

F 641

P1 Prof. Lunazzi

04.09.01

**P1.1)** A nitidez de uma radiografia pode ser aumentada:

- a) reduzindo o tamanho da fonte.
- b) afastando a fonte.

Verifique se ha algum aumento na intensidade disponível em algum desses dois casos quando posicionamos de maneira a obter sempre o mesmo valor do circulo de nitidez.

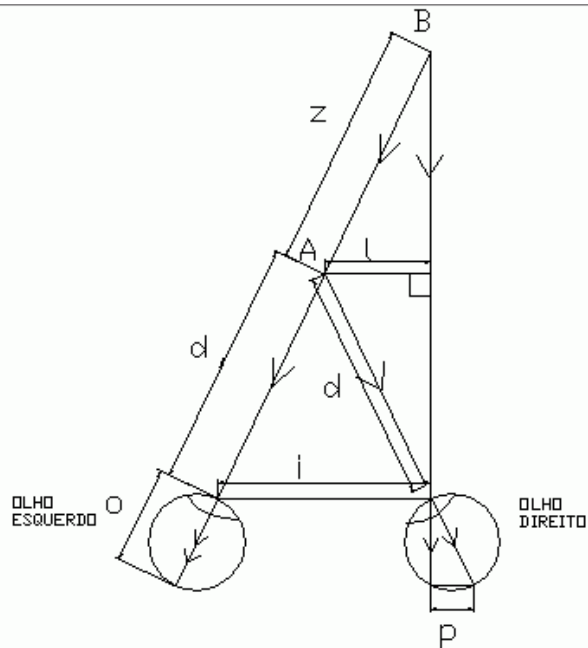
**P2.1)**

A figura mostra a situação dos olhos de um observador frente a um objeto tridimensional composto dos pontos luminosos A e B, separados pela distância  $z$  e à distância  $d$  do observador. As imagens desses pontos na retina do olho esquerdo, pelo acaso na posição do olho, coincidem no mesmo ponto. Mas no olho direito, estão separadas pela distância  $p$ . Obtenha o valor de  $p$  em função dos dados indicados na figura:  $z, o, i, d$  Para se familiarizar melhor com os elementos da figura correspondendo à situação real, podemos chamar de:

$i$  = distância interpupilar

$p$  = paralaxe

$o$  = diâmetro do globo ocular



**P1.3)** A definição de imagem real diz que é aquela para a qual os raios estão fisicamente se encontrando na posição da imagem. Pode uma imagem virtual estar dentro de uma lente, mesmo que os raios que atravessam a lente vão se encontrar com os raios virtuais da imagem?

**P1.4)** A figura 5.112 foi tomada de um texto de física. O que tem de errado?.

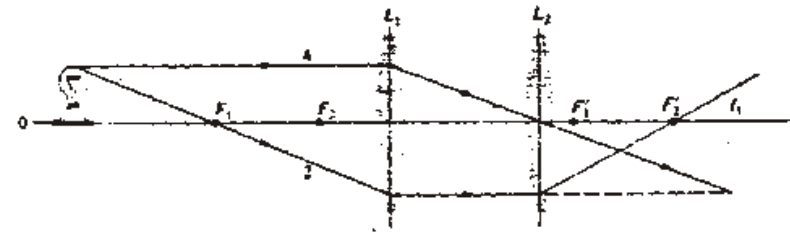


Fig. 5.112

**P1.5)** Encontre a distância focal e a localização dos pontos principais de uma lente "menisco" de focal positiva com raios 6, 10, espessura 3 e  $n = 1,5$ . Uma lente chamada "menisco" é aquela onde os raios de curvatura tem o mesmo sentido, como uma lente de contato.

**P1.6)** Derive a condição de estabilidade para o ressonador óptico assimétrico, eq. 10.33 do Fowles. Veja o texto do Fowles a respeito.

**10.5 Periodic Lens Waveguides and Optical Resonators**

The method of ray matrices will now be employed to analyze the behavior of a ray that passes through a series of many lenses. Also, since a curved mirror of radius  $r$  is optically equivalent to a lens of focal length  $f = r/2$ , the same analysis applies to a ray that is reflected back and forth between two curved mirrors. For simplicity we consider the symmetrical case in which the lenses (mirrors) are identical, of focal length  $f$ , and uniformly spaced a distance  $d$  apart as shown in Figure 10.7.

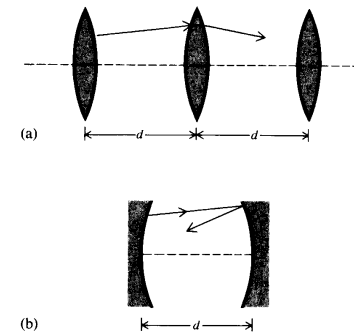


Figure 10.7. Symmetrical periodic lens waveguide (a), and optical resonator (b).

