Relatório Final de F809

Trilho de Ar

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1 Introdução

No início da década de 60 foram introduzidos dispositivos com colchão de ar [1] para o ensino das leis da mecânica física na ausência de atrito, que geralmente diminui consideravelmente a precisão das medidas de espaço e tempo. O registro dessas grandezas dos corpos em movimento tem sido feito geralmente com marcas pontuais deixadas por descargas elétricas periódicas de alta tensão em papel térmico do tipo empregado em fax, método que funciona muito bem exceto pela alta tensão que muitas vezes inibe iniciativas dos alunos na operação do equipamento.

2 Descrição

No presente projeto é apresentado um novo dispositivo de registro espaço-tempo do movimento linear e sem atrito de um carro sobre um trilho com colchão de ar. O princípio do dispositivo é o mesmo de uma impressora a jato de tinta acionado por um elemento piezoelétrico. Consta essencialmente de um gerador de pulsos elétricos periódicos e alimentado por uma pilha de nove volts. Estes pulsos atuam sobre uma lâmina piezoelétrica colada em uma lâmina metálica fina que flexiona com cada pulso recebido. Esta lâmina metálica fecha uma cavidade cônica cheia de tinta. Um orifício no vértice da cavidade cônica lança uma gotícula de tinta toda vez que o piezoelétrico recebe um pulso elétrico. Papel comum estendido logo acima do orifício registra o ponto deixado pela gotícula. O dispositivo registrador é portátil, cabendo inteiramente em um carro de 15 cm de comprimento, não havendo nenhum contato físico com as partes imóveis do soprador formador do colchão de ar.

Além de ser totalmente inofensivo pela ausência de alta tensão, este registrador permite usar cores diferentes para trajetória de cada carro, uma propriedade interessante para experimentos em que pode ocorrer mistura dos pontos de registro. O carro pode ser visto na figura 1.

O trilho consiste em duas lâminas em V com 1,2m de comprimento e possui cerca de 600 furos, distribuidos em 3 fileiras em cada lâmina e pode ser visto na figura 2. O soprador constitui em um aspirador de pó comum ligado reversamente.

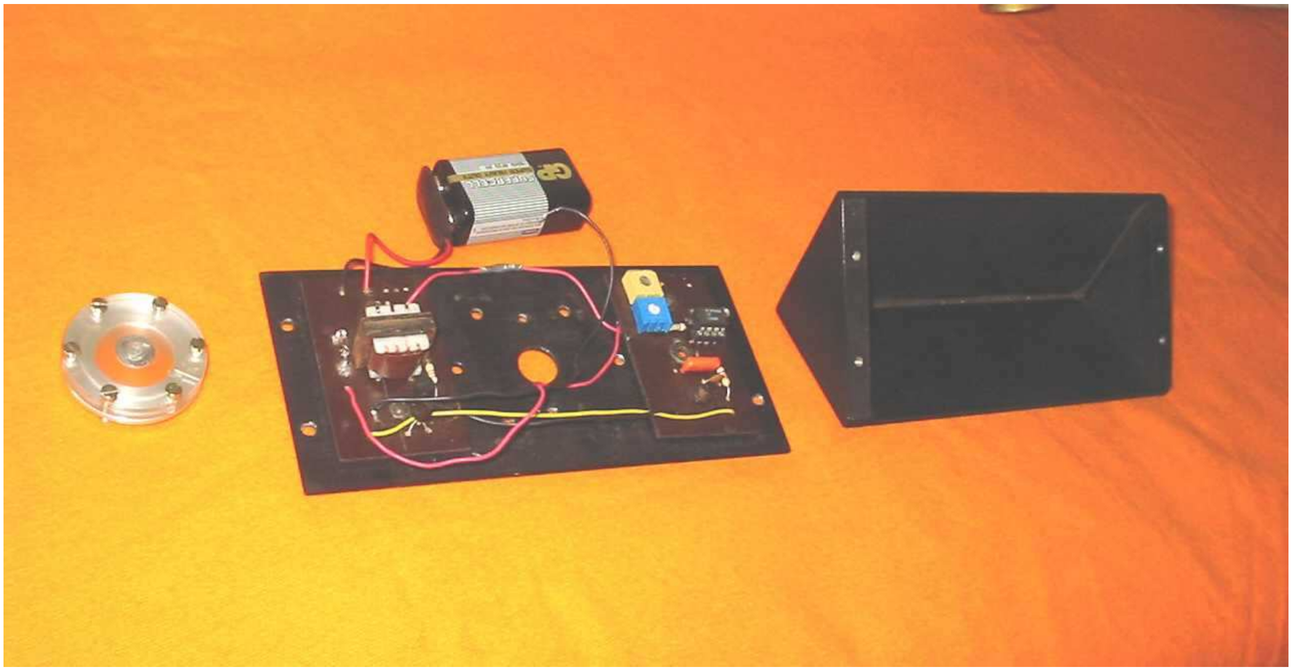


Figura 1 – Carro deslizante.

Figura 2 – Trilho de ar.

3 Experimento

Para testar o trilho de ar, podemos calcular a aceleração da gravidade e comparar seu valor com a literatura. Para tal podemos inclinar o trilho em um ângulo de θ graus, logo o carro sofrerá uma aceleração em módulo g.sen(θ), onde θ é o ângulo com a horizontal e g é a aceleração da gravidade. Como o jateador deixa uma marca no papel a cada 0,1 segundos, é possível obter um gráfico de posição versus tempo. Com a mecânica clássica temos que para uma aceleração constante, segue a seguinte relação:

$$x(t) = x_0 + v_0.t + a.t^2/2,$$

portanto ajustando os pontos do gráfico com uma função de segundo grau é obtido a/2 que no nosso caso é g.sen(θ)/2.

4 Resultados

O trilho foi posicionado com duas diferentes inclinações e foi solto o carrinho, marcando a folha de papel. Os resultados podem ser observados nas figuras 3 e 4.

