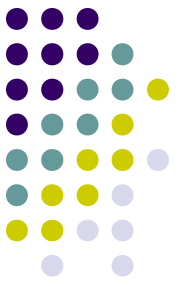


F-315 (Mecânica Geral I)

Aula 29



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http://www.ifi.unicamp.br/~mtamash/f315_mecgeral_i

Slides do prof. Antonio Vidiella Barranco:

<http://www.ifi.unicamp.br/~vidiella/aulas.html>

Forças não-conservativas (forças de fricção)

Função dissipação de Rayleigh \mathcal{R}

$$\mathcal{R} \equiv \frac{1}{2} \sum_{i,j} f_{ij} \dot{q}_i \dot{q}_j, \quad \text{tensor de fricção/dissipação } \{f_{ij}\}$$

$$\text{Forças de fricção: } \mathbf{Q}_j^{\text{fric}}(\dot{\mathbf{q}}) = -\frac{\partial \mathcal{R}}{\partial \dot{q}_j} \rightarrow \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j} \right) - \frac{\partial \mathcal{L}}{\partial q_j} = \mathbf{Q}_j^{\text{fric}}$$

$$\dot{W}^{\text{fric}} = \sum_i \mathbf{Q}_i^{\text{fric}} \dot{q}_i = -\sum_{i,j} f_{ij} \dot{q}_i \dot{q}_j = -2\mathcal{R}$$

$$\dot{E} = \dot{T} + \dot{V} = \dot{V} + \sum_i \left(\frac{\partial T}{\partial q_i} \dot{q}_i + \frac{\partial T}{\partial \dot{q}_i} \ddot{q}_i \right). \quad \text{O último termo fica:}$$

$$\begin{aligned} \sum_i \frac{\partial T}{\partial \dot{q}_i} \ddot{q}_i &= \frac{d}{dt} \left(\sum_i \frac{\partial T}{\partial \dot{q}_i} \dot{q}_i \right) - \sum_i \dot{q}_i \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) = 2\dot{T} - \sum_i \dot{q}_i \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) \\ &= 2\dot{T} - \sum_i \mathbf{Q}_i^{\text{fric}} \dot{q}_i - \sum_i \frac{\partial \mathcal{L}}{\partial q_i} \dot{q}_i = 2(\dot{T} + \mathcal{R}) - \sum_i \frac{\partial T}{\partial q_i} \dot{q}_i + \sum_i \frac{\partial V}{\partial q_i} \dot{q}_i \end{aligned}$$

Note que $\dot{V} = \frac{\partial V}{\partial t} + \sum_i \frac{\partial V}{\partial q_i} \dot{q}_i$

$$\sum_i \frac{\partial T}{\partial \dot{q}_i} \ddot{q}_i = 2(\dot{T} + \mathcal{R}) - \sum_i \frac{\partial T}{\partial q_i} \dot{q}_i + \dot{V} - \frac{\partial V}{\partial t}, \text{ ou seja}$$

$$\dot{T} + \dot{V} = 2(\dot{T} + \dot{V} + \mathcal{R}) - \frac{\partial V}{\partial t}$$

Variação da energia total: Greiner eq. (17.25)

$$\dot{E} = \dot{T} + \dot{V} = \dot{W}^{\text{fric}} + \frac{\partial V}{\partial t} = -2\mathcal{R} + \frac{\partial V}{\partial t}$$

Exemplo: queda em campo gravitacional com fricção

$$\mathcal{L} = \frac{1}{2}m\dot{\mathbf{r}}^2 - mgz, \quad \mathcal{R} = \frac{1}{2}b\dot{\mathbf{r}}^2 = \frac{1}{2}b(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

Equações de Euler-Lagrange: $\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j} \right) - \frac{\partial \mathcal{L}}{\partial q_j} = -\frac{\partial \mathcal{R}}{\partial \dot{q}_j} = -b\dot{q}_j$

$$\rightarrow m\ddot{x} = -b\dot{x}, \quad m\ddot{y} = -b\dot{y}, \quad m\ddot{z} = -mg - b\dot{z}$$

Partícula carregada em campo eletromagnético

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\Phi(\mathbf{r}, t) - \frac{\partial}{\partial t}\mathbf{A}(\mathbf{r}, t), \quad \mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t)$$

$$\mathcal{L}(\mathbf{r}, \dot{\mathbf{r}}, t) = \frac{1}{2}m\dot{\mathbf{r}}^2 - V(\mathbf{r}, \dot{\mathbf{r}}, t), \quad V(\mathbf{r}, \dot{\mathbf{r}}, t) = q\Phi(\mathbf{r}, t) - q\dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{r}, t)$$

$$\mathbf{p} \equiv \nabla_{\dot{\mathbf{r}}} \mathcal{L} \equiv \left(\hat{\mathbf{x}} \frac{\partial}{\partial \dot{x}} + \hat{\mathbf{y}} \frac{\partial}{\partial \dot{y}} + \hat{\mathbf{z}} \frac{\partial}{\partial \dot{z}} \right) \mathcal{L} = m\dot{\mathbf{r}} + q\mathbf{A}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\dot{\mathbf{r}} \cdot \nabla) + (\ddot{\mathbf{r}} \cdot \nabla_{\dot{\mathbf{r}}}), \quad \dot{\mathbf{p}} = \frac{d\mathbf{p}}{dt} = m\ddot{\mathbf{r}} + q\frac{\partial \mathbf{A}}{\partial t} + q(\dot{\mathbf{r}} \cdot \nabla)\mathbf{A}$$

$$\dot{\mathbf{p}} = \nabla \mathcal{L} = -q\nabla\Phi + q[(\dot{\mathbf{r}} \cdot \nabla)\mathbf{A} + \dot{\mathbf{r}} \times (\nabla \times \mathbf{A})], \text{ utilizando}$$

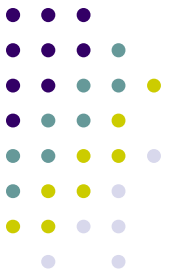
$$\nabla(\mathbf{a} \cdot \mathbf{b}) = (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a} + \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a})$$

Força de Lorentz: G eq. (1.60), S eq. (9.160)

$$\mathbf{F} = m\ddot{\mathbf{r}} = -q\left(\nabla\Phi + \frac{\partial \mathbf{A}}{\partial t}\right) + q\dot{\mathbf{r}} \times (\nabla \times \mathbf{A}) = q(\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B})$$

$$\mathcal{H}(\mathbf{r}, \mathbf{p}, t) = \mathbf{p} \cdot \dot{\mathbf{r}} - \mathcal{L} = \frac{1}{2m} [\mathbf{p} - q\mathbf{A}(\mathbf{r}, t)]^2 + q\Phi(\mathbf{r}, t)$$

Curiosidades envolvendo campos eletromagnéticos



Indução de correntes de Foucault

<https://www.youtube.com/watch?v=N7tli71-AjA>

Levitação diamagnética

<https://www.youtube.com/watch?v=A5pZZJ23rDM>

<https://www.youtube.com/watch?v=dIRPVqnQro4>

<https://www.youtube.com/watch?v=WtmLzXn2d8g>

Motor monopolar

<https://www.youtube.com/watch?v=7SADAnt3hpA>

<https://www.youtube.com/watch?v=voHz6sxxQ2Q>

<https://www.youtube.com/watch?v=oPzJr1jjHnQ>

<https://www.youtube.com/watch?v=xMLWiA5ApUU>

Trem eletromagnético

<https://www.youtube.com/watch?v=J9b0J29OzAU>

<https://www.youtube.com/watch?v=Y1MDOerruDU>