F-315 (Mecânica Geral I)
Aula 29

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Slides do prof. Antonio Vidiella Barranco:
http://www.ifi.unicamp.br/~vidiella/aulas.html

Forças não-conservativas (forças de fricção)
Função dissipação de Rayleigh $\mathcal{R}$

$\mathcal{R} \equiv \frac{1}{2}\sum_{i,j} f_{ij}\dot{q}_i\dot{q}_j$, tensor de fricção/dissipação $\{f_{ij}\}$

Forças de fricção: $Q_j^{\text{fric}}(\dot{\boldsymbol{q}}) = -\frac{\partial\mathcal{R}}{\partial\dot{q}_j} \;\rightarrow\; \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\mathscr{L}}{\partial\dot{q}_j}\right) - \frac{\partial\mathscr{L}}{\partial q_j} = Q_j^{\text{fric}}$

$$\dot{W}^{\text{fric}} = \sum_i Q_i^{\text{fric}}\dot{q}_i = -\sum_{i,j} f_{ij}\dot{q}_i\dot{q}_j = -2\mathcal{R}$$

$\dot{E} = \dot{T} + \dot{V} = \dot{V} + \sum_i\left(\frac{\partial T}{\partial q_i}\dot{q}_i + \frac{\partial T}{\partial\dot{q}_i}\ddot{q}_i\right)$. O último termo fica:

$$\sum_i \frac{\partial T}{\partial\dot{q}_i}\ddot{q}_i = \frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_i \frac{\partial T}{\partial\dot{q}_i}\dot{q}_i\right) - \sum_i \dot{q}_i\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial\dot{q}_i}\right) = 2\dot{T} - \sum_i \dot{q}_i\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\mathscr{L}}{\partial\dot{q}_i}\right)$$

$$= 2\dot{T} - \sum_i Q_i^{\text{fric}}\dot{q}_i - \sum_i \frac{\partial\mathscr{L}}{\partial q_i}\dot{q}_i = 2(\dot{T} + \mathcal{R}) - \sum_i \frac{\partial T}{\partial q_i}\dot{q}_i + \sum_i \frac{\partial V}{\partial q_i}\dot{q}_i$$

Note que $\dot{V} = \frac{\partial V}{\partial t} + \sum_i \frac{\partial V}{\partial q_i} \dot{q}_i$

$$\sum_i \frac{\partial T}{\partial \dot{q}_i} \ddot{q}_i = 2(\dot{T} + \mathcal{R}) - \sum_i \frac{\partial T}{\partial q_i} \dot{q}_i + \dot{V} - \frac{\partial V}{\partial t}, \text{ ou seja}$$

$$\dot{T} + \dot{V} = 2(\dot{T} + \dot{V} + \mathcal{R}) - \frac{\partial V}{\partial t}$$

Variação da energia total: Greiner eq. (17.25)

$$\dot{E} = \dot{T} + \dot{V} = \dot{W}^{\text{fric}} + \frac{\partial V}{\partial t} = -2\mathcal{R} + \frac{\partial V}{\partial t}$$

Exemplo: queda em campo gravitacional com fricção

$$\mathcal{L} = \frac{1}{2} m \dot{\boldsymbol{r}}^2 - mgz, \quad \mathcal{R} = \frac{1}{2} b \dot{\boldsymbol{r}}^2 = \frac{1}{2} b(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

Equações de Euler-Lagrange: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = -\frac{\partial \mathcal{R}}{\partial \dot{q}_j} = -b\dot{q}_j$

$$\rightarrow m\ddot{x} = -b\dot{x}, \quad m\ddot{y} = -b\dot{y}, \quad m\ddot{z} = -mg - b\dot{z}$$

