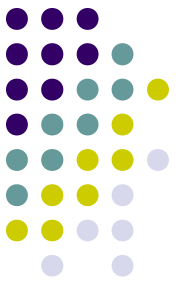


F-315 (Mecânica Geral I)

Aula 19



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Slides do prof. Antonio Vidiella Barranco:

<http://www.ifi.unicamp.br/~vidiella/aulas.html>

Força e campo gravitacional: $\mathbf{F}(\mathbf{r}) = m\mathbf{g}(\mathbf{r})$

$$\begin{aligned} \text{Lei de Gauss: } \oint_{S=\partial V} \mathbf{g} \cdot \hat{\mathbf{n}} \, dS &= \int_V \nabla \cdot \mathbf{g} \, d^3\mathbf{r} = -4\pi GM \\ &= -4\pi G \int_V \rho(\mathbf{r}) \, d^3\mathbf{r} \quad \rightarrow \quad \nabla \cdot \mathbf{g}(\mathbf{r}) = -4\pi G\rho(\mathbf{r}) \end{aligned}$$

Campo gravitacional de massa puntiforme na origem:

$$\rho(\mathbf{r}) = M\delta^3(\mathbf{r}) \quad \rightarrow \quad \mathbf{g}(\mathbf{r}) = -G \int_{V'} \rho(\mathbf{r}') \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} \, d^3\mathbf{r}' = -\frac{GM}{r^2} \hat{\mathbf{r}}$$

Potencial gravitacional $\Phi(\mathbf{r})$: $\mathbf{g}(\mathbf{r}) = -\nabla\Phi(\mathbf{r})$

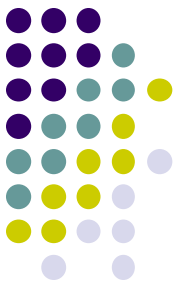
Campo gravitacional é conservativo: $\nabla \times \mathbf{g}(\mathbf{r}) = \mathbf{0}$

Equação de Poisson:

$$-\nabla \cdot \mathbf{g}(\mathbf{r}) = \nabla \cdot \nabla\Phi(\mathbf{r}) =$$

TM eq. (5.38); S eq. (6.33)

$$\nabla^2\Phi(\mathbf{r}) = 4\pi G\rho(\mathbf{r})$$

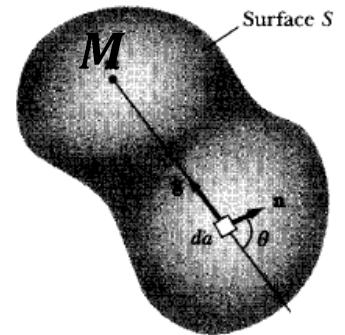


Força gravitacional

Lei de Gauss:

fluxo de \vec{g} através de uma superfície fechada S

Para a massa M no interior de superfície:



$$\oiint \vec{g} \cdot \hat{n} da = -GM \oiint \frac{da'}{r^2} = -GM4\pi$$

M no exterior da superfície: $\oiint \vec{g} \cdot \hat{n} da = 0$

Obter $\rho(r)$ para uma esfera de raio R e massa M tal que \mathbf{g} independe de r , para $r < R$. Se $\mathbf{g}(\mathbf{r})$ para $r < R$ é constante, $\mathbf{g}(\mathbf{r}) = -\nabla\Phi(\mathbf{r}) = -C\hat{\mathbf{r}}$, então o potencial gravitacional deve ser esfericamente simétrico

$\Phi = \Phi(r)$ e satisfazer $\frac{\partial\Phi}{\partial r} = C = |\mathbf{g}|$.

