47 ± 0.02	0.5	0.3	0.42	0.44	0.455 ± 0.01
Pb	0.48 ± 0.01	0.5	0.3	0.47	0.47	0.485 ± 0.005
Ti		0.5		0.25	0.2	0.145 ± 0.17
Zr	0.0	0.5		0.30	0.35	0.15 ± 0.17
V		0.5		0.41	0.15	0.25 ± 0.125
Ta		0.5		0.42	0.35	0.35 ± 0.075
Mo	0.33 ± 0.05	0.5	0.15	0.3	0.35	0.35 ± 0.075
Ru	0.0 ± 0.10	0.5	0.0	0.35	0.0	0.065 ± 0.15
Os	0.20 ± 0.05	0.5	0.1	0.25	0.1	0.225 ± 0.10
Ir		0.5		0.3	-0.2	-0.015 ± 0.17
Hf		0.5		0.5	0.3	0.1 ± 0.2
Re	0.39 ± 0.01	0.5		0.41	0.3	0.355 ± 0.05
U(α)	-2.2 ± 0.2	0.5	[(*32a*); see also (*198a*)].			

[a] In some cases the values for β_{SWI} and β_{MA} have been calculated by Garland using their models, respectively. (See Garland for references and (*27*) for Zn and (*32* for Zr, Mo, Re, Ru, and Os.)

Calor específico x T

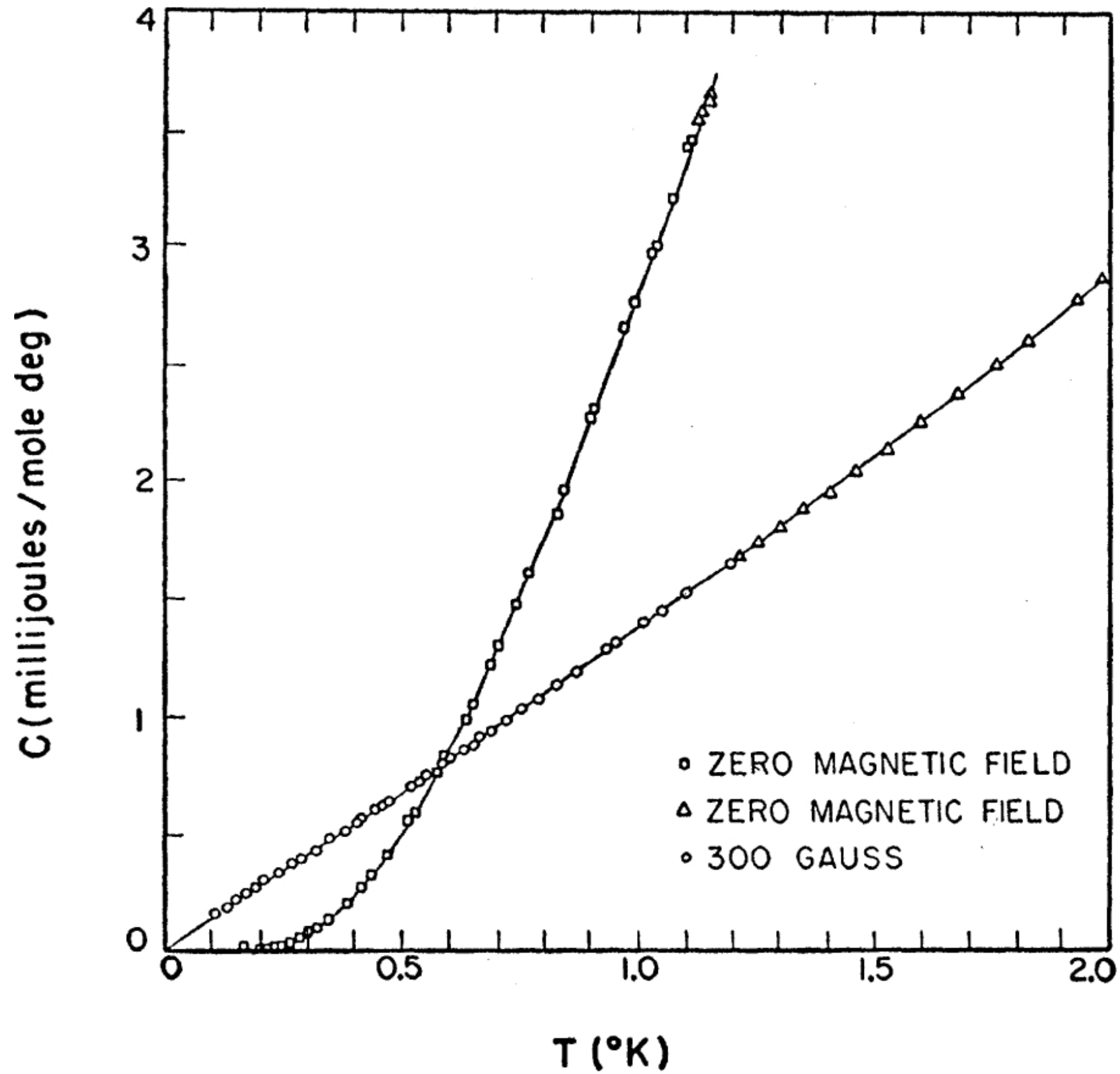


TABLE IV

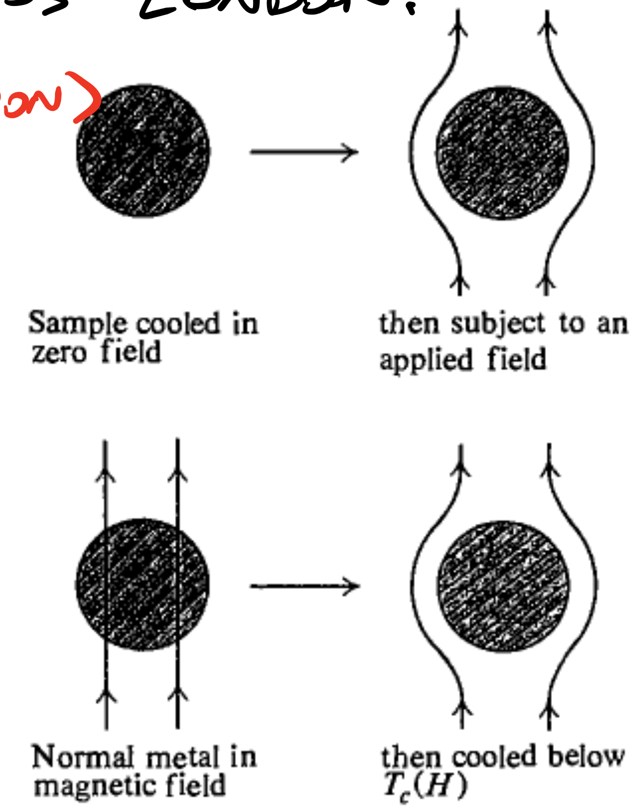
Measured Values of Two Thermodynamic Functions

