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Magnetic Energy and Effective Inertial Mass of the Conduction Electrons in Circuit Theory

Contents

Abstract

In this work we calculate the electrodynamic energy of conductors carrying slowly time-varying currents. We show that this energy can be quantitatively identified with the classical kinetic energy of the current-carrying electrons with effective intertial mass proportional to the self-inductance of the conductor.

Keywords: Circuit theory, Weber's electrodynamics, effective inertial mass.

1. Introduction

There has been a recent theoretical and experimental interest in the study of the effective inertial mass of electrons which can be different from their free inertial mass of 9.1×10^{-31} kg, [1–6]. These attempts utilize Weber's force between charged bodies, [7–13].

Here we consider the interactions between the conduction electrons and the positive stationary lattice of the current carrying conductor. The goal is to establish a connection between these interactions and the self-inductance of the conductor. In this approach the self-inductance is seen as a measure of the effective inertial mass of the conduction electrons. We utilize Weber's energy of interaction between charges and compare these results with those given by classical electrodynamics (utilizing Darwin's energy, from which the Liénard-Schwarzschild force can be derived, [14], [15, pp. 150-151], [16, Section 12.7, pp. 593–595]). We wish to find a connection between these interaction energies and the kinetic energy of the conduction electrons. The attempt is to clarify the relation between a global variable describing an electric circuit, namely, its self-induction L, and the microscopic variables related to the underlying physics, that is, the effective inertial mass of the conduction electrons.

As is well known, if there is a current i flowing through a conductor with self-inductance L, the classical magnetic energy U_B of the system is given by:

$$U_B = \frac{1}{2} L i^2 \ . \tag{1}$$

Suppose that the current flows through the cross-section A of the conductor with an average drifting velocity v. Then i can be substituted by $i = \rho A v$, where ρ is the volumetric charge density of conduction electrons in the conductor. The magnetic energy can then be rewritten as:

$$U_B = \frac{1}{2} L (\rho A v)^2 = \frac{1}{2} (L \rho^2 A^2) v^2 = N \frac{m_{\text{eff}} v^2}{2} \ . \tag{2}$$

Here we defined $m_{\text{eff}} \equiv L\rho^2 A^2 / N$ as the effective mass of a conduction electron and N is the total number of electrons in the current flow. This yields the magnetic energy U_B as an effective kinetic energy.

But this effective kinetic energy cannot be identified with the usual kinetic energy $T_{\rm usual} = Nmv^2/2$ (where $m = 9.1 \times 10^{-31}$ kg is the mass of the electron) for two reasons. The first reason is that for fixed N and v it is known that $U_B = Li^2/2$ depends on the geometry of the circuit. On the other hand, $T_{\rm usual}$ does not yield this dependence on the geometry, as it has the same value for all the shapes of a circuit. For instance, by bending a wire without changing the current i, the self-inductance L changes, the same happening with U_B, while $T_{\rm usual}$ remains the same. The second reason is that for typical situations $T_{\rm usual} \ll U_B$, which prevents the identification of these two concepts. Consider for instance a cylindrical copper wire of length ℓ and radius $r \ll \ell$ carrying a current i. Its self-inductance is given by $L = (\mu_0 \ell/2\pi) \ln(\ell/r)$, [17, p. 35]. This means that $U_B = Li^2/2 \approx 7.6 \times 10^{-7}$ J if $\ell = 1$ m, $r = 0.5$ mm and $i = 1$ A. In a copper wire we have typically one conduction electron per atom and 8.5×10^{22} atoms per cm^3. This yields $T_{\rm usual} \approx 2.7 \times 10^{-16}$ J $\ll U_B$. This means that the magnetic energy cannot be identified with the usual kinetic energy of the conduction electrons.

Despite this fact, our goal in this work is to derive the magnetic energy U_B as an *effective* kinetic energy of the conduction electrons due to their electrodynamic interactions with the conductor. To show this and to have a better understanding of the physical concepts behind this approach we compare two formulations, namely, Weber's electrodynamics and classical electromagnetism.