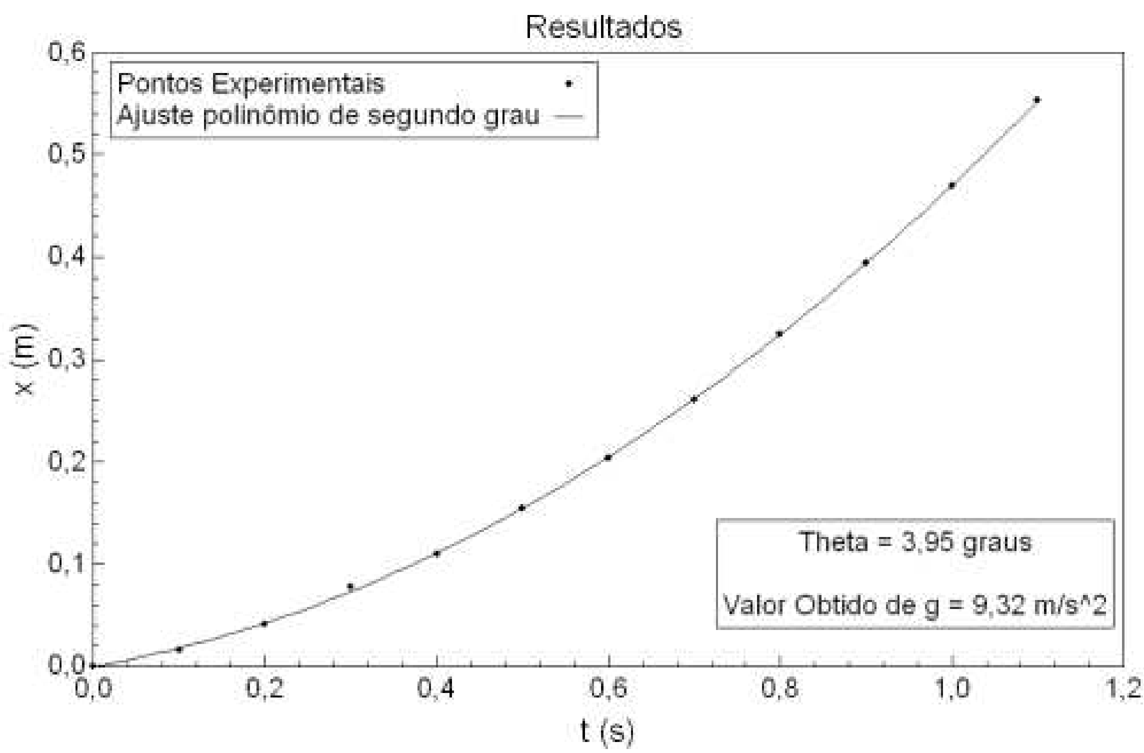


Figura 3 – Gráfico posição versus tempo, ajustado por um polinômio de grau 2.

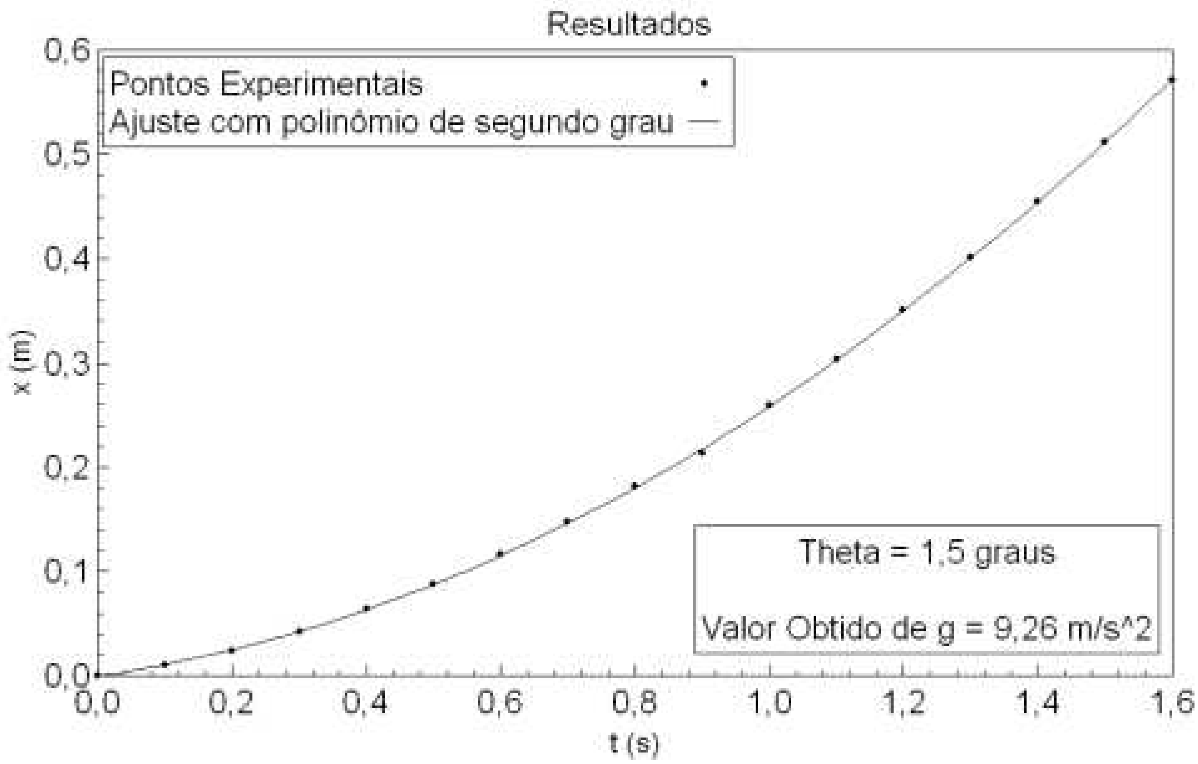


Figura 4 – Gráfico posição versus tempo, ajustado por um polinômio de grau 2.