	$\left(\frac{C_s - C_n}{C_n}\right)_{T_c}$	Ref.	$\frac{\gamma T_c^2}{V_M H_c^2(0)}$[a]	Ref.
BCS	1.43[b]	(*1*[b],*4*)	0.168	
		(*166*		
Al	1.29[c]–1.59[c]	(*167*,*168*)	0.171	(*167*)
		(*148*,*170*)		
		(*171*)		
	1.45[c] (av.)		0.170	(*170*[d])
Cd	1.32	(*172*)	0.177	(*172*)
	1.40	(*173*)		
Ga	1.44, 1.41	(*172*,*174*)	0.169, 0.170	(*172*,*174*)
	1.40	(*175*)[e]	0.173	(*175*)
Hg	2.37	(*169*,*176*)	0.134	(*165*)
		(*165*)		
In	1.73	(*156*)	0.157,0.150	(*152*,*165*)
La (hcp)	1.5	(*177*)		
Mo	1.28	(*186*)	0.182	(*148*)
Nb	1.87 (calorimetric)	(*178*)	0.157	(*179*)
	2.0 (magnetic)	(*179*)		
Pb	2.71	(*180*,*176*)	0.134	(*187*[d])
Sn	1.60	(*147*,*181*)	0.161	(*165*)
			0.164	(*152*)
Ta	1.59	(*147*,*181*,*182*)	0.161	(*181*,*182*)
Tl	1.50	(*183*)	0.161	(*181*)
U(β)	1.36, 1.52	(*184*)		
V	1.49	(*162*)	0.170	(*162*)
Zn	1.30	(*174*)	0.177	(*174*)
	1.24	(*185*)		

O efeito Meissner-Ochsenfeld

TEORIA FENOMENOLÓGICA DOS IRMÃOS LONDON:

(1935): $\vec{\nabla}\times\vec{j} = -\Gamma\vec{B}$ (EQ. DE LONDON)



LEI DE AMPÈRE:

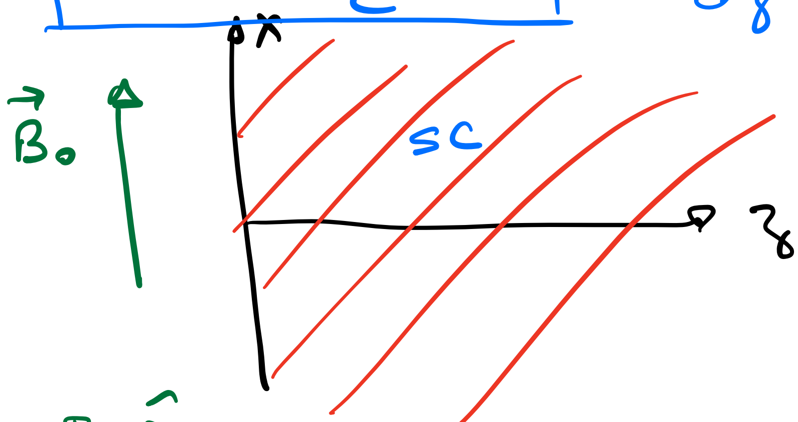
$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{j} \Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \frac{4\pi}{c}\vec{\nabla}\times\vec{j}$$

$$\Rightarrow -\nabla^2\vec{B} + \vec{\nabla}(\cancelto{0}{\vec{\nabla}\cdot\vec{B}}) = -\frac{4\pi\Gamma}{c}\vec{B}$$

$$\Rightarrow \boxed{\nabla^2\vec{B} = \frac{4\pi\Gamma}{c}\vec{B}} \Rightarrow \frac{d^2B_x}{dz^2} = \frac{4\pi\Gamma}{c}B_x$$

$$\Downarrow$$

$$B_x(z) = \cancel{A}\,e^{+z/\lambda} + C e^{-z/\lambda} \qquad \lambda = \sqrt{\frac{c}{4\pi\Gamma}}$$



$$\Rightarrow \boxed{B_x(z) = B_0 e^{-z/\lambda}}$$

$\vec{B}_0 = B_0\hat{x}$

$\vec{B}(z) = B_x(z)\hat{x}$

λ = COMPRIMENTO DE PENETRAÇÃO

EQ. DE LONDON: $\vec{j} = -\Gamma \vec{A}$ $\Longrightarrow$ $\vec{\nabla}\cdot\vec{A} = 0$ (GAUGE DE LONDON)

$\Rightarrow \vec{\nabla}\cdot\vec{j} = 0$ (CORR. ESTAC.)

TRANSFORMANDO EM FOURIER:

$$\vec{j}(\vec{k}) = -\Gamma \vec{A}(\vec{k}) \quad ; \quad \vec{k}\cdot\vec{A}(\vec{k}) = 0$$

SE: $k \ll \frac{T_c}{v_F}$ (LIMITE DE LONDON) $\Rightarrow$ $\vec{A}(\vec{k})$ NÃO DEPENDE DE $\vec{k}$. VAMOS ASSUMIR ESSA CONDIÇÃO.

NO OUTRO LIMITE: $k \gg \frac{T_c}{v_F}$ O DESENVOLVIMENTO A SEGUIR PRECISA SER MODIFICADO. NO ENTANTO, ESSA SEGUNDA SITUAÇÃO SÓ OCORRE PARA ALGUNS SCs DO TIPO I E EM BAIXAS TEMPERATURAS.

$$A(\vec{k}) = A(\vec{0}) = \text{CONST.}$$

EM TERMOS DE $\vec{A}$:

$$\vec{p} \longrightarrow \vec{p} - \frac{q}{c}\vec{A} = \vec{p} + \frac{e}{c}\vec{A} \quad (e>0)$$

$$\frac{\vec{\nabla}}{i} \longrightarrow \frac{\vec{\nabla}}{i} + \frac{e}{c}\vec{A}$$

CORRENTE ELÉTRICA: $-\frac{e}{m}\frac{\vec{\nabla}}{i} \longrightarrow -\frac{e}{m}\left[\frac{\vec{\nabla}}{i} + \frac{e}{c}\vec{A}\right]$

NO ESPAÇO $\vec{k}$: $\frac{1}{V}\sum_{\vec{k},\sigma}\left(-\frac{e}{m}\right)\left[\vec{k} + \frac{e}{c}\vec{A}\right] c^{\dagger}_{k\sigma} c_{\vec{k}\sigma} = \vec{j}$

HAMILTONIANO: $T = \sum_{\sigma}\int d^3r \, \frac{\psi^{\dagger}_{\sigma}(\vec{r})}{2m}\left[\frac{\vec{\nabla}}{i} + \frac{e}{c}\vec{A}\right]^2 \psi_{\sigma}(\vec{r})$

$$H_0 = \sum_{\vec{k},\sigma}\left[\underbrace{\left(\frac{k^2}{2m} - \mu\right)}_{\epsilon(\vec{k})} + \frac{e}{mc}\vec{k}\cdot\vec{A} + \frac{e^2}{2mc^2}\vec{A}^2\right] c^{\dagger}_{k\sigma} c_{k\sigma}$$