2. Weber's Potential Energy

Consider two charged particles q_1 and q_2, located in $\vec{r}_1$ and $\vec{r}_2$, with velocities $\vec{v}_1$ and $\vec{v}_2$ relative to the origin O of an inertial system M, respectively. Weber's potential energy of interaction between these charges is given by, [11, p. 61]:

$$U_W = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r_{12}} \left(1 - \frac{\dot{r}_{12}^2}{2c^2} \right) , \qquad (3)$$

where $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2 is the electric permittivity of the vacuum, $r_{12} = |\vec{r}_1 - \vec{r}_2|$, $\dot{r}_{12} = dr_{12}/dt$ and $c = 3 \times 10^8$ m/s is the velocity of light in vacuum. The term $\dot{r}_{12}$ can be rewritten as $\dot{r}_{12} = \hat{r}_{12} \cdot \vec{v}_{12}$, where $\hat{r}_{12} = \vec{r}_{12}/r_{12}$ and $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$. The Hamiltonian energy H_W is given by $H_W = T + U_W$, where T is the kinetic energy of the system.

We will calculate the energy in four different situations, following the approach of [2]: (A) a cylindrical wire of length ℓ and radius $r \ll \ell$, with an uniform longitudinal volume current density $\vec{J} = J\hat{z}$ flowing over its cross section; (B) a straight rectangular strip of length ℓ and side $d \ll \ell$, with an uniform longitudinal surface density $\vec{K} = K\hat{z}$ flowing over its surface; (C) a hollow cylinder with length ℓ and radius $r \ll \ell$, with an uniform longitudinal surface density $\vec{K} = K\hat{z}$ flowing over its surface; (D) the same cylinder as in (C), but with an uniform surface density $\vec{K} = K\hat{\varphi}$ (azimuthal current) flowing over its surface.

First we consider case (A). That is, a straight cylindrical wire with length ℓ and radius $r \ll \ell$, with an uniform longitudinal volume current density $\vec{J} = J\hat{z}$ flowing over its cross section. The axis of the cylinder is along the z axis. For a resistive circuit carrying steady currents there are surface charges spread along its surface, [18–23]. Despite this fact we consider in a simplified model the wire to be essentially neutral at all points, $dq_- = -dq_+$, and utilize cylindrical coordinates (r, φ, z). We also consider the situation of slowly time-varying current, so that at a given instant of time all conduction electrons have the same average drifting velocity $\vec{v}_1 = v_1\hat{z}$.

In both calculations (utilizing Weber's electrodynamics and utilizing Darwin's energy or classical electromagnetism) we take into account only this drifting velocity of the conduction electrons, which is a statistical non-equilibrium average. We do not take into account the thermal random velocities because we are interested here only in the effective kinetic energy of the conduction electrons which is a function of the current intensity in the current carrying conductor. A full statistical treatment taking into account the thermal random velocities of the conduction electrons is beyond the scope of this paper.

In each current element j there are positive and negative charges, dq_{j+} and dq_{j-}, respectively. Due to the assumed charge neutrality of each current element we have $dq_{j-} = -dq_{j+}$. When we consider the interaction between two neutral current elements, $j = 1$ and $j = 2$, we need to take into account four interactions: (I) stationary positive ions dq_{1+} interacting with stationary positive ions dq_{2+}; (II) mobile conduction electrons dq_{1-} interacting with mobile conduction electrons dq_{2-}; (III) stationary positive ions dq_{1+} interacting with mobile conduction electrons dq_{2-}; and (IV) mobile conduction electrons dq_{1-} interacting with stationary positive ions dq_{2+}. We consider each of these interactions separately, beginning with interaction (I). From Eq. (3) and the fact that the ions are at rest relative to one another it remais only a repulsive Coulombian interaction. The same happens with interaction (II) due to the supposition that all electrons move with the same drifting velocity, namely, $\vec{v}_{1-} = \vec{v}_{2-} = v_1\hat{z}$. This means that $\vec{v}_{1-} - \vec{v}_{2-} = 0$, so that $\dot{r}_{12} = 0$. This reduces Weber's potential energy given by Eq. (3) to a Coulombian repulsion. Due to the assumed charge neutrality of the wire we have $dq_{1-} = -dq_{1+}$ and $dq_{2-} = -dq_{2+}$. This means that the Coulombian attraction represented by the first term on the right hand side of Eq. (3) for interactions (III) and (IV) will yield two attractions that will cancel the Coulombian repulsions given by interactions (I) and (II). The only components which

