



# Non-Fermi liquid behavior as a consequence of Kondo disorder

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## Abstract

We describe a route based on disorder towards non-Fermi liquid behavior in Kondo alloys. Disorder in the  $f$ -parameters leads to a distribution of local Kondo temperatures,  $P(T_K)$ . When this distribution is such that  $P(0) \neq 0$ , the system exhibits non-Fermi liquid behavior, due to the presence of dilute low- $T_K$  spins which are unquenched at finite temperatures.

*Keyword:* Non-Fermi liquid system

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There are by now several  $f$ -electron metals which show non-Fermi liquid (NFL) behavior [1]. Typically, they show  $C_V(T)/T \sim \ln(\Gamma/T)$ ,  $\chi(T) \sim \ln(\Gamma/T)$  or  $T^{-\alpha}$  and  $\rho(T) \sim \rho_0(1 - T/T_0)$ . The origin of the NFL behavior in these alloys remains controversial.

In this article, we will show a route towards NFL behavior which relies on the disordered nature of these alloys [2]. We were motivated by the Cu NMR study of Ref. [3], which showed that the broad NMR lines in  $\text{UCu}_{5-x}\text{Pd}_x$  ( $x = 1$  and  $x = 1.5$ ) could only be explained by the presence of microscopic disorder of a short-range nature. A simple disorder model was then used to describe both the thermodynamics and the NMR line widths.

Our point of departure is a disordered Anderson lattice Hamiltonian

$$H = \sum_{ij\sigma} -t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + \sum_{j\sigma} E_j^\dagger f_{j\sigma}^\dagger f_{j\sigma} + \sum_{j\sigma} (V_j c_{j\sigma}^\dagger f_{j\sigma} + \text{h.c.}) + U \sum_j n_{fj\uparrow} n_{fj\downarrow}. \quad (1)$$

We will take  $U \rightarrow \infty$  in the remainder of the paper. We assume the parameters  $E_j^\dagger$  and  $V_j$  are distributed according to some general distribution

functions  $P_1(E^f)$  and  $P_2(V)$ , respectively, which are *not* assumed to be very broad.

We will focus on the dynamical mean field theory of correlations and disorder [4], which becomes exact in the limit of large coordination. The problem is then reduced to the solution of an *ensemble* of impurity problems in a self-consistently determined conduction electron bath  $\Delta(\omega)$ , which is defined by the following effective action and self-consistency condition (for a semi-circular density of states):

$$S_j^{\text{imp}} = T \sum_{\omega, \sigma} \left[ f_{j\sigma}^\dagger (-i\omega_n + E_j^f + V_j^2 \Delta(i\omega_n)) f_{j\sigma} \right], \quad (2)$$

$$\Delta(\omega) = \frac{1}{\omega + \mu - t^2 \bar{G}_c(\omega)}, \quad (3)$$

$$\bar{G}_c(\omega) = \left\langle \frac{1}{\omega + \mu - t^2 \bar{G}_c(\omega) - \frac{V_j^2}{\omega - E_j^f - \Sigma_j^f(\omega)}} \right\rangle^{\text{av}}, \quad (4)$$

where  $\mu$  is the chemical potential,  $t$  is the hopping amplitude,  $\bar{G}_c(\omega)$  is the self-averaged conduction electron Green's function,  $\langle \dots \rangle^{\text{av}}$  denotes the disorder average and  $\Sigma_j^f(\omega)$  is the  $f$ -self-energy under the action (2). Details of the equations will be given elsewhere [2].

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Each impurity problem (2) is governed by the Kondo temperature scale given, in the Kondo limit, by

$$T_{Kj} = De^{-1/\lambda_j} \quad (\lambda_j \equiv 2\rho_0 V_j^2 / |E_j^f|), \quad (5)$$

where  $\rho_0 = \text{Im } \Delta(0)$ . The distributions of f-parameters  $P_1(E^f)$  and  $P_2(V)$  will give rise to a distribution of Kondo temperatures  $P(T_K)$ . It can be shown that [2], to a high degree of accuracy, the thermodynamic response of the disordered lattice will be given by an average over the single-impurity results with  $P(T_K)$ . Due to the strong exponential dependence in Eq. (5), a moderately broad distribution of  $\lambda_j$ 's will give rise to a *rather broader* distribution  $P(T_K)$ . Fig. 1 depicts the  $P(T_K)$  appropriate for  $\text{UCu}_{5-x}\text{Pd}_x$  according to Ref. [3]. Note that  $P(0) \neq 0$  for both distributions. This has profound effects on the behavior of the system.