TRANSFORMANDO PARA A NOTAÇÃO DE NAMBU:

$$H = \sum_{\vec{k}} \Psi^{\dagger}_{\vec{k}}\left[\epsilon(\vec{k})\tau_3 + \frac{e}{mc}\vec{k}\cdot\vec{A}\,\tau_0 + \frac{e^2}{2mc^2}\vec{A}^2\tau_3 + \Delta\tau_1\right]\Psi_{\vec{k}}$$

$$\vec{j} = \frac{1}{V}\sum_{\vec{k}} \Psi^\dagger_{\vec{k}} \left[-\frac{e}{m}\vec{k}\,\tau_0 - \frac{e^2}{mc}\vec{A}\,\tau_3 \right] \Psi_{\vec{k}}$$

$$j_i = \frac{1}{V}\sum_{\vec{k}} \Psi^\dagger_{\vec{k}} \left[-\frac{e}{m} k_i\,\tau_0 - \frac{e^2}{mc} A_i\,\tau_3 \right] \Psi_{\vec{k}}$$

VAMOS QUERER : $\langle j_i \rangle$ NA PRESENÇA DE $\vec{A}$ EM ORDEM LINEAR EM $\vec{A}$:

$$\langle j_i \rangle = j_i^p + j_i^d$$

$$j_i^p = \frac{1}{V}\sum_{\vec{k}} \left(-\frac{e}{m} k_i \right) \langle \Psi^\dagger_{\vec{k}} \tau_0 \Psi_{\vec{k}} \rangle$$

ESSE VALOR MÉDIO DEVE SER OBTIDO EM ORDEM LINEAR EM $\vec{A}$

$$j_i^d = \frac{1}{V}\sum_{\vec{k}} \left(-\frac{e^2}{mc} A_i \right) \langle \Psi^\dagger_{\vec{k}} \tau_3 \Psi_{\vec{k}} \rangle$$

= JÁ ESTÁ EM ORDEM LINEAR EM A_i

PARA j_i^d, SÓ PRECISO $\langle \Psi^\dagger_{\vec{k}} \tau_3 \Psi_{\vec{k}} \rangle$ EM ORDEM ZERO

MAS: $\Psi^\dagger_{k} \tau_3 \Psi_{k} = \sum_\sigma a^\dagger_{\vec{k}\sigma} a_{\vec{k}\sigma} \Rightarrow \sum_{\vec{k}} \langle \Psi^\dagger_{k} \tau_3 \Psi_{k} \rangle = N$

$$\Rightarrow \boxed{j_i^d = -\frac{N}{V}\frac{e^2}{mc} A_i}$$

$$j_i^P = \frac{1}{V}\sum_{\vec{k}}\left(-\frac{e}{m}k_i\right)\langle \Psi_{\vec{k}}^{\dagger}\tau_0\Psi_{\vec{k}}\rangle$$

PARA O SISTEMA DESCRITO POR:

$$H = \sum_{\vec{k}} \Psi_{\vec{k}}^{\dagger}\left[\epsilon(\vec{k})\tau_3 + \frac{e}{mc}\vec{k}\cdot\vec{A}\,\tau_0 + \frac{e^2}{2mc^2}\vec{A}^2\tau_3 + \Delta\tau_1\right]\Psi_{\vec{k}}$$

PODE SER IGNORADO EM ORDEM LINEAR

$$H = \sum_{\vec{k}} \Psi_{\vec{k}}^{\dagger}\left[\epsilon(\vec{k})\tau_3 + \Delta\tau_1 + \frac{e}{mc}\vec{k}\cdot\vec{A}\,\tau_0\right]\Psi_{\vec{k}}$$

AGORA NOTAMOS QUE A CORREÇÃO $O(\vec{A})$ É PROPORCIONAL À IDENTIDADE NO ESPAÇO DE NAMBU. LOGO, A TR. DE BOGOLIUBOV NÃO É AFETADA POR ELA.

$$\Rightarrow H = \sum_{\vec{k}} B_{\vec{k}}^{\dagger}\left[\underset{\sim}{h}(\vec{k}) + \delta E(\vec{k})\mathbb{1}\right]B_{\vec{k}} \qquad \delta E(\vec{k}) = \frac{e}{mc}(\vec{k}\cdot\vec{A})$$

$$= \sum_{\vec{k}\sigma}\left[E(\vec{k}) + \delta E(\vec{k})\right] b_{\vec{k}\sigma}^{\dagger} b_{\vec{k}\sigma} \qquad E(\vec{k}) = \sqrt{\epsilon^2(k) + \Delta^2}$$

$$j_i^p = \frac{1}{V}\sum_{\vec k}\left(-\frac{e}{m}k_i\right)\langle \Phi_{\vec k}^\dagger \tau_0 \Phi_{\vec k}\rangle$$

$$= -\frac{e}{mV}\sum_{\vec k} k_i \langle B_k^\dagger B_{\vec k}\rangle = -\frac{e}{mV}\sum_{\vec k\sigma} k_i \langle b^\dagger_{\vec k\sigma} b_{\vec k\sigma}\rangle$$

$$= -\frac{2e}{mV}\sum_{\vec k} k_i\, f(E(k)+\delta E(\vec k))$$

$$\cong -\frac{2e}{mV}\sum_{\vec k} k_i\left[\cancelto{0}{f(E(\vec k))} + f'(E(\vec k))\,\delta E(\vec k)\right]$$

$$\boxed{j_i^p = -\frac{2e^2}{m^2 cV}\sum_{\vec k} k_i k_j A_j\, f'(E(\vec k))}$$

JUNTANDO TUDO:

$$j_i = j_i^p + j_i^d = -\frac{Ne^2}{Vmc}\left[\delta_{ij} + \left(\frac{\cancel{V}\,\cancel{mc}}{N\cancel{e^2}}\right)\frac{2\cancel{e^2}}{m^{\cancel{2}}\cancel{cV}}\sum_{\vec k} k_i k_j f'(E)\right]A_j$$

$$= -\frac{ne^2}{mc}\left[\delta_{ij} + \frac{2}{Nm}\sum_{\vec k} k_i k_j f'(E(\vec k))\right]A_j$$

$$\frac{2}{Nm}\sum_{\vec k} k_i k_j f'(E(\vec k)) = \frac{2}{Nm} V \underbrace{\int \frac{d^3k}{(2\pi)^3} k_i k_j f'(E(\vec k))}_{=0 \text{ SE } i\neq j}$$

$$= \frac{2}{nm}\delta_{ij}\int\frac{d^3k}{(2\pi)^3} k_i^2 f'(E(\vec k)) \qquad \text{(INDEPENDE DE } i\text{)}$$

$$= \frac{2}{3nm}\delta_{ij}\int \frac{d^3k}{(2\pi)^3} k^2 f'(E(\vec k))$$

$$E(\vec k) = \sqrt{\epsilon^2(k)+\Delta^2}$$

$$= \frac{\cancel{2}}{3nm}\,\frac{1}{\cancel{2}\pi^2}\,\delta_{ij}\int k^4\, dk\, f'(E(\vec k)) =$$