will remain from interactions (III) and (IV) will be given by the velocity component of Eq. (3), namely, $-dq_1 dq_2 \dot{r}_{12}^2/8\pi\epsilon_0 r_{12} c^2$. In particular we need to consider the interaction between mobile conduction electrons (an element of charge $dq_1 = -\rho r_1 dr_1 d\varphi_1 dz_1$ located at $\vec{r}_1 = r_1\cos\varphi_1\hat{x} + r_1\sin\varphi_1\hat{y} + z_1\hat{z}$, moving with velocity $\vec{v}_1 = v_1\hat{z}$) and the stationary positive lattice (an element of charge $dq_2 = +\rho r_2 dr_2 d\varphi_2 dz_2$ located at $\vec{r}_2 = r_2\cos\varphi_2\hat{x} + r_2\sin\varphi_2\hat{y} + z_2\hat{z}$, which is at rest relative to the laboratory, that is, $\vec{v}_2 = 0$), where ρ is the conduction charge density inside the wire.

By considering the four interactions above what remains from Weber's energy is then given by:

$$U_W = \frac{\mu_0\rho^2}{8\pi} v_1^2 I_1 \ , \tag{4}$$

where $\mu_0 = 1/(\epsilon_0 c^2)$. From Eq. (3) the integral I_1 is given by (defining $a^2 \equiv r_1^2 + r_2^2 - 2r_1 r_2\cos(\varphi_2 - \varphi_1)$):

$$\begin{aligned} I_1 \equiv & \int_0^r r_1 dr_1 \int_0^r r_2 dr_2 \int_0^{2\pi} d\varphi_1 \int_0^{2\pi} d\varphi_2 \\ & \times \int_{-\ell/2}^{\ell/2} dz_1 \int_{-\ell/2}^{\ell/2} \frac{dz_2 (z_2 - z_1)^2}{[a^2 + (z_2 - z_1)^2]^{3/2}} \\ & = \int_0^r r_1 dr_1 \int_0^r r_2 dr_2 \int_0^{2\pi} d\varphi_1 \\ & \times \int_0^{2\pi} d\varphi_2 \Bigg(4\sqrt{a^2} - 4\sqrt{a^2 + \ell^2} \\ & + \ell \ln \frac{\ell + \sqrt{a^2 + \ell^2}}{-\ell + \sqrt{a^2 + \ell^2}} \Bigg) \ . \end{aligned} \tag{5}$$

In this point we utilize the approximation $\ell \gg r$, resulting:

$$I_1 \approx 2\pi^2 r^4 \ell \left(\ln\frac{2\ell}{r} - \frac{7}{4} \right) \approx 2\pi^2 r^4 \ell \ln\frac{\ell}{r} \ . \tag{6}$$

The energy U_W is then found to be given by:

$$U_W = \frac{1}{2}\left(\frac{\mu_0\rho^2\pi r^4 \ell}{2} \ln\frac{\ell}{r} \right) v_1^2 \ . \tag{7}$$

This can be arranged in a more interesting manner. Utilizing the self-inductance of this wire, $L = (\mu_0\ell/2\pi)\ln(\ell/r)$, and the total number of conduction electrons $N = \rho\pi r^2\ell/e$, where $-e = -1.6 \times 10^{-19}$ C is the electron's charge, the energy U_W can be rewritten as:

$$U_W = \frac{1}{2} L (\rho\pi r^2 v_1)^2 = N \frac{m_{\text{eff}} v_1^2}{2} \ , \tag{8}$$

where $m_{\text{eff}} \equiv (e\rho A/\ell)L$ is the effective electron mass. The order of magnitude of the effective mass can be obtained considering a typical copper wire, with $\ell \approx 1$ m and $r \approx 1$ mm, yielding $m_{\text{eff}} \approx 10^{-20}$ kg. As the free inertial mass of the electron is given by $m = 9 \times 10^{-31}$ kg, we have $m_{\text{eff}} \gg m$.