Os valores estimados da aceleração da gravidade se mostraram razoáveis, em comparação com o valor da literatura de 9,8 m/s². Uma provável fonte de erro é o valor de ângulo utilizado, já que como o ângulo da inclinação do trilho foi baixo, um pequeno desnível da própria mesa onde o trilho estava apoiado influenciaria fortemente a estimativa da gravidade. Tal afirmação pode ser confirmada ao calcularmos para o valor obtido de “a” (o parâmetro ajustado) qual seria o ângulo que obteríamos g = 9,8 m/s² e obtivemos 3,70° quando inclinamos de 3,95° e obtivemos 1,4° quando inclinamos de 1,5°, ou seja, uma diferença de 0,3° para uma inclinação e 0,1° para outra.

Portanto para aumentar a precisão da medida e poder realmente estudar apenas o efeito do atrito no trilho de ar, se faz necessário o uso de uma inclinação mais acentuada, onde um erro da ordem de décimos de graus se faz desprezível.

Comentários do Coordenador do Curso

Projeto:
"Projeto aprovado-PA.Bom trabalho!"

Relatório Parcial:
"O RP não está disponível, aparece sim com um síbolo de formato texto, mas não vem se clicamos nele, tem de ser recolocado. Também, não está otíulo seguindo o formato das instruções: Nome do aluno seguido do nome do orientador+RP."

"Falta a figura XX citada, que daria apóio à afirmação de que algo foi feito, coloque-a logo."

"Nota 10."

Relatório Final:

Referências

[1] H.V. Neher and R.B. Leighton, American Journal of Physics 31, 255 (1963).

[2] http://educar.sc.usp.br/licenciatura/2002/trilho_de_ar/tda.htm

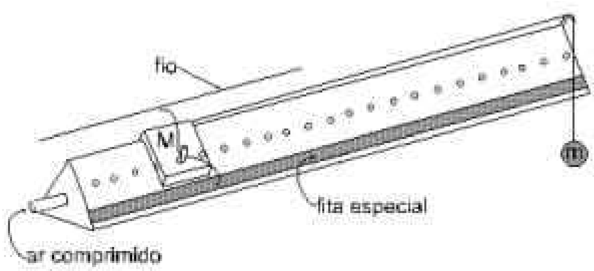
[3] http://lectureonline.cl.msu.edu/~mmp/kap6/cd157a.htm

[4] http://badger.physics.wisc.edu/lab/manual/node27.html

Referência [2]

TRILHO DE AR

Nesta análise de movimento utilizaremos um trilho de ar. Este consiste de um trilho com orifícios laterais por onde o ar, proveniente de um compressor, escapa. O colchão de ar que se forma impede o contato entre as superfícies, eliminando o atrito. Também utilizaremos um carrinho de massa M = 417,7g, um corpinho de massa m = 57,13g e outra m1 = 12,76g, um sistema de roldanas por onde passa o fio que se prende ao carrinho e ao corpinho, montando o sistema mostrado na figura abaixo.



Usaremos um carrinho de massa $\mathcal{M}_1$ *puxado por um objeto de massa muito pequena (m) se comparada com* $\mathcal{M}_1$.

Procedimento: A massa m é segura de modo que o sistema fique em repouso. Libere o sistema marcando a posição do carrinho em função do tempo. Como a variação do sitema é constante, um gráfico de vxt deve ser uma reta da qual pode-se obter a aceleração. Através do programa SAM marcamos a posição do carrinho em instantes sucessivos.

Veja a fotografia estroboscópica do carrinho se movimentando no trilho de ar. Observe as sucessivas posições do carrinho e o movimento uniformemente variado ao longo da trajetória.

Resultados obtidos: A tabela a seguir apresenta as posições do carrinho em instantes sucessivos marcados através do programa SAM.

S (cm)	t (s)	Δ S(cm)	Δ t (s)	V (cm/s)	Δ V (cm/s)	a (cm/s^2)
0	0					
		3.19	0.2	15.95		
3.19	0.2				15.6	78
		10.31	0.2	38.55		
9.55	0.4				23	115
		10.91	0.2	54.8		
20.46	0.6				20.2	101
		15	0.2	75		
35.46	0.8				20.45	102.25
		19.09	0.2	95.45		
54.55	1				22.9	113.5
		23.63	0.2	118.15		
78.18	1.2					

Veja os gráficos do S x t, V x t e a x t. Observe o comportamento do movimento uniformemente variado, com a aceleração constante e em torno de 101.95 cm/s^2.

O valor da aceleração média calculado a partir da tabela já convetida em metros é 1,02 m/s^2. Tendo o valor da aceleração e sabendo a massa do carrinho ($M = 0{,}4177kg$) e do corpinho ($m = 0{,}05713kg$), podemos calcular a tração no fio. Lembrando que o fio é inextensível e de massa desprezível e que o trilho de ar, praticamente, acaba com o atrito.

Temos duas equações para calcular a tensão no fio. Uma equação é obtida analisando as forças que atuam sobre o carro $T = M \cdot a$ e a outra é analisando as forças que atuam no corpinho $P - T = m.a$. Escolhendo uma das equações encontraremos o valor da tensão no fio. Vamos escolher a primeira

$$T = M.a = 0.4177 \cdot 1{,}02 = 0{,}43N$$

O valor da aceleração calculado experimentalmente, é $1{,}02\ m/s^2$ e o valor teórico será calculado logo abaixo:

Para obter essas equações observe a figura no ínicio da página, veja as forças que atuam no corpo m e no corpo M. As equações destes corpos são as seguintes: $T = M \cdot a$ (1) $P - T = m.a$ (2) somando essas duas equações temos que $P = a\,(M + m) = mg = a\,(M + m)$ $a = mg/M + m$ Onde g é aceleração da gravidade e seu valor é $9{,}81\ m/s^2$ logo, $a = 1{,}17\ m/s^2$

Comparando o valor teórico com o valor experimental temos um erro relativo da ordem de 10%. Essa diferença de valores ocorreu devido a erros de medida do experimentador durante a calibração.