POR CAUSA DE $f'(x)$, A INTEGRAL É DOMINADA PELA REGIÃO $k \sim k_F$:

$$= \frac{k_F^4}{3\pi^2 nm}\delta_{ij}\int dk\, f'(E(\vec k)) \qquad \left(dk = \frac{m}{k_F}\, d\epsilon\right)$$

$$= \left(\frac{k_F^3}{3\pi^2}\right)\frac{1}{n}\,\delta_{ij}\int_{-\infty}^{+\infty} d\epsilon\, f'\left[\sqrt{\epsilon^2+\Delta^2}\right] = \delta_{ij}\int_{-\infty}^{+\infty} d\epsilon\, f'\left[\sqrt{\epsilon^2+\Delta^2}\right]$$

$$\vec{J} = -\frac{e^2}{mc} n \left[1 + \int_{-\infty}^{+\infty} d\epsilon\, f'\left[\sqrt{\epsilon^2+\Delta^2}\right] \right] \vec{A} \equiv -\underbrace{\frac{e^2}{mc} n_s(T)}_{\Gamma} \vec{A}$$

ONDE:

$$\boxed{\frac{n_s(T)}{n} = 1 + \int_{-\infty}^{\infty} d\epsilon\, f'\left[\sqrt{\epsilon^2+\Delta^2}\right]}$$

$$\lambda^{-2}(T) = \frac{4\pi}{c}\Gamma = \frac{4\pi e^2}{mc^2} n_s(T)$$

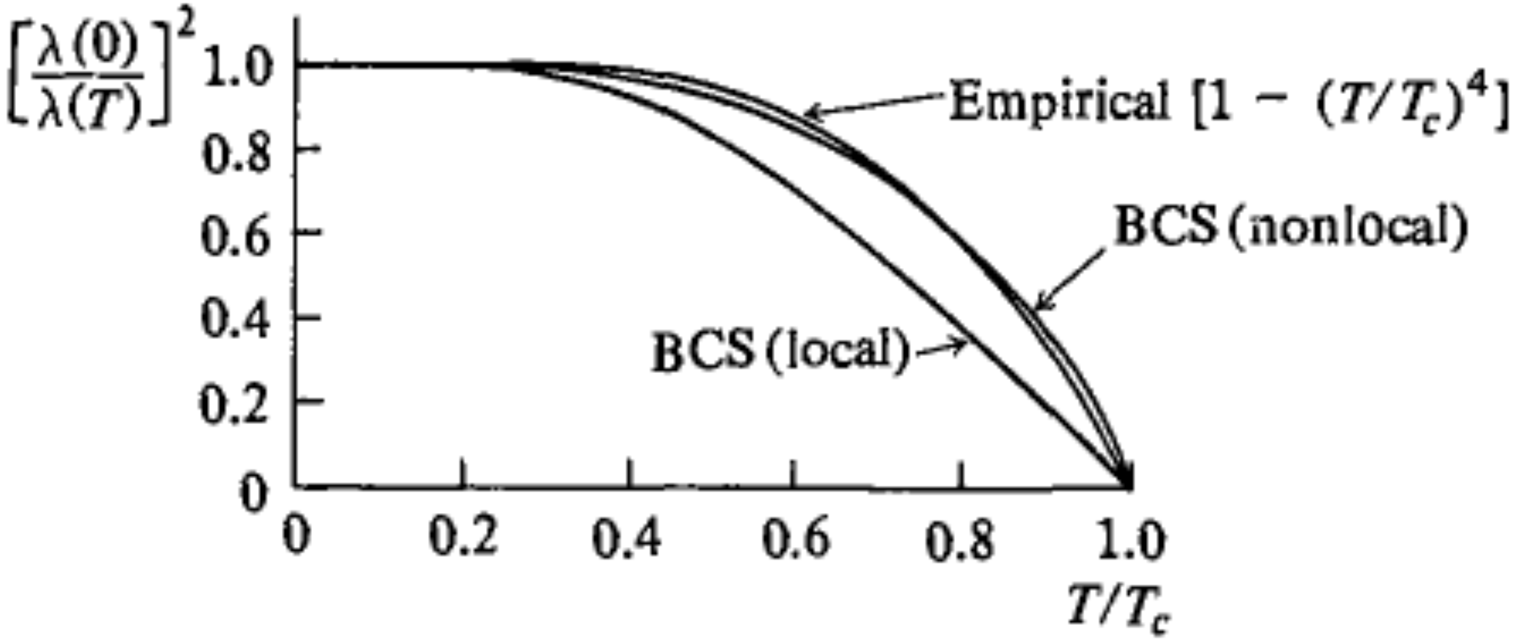
$T \simeq 0$: $f'(x) \to -\delta(x) \Rightarrow \int_{-\infty}^{+\infty} d\epsilon\, \delta\left[\sqrt{\epsilon^2+\Delta^2}\right] = 0$

$$\Rightarrow n_s(0) = n$$

$T \simeq T_c$: $\int_{-\infty}^{+\infty} d\epsilon\, f'\left[|\epsilon|\right] = \left[\int_{-\infty}^{0} + \int_{0}^{\infty}\right] d\epsilon\, f'(|\epsilon|) =$

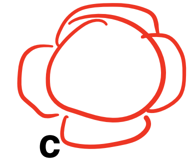
$$= \int_{+\infty}^{0} [-d\epsilon]\, f'(\epsilon) + \int_{0}^{\infty} d\epsilon\, f'(\epsilon) = 2\int_{0}^{\infty} f'(\epsilon)\, d\epsilon = 2\, f(\epsilon)\Big|_{0}^{\infty} = -1$$

$$\Rightarrow n_s(T_c) = 0$$



$\left[\frac{\lambda(0)}{\lambda(T)}\right]^2$
1.0
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
T/T_c
Empirical $[1 - (T/T_c)^4]$
BCS (nonlocal)
BCS (local)