For case (B) a similar integration leads to:

$$U_W = \frac{\mu_0\sigma^2\ell d^2}{4\pi} \ln\frac{\ell}{d} v_1^2 = N \frac{m_{\text{eff}} v_1^2}{2} \ , \tag{9}$$

with $L = (\mu_0\ell/2\pi)\ln(\ell/d)$ and $N = \sigma\ell d/e$. Here $-\sigma$ is the surface charge density of the conduction electrons. In this case we have $m_{\text{eff}} \equiv (e\sigma d/\ell)L$.

For case (C) we obtain:

$$U_W = \mu_0\sigma^2\pi\ell r^2 \ln\frac{\ell}{r} v_1^2 = N \frac{m_{\text{eff}} v_1^2}{2} \ , \tag{10}$$

with $L = (\mu_0\ell/2\pi)\ln(\ell/r)$ and $N = \sigma 2\pi r\ell/e$. In this case $m_{\text{eff}} \equiv (e\sigma P/\ell)L$, where $P = 2\pi r$ is the length of the cross section. That is, the perimeter of the conductor through which the current flows.

An analogous calculation yields for case (D) the following relation:

$$U_W = \frac{\mu_0\sigma^2\pi\ell r^2}{2} v_1^2 = N \frac{m_{\text{eff}} v_1^2}{2} \ , \tag{11}$$

with $L = (\mu_0\pi r^2/\ell)$ and $m_{\text{eff}} \equiv (e\sigma\ell/2\pi r)L$.

Eqs. (8) to (11) yielded the same general result, namely:

$$U_W = N \frac{m_{\text{eff}} v_1^2}{2} \ , \tag{12}$$

with the appropriate effective inertial energy for each geometry.

Instead of integrating Weber's potential energy, it is also possible to work with the Lagrangian formulation. Weber's Lagrangian energy, as first obtained by Carl Neumann, is given by, [11, p. 68]:

$$S_W = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{1}{r_{12}} \left(1 + \frac{\dot{r}_{12}^2}{2c^2} \right) \ . \tag{13}$$

Note the change of sign in front of the term with $1/c^2$ when we compare Eqs. (3) and (13). Weber's Lagrangian is given by $L_W = T - S_W$, with T being the kinetic energy of the system. The Hamiltonian H_W and the conserved energy E_W on the other hand, are given by $H_W = E_W = T + U_W$.

Comparing Eqs. (3) and (13) indicate that the Lagrangian energy for these four cases according to Weber's expression can be written as:

$$S_W = -N \frac{m_{\text{eff}} v_1^2}{2} \ , \tag{14}$$

with the appropriate effective inertial mass for each geometry.

3. Darwin's Energy

To compare Weber's electrodynamics with classical electromagnetism we now consider Darwin's energy, [14], [15, pp. 150-151] and [16, Section 12.7, pp. 593–595]. We perform the same calculations as above but

utilizing Darwin's expression for the interaction energy between the charges. From this potential, we can derive the Liénard-Schwarzschild force utilizing the Lagrangian formalism. Darwin's energy is given by:

$$U_D = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{r_{12}} \times \left[1 + \frac{\vec{v}_1\cdot\vec{v}_2 + (\vec{v}_1\cdot\hat{r}_{12})(\vec{v}_2\cdot\hat{r}_{12})}{2c^2}\right] . \quad (15)$$

The Hamiltonian H_D and the conserved energy E_D for classical electromagnetism are given by $H_D = E_D = T + U_D$, where T is the kinetic energy. This is not to be confused with Darwin's Lagrangian energy:

$$S_D = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{1}{r_{12}} \times \left[1 - \frac{\vec{v}_1\cdot\vec{v}_2 + (\vec{v}_1\cdot\hat{r}_{12})(\vec{v}_2\cdot\hat{r}_{12})}{2c^2}\right] , \quad (16)$$

so that the Lagrangian is $L_D = T - S_D$.