Para $r < R$ vale a equação de Poisson:

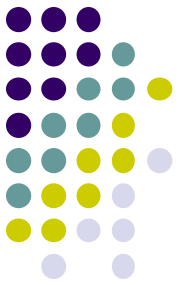
$$\nabla^2\Phi(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial\Phi}{\partial r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} (Cr^2) = 4\pi G\rho(r)$$

$$2Cr = 4\pi r^2 G\rho(r), \quad C \text{ obtém-se de } \int_{V'} \rho(\mathbf{r}') d^3\mathbf{r}' = M$$

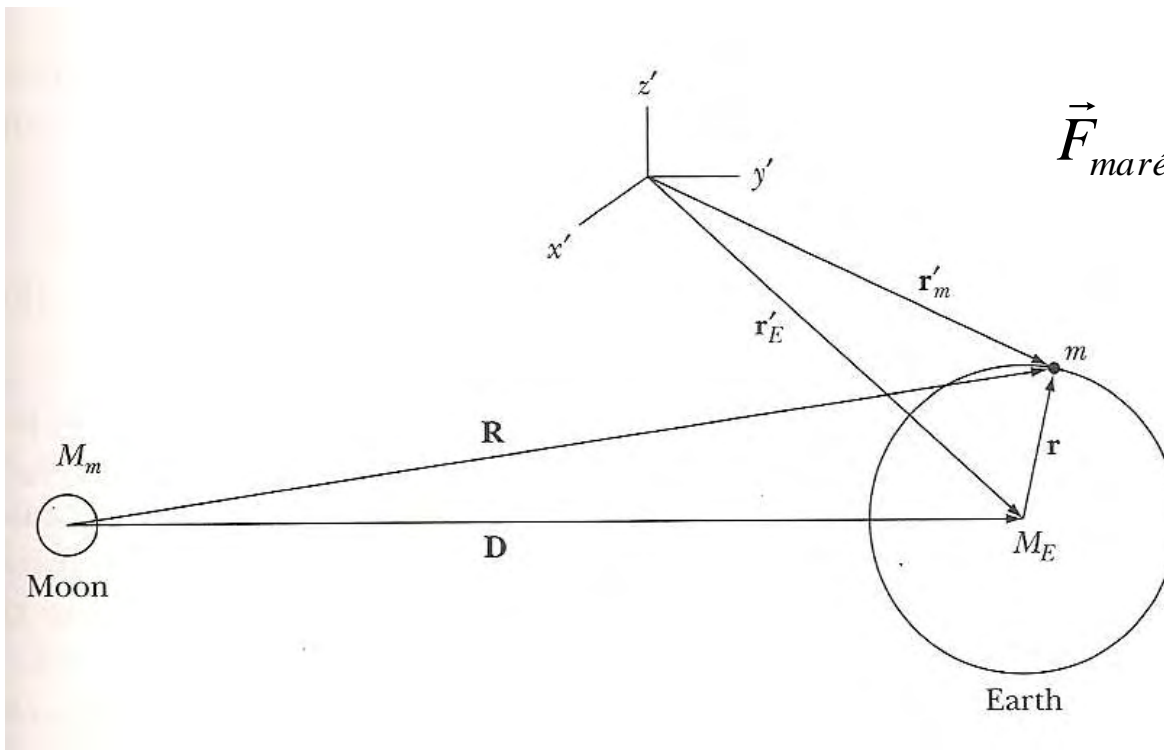
$$\rho(r < R) = \frac{|\mathbf{g}|}{2\pi Gr}, \quad |\mathbf{g}| = \frac{GM}{R^2}, \quad \Phi(r < R) = |\mathbf{g}|(r - 2R)$$

Forças de maré

TM seção 5.5, pgs. 198-202



Considere um elemento de massa m na superfície da Terra, sob a ação da Lua



$$\vec{F}_{\text{maré}} = -GM_L m \left(\frac{\hat{e}_R}{R^2} - \frac{\hat{e}_D}{D^2} \right)$$

TM seção 5.5 (forças de maré), pgs. 198–202

$$\ddot{\mathbf{r}}'_m = -\frac{GM_T}{r^2} \hat{\mathbf{r}} - \frac{GM_L}{R^2} \hat{\mathbf{R}} \quad (\text{aceleração de } m \text{ devido à Terra e à lua})$$

$$\ddot{\mathbf{r}}'_T = -\frac{GM_L}{D^2} \hat{\mathbf{D}} \quad (\text{aceleração do CM da Terra devido à lua})$$