$\Delta(\vec{k}) \propto (k_x^2 - k_y^2)$



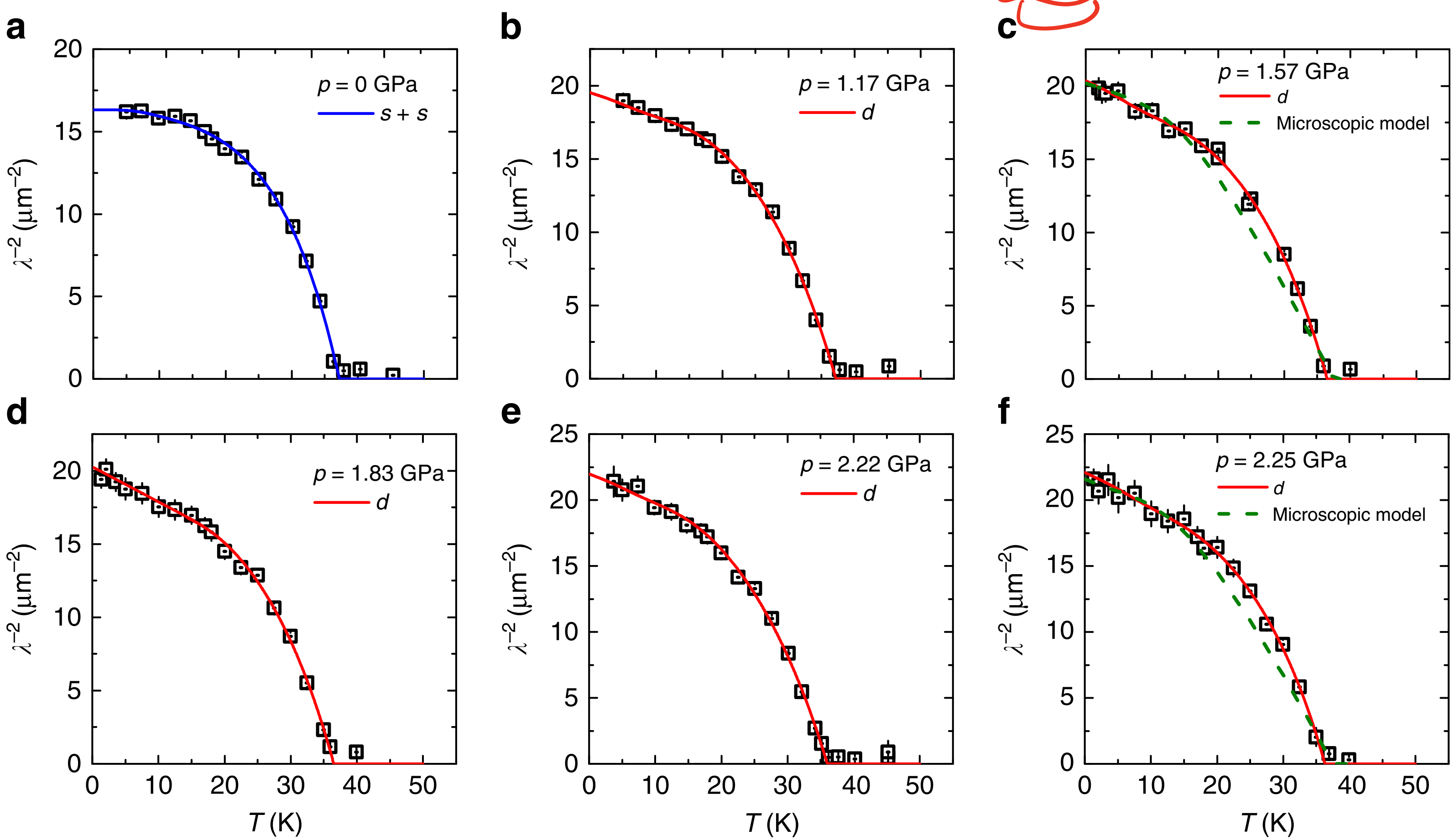


Figure 3 | Inverse-squared magnetic penetration depth. The temperature dependence of λ^{-2} measured at various applied hydrostatic pressures for $Ba_{0.65}Rb_{0.35}Fe_2As_2$. The solid line for $p = 0$ GPa corresponds to a two-gap *s*-wave model (**a**) and the solid lines for finite pressure represent a fits to the data using a multiband *d*-wave model (**b**–**f**). The dashed lines in **c** and **f** represent fits to the data using the microscopic model. The error bars are calculated as the s.e.m.

Direct evidence for a pressure-induced nodal superconducting gap in the $Ba_{0.65}Rb_{0.35}Fe_2As_2$ superconductor, Nat. Comm. **6**, 8863 (2015).

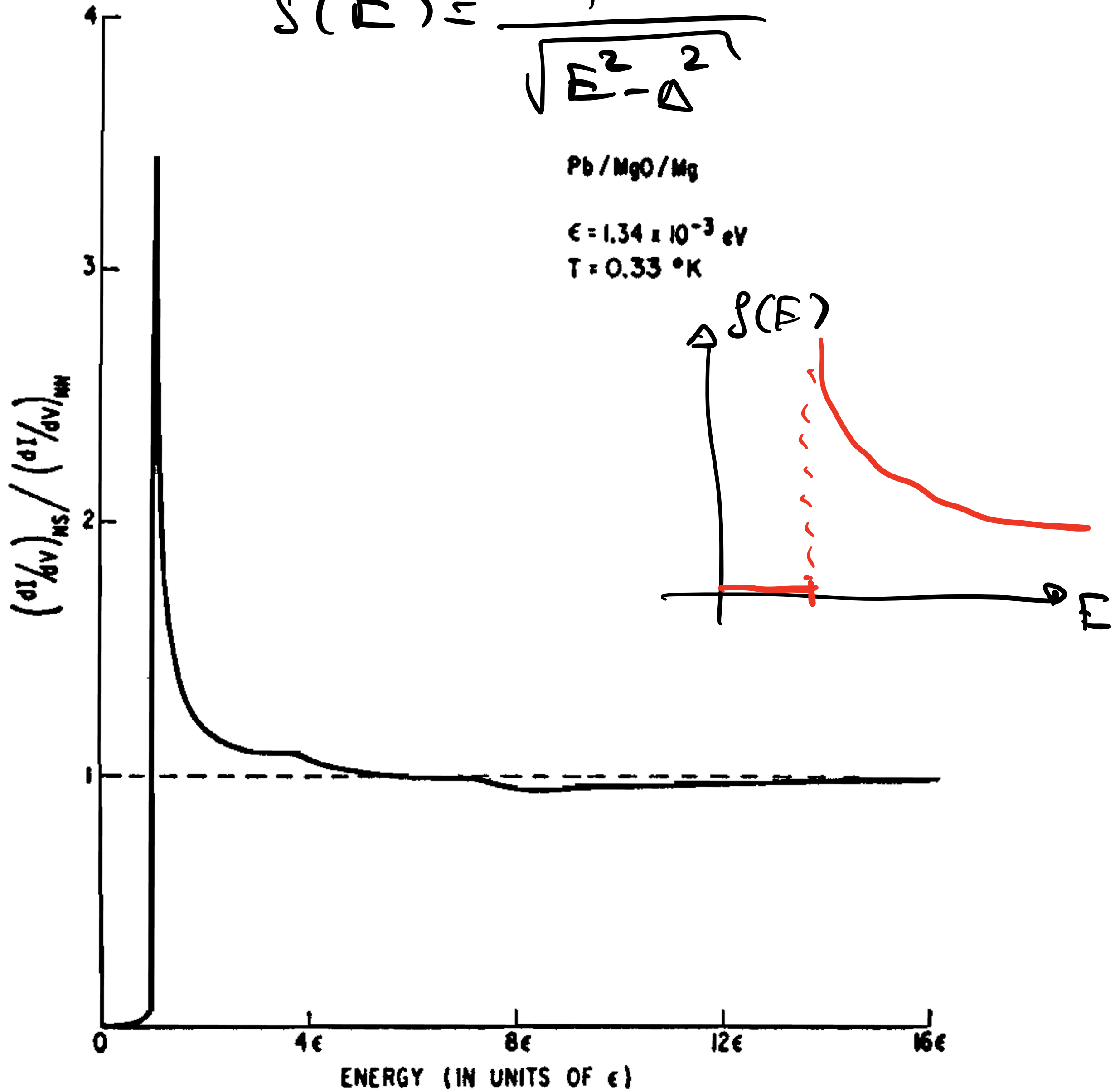
“The smoking gun”

Como sabemos que fônons são o mecanismo para os supercondutores clássicos?

$$\Delta \rightarrow \Delta(\omega)$$

$$\rho(E) = \frac{1}{\sqrt{E^2 - \Delta^2}}$$

Densidade de estados (determinada por experimentos de tunelamento)



$$\rho(\epsilon)\, d\epsilon = \rho(E)\, dE$$

$$\frac{d\epsilon}{dE} = \left(\frac{dE}{d\epsilon}\right)^{-1} = \frac{1}{\sqrt{\epsilon^2 + \Delta^2}}$$

Fig. 14. Differential conductance of a Pb–Mg junction vs. voltage, showing the superconducting density of states of Pb (*64*).