Referência [3]

Virtual Air Track

So far, we have only studied some limited cases of totally elastic or totally inelastic collisions. Actually, there are many other cases that you can construct by varying the masses and initial velocities of the two colliding carts on the air track. To enable you to play with some of these, we have constructed the ***virtual air track*** applet below.

(Neste local há uma simulação em Java, que não pode ser transportada no relatório).

This applet simulates collisions in one dimension. To run a simulation select one of the scenarios in the drop down menu and hit ***Go***. Press ***Stop*** at any time to pause the simulation. Pressing ***Go*** will restart it at that point. To create your own simulation, choose that option from the drop down menu, and two windows will appear that will allow you to edit the carts' properties. You may also edit their initial velocities and masses in the appropriate fields at the top, drag the carts to any position on the track, and edit α, a measure of the elasticity of a collision. An α value of 1 indicates an elastic collision, in which no kinetic energy is lost. Balls on a billiard table exhibit collisions that are nearly elastic. An α value of zero is a perfectly inelastic collision, in which the two colliding objects stick together after colliding. Colliding blobs of putty would exhibit perfectly inelastic collisions. Much of the energy in such a collision is converted to internal vibrations or heat.

Referência [4]

MC-15b Simple Harmonic Motion and Resonance (Air Track)

Launch Expt. I-II   Launch Expt. IV   Launch Expt. V   Launch Expt. VI

OBJECTIVES:

1. To study the period of Simple Harmonic Motion (SHM) as a function of oscillation amplitude.
2. To study the period of SHM as a function of oscillating mass.
   Expected result: Period($\equiv T = 2\pi/\omega = 1/f$)
   is proportional to $\sqrt{\text{Mass/Spring Constant}(k)}$
3. To demonstrate Hooke's Law, $F = -kx$
4. To observe the relationships between the potential and kinetic energy

THEORY:

The restoring force (F) on an object attached to a ``simple'' one-dimensional spring is proportional to the displacement from equilibrium and has the form, $F = -k(x - x_0)$, where k is the spring constant (or stiffness in N/m), x_0 is the equilibrium position (i.e., no net force) and x is the position of the object. This is Hooke's Law. Remember that the simple harmonic oscillator is a good approximation to physical systems in the real world, so we want to understand it well. That's the purpose of this lab!

The expression $F = ma = -k(x - x_0)$ is a 2nd order differential equation with

$$a = \frac{d^2x'}{dt^2} = -\frac{k}{m}(x')$$

where $x' \equiv x - x_0$. The most general solution for this expression is often given as

$$x'(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

where $\omega_0 \equiv \sqrt{k/m}$. ω_0 is defined as the *natural frequency* for the *undamped* harmonic oscillator. A and B are arbitrary initial displacement parameters. Alternatively the solution is often specified as

$$x'(t) = C\sin(\omega_0 t + \phi) \qquad \text{or} \qquad x'(t) = C\cos(\omega_0 t + \phi)$$

where ϕ is the starting phase and C is the displacement. It is also possible to write down these solutions using complex numbers as in

$$x'(t) = A' e^{i\omega_0 t} + B' e^{-i\omega_0 t} \quad .$$

The relative merit of these equivalent expressions will become clearer in EXPTS. V and VI.

FUNDAMENTAL CONCEPTS:

1. The solution to the undamped (i.e., no frictional or drag forces) harmonic oscillator is time dependent and periodic. When the $\omega_0 t$ term varies by 2π (or one period $T = 1/f = 2\pi/\omega_0$) both the position, $x'(t+T) = x'(t)$, and velocity, $v'(t+T) = v'(t)$ return to their previous values.
2. Total energy (TE) is conserved in SHM motion. As time evolves kinetic energy (KE) is transferred to (and from) potential energy (PE). Thus at any time t:

$$TE = \text{constant} = KE(t) + PE(t) = \frac{1}{2}m[v'(t)]^2 + \frac{1}{2}k[x'(t)]^2 \quad .$$

APPARATUS:

Basic equipment: Air track, assorted slotted masses, air supply, hose, adjustable stop, glider, springs, timer, photogate & support stand, knife edge assembly.

Computer equipment: Personal computer set to the MC15A lab manual web-page; PASCO interface module; photogate sensor and extension jack, PASCO sonic position sensor, speaker with driver stem, power amplifier module.

PRECAUTIONS:

1. See MC-14b

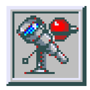
2. **When setting up the springs, use the adjustable end stop on the air track so the springs are stretched properly: don't stretch them beyond their elastic limit and don't let them sag and drag on the track**.
3. *Keep the amplitudes small enough that a slack spring doesn't touch the track nor a stretched spring exceed its elastic limit*.

SUGGESTIONS: To measure the period of the oscillating glider:

1. Locate the photogate so the glider just cuts off beam when glider is in the equilibrium position ($x - x_0 = 0$). The photogate phone jack should be in the first PASCO interface position.

   Choose two springs having a similar length and refer to the above precaution. Turn on the air supply adjust the blower speed to minimize frictional forces.

2. To initiate the PASCO interface software click the computer mouse when centered on the telescope icon in the ``toolkit'' area below. There will be a just a single table for recording the measured period.

 Calculator Unit conversion  aunch PASCO applicatio

3. Start the glider by displacing it from equilibrium and then releasing it. Then start the data acquisition by clicking the REC button button. Let the glider oscillate for about 10 periods. Calculated the mean and standard deviation by simply clicking on the statistics icon (i.e. Σ) on the data table.

