Surface charges and fields in stationary conductors with steady currents

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Abstract. Is there a force between an external stationary charge and a resistive stationary conductor carrying a steady current? The answer to this question is positive. In this work we present the main results of this interaction. We present experiments published in the literature which measured these effects. We also show the analytical solution for the most common situations, namely: straight wires, strips and toroidal conductors. This force is due to charges spread along the surface of the current carrying conductor. This distribution of surface charges is maintained by the battery, and keep the current flowing along the conductor. This had been pointed out by Kirchhoff and Weber. These surface charges keep the potential gradient along the resistive circuit. They also create an electric field inside and outside the conductor. With this approach we show that there are no fundamental differences between electrostatics and current-carrying conductors.

Keywords: Electric potential, electric field outside steady currents, surface charges
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INTRODUCTION

Consider a stationary resistive wire, in the form of a closed loop circuit, connected to a battery which generates a voltage V between its terminals. If the wire carries a steady current I, will the resistive wire exert a force on a stationary charge q located nearby? Do some components of this force depend on the voltage generated by the battery? These questions can be rephrased in terms of electric fields: does the wire generate an electric field in the surrounding space, depending on the battery voltage? Is the wire carrying a steady current charged along its surface, or inside it?

These fundamental questions have been answered by many physicists as "no," and this opinion has been held for a long time. We show that conducting wires with steady currents are not neutral along its surface, and there is in fact an electric field outside the conductor, proportional to the voltage of the battery.

In this work we show theoretically the existence of a force upon the stationary external charge exerted by a resistive wire connected to a battery and carrying a steady current when there is no motion between the test charge and the wire. We also compare the theoretical calculations with the experimental results which proved the existence of this force. Detailed results can be found in the book [1], which can be freely downloaded in PDF format.

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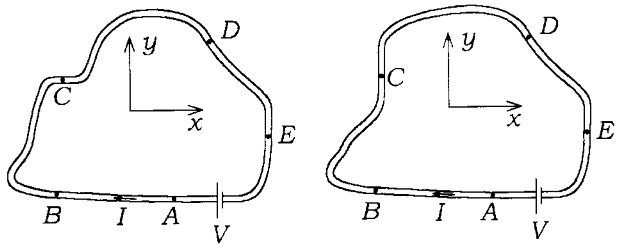

FIGURE 1. A wire is bent around point C, changing the direction of the electric field.

ELECTRIC FIELD INSIDE AND OUTSIDE THE WIRE

Consider a resistive wire of finite conductivity connected to a battery V carrying a steady current I, as in the left side of Figure 1. Where are the charges which generate the electric field at each point along the wire located? It might be thought that this electric field is due to the battery, but this is not the complete answer. We can see that the battery does not generate the electric field at all points along the wire looking at the right side of Figure 1. At a specific point C we bend the wire, changing the direction of the electric field. However, at the other points there is no change in the electric field. The battery can not be responsible for such local change. This was Weber's and Kirchhoff's idea [2].

What creates the electric field inside the wire are surface charges, distributed as a continuous gradient density along the length of the wire, more positive towards the positive terminal of the battery, and increasingly negative towards the negative terminal.

CYLINDRIC WIRE

Consider a long straight wire of circular cross-section, of length ℓ and radius $a \ll \ell$. The uniform longitudinal electric field inside the wire keeps the steady current flowing, and is generated by surface charges along the conductor. By supposing a linear distribution [3], we can calculate the electric potential and the electric field. The mathematical treatment is detailed in [4]. Figure 2 shows the electric field lines along a longitudinal cross-section of the wire.

The solution is valid in the case of a solid cylinder or a hollow cylindrical shell. The potential is linear along the cylindrical surface, and obeys Laplace's equation in the space surrounding the conductor. Similar results can be obtained for a conducting strip (of length ℓ and width $a \ll \ell$) [5].

TOROIDAL CONDUCTOR

Consider a resistive toroidal conductor with a steady current I in the azimuthal direction, flowing along the circular loop. There is a battery located in $\varphi = \pm\pi$, as indicated in Figure 3. This case is detailed in [6].