Densidade de estados: teoria X exp.

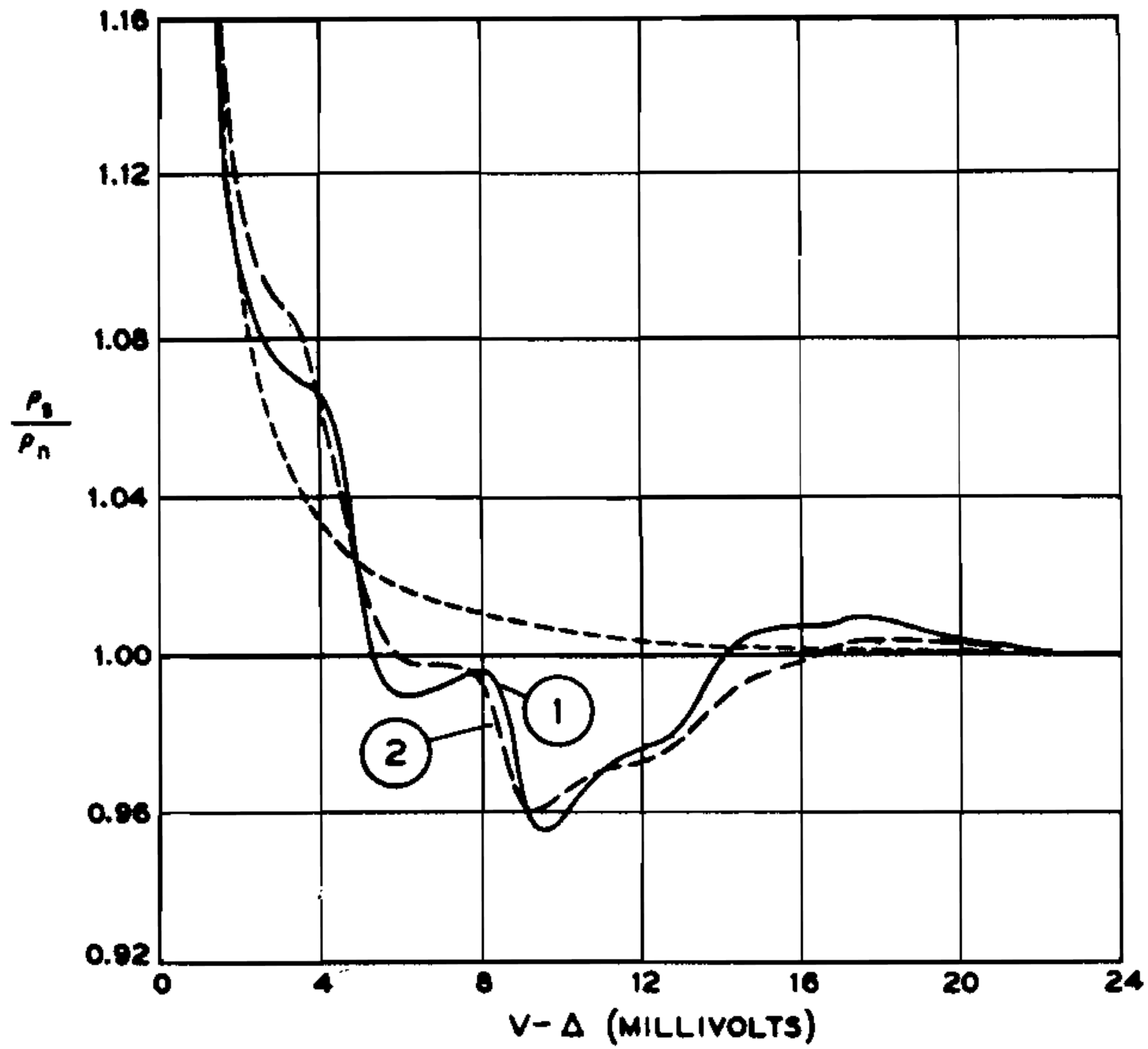


Fig. 15. Comparison of (1) the density of states vs. energy as calculated by Schrieffer et al. (solid line); (2) the measured tunneling characteristic (long-dashed line); and the BCS theory (short-dashed line) (*70*).

Outras propriedades

- Condutividade óptica: $\sigma(\omega)$
  - Gap supercondutor
- NMR:
  - Knight shift: gap supercondutor
  - $1/T_1$: pico de Hebel-Slichter
- Atenuação de ultra-som:
  - Gap supercondutor
- Tunelamento normal (SIN):
  - Densidade de estados
- Tunelamento SIS:
  - Efeito Josephson