The main difference between Darwin's energy (15) and Weber's energy (3) is that in Weber's energy the velocities appear only in their relational form ($\dot{r}_{12}^2 = (\hat{r}_{12}\cdot\vec{v}_{12})^2 = [\hat{r}_{12}\cdot(\vec{v}_1 - \vec{v}_2)]^2$). Darwin's energy, on the other hand, depends on the product of the velocity of each particle involved in the interaction. To see the difference between the two approaches we consider a fixed positive ion of the lattice with $\vec{v}_1 = 0$ interacting with a mobile conduction electron $\vec{v}_2 \neq 0$. In this case the velocity components of Eqs. (15) and (16) go to zero, yielding only a Coulombian interaction between these charges. On the other hand, as $\dot{r}_{12} \neq 0$ in this case, the velocity components of Eqs. (3) and (13) do not go to zero and the Weberian interaction for these charges is different from the Coulombian interaction. The opposite happens when we consider the interaction of two conduction electrons moving relative to the lattice with the same velocity, $\vec{v}_1 = \vec{v}_2 \neq 0$. In this case $\dot{r}_{12} = 0$ so that the Weberian interaction reduces to the Coulombian potential energy, while the velocity components of Eqs. (15) and (16) remain different from zero. This results in different interpretations of the phenomenom of a conductor carrying a current. By disregarding the Coulombian interaction which is common to Weber's electrodynamics and to Darwin's energy, the difference between the two approaches can be stated as follows: Classical electromagnetism (as represented by Darwin's energy) deals only with interactions among conduction electrons, while Weber's electrodynamics deals only with an interaction between the conduction electrons and the stationary lattice.

We calculate Darwin's energy for each of the four cases treated with Weber's energy. We begin with case (A). Once more we consider in each current element j positive and negative charges, dq_{j+} and dq_{j-}, respectively. Due to the assumed charge neutrality of the current elements we have $dq_{j-} = -dq_{j+}$. When we consider the interaction between two neutral current elements, $j = 1$ and $j = 2$, we need to take into account four interactions: (I) stationary positive ions dq_{1+} interacting with stationary positive ions dq_{2+}; (II) mobile conduction electrons dq_{1-} interacting with mobile conduction electrons dq_{2-}; (III) stationary positive ions dq_{1+} interacting with mobile conduction electrons dq_{2-}; and (IV) mobile conduction electrons dq_{1-} interacting with stationary positive ions dq_{2+}. We consider each of these interactions separately, beginning with interaction (I). From Eq. (15) and the fact that the ions are at rest relative to one another it remais only a repulsive Coulombian interaction. In interaction (II) we have from Eq. (15) a Coulombian repulsion plus a velocity component given by

$$-dq_{1-}dq_{2-}[\vec{v}_{1-}\cdot\vec{v}_{2-} + (\vec{v}_{1-}\cdot\hat{r}_{12})(\vec{v}_{2-}\cdot\hat{r}_{12})]/8\pi\epsilon_0 r_{12}c^2.$$

For interactions (III) and (IV) we have from Eq. (15) only a Coulombian attraction, as the velocity component goes to zero due to the fact that we are assuming a stationary positive lattice. This Coulombian attraction of interactions (III) and (IV) cancels the Coulombian repulsion of interactions (I) and (II). This means that from these four interactions based on Darwin's energy it only remains the velocity component of Eq. (15) arising from the interaction of the mobile electrons of element $j = 1$ with the mobile electrons of element $j = 2$. In particular we need to consider the interaction between mobile conduction electrons of the first element (an element of charge

$$dq_1 = -\rho r_1 dr_1 d\varphi_1 dz_1$$

located at

$$\vec{r}_1 = r_1\cos\varphi_1\hat{x} + r_1\sin\varphi_1\hat{y} + z_1\hat{z},$$

moving with velocity $\vec{v}_1 = v_1\hat{z}$) and the mobile conduction electrons of the second element (an element of charge $dq_2 = -\rho r_2 dr_2 d\varphi_2 dz_2$ located at $\vec{r}_2 = r_2\cos\varphi_2\hat{x} + r_2\sin\varphi_2\hat{y} + z_2\hat{z}$, moving with velocity $\vec{v}_2 = v_2\hat{z}$), where ρ is the conduction charge density inside the wire.