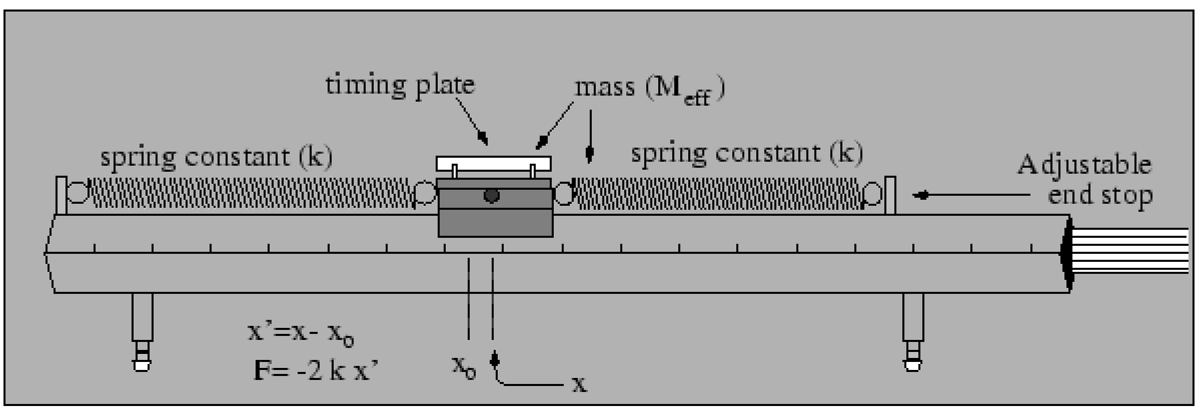


Figure 1: Sketch of the air track configuration for SHM.

UNDAMPED SIMPLE HARMONIC MOTION:

1. Using sketches in your lab book, both at equilibrium and after a displacement from equilibrium, show that the effective force constant for 2 identical springs of force constant k on either side of an oscillating mass is $2k$.

2. **EXPERIMENT I:** Show experimentally that the period is independent of the amplitude. Try amplitudes of *approximately* 10 cm, 20 cm, and 30 cm. (Friction may be a problem at very small amplitudes).

3.

EXPERIMENT II: By adding mass to the glider, study the period versus total oscillating mass. The latter must include a correction for the oscillating springs whose effective oscillating mass (see the note below) is approximately m_s/3 where m$_s$ is the mass of the *two* springs. Explain, in words, how this correction may be qualitatively justified (Why not m_s or $m_s/2$?) By inspection of your T vs $[M+(m_s/3)]$ curve, what function of T might yield a straight line when plotted versus $[M+(m_s/3)]$? Prepare this plot. Calculate the effective force constant, k', (equal to k_1+k_2, the two springs will actually differ slightly) of the system from the slope of this straight line graph. (Remember $T=2\pi\sqrt{M_{\rm eff}/k'}$ where $M_{\rm eff}\simeq$ M + m$_s$/3.) Estimate the uncertainty in k' by solving for k' at each glider mass and calculating the standard deviation of the mean.

NOTE: Theoretically $m_{\rm eff}=m/3$ only for $M/m_s=\infty$ For $M=0$, the effective mass is $4m/\pi^2=0.405$ m. However for M/m_s = 5, the effective mass is already $\simeq 0.336\ m_s$. See Fig. 1 of J.G. Fox and J. Makanty, *American Journal of Physics*, **38**, 98 (1970).

4.

EXPERIMENT III : Also measure k directly by hanging the two springs vertically on a ``knife edge'' assembly in the laboratory. Record the stretch produced by a series of weights, *but do not exceed the elastic limit of either spring!* Graph F vs y for each spring and obtain a best-fit straight line. The slope should be the spring constant. Compare this value of k' with that obtained in part #3.

5.

QUESTION: Assuming the spring constant doubles, how would T vary?

FURTHER INVESTIGATIONS OF ``UNDAMPED'' SIMPLE HARMONIC MOTION:

Up to this point you have characterized the SHM of a spring-mass assembly in terms of only a single parameter T (the period). One of the possible time-dependent expressions for describing the motion was

$$x'(t)=A\cos(\omega_0 t+\phi_0) \quad \text{and} \quad v'(t)=-A\omega_0\sin(\omega_0 t+\phi_0)$$

or

$$v'(t) = A\omega_0 \cos(\omega_0 t + \phi_0 - \frac{\pi}{2})$$

where A is the amplitude (displacement from equilibrium), ω_0 is the *natural* angular frequency and ϕ_0 is the starting phase. Notice that the velocity has a *phase* shift of $\frac{\pi}{2}$ relative to the displacement.

Equally characteristic of SHM is the process of energy transference: kinetic energy of motion is transferred into potential energy (stored in the spring) and back again. Friction is an ever present energy loss process so that the total energy always diminishes with time.

To capture this rather rapid cyclic process we will again use the PASCO interface while replacing the photogate sensor with the sonic position sensor.

EXPERIMENT IV: Measuring the x vs t behavior:

1. Remove the 10 cm timing plate and plug the aluminum vane into the central banana plug position with the vane perpendicular to the long axis of the glider. The mass of this arrangement will now have changed slightly; predict the new natural frequency.

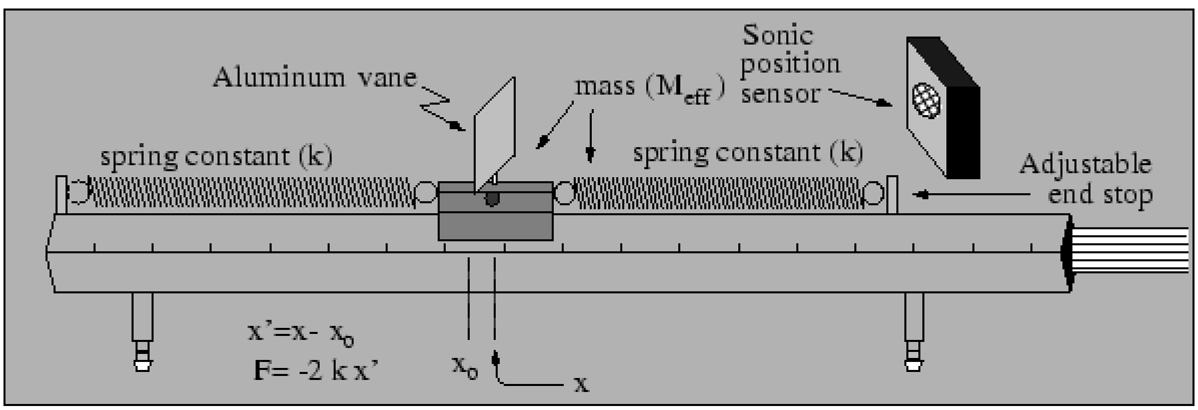


Figure 2: Sketch of the air track with the position sensor.