Partícula carregada em campo eletromagnético

$$\boldsymbol{E}(\boldsymbol{r},t) = -\nabla\Phi(\boldsymbol{r},t) - \frac{\partial}{\partial t}\boldsymbol{A}(\boldsymbol{r},t), \quad \boldsymbol{B}(\boldsymbol{r},t) = \nabla\times\boldsymbol{A}(\boldsymbol{r},t)$$

$$\mathscr{L}(\boldsymbol{r},\dot{\boldsymbol{r}},t) = \frac{1}{2}m\dot{\boldsymbol{r}}^2 - V(\boldsymbol{r},\dot{\boldsymbol{r}},t), V(\boldsymbol{r},\dot{\boldsymbol{r}},t) = q\Phi(\boldsymbol{r},t) - q\dot{\boldsymbol{r}}\cdot\boldsymbol{A}(\boldsymbol{r},t)$$

$$\boldsymbol{p} \equiv \nabla_{\dot{\boldsymbol{r}}}\mathscr{L} \equiv \left(\hat{\boldsymbol{x}}\frac{\partial}{\partial\dot{x}} + \hat{\boldsymbol{y}}\frac{\partial}{\partial\dot{y}} + \hat{\boldsymbol{z}}\frac{\partial}{\partial\dot{z}}\right)\mathscr{L} = m\dot{\boldsymbol{r}} + q\boldsymbol{A}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + (\dot{\boldsymbol{r}}\cdot\nabla) + (\ddot{\boldsymbol{r}}\cdot\nabla_{\dot{\boldsymbol{r}}}), \quad \dot{\boldsymbol{p}} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = m\ddot{\boldsymbol{r}} + q\frac{\partial\boldsymbol{A}}{\partial t} + q(\dot{\boldsymbol{r}}\cdot\nabla)\boldsymbol{A}$$

$$\dot{\boldsymbol{p}} = \nabla\mathscr{L} = -q\nabla\Phi + q\left[(\dot{\boldsymbol{r}}\cdot\nabla)\boldsymbol{A} + \dot{\boldsymbol{r}}\times(\nabla\times\boldsymbol{A})\right], \text{ utilizando}$$

$$\nabla(\boldsymbol{a}\cdot\boldsymbol{b}) = (\boldsymbol{a}\cdot\nabla)\boldsymbol{b} + (\boldsymbol{b}\cdot\nabla)\boldsymbol{a} + \boldsymbol{a}\times(\nabla\times\boldsymbol{b}) + \boldsymbol{b}\times(\nabla\times\boldsymbol{a})$$

Força de Lorentz: G eq. (1.60), S eq. (9.160)

$$\boldsymbol{F} = m\ddot{\boldsymbol{r}} = -q\left(\nabla\Phi + \frac{\partial\boldsymbol{A}}{\partial t}\right) + q\dot{\boldsymbol{r}}\times(\nabla\times\boldsymbol{A}) = q(\boldsymbol{E} + \dot{\boldsymbol{r}}\times\boldsymbol{B})$$

$$\mathscr{H}(\boldsymbol{r},\boldsymbol{p},t) = \boldsymbol{p}\cdot\dot{\boldsymbol{r}} - \mathscr{L} = \frac{1}{2m}\left[\boldsymbol{p} - q\boldsymbol{A}(\boldsymbol{r},t)\right]^2 + q\Phi(\boldsymbol{r},t)$$

Curiosidades envolvendo campos eletromagnéticos

Indução de correntes de Foucault

https://www.youtube.com/watch?v=N7tIi71-AjA

Levitação diamagnética

https://www.youtube.com/watch?v=A5pZZJ23rDM

https://www.youtube.com/watch?v=dIRPVqnQro4

https://www.youtube.com/watch?v=WtmLzXn2d8g

Motor monopolar

https://www.youtube.com/watch?v=7SADAnt3hpA

https://www.youtube.com/watch?v=voHz6sxwQ2Q

https://www.youtube.com/watch?v=oPzJr1jjHnQ

https://www.youtube.com/watch?v=xMLWiA5ApUU

Trem eletromagnético

https://www.youtube.com/watch?v=J9b0J29OzAU

https://www.youtube.com/watch?v=Y1MDOerruDU