For case (A) and by taking into account the four interactions above, the integration of Darwin's energy (15) is then given by:

$$U_D = \frac{\mu_0\rho^2}{8\pi} v_1 v_2 I_2 . \quad (17)$$

The integral I_2 which appears here is given by, from

Eq. (15) and the considerations above:

$$I_2 \equiv \int_0^r r_1 dr_1 \int_0^r r_2 dr_2 \int_0^{2\pi} d\varphi_1 \int_0^{2\pi} d\varphi_2 \times \int_{-\ell/2}^{\ell/2} dz_1 \int_{-\ell/2}^{\ell/2} dz_2 \left[\frac{(z_2 - z_1)^2}{r_{12}^3} + \frac{1}{r_{12}} \right] = \int_0^r r_1 dr_1 \int_0^r r_2 dr_2 \int_0^{2\pi} d\varphi_1 \times \int_0^{2\pi} d\varphi_2 \left(6\sqrt{a^2} - 6\sqrt{a^2+\ell^2} + 2\ell \ln \frac{\ell + \sqrt{a^2+\ell^2}}{-\ell + \sqrt{a^2+\ell^2}} \right)$$

$$\approx \; 4\ell\pi^2 r^4 \left(\ln \frac{2\ell}{r} - \frac{5}{4} \right) \; \approx \; 4\ell\pi^2 r^4 \ln \frac{\ell}{r} \;\; , \quad (18)$$

where we defined once more $a^2 \equiv r_1^2 + r_2^2 - 2r_1 r_2 \cos(\varphi_1 - \varphi_2)$ and utilized in the last equation the approximation $\ell \gg r$. Due to the fact that the interaction is between conduction charges only, we maintained the term $v_1 v_2$ in the energy (17), although we are considering the case in which $v_2 = v_1 = v$. This will be discussed in the next section of this article.

The energy for this case becomes:

$$U_D = \frac{\mu_0 \rho^2 \pi r^4 \ell}{2} \ln \frac{\ell}{r} v_1 v_2 = N m_{\text{eff}} v_1 v_2 \;\; , \quad (19)$$

with $m_{\text{eff}} = e\rho AL/\ell$. So U_D is almost the same as Eq. (8), except for the factor 1/2.

For cases (B), (C) and (D) we obtain by similar calculations, respectively:

$$U_D = \frac{\mu_0 \sigma^2 \ell d^2}{2\pi} \ln \frac{\ell}{d} v_1 v_2 = N m_{\text{eff}} v_1 v_2 \;\; , \quad (20)$$

$$U_D = 2\mu_0 \sigma^2 r^2 \pi \ell \ln \frac{\ell}{r} v_1 v_2 = N m_{\text{eff}} v_1 v_2 \;\; , \quad (21)$$

$$U_D = \mu_0 \sigma^2 r^2 \pi \ell v_1 v_2 = N m_{\text{eff}} v_1 v_2 \;\; , \quad (22)$$

where the effective masses in Eqs. (20) to (22) are the same as the analogous ones obtained previously with Weber's energy for each case.

Eqs. (19) to (22) can be written as

$$U_D = N m_{\text{eff}} v_1 v_2 \;\; , \quad (23)$$

with the appropriate effective inertial mass for each geometry.

By comparing Eqs. (15), (16) and (19) to (22) we obtain that in the four geometries considered here, Darwin's Lagrangian energy can be written as:

$$S_D = -N m_{\text{eff}} v_1 v_2 \;\; , \quad (24)$$

with the appropriate effective inertial mass for each case.

4. Discussion and Conclusion

The velocity-dependent potentials we have used so far are also called generalized potentials. From them we can obtain the net force $\vec{F}_1$ applied on charge q_1 utilizing the standard approach, namely:

$$F_{1x} = -\frac{\partial S}{\partial x_1} + \frac{d}{dt} \frac{\partial S}{\partial v_{1x}} \;\; , \quad (25)$$

where x_1 is the x component of the position vector of the charge q_1, and v_{1x} is the x component of the velocity of the charge q_1. The y and z components can be obtained analogously. For both Weber's and Darwin's potentials, we obtain from Eq. (25) applied to Eqs. (8) to (14) and (19) to (24):

$$\vec{F}_1 = -m_{\text{eff}} \frac{d\vec{v}_1}{dt} = -m_{\text{eff}} \vec{a}_1 \;\; . \quad (26)$$

For Darwin's energy we utilized the fact that $\vec{v}_1 = \vec{v}_2 = \vec{v}$, after the derivation above.