2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help (or raising the vane up slightly as well).

3. CLICK on the telescope icon below to initiate the PASCO © interface software.

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4. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition. The graph will simultaneously display both absolute position and velocity versus time.

5. Practice a few times to make sure you can obtain smoothly varying sinusoidal curves. Then run

the data acquisition for just over ten cycles and use the cross-hair feature to read out the time increment for ten full cycles. How does your prediction check out? Determine the initial phase (i.e., at $t=0$). Record the equilibrium position (x_0) as well.

6. Use the magnification option of the PASCO software to better view a single full cycle by clicking on the magnifying glass icon (in the graph window) and then select two points in the graph using a CLICK and DRAG motion of the mouse.
7. Print out (click in graph region and then type ALT, CTRL-P) or, alternatively, sketch the position and velocity curves in your lab book identifying key features in the time dependent curves. In particular identify the characteristic(s) which demonstrate the $\pi/2$ phase difference between the velocity and displacement curves.
8. Using the PASCO cross-hair option to read out the relevant time, position and velocity, make a table as below and fill in the missing entries (identify units).

time	phase($\omega_0 t + \phi_0$)	$x'(t)$	$v'(t)$	KE	PE	TE
	0					
	$\pi/4$ (45 deg)					
	$\pi/2$ (90 deg)					
	$3\pi/4$ (135 deg)					
	π (180 deg)					

9. Is the total energy a constant of the motion?
10. How much displacement amplitude and energy are lost after five full cycles? What is the approximate friction coefficient?

OPTIONAL INVESTIGATIONS OF ``UNDER-DAMPED" SIMPLE HARMONIC MOTION:

In the real world friction is an ever present process. In the case of SHM friction friction can have a profound effect. Damping of unwanted vibrations is important in a myriad of situations. (Imagine what driving a car would be like if there were no shock absorbers!)

Introducing friction can be done by simply adding one more term in the force expression, $F_{\text{drag}} \equiv -Rv$, a drag force which is proportional to the velocity where R is the drag coefficient [units of kg m/s or N/(m/s)] . This is appropriate for motion thru a viscous fluid but it is really only a rough approximation for the frictional forces in the air track. The modified force expression now becomes

$$F = ma = -kx' - Rv' \quad \text{or} \quad 0 = \frac{d^2x'}{dt^2} + \frac{R}{m}\frac{dx'}{dt} + \omega^2 x'$$

Since energy is continually lost solutions of this expression will be *time dependent* but NOT periodic. Adding this ``simple" term dramatically complicates the process of finding appropriate solutions. The most general form of the solution is

$$x' = e^{-(R/2m)t} \left[Ae^{+\sqrt{(\frac{R^2}{4m^2} - \omega_0^2)}\;t} + Be^{-\sqrt{(\frac{R^2}{4m^2} - \omega_0^2)}\;t} \right]$$

which is quite formidable. Since the air track drag is low (i.e., R is relatively small) the solution is said to be *underdamped* and oscillatory when $\omega_0^2 > R^2/4m^2$. The solution in this case becomes:

$$x' = Ce^{-(R/2m)t} \; \cos(\omega_1 t + D) \quad \text{where} \quad \omega_1 \equiv \omega_0^2 - \frac{R^2}{4m^2}$$

where ω_1 is the *natural frequency* of the *underdamped* system and C and D are the initial displacement and phase. Because the frequency is lowered, the period lengthens. This is consistent with one's intuition; drag works against oscillatory motion.

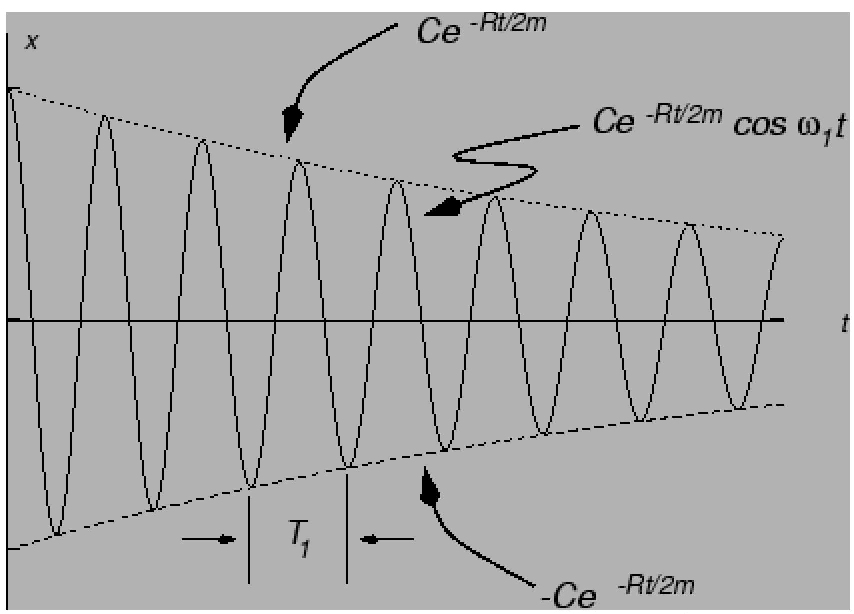

Figure 3: Sketch of underdamped harmonic motion with $\omega_0^2 > R^2/4m^2$.

EXPERIMENT V:

1. Reduce the air flow to the air track so that the amplitude of the glider motion diminishes by 50% (or so) in a few minutes. Typically the lowest blower setting will work well enough. (Make sure the other group is also ready.)

2. CLICK on the telescope icon below to initiate the next PASCO © interface application. In addition to the graph there will be a ``two" column table displaying the time and position then the time and velocity.