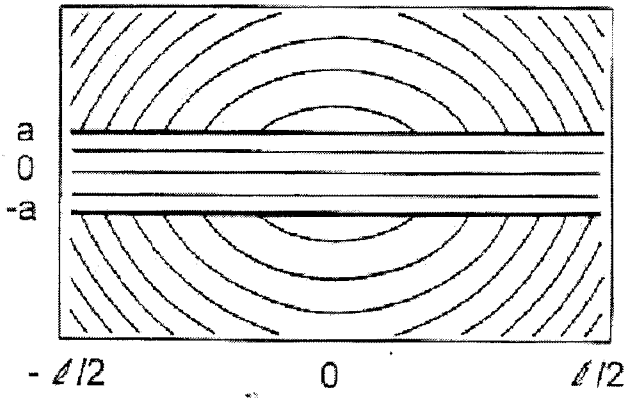

FIGURE 2. Electric field lines along a longitudinal cross-section of the wire. The wire surface is represented by the two horizontal thick lines.

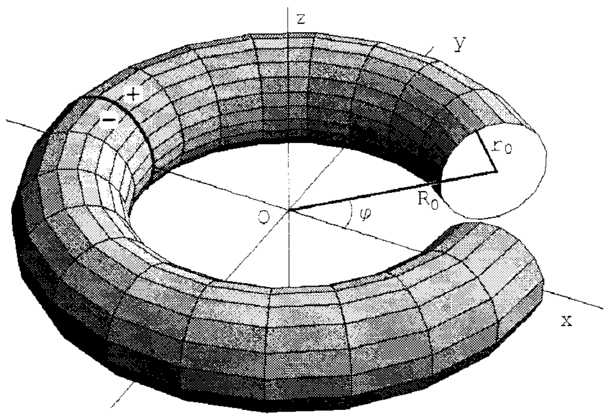

FIGURE 3. Toroidal conductor with a steady azimuthal current. The battery is indicated by the signs "+" and "-."

We solve Laplace's equation in toroidal coordinates, using the azimuthally linear potential along the conductor surface as a boundary condition. The equipotential lines are shown in Figure 4. The surface charges along the conductor, in this case, is not linear as in the case of a straight conductor.

Experiments performed by Jefimenko mapped the electric field lines [7]. He utilised a transparent conducting ink to make a two-dimensional printed circuit on glass plates of 10 × 12 inches. After the power supply (of about 10^4 V) was turned on, he spread some fine grass seeds over the plate and conducting system. The seeds lined up in the direction of the electric field over and outside the conductors.

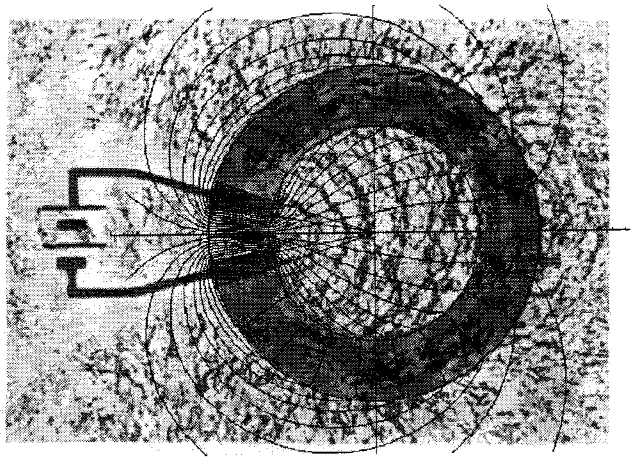

FIGURE 4. Equipotential lines in the plane of the toroid. This image was superimposed on an experimental plate by Jefimenko.

CONCLUSION

There is a non-null electric field outside stationary conductors carrying steady currents. This electric field is proportional to the battery *emf* and is generated by surface charges along the conductor. This approach shows a very important connection between electrostatics and magnetostatics (stationary currents). A related topic is to consider in detail the behavior of surface charges and the corresponding external field in the transition from steady-currents to low and high frequency circuits with alternating currents [8]. Another is a possible connection between the external electric field around a resistive cylindrical conductor carrying a steady current and the Aharonov-Bohm effect, as discussed in [9].

For further discussion, with many more examples, exact mathematical expressions and references, see the book [1].

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