F-315 (Mecânica Geral I)
Aula 11

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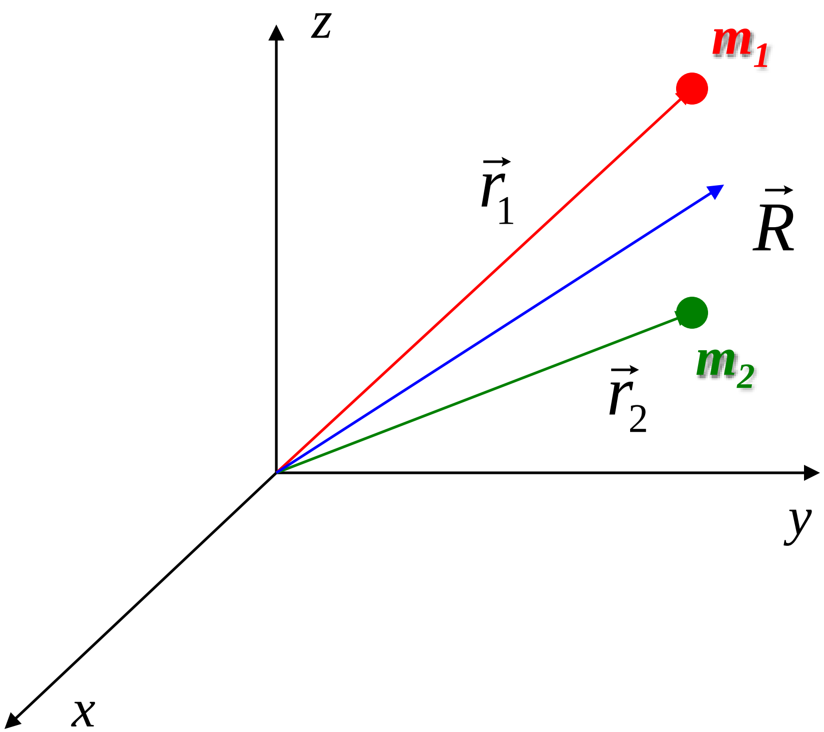
Slides do prof. Antonio Vidiella Barranco:
http://www.ifi.unicamp.br/~vidiella/aulas.html

Dinâmica de um sistema de partículas

Coordenada do centro de massa



$$\vec{R} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$$

S 4.14

TM 9.3

Sistema <u>discreto</u> de partículas com massas

$$m_1, m_2, \ldots m_N$$

S 4.15

TM 9.2

$$M = \sum_{i=1}^{N} m_i$$

Dinâmica de um sistema de partículas

Movimento da i-ésima partícula: separação entre forças externas e forças internas ao sistema

S 4.2

TM 9.7

$$m_i \ddot{\vec{r}}_i = \sum_j \vec{F}_{ij} + \vec{F}_i$$

Forças externas

Forças internas

2ª lei de Newton para a partícula i: $\dot{\boldsymbol{p}}_i = \boldsymbol{F}_i^{\text{ext}} + \sum_{j \neq i} \boldsymbol{f}_{ij}^{\text{int}}$

Momento linear da partícula i: $\boldsymbol{p}_i = m_i \dot{\boldsymbol{r}}_i$

Momento linear do centro de massa (CM): $\boldsymbol{P} \equiv \sum_i \boldsymbol{p}_i$

$$\dot{\boldsymbol{P}} = \sum_i m_i \ddot{\boldsymbol{r}}_i = M\ddot{\boldsymbol{R}} = \sum_i \boldsymbol{F}_i^{\text{ext}} + \sum_{i,j \neq i} \boldsymbol{f}_{ij}^{\text{int}}$$

TM eq. (9.1); S eq. (4.9)

Forma *fraca* da 3ª lei de Newton: $\boldsymbol{f}_{ij}^{\text{int}} = -\boldsymbol{f}_{ji}^{\text{int}}$

Forma *forte* da 3ª lei de Newton: $\boldsymbol{f}_{ij}^{\text{int}} \parallel (\boldsymbol{r}_i - \boldsymbol{r}_j)$

As forças exercidas mutuamente por duas partículas i e j, em adição a serem iguais e opostas, devem estar *sobre a reta* que une as partículas $\rightarrow$ $\boldsymbol{f}_{ij}^{\text{int}} \times (\boldsymbol{r}_i - \boldsymbol{r}_j) = \boldsymbol{0}$