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}'_m - \ddot{\mathbf{r}}'_T = -\frac{GM_T}{r^2} \hat{\mathbf{r}} - GM_L \left(\frac{1}{R^2} \hat{\mathbf{R}} - \frac{1}{D^2} \hat{\mathbf{D}} \right)$$

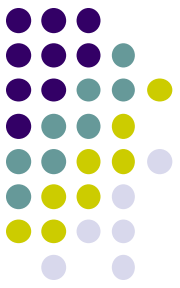
$$\mathbf{F}_{\text{maré}} \equiv -GM_L m \left(\frac{1}{R^2} \hat{\mathbf{R}} - \frac{1}{D^2} \hat{\mathbf{D}} \right), \quad R^2 = r^2 + D^2 + 2rD \cos \theta$$

$$\mathbf{R} = \mathbf{r} + \mathbf{D} = (r \cos \theta + D) \hat{\mathbf{x}} + r \sin \theta \hat{\mathbf{y}}$$

$$\frac{1}{R^3} = (r^2 + D^2 + 2rD \cos \theta)^{-3/2} = \frac{1}{D^3} \left(1 - \frac{3r}{D} \cos \theta + \dots \right)$$

$$F_x = -GM_L m \left(\frac{r \cos \theta + D}{R^3} - \frac{1}{D^2} \right) \approx \frac{2GM_L m}{D^3} r \cos \theta \quad \text{TM eq. (5.54a)}$$

$$F_y = -\frac{GM_L m}{R^3} r \sin \theta \approx -\frac{GM_L m}{D^3} r \sin \theta \quad \text{TM eq. (5.54b)}$$



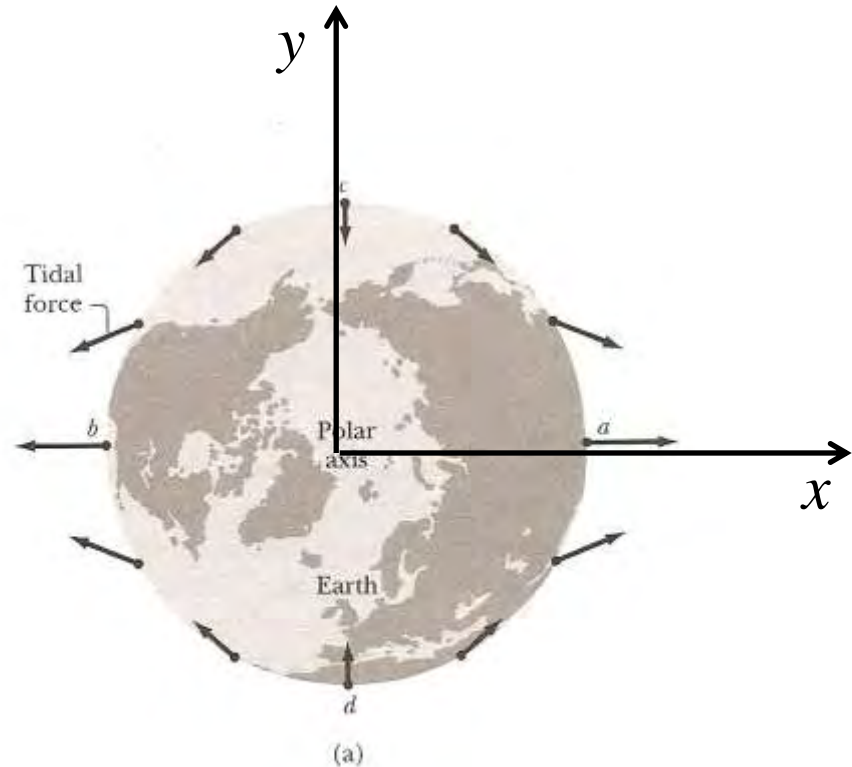
Forças de maré

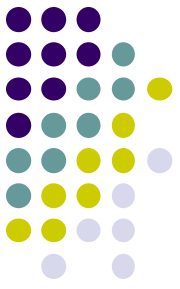
Força resultante no ponto a, visto que $r \ll D$