Condutividade óptica

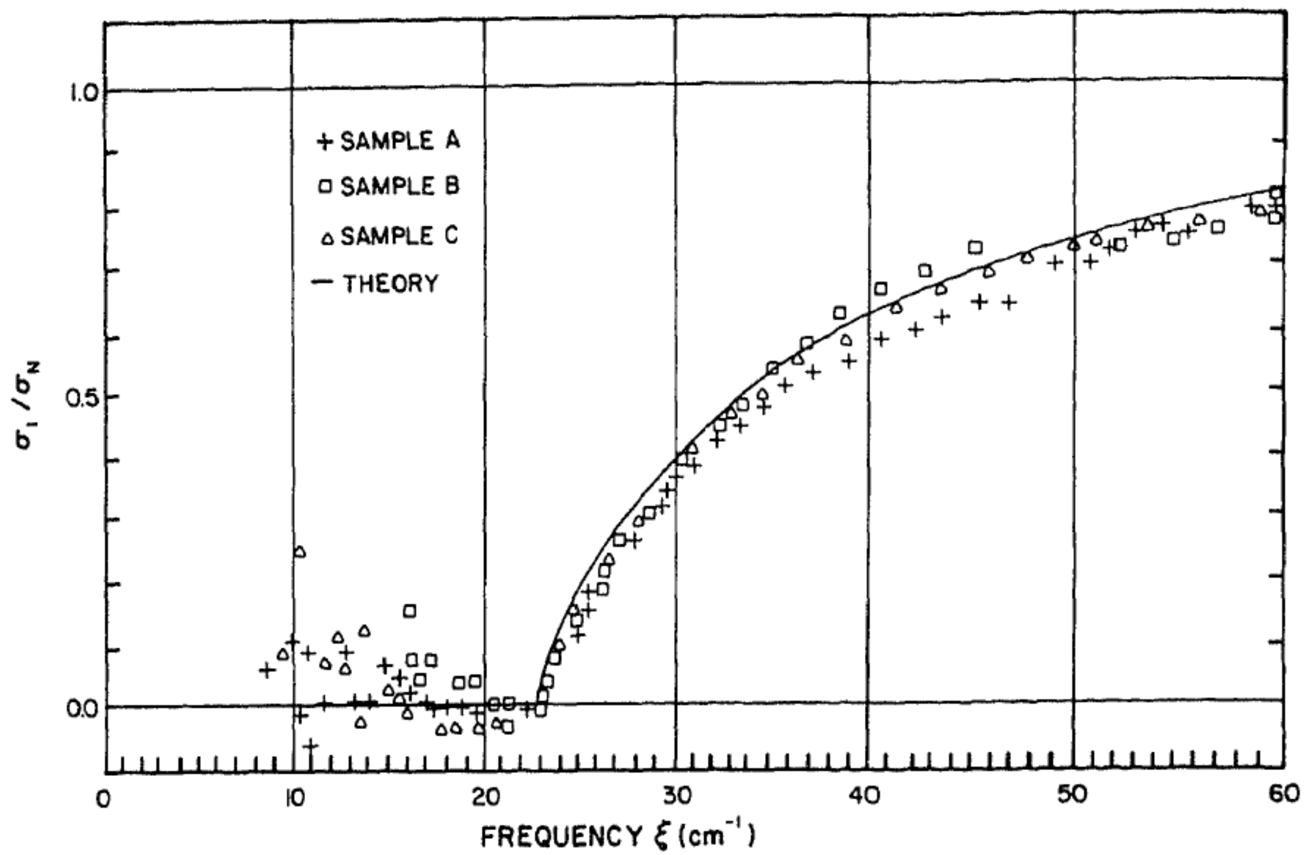


FIGURE 10.18 Far-infrared absorption in superconducting lead at 2 K. The solid line is BCS theory, and points are experimental. Source: Palmer and Tinkham (1968) (used with permission).

$1/T_1$ e atenuação de ultra-som

→ PICO DE HEBEL-SLICHTER

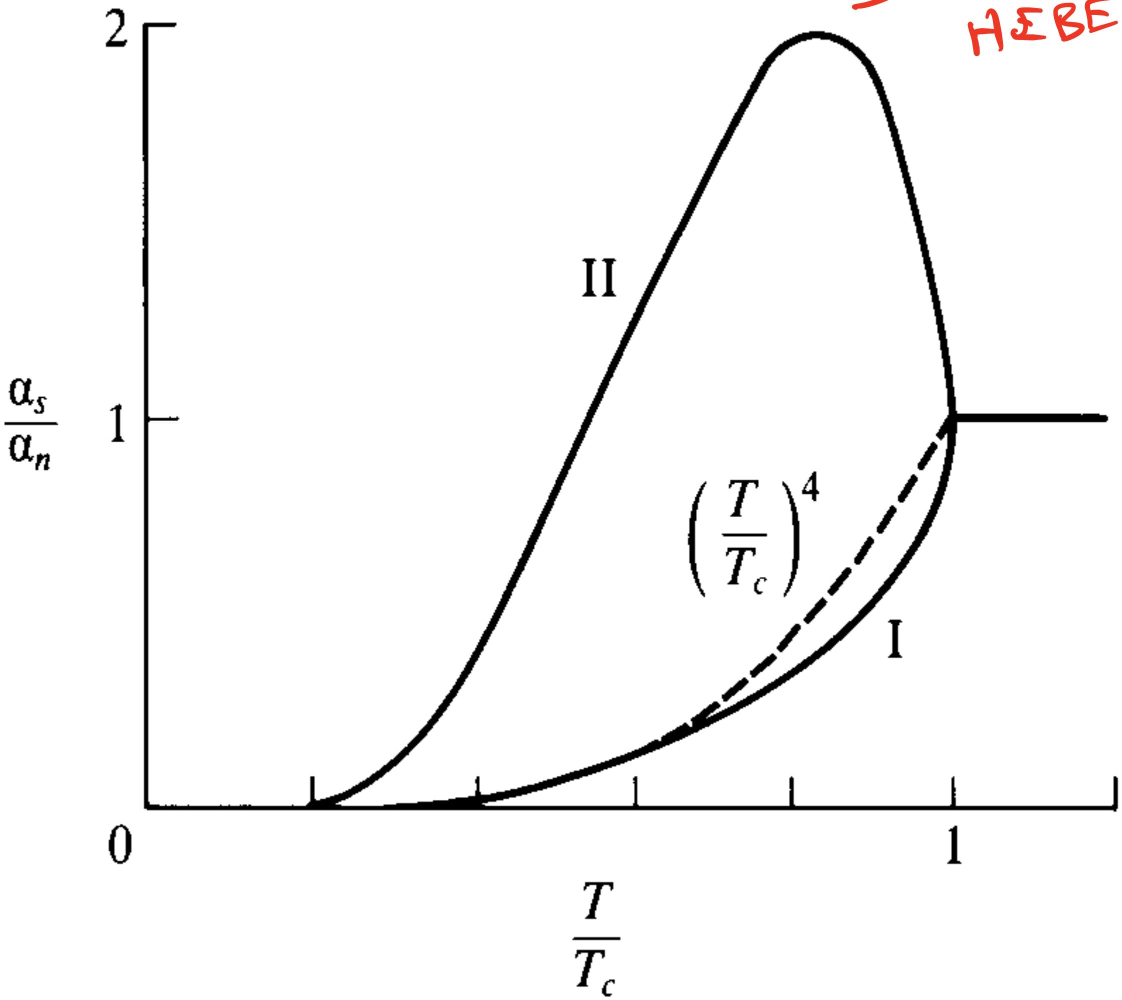
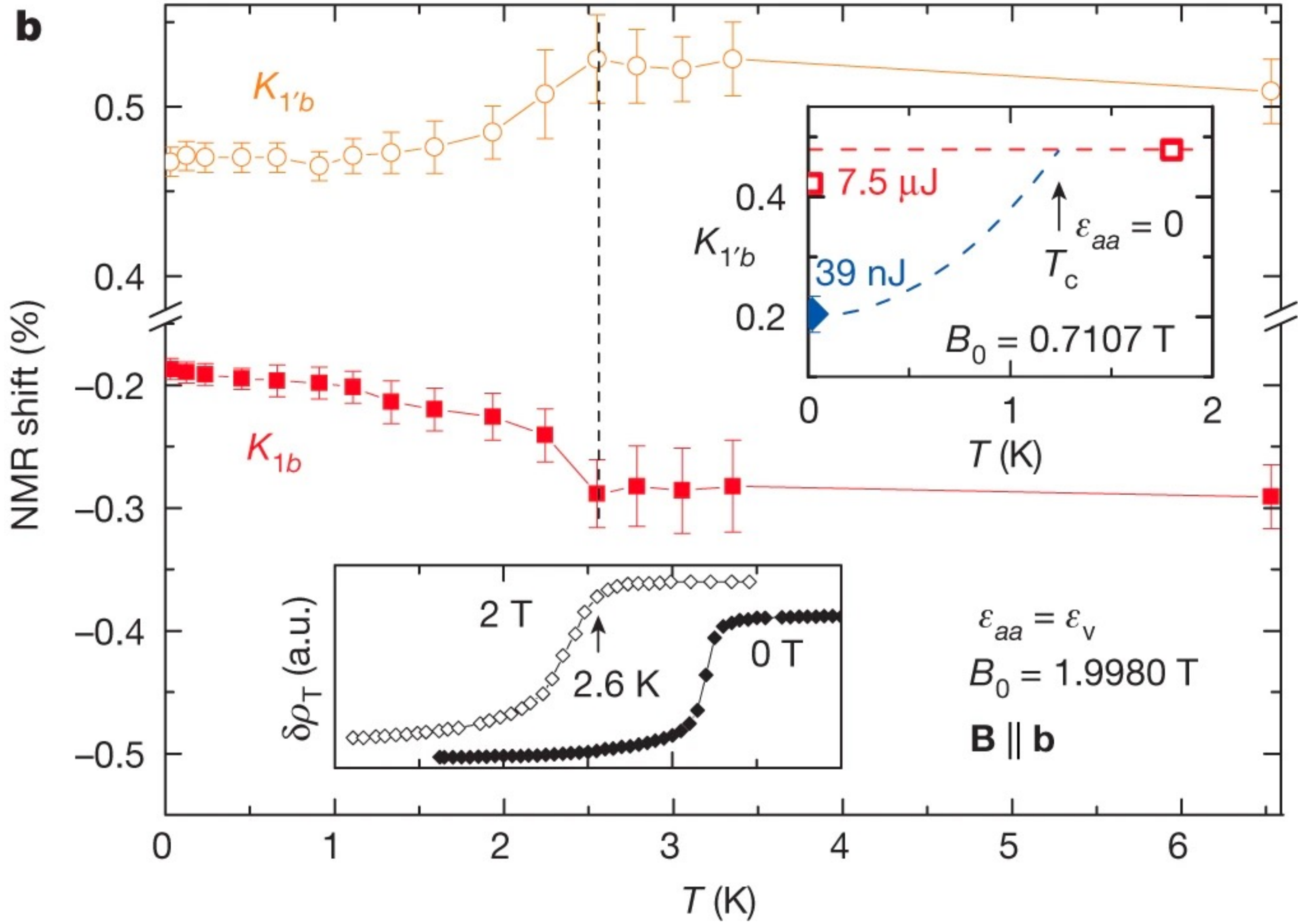