Let us consider for example the susceptibility  $\chi$ . For a single impurity, it obeys a scaling form  $T_K \chi(T) \propto f(T/T_K)$ , where [5]

$$f(x) \approx \begin{cases} \alpha - \beta x^2 & x \ll 1; \\ \frac{\gamma}{x} (1 - \frac{1}{\ln x}) & x \gg 1, \end{cases} \quad (6)$$

$\alpha$ ,  $\beta$  and  $\gamma$  being universal numbers. Since  $P(0) \neq 0$ , one can expand  $P(T_K) = P_0 + P_1 T_K + \dots$  for  $T_K$  up to an arbitrary cutoff  $\Gamma$ . Thus, taking the average

$$\begin{aligned} \langle \chi(T) \rangle^{\text{av}} &\propto \int_0^\infty \frac{dT_K}{T_K} P(T_K) f\left(\frac{T}{T_K}\right) \\ &\approx \int_0^{\Gamma/T} \frac{dy}{y} P_0 f(1/y) \sim \alpha P_0 \ln\left(\frac{\Gamma}{T}\right), \end{aligned} \quad (7)$$

since the integral is dominated by its upper limit. This log-divergent NFL susceptibility (and similarly for  $C_V(T)/T$ ) is observed in  $\text{UCu}_{5-x}\text{Pd}_x$ . As seen in Fig. 1, at a temperature  $T$ , there are always a few f-sites with  $T_K \ll T$ . While the remainder of sites have undergone quenching and effectively fallen out of the problem, these low- $T_K$  spins remain unquenched and dominate the low temperature behavior. The response is dominated by rare events at the tail of the distribution function, rather than by the average spin, a situation commonly called a Griffiths phase [6]. If  $P(0) = 0$ , all spins freeze out at some finite temperature and Fermi liquid behavior is recovered.

Whereas thermodynamic quantities are given by the average over single-impurity results, transport properties are more subtle. Indeed, the well-known onset of coherence at low temperatures, typical of clean

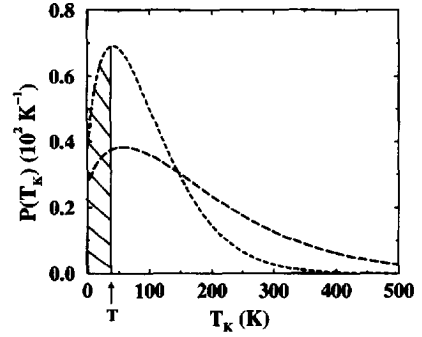


Fig. 1. Experimentally determined  $P(T_K)$  for  $\text{UCu}_{5-x}\text{Pd}_x$ :  $x = 1$  (long-dashed line),  $x = 1.5$  (dashed line) (from Ref. [3]). The shaded area below  $T$  represents the low- $T_K$  spins which remain unquenched at temperature  $T$ .

heavy fermion materials bears no resemblance to the incoherent single-impurity resistivity. However, the dynamical mean field theory is able to account for coherence in a natural way, while at the same time exhibiting incoherent scattering at intermediate and high temperatures [7]. Besides, sufficient disorder is able to completely destroy coherence in the low temperature transport [2]. Indeed, by using the slave boson mean field theory to solve the *ensemble* of impurity problems at  $T = 0$ , we have shown that the strong f-site correlations lead to an *enhancement* of the effective disorder experienced by the conduction electrons [2].

Once the disorder strength is large enough, all trace of coherence is destroyed. The resistivity is then a monotonically decreasing function of temperature, much like the behavior of a system of dilute Kondo impurities. But the temperature dependence can be non-trivial, especially at low temperatures. To see that, let us analyze the conduction electron self-energy in the dynamical mean field theory. It can be shown that [2]

$$\Sigma_c(\omega) = \frac{\langle T_j^{\text{imp}}(\omega) \rangle^{\text{av}}}{\bar{G}_c(\omega)(\omega + \mu - t^2 \bar{G}_c(\omega))}, \quad (8)$$

where  $T_j^{\text{imp}}(\omega)$  is the  $T$ -matrix of site  $j$ . At low temperatures, the temperature dependence of  $\Sigma_c(\omega)$  will determine the DC conductivity through the Kubo formula (within this approach there are no vertex corrections).

The temperature dependence of  $\Sigma_c(\omega)$  strongly depends on the structure of  $P(T_K)$  at low  $T_K$ 's. Fig. 2 shows the imaginary part of the  $T$ -matrix averaged

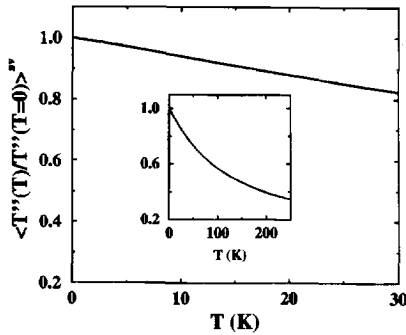


Fig. 2. Imaginary part of the single impurity  $T$ -matrix averaged with the  $P(T_K)$  for  $\text{UCu}_{3.5}\text{Pd}_{1.5}$ , from Ref. [3]. The inset shows the same quantity over a wider temperature range.

over disorder. The leading temperature dependence is linear and leads to a resistivity  $\rho(T) \approx \rho_0 - AT$ , as observed in  $\text{UCu}_{5-x}\text{Pd}_x$  [8]. The condition for NFL behavior is again  $P(0) \neq 0$ . As long as  $P(0) \neq 0$ , the linear behavior of Fig. 2 is observed. If  $P(0) = 0$  or negligible, Fermi liquid behavior is recovered. Again, the anomalous NFL behavior is due to the gradual

unquenching of low- $T_K$  spins. At a temperature  $T$ , spins with  $T_K \ll T$  enter their high temperature perturbative regime and effectively cease to contribute to the overall scattering. Since they form a *dilute* system of excitations at low  $T$ , their effect is additive and an average over their subtracted contribution can be performed.

In conclusion, we have presented a route to NFL behavior driven by Kondo disorder, whereby a dilute system of low- $T_K$  spins dominates the low temperature response and leads to the anomalous behavior.

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