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3. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition and record enough cycles to see the amplitude diminish by two-thirds.
4. Repeat the experiment at full blower speed for approximately the same period of time. Can you distinguish the difference between ``ω_0" and ω_1 (T_0 and T_1)?

   HINT: The graph display for the position is configured to overlay the data sets. Use the magnifying glass icon feature to examine the relative phase difference at early time and late time.
5. If so, from the formula $\omega_1^2 = \omega_0^2 - R^2/4m^2$ determine R.
6. For the reduced air flow data, select six representative times using the table (identifying where the velocity changes sign) and make a table of t vs maximum displacement. Use these points in the graphical analysis package and fit these points to the expression $C \exp -(R/2m)t$ or, in terms of the explicit analysis formula, $y = C * \exp[-B * (x - x_0)]$.
7. Does this value compare favorably with the results of item 4? How good or poor is the assumption that the drag force is proportional to the velocity?
8. Sketch out an approximate curve of x vs t if R were significantly larger. Which R would be more appropriate for absorbing and dissipating a physical ``shock" (and why)?

OPTIONAL INVESTIGATIONS OF RESONANCE:

One of the most important situations of the harmonic oscillator is that of FORCED, damped harmonic motion. In one-dimension the applied force is typically sinusoid and when ω approaches the natural frequency of the system (nominally ω_1 and, if $R^2/4m^2 \ll 1$, also ω_0) the energy of the driver is additively coupled to the moving mass (e.g., a glider) and resonance occurs. Resonance is a *very* important aspect of the world around us and many mechanical and electronic devices employ resonant behavior as a fundamental aspect of their operation (e.g., musical instruments, radios, televisions).

Along with the frictional drag (Rv' where R is the drag coefficient) one more force term must be added, that of the mechanical driver, with $F_{\text{driver}} = F_d \ \cos(\omega t)$. The new force expression is conventionally written as:

$$F = ma = -kx' - Rv' + F_d \cos\omega t \quad \text{or} \quad \frac{F_d}{m}\cos\omega t = \frac{d^2x'}{dt^2} + \frac{R}{m}\frac{dx'}{dt} + \omega^2 x' \ .$$

Since the system energy is lost through friction and may be gained through the driver action, solutions of this expression will be *time dependent* but with both *transient* and *steady-state* attributes. In many instances resonant systems respond so quickly that one only views the steady-state behavior. In this lab you will be able to observe BOTH the *transient* and *steady-state* processes.

As you may expect the most complete solution of this new differential equation has a rather complicated form and so is not reproduced here. Since we are interested only in resonance we can simplify the expression by assuming solutions that apply to the *underdamped case* (those with oscillatory behavior). Thus the analytic solution reduces to:

$$x' = Ce^{-(R/2m)t}\ \cos(\omega_1 t + D) + \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}}\cos(\omega t - \phi)$$

where the first term is the *transient* behavior, identical to that of the simple damped harmonic oscillator (described in the last section) and the second term is the *steady-state* solution. At large times t the first terms dies out exponentially so that x' is approximated by only

$$x'((R/2m)t \gg 1) = \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}}\cos(\omega t - \phi)$$

where ω_0 is the natural frequency of the undamped harmonic oscillator, ω is the mechanical driver frequency, R is the drag coefficient and ϕ is a measure of the phase difference between the driver motion and the glider motion.

In this lab we will only investigate the nature of the glider displacement with driver frequency (ω) in the vicinity of the resonant frequency. Thus the only relationship of interest becomes $x' \propto [m^2(\omega_0^2 - \omega^2)^2 + \omega^2]^{-1/2} \equiv Z^{-1/2}$. Z is a minimum when the driver frequency is set to $\omega_0^2 - R^2/(2m^2)$ or $\omega_1^2 - R^2/(4m^2)$ which is defined to be ω_2. Since $R^2/2m^2$ is small (the air-track is a low friction experiment) ω_0 is nearly the *same* and, in addition, x' will be sharply peaked about ω_0.

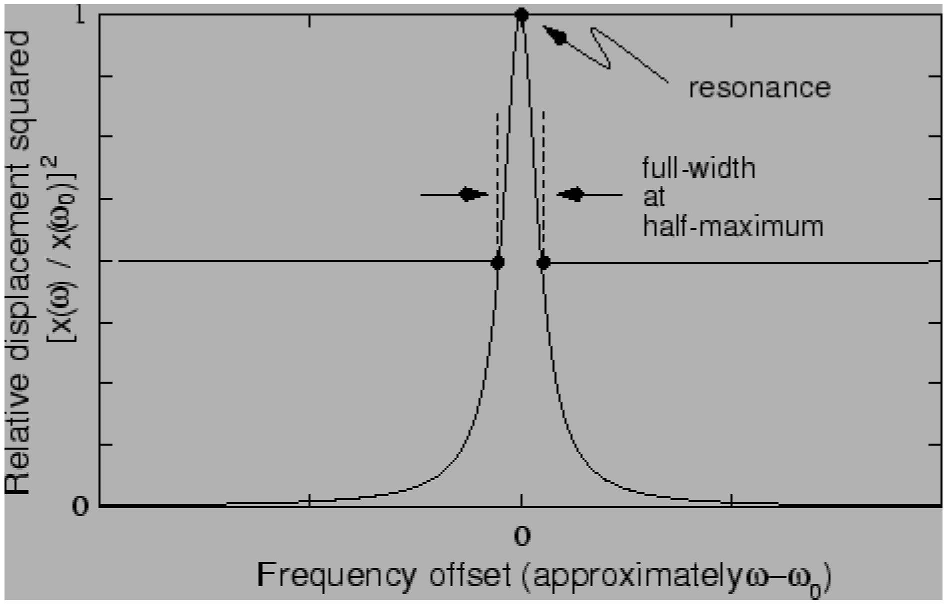

Figure 4: Sketch of the relative maximum displacement squared vs driver frequency.