$$F_x \approx \frac{2GM_L m}{D^3} r$$

No ponto c,

$$F_y \approx -\frac{GM_L m}{D^3} r$$





Forças de maré

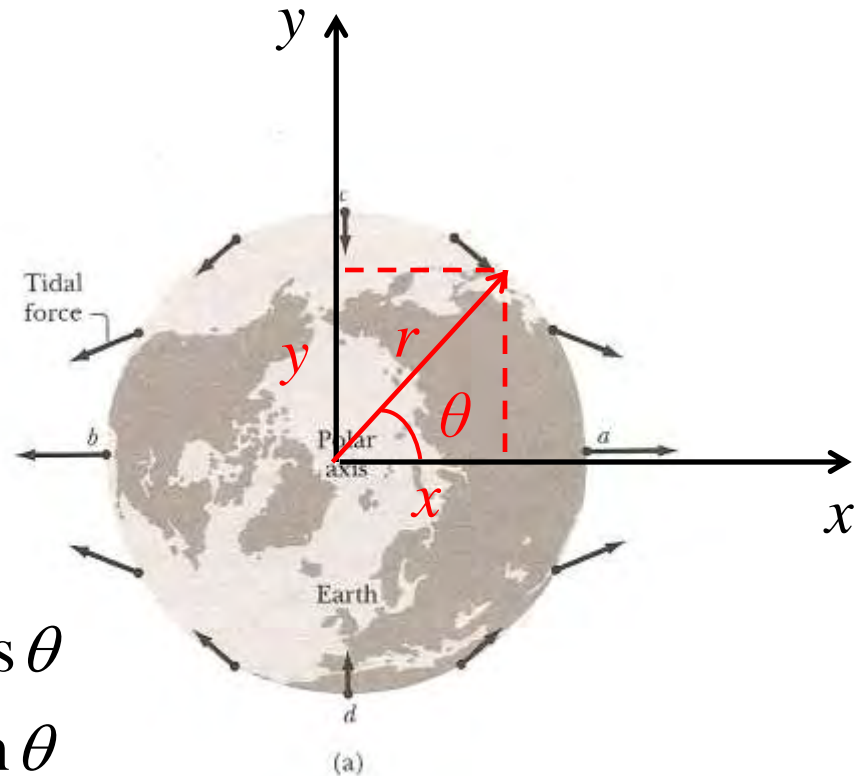
Para um ponto genérico na superfície,

$$F_x \approx \frac{2GM_L m}{D^3} x$$

$$F_y \approx -\frac{GM_L m}{D^3} y$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$



Estimativas numéricas (índice S refere-se ao Sol)

Aceleração da gravidade terrestre: $g \equiv \frac{GM_T}{r^2} \approx 10 \text{ m/s}^2$

Razões de massas: $\frac{M_L}{M_T} \approx \frac{1}{81,3}$, $\frac{M_S}{M_T} \approx 332.946$

Distâncias: $r \approx 6.371 \text{ km}$; $D \approx 384.400 \text{ km}$ (média),
356.500 km (perigeu), 406.700 km (apogeu);

$D_S \approx 1,471 \times 10^8 \text{ km}$ (periélio), $1,521 \times 10^8 \text{ km}$ (afélio)

Valor máximo da força de maré lunar: $F_x^{\max} = \frac{2GM_L m r}{D^3}$

$= 2mg \frac{M_L}{M_T} \left(\frac{r}{D}\right)^3 \approx 1,1 \times 10^{-7} mg$ (média), $1,4 \times 10^{-7} mg$

(perigeu), $9,5 \times 10^{-8} mg$ (apogeu); $\frac{F_L}{F_S} = \frac{M_L}{M_S} \left(\frac{D_S}{D}\right)^3 \approx$

entre 1,75 (periélio/apogeu) e 2,87 (afélio/perigeu)