Consider a ray that starts out at the surface of one lens or mirror, travels a distance  $d$  to the next lens or mirror, and is refracted by the lens or reflected by the mirror. The transformation of the ray is then given by the following equation:

## 10.5 · PERIODIC LENS WAVEGUIDES AND OPTICAL RESONATORS

$$\begin{bmatrix} \rho_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \rho_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & d \\ -\frac{1}{f} & 1 - \frac{d}{f} \end{bmatrix} \begin{bmatrix} \rho_1 \\ \theta_1 \end{bmatrix} \quad (10.24)$$

At this point we consider the following question: Is there any initial ray vector such that the output ray vector is equal to the initial ray vector multiplied by a constant factor? In other words, do there exist any solutions of the equation

$$\begin{bmatrix} \rho_2 \\ \theta_2 \end{bmatrix} = \lambda \begin{bmatrix} \rho_1 \\ \theta_1 \end{bmatrix}$$

The same question could be asked in the case of any optical system having an overall ray matrix as expressed by Equation (10.22). The mathematical problem is one of finding solutions to the equation

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \rho_1 \\ \theta_1 \end{bmatrix} = \lambda \begin{bmatrix} \rho_1 \\ \theta_1 \end{bmatrix}$$

or, equivalently,

$$\begin{bmatrix} M_{11} - \lambda & M_{12} \\ M_{21} & M_{22} - \lambda \end{bmatrix} \begin{bmatrix} \rho_1 \\ \theta_1 \end{bmatrix} = 0 \quad (10.25)$$

We have here the same sort of mathematical problem as that discussed earlier in Chapter 2 in connection with the eigenvectors of Jones matrices. Solutions of Equation (10.25) define the eigenvectors optical system. The secular determinant is

$$\begin{vmatrix} M_{11} - \lambda & M_{12} \\ M_{21} & M_{22} - \lambda \end{vmatrix} = 0 \quad (10.26)$$

Returning to the problem of the symmetrical resonator or periodic waveguide, the secular determinant is

$$\begin{vmatrix} 1 - \lambda & d \\ -\frac{1}{f} & 1 - \frac{d}{f} - \lambda \end{vmatrix} = 0 \quad (10.27)$$

which reduces to

$$\lambda^2 + \lambda \left( 2 - \frac{d}{f} \right) + 1 = 0$$

Let us introduce the abbreviation  $\alpha = 1 - d/2f$ . The roots of the above quadratic equation are then found to be

$$\lambda = \alpha \pm \sqrt{\alpha^2 - 1} = e^{\pm i\phi} \quad |\alpha| > 1 \quad (10.28)$$

or

$$\lambda = \alpha \pm i \sqrt{1 - \alpha^2} = e^{\pm i\phi} \quad |\alpha| < 1 \quad (10.29)$$

in which  $\phi$  is a real number.

## 304 RAY OPTICS

Now suppose that a given ray vector is an eigenvector of the system, and that this ray passes through a number  $N$  of repeated reflections or refractions in the optical system. The final output ray vector is then

$$\begin{bmatrix} \rho_N \\ \theta_N \end{bmatrix} = \lambda^N \begin{bmatrix} \rho_1 \\ \theta_1 \end{bmatrix} \quad (10.30)$$

It is clear that the ray trajectory will remain *stable* or stay close to the axis of the optical system if the condition expressed by Equation (10.29) holds, because in that case  $\lambda^N = e^{\pm iN\phi}$  and  $|\lambda^N| = 1$ . In the *unstable* case of Equation (10.28) the trajectory will diverge. In terms of the optical parameters the stability criterion  $|\alpha| < 1$  gives

$$0 < d < 4f \quad (10.31)$$

or in the case of the optical resonator

$$0 < d < 2r \quad (10.32)$$

The focal length must be positive (converging lens or mirror) and the separation must be less than four times the focal length, or twice the radius of curvature of the resonator mirrors. In the *confocal* configuration,  $d = 2f = r$ . This satisfies the stability requirement and is often the configuration employed for laser resonators.

For the unsymmetrical optical resonator or periodic waveguide consisting of two mirrors of focal length  $f_1$  and  $f_2$  separated by a distance  $d$ , or by alternate lenses of focal length  $f_1$  and  $f_2$  all separated by equal distances  $d$ , a similar procedure to that outlined above yields the following stability criterion:

$$0 < \alpha_1 \alpha_2 < 1 \quad (10.33)$$

in which

$$\alpha_1 = 1 - \frac{d}{2f_1} \quad \alpha_2 = 1 - \frac{d}{2f_2} \quad (10.34)$$

The calculation is left as an exercise. See problems 10.6 and 10.7.