Dinâmica de um sistema de partículas

Momento linear:

$$\frac{d\vec{P}}{dt} = M\ddot{\vec{R}} = \sum_i \vec{F}_i$$

S 4.18

TM 9.13

O momento linear é <u>cte</u> para força externa resultante nula

O sistema de partículas é dinamicamente equivalente a um sistema de massa $M = \sum_i m_i$ localizada na posição do centro de massa do sistema, $\vec{R}$

Momento linear do centro de massa (CM): $\boldsymbol{P} \equiv \sum_i \boldsymbol{p}_i$

$$\dot{\boldsymbol{P}} = \sum_i m_i \ddot{\boldsymbol{r}}_i = M\ddot{\boldsymbol{R}} = \sum_i \boldsymbol{F}_i^{\text{ext}} + \sum_{i,j\neq i} \boldsymbol{f}_{ij}^{\text{int}}$$

Devido à forma *fraca* da 3ª lei de Newton $\boldsymbol{f}_{ij}^{\text{int}} = -\boldsymbol{f}_{ji}^{\text{int}}$:

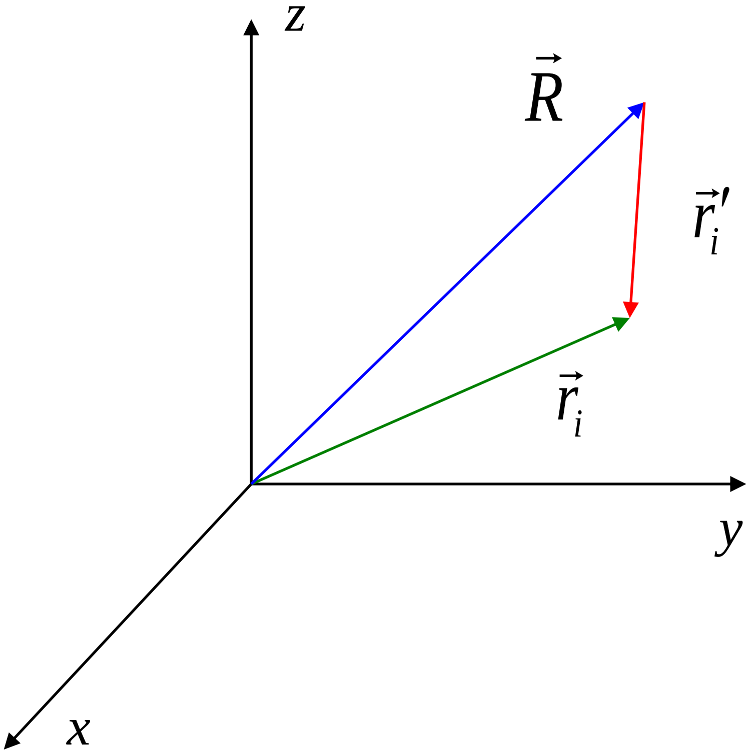
$$\sum_{i,j\neq i} \boldsymbol{f}_{ij}^{\text{int}} = \frac{1}{2} \sum_{i,j\neq i} (\boldsymbol{f}_{ij}^{\text{int}} + \boldsymbol{f}_{ji}^{\text{int}}) = \sum_{i,j<i} (\boldsymbol{f}_{ij}^{\text{int}} + \boldsymbol{f}_{ji}^{\text{int}}) = \boldsymbol{0}$$

TM eq. (9.13); S eqs. (4.7) e (4.18)

$$\dot{\boldsymbol{P}} = M\ddot{\boldsymbol{R}} = \sum_i \boldsymbol{F}_i^{\text{ext}}$$

Dinâmica de um sistema de partículas

Momento angular:



O momento angular pode ser decomposto da seguinte forma

$$\vec{L} = \vec{R} \times \vec{P} + \sum_i \vec{r}_i' \times \vec{p}_i'$$

S 4.129
TM 9.23

onde

$$\vec{P} = M\dot{\vec{R}}$$

Momento angular total do sistema: $\boldsymbol{L} \equiv \sum_i \boldsymbol{r}_i \times \boldsymbol{p}_i$

Usando as coordenadas relativas $\boldsymbol{r}_i = \boldsymbol{R} + \boldsymbol{r}'_i$

$$\boldsymbol{L} = \sum_i m_i \boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i = \sum_i m_i (\boldsymbol{R} + \boldsymbol{r}'_i) \times (\dot{\boldsymbol{R}} + \dot{\boldsymbol{r}}'_i)$$
$$= \boldsymbol{R} \times \dot{\boldsymbol{R}} \sum_i m_i + \boldsymbol{R} \times \sum_i m_i \dot{\boldsymbol{r}}'_i + \sum_i m_i \boldsymbol{r}'_i \times \dot{\boldsymbol{R}} + \sum_i \boldsymbol{r}'_i \times m_i \dot{\boldsymbol{r}}'_i$$