FIGURE 3.9
Temperature dependence of low-frequency absorption processes obeying case I and II coherence factors, compared with the $(T/T_c)^4$ dependence that might be expected for *all* processes from a simple two-fluid model. The curve for case I applies to ultrasonic attenuation, and it is a well-defined low-frequency limit. The curve for case II, which applies to nuclear relaxation or electromagnetic absorption, has no well-defined low-frequency limit unless gap anisotropy or level broadening is taken into account. The curve drawn here corresponds to a broadening of about $0.02\Delta(0)$.

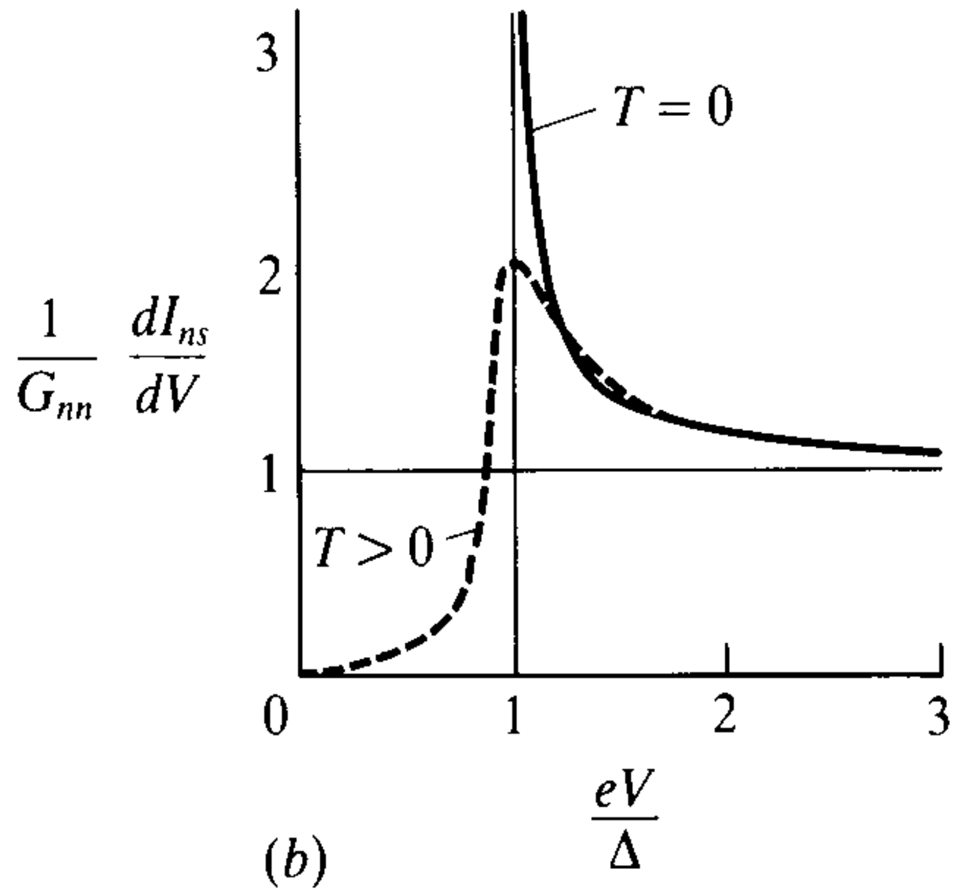
Knight shift

Há muito tempo que se supôs se tratar de um supercondutor tripleto, que não teria uma supressão do Knight shift abaixo de T_c.
O experimento ao lado indica supressão e, portanto, singleto.



Constraints on the superconducting order parameter in Sr_2RuO_4 from oxygen-17 nuclear magnetic resonance
A. Pustogow et al., Nature **574**, 72 (2019).

Tunelamento SIN



Outras propriedades

- Dedução da teoria de Ginzburg-Landau a partir de BCS (Gorkov)

$$F=\int d^3x\left\{\frac{1}{2m}\left|\left[\frac{\hbar\boldsymbol{\nabla}}{i}+2e\mathbf{A}\left(\mathbf{r}\right)\right]\Psi\left(\mathbf{r}\right)\right|^2+a\left|\Psi\left(\mathbf{r}\right)\right|^2+\frac{b}{2}\left|\Psi\left(\mathbf{r}\right)\right|^2+\frac{\mathbf{H}^2\left(\mathbf{r}\right)}{8\pi}\right\}$$

- Estado misto, rede de Abrikosov
- Quantização do fluxo magnético.