Weber's energy (3) and Darwin's energy (15) differ by a factor of 1/2. The same can be said of the final results obtained from Weber's electrodynamics and from classical electromagnetism, namely, Eqs. (12) and (23). Despite this fact, the expressions for the forces in these two approaches are exactly the same, that is, Eq. (26). That the forces give the same result had already been obtained in [2]. Beyond the difference of 1/2 in the expressions for Weber's and Darwin's energies, these energies have completely different interpretations. Darwin's energy is due only to the interactions among the conduction electrons, and all other interactions (between conduction electrons and the stationary positive lattice, and among the charges of the positive lattice with one another) are zero. This happens due to the explicit velocities that appear in Eq. (15). Weber's energy is due only to a non-zero interaction between the conduction electrons and the positive lattice (where there is a non-zero relational velocity, $\dot{r}_{12} \neq 0$), while all other interactions are null.

Another important aspect to discuss is the different effective inertial masses obtained in cases (C) and (D). In these cases the conductor has the same size and form, but the currents are orthogonal to one another (in case (C) the current flows along the longitudinal $\hat{z}$ direction, while in case (D) the current flows along the azimuthal $\hat{\varphi}$ direction). Notice that Weber's effective masses for these cases differ considerably, respectively, $m_{\text{eff}} = \mu_0 e\sigma r \ln(\ell/r)$ and $m_{\text{eff}} = \mu_0 e\sigma r/2$. This anisotropy in the effective intertial mass of a test charge had already been noticed before, [24] and [11, p. 189–190].

We can write the results of the effective mass for Weber's energy in the general form given below for volumetric and surface currents, respectively:

$$m_{\text{eff}} = \frac{e\rho A}{\ell'} L \;\; , \qquad \text{or,} \qquad m_{\text{eff}} = \frac{e\sigma P}{\ell'} L \;\; , \quad (27)$$

where A is the cross section of the volumetric current, P is the perimeter of the cross section of the surface currents and ℓ' is the total length of the conductor

along the direction of the current. In the same way, the Weberian energies can be written in the general form:

$$U_W = N\frac{m_{\text{eff}}v^2}{2} = \frac{1}{2}Li^2 \ . \qquad (28)$$

We can interpret Eq. (28) as follows: the kinetic energy of conduction electrons in Weber's electrodynamics is quantitatively identified with the classical magnetic energy (which is not the case for Darwin's energy, Eq. (23)). Moreover, this shows a connection between the self-inductance of a circuit and the effective inertial mass of its conduction electrons. This interpretation offers a new insight to the microscopic theory of conduction. In particular it indicates that the collective behaviour of the conduction electrons in slowly varying current carrying conductors represented by the magnetic energy $U_B = Li^2/2$ can be seen as an effective kinetic energy of the mobile electrons arising from their electrodynamic interaction with the stationary positive lattice of the conductor through Weber's force law, $U_B = Nm_{\text{eff}}v^2/2$. According to Mach's principle the usual kinetic energy $mv^2/2$ of any particle arises from its gravitational interaction with the distant bodies in the cosmos, [12, 25–29]. The quantitative results presented in this paper yielding $Li^2/2 = Nm_{\text{eff}}v^2/2$ indicate that also the magnetic energy arises from an electrodynamic interaction of the mobile electrons with the stationary positive ions of the lattice. As m_{eff} was found in all cases much greater than the usual free electron mass $m = 9.1 \times 10^{-31}$ kg, this explains why we can usually neglect the usual inertial mass when dealing with current carrying conductors. The results obtained here indicate the correctness of Mach's principle as applied to electromagnetism, by showing how to derive the magnetic energy as a dynamical effect arising from the interacting of the conduction electrons when they move relative to the stationary positive lattice, [30].

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