Visto que $M\boldsymbol{R} = \sum_i m_i \boldsymbol{r}_i = \sum_i m_i (\boldsymbol{R} + \boldsymbol{r}'_i) = M\boldsymbol{R} + \sum_i m_i \boldsymbol{r}'_i$

$$\rightarrow \sum_i m_i \boldsymbol{r}'_i = \sum_i m_i \dot{\boldsymbol{r}}'_i = \boldsymbol{0}$$

TM eq. (9.23); S eq. (4.129)

$$\boldsymbol{L} = \boldsymbol{R} \times \boldsymbol{P} + \sum_i \boldsymbol{r}'_i \times \boldsymbol{p}'_i$$

Dinâmica de um sistema de partículas

Momento angular:

$$\frac{d\vec{L}}{dt} = \sum_i \left(\vec{r}_i \times \vec{F}_i\right) = \sum_i \vec{\tau}_i$$

S 4.28

TM 9.30

O momento angular é <u>constante</u> para torque externo resultante nulo.

Obs: Esse resultado é válido para forças internas do tipo central, que obedecem à forma forte da terceira lei de Newton.

Momento angular total do sistema: $\boldsymbol{L} \equiv \sum_i \boldsymbol{r}_i \times \boldsymbol{p}_i$

$$\dot{\boldsymbol{L}} = \sum_i \boldsymbol{r}_i \times \dot{\boldsymbol{p}}_i = \sum_i \boldsymbol{r}_i \times m_i \ddot{\boldsymbol{r}}_i = \sum_i \boldsymbol{r}_i \times \boldsymbol{F}_i^{\text{ext}} + \sum_{i,j\neq i} \boldsymbol{r}_i \times \boldsymbol{f}_{ij}^{\text{int}}$$

$$\sum_{i,j\neq i} \boldsymbol{r}_i \times \boldsymbol{f}_{ij}^{\text{int}} = \sum_{i,j<i} (\boldsymbol{r}_i \times \boldsymbol{f}_{ij}^{\text{int}} + \boldsymbol{r}_j \times \boldsymbol{f}_{ji}^{\text{int}}) = \sum_{i,j<i} (\boldsymbol{r}_i - \boldsymbol{r}_j) \times \boldsymbol{f}_{ij}^{\text{int}} = \boldsymbol{0}$$

TM eq. (9.30); S eq. (4.28)

$$\dot{\boldsymbol{L}} = \sum_i \boldsymbol{\tau}_i, \quad \boldsymbol{\tau}_i \equiv \boldsymbol{r}_i \times \boldsymbol{F}_i^{\text{ext}}$$

Torque $\boldsymbol{\tau}_i$ associado à força externa $\boldsymbol{F}_i^{\text{ext}}$

Dinâmica de um sistema de partículas

Energia:

$$T=\frac{1}{2}\sum_i m_i v_i^2=\frac{1}{2}\sum_i m_i v'^2_i+\frac{1}{2}M\dot{\vec{R}}\cdot\dot{\vec{R}}$$

S 4.127
TM 9.39

Também

$$\frac{d\left(T+V_{\text{int}}\right)}{dt}=\sum_i\left(\dot{\vec{r}}_i\cdot\vec{F}_i\right)$$

S 4.40
TM 9.53

Energia cinética total $T \equiv \sum_i T_i\,,\quad T_i \equiv \frac{1}{2} m_i \dot{\boldsymbol{r}}_i^2$

$$\dot{\boldsymbol{r}}_i^2 = (\dot{\boldsymbol{R}} + \dot{\boldsymbol{r}}_i')^2 = \dot{\boldsymbol{R}}^2 + \dot{\boldsymbol{r}}_i'^2 + 2\dot{\boldsymbol{R}} \cdot \dot{\boldsymbol{r}}_i'$$

$$T = \frac{1}{2}\dot{\boldsymbol{R}}^2 \sum_i m_i + \frac{1}{2}\sum_i m_i \dot{\boldsymbol{r}}_i'^2 + \dot{\boldsymbol{R}} \cdot \sum_i m_i \dot{\boldsymbol{r}}_i'$$

TM eq. (9.39); S eq. (4.127)

$$T = T_{\text{CM}} + T',\quad T_{\text{CM}} \equiv \frac{1}{2} M \dot{\boldsymbol{R}}^2,\quad T' \equiv \frac{1}{2}\sum_i m_i \dot{\boldsymbol{r}}_i'^2$$

Energia potencial interna $V_{\text{int}} = \sum_{i,j<i} V_{ij}^{\text{int}},\ \boldsymbol{f}_{ij}^{\text{int}} = -\boldsymbol{\nabla}_i V_{ij}^{\text{int}}$