SUGGESTED PROCEDURE:

1. **EXPERIMENT VI:** Make sure the 10 cm timing plate is replaced by the aluminum vane (in the center banana plug position) with the vane perpendicular to the long axis of the glider. Predict the natural frequency for this new arrangement.

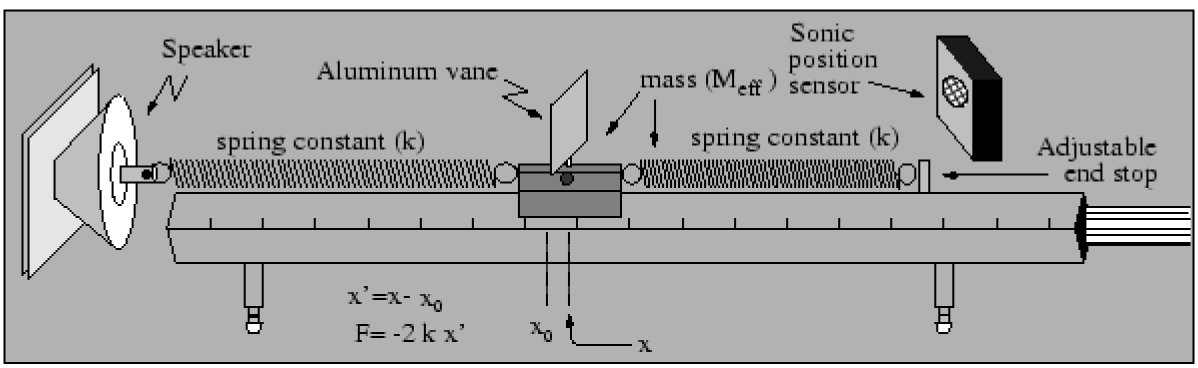

Figure 5: Sketch of the air track with the position sensor and speaker.

2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help (or raising the vane up slightly as well).
3. Detach the fixed spring end stop and place the speaker as shown in the figure above with the spring looped through the small slot in the speaker driver stem using the same considerations for the spring extension as in the previous experiments.
4. Make sure the speaker power leads are plugged into the amplifier module and that its power is turned on. Also verify that the DIN-9 pin connector (from the amplifier module) is plugged into the A position in the PASCO interface module.
5. CLICK on the telescope icon below to initiate the PASCO interface software.

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6. Make sure the speaker driver window has been switched to the **Off** position. With maximum blower airflow, displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition. The graph will simultaneously display glider position and velocity versus time and the amplifier current (which should be zero). Practice a few times to make sure you can obtain smoothly varying sinusoidal curves.

7. Reduce the air flow to the air track so that the amplitude of the glider motion diminishes by 50% (or so) in a few minutes. Typically the lowest blower setting will work well enough. (Make sure the other group is also ready.)

8. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition and record enough cycles to see the amplitude diminish by 90%. Compare your data to Fig. 3 and verify that your glider has a similar transient behavior. Determine the *natural frequency* of the system and compare to your prediction. (This frequency is actually ω_1 but it is, for this low friction set-up, nearly the same as either ω_2 or ω_0.) To observe the steady-state properties of resonance you will have to wait for times *longer* than those required for this step.

9. Moving to the PASCO Signal Generator window (shown in Fig. 6), set the amplifier frequency (which is in Hertz or cycles per second) to the closest to resonance (0.01 Hz steps) and engage the speaker but CLICKing the **Auto** icon. The driver output should use the sinusoid AC waveform and, if the overload light on the Power Amplifier flashes on, reduce the voltage setting slightly. Start the data acquisition (and speaker motion) by CLICKing on the REC button and record data until you achieve steady-state behavior.

   NOTE: The nominal step sizes for adjusting the amplifier frequecy and voltage are *very* large. To reduce the step size depress either the CTRL key (1 Hz), the ALT key (0.1 Hz), or both (0.01 Hz) while simultaneously CLICKing the Up or Down arrows.

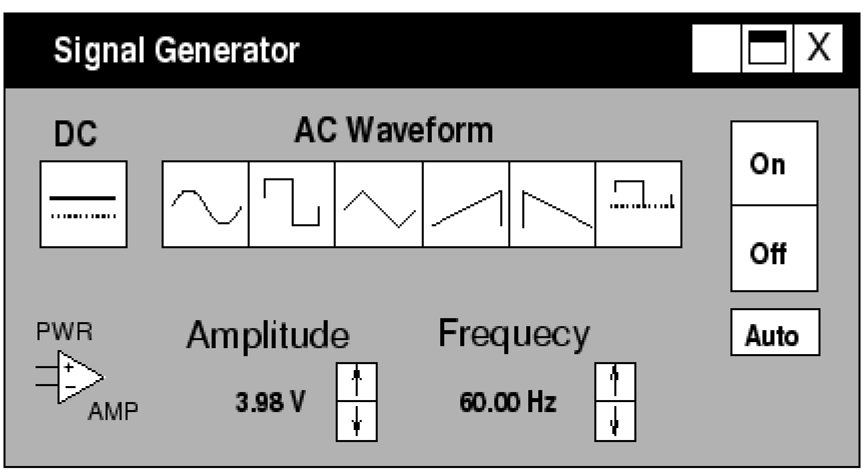

Figure 6: Sketch of the PASCO software Signal Generator window.

10. Set the amplifier frequency 0.01 Hz steps above and below resonance and record data until you achieve steady-state behavior. Repeat for 0.10 Hz and (if time permits) 0.40 Hz steps. Plot out a

few representative sets of data.

11. Determine the maximum steady-state displacement of the glider for each of the measured frequencies and plot the relative amplitude squared [($x'(\omega)/x'(\omega_0))^2$] vs frequency offset ($\omega - \omega_0$). Estimate the full width at half maximum for this curve. This value should be equal to R/m.

12. Discuss the nature of this resonance curve. If you adjust the R/m ratio to further sharpen the resonance curve can you identify a compensating complication if you are interested in achieving steady-state behavior? If the speaker were attached to an amplifier playing audible music (nominally 20-20,000 Hz), what do you think this the mass will do?

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