Energia potencial externa $V_{\text{ext}} = \sum_i V_i^{\text{ext}},\ \boldsymbol{F}_i^{\text{ext}} = -\boldsymbol{\nabla}_i V_i^{\text{ext}}$

$$\dot{T}_i = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}m_i\dot{\boldsymbol{r}}_i^2\right) = \dot{\boldsymbol{r}}_i\cdot m_i\ddot{\boldsymbol{r}}_i = \dot{\boldsymbol{r}}_i\cdot\boldsymbol{F}_i^{\text{ext}} + \sum_{j\neq i}\dot{\boldsymbol{r}}_i\cdot\boldsymbol{f}_{ij}^{\text{int}}$$

$$\dot{T} = \sum_i \dot{T}_i = \sum_i \dot{\boldsymbol{r}}_i\cdot\boldsymbol{F}_i^{\text{ext}} + \sum_{i,j\neq i}\dot{\boldsymbol{r}}_i\cdot\boldsymbol{f}_{ij}^{\text{int}}$$

$$\sum_{i,j\neq i}\dot{\boldsymbol{r}}_i\cdot\boldsymbol{f}_{ij}^{\text{int}} = \sum_{i,j<i}\left(\dot{\boldsymbol{r}}_i\cdot\boldsymbol{f}_{ij}^{\text{int}} + \dot{\boldsymbol{r}}_j\cdot\boldsymbol{f}_{ji}^{\text{int}}\right) = \sum_{i,j<i}\left(\dot{\boldsymbol{r}}_i - \dot{\boldsymbol{r}}_j\right)\cdot\boldsymbol{f}_{ij}^{\text{int}}$$ **TM 9.43**

$$\dot{V}_{\text{int}} = \sum_{i,j<i}\dot{V}_{ij}^{\text{int}} = \sum_{i,j<i}\left(\dot{\boldsymbol{r}}_i\cdot\boldsymbol{\nabla}_i V_{ij}^{\text{int}} + \dot{\boldsymbol{r}}_j\cdot\boldsymbol{\nabla}_j V_{ij}^{\text{int}}\right)$$ **TM 9.45**

$$= -\sum_{i,j<i}\left(\dot{\boldsymbol{r}}_i - \dot{\boldsymbol{r}}_j\right)\cdot\boldsymbol{f}_{ij}^{\text{int}},\quad \boldsymbol{f}_{ij}^{\text{int}} = -\boldsymbol{\nabla}_i V_{ij}^{\text{int}} = \boldsymbol{\nabla}_j V_{ij}^{\text{int}} = -\boldsymbol{f}_{ji}^{\text{int}}$$

S eq. (4.40)

$$\dot{T} + \dot{V}_{\text{int}} = \sum_i \dot{\boldsymbol{r}}_i\cdot\boldsymbol{F}_i^{\text{ext}} = -\sum_i \dot{\boldsymbol{r}}_i\cdot\boldsymbol{\nabla}_i V_i^{\text{ext}} = -\dot{V}_{\text{ext}}$$

TM eq. (9.53)

$$\dot{E} \equiv \dot{T} + \dot{V}_{\text{int}} + \dot{V}_{\text{ext}} = 0$$

E é a energia mecânica total

$$\dot{T} = \dot{T}_{\text{CM}} + \dot{T}' = \sum_i \dot{\boldsymbol{r}}_i \cdot \boldsymbol{F}_i^{\text{ext}} - \dot{V}_{\text{int}}$$

$$\dot{T}_{\text{CM}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}M\dot{\boldsymbol{R}}^2\right) = \dot{\boldsymbol{R}} \cdot M\ddot{\boldsymbol{R}} = \dot{\boldsymbol{R}} \cdot \sum_i \boldsymbol{F}_i^{\text{ext}}$$

$$\dot{T}' = \dot{T} - \dot{T}_{\text{CM}} = \sum_i (\dot{\boldsymbol{r}}_i - \dot{\boldsymbol{R}}) \cdot \boldsymbol{F}_i^{\text{ext}} - \dot{V}_{\text{int}} = \sum_i \dot{\boldsymbol{r}}_i' \cdot \boldsymbol{F}_i^{\text{ext}} - \dot{V}_{\text{int}}$$

Conservação de energia em termos dos $\{\dot{\boldsymbol{r}}_i'\}$

$$\dot{T}' + \dot{V}_{\text{int}} = \sum_i \dot{\boldsymbol{r}}_i' \cdot \boldsymbol{F}_i^{\text{ext}}$$