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Nonlocal Forces of Inertia in Cosmology

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This paper reviews the origin of inertia according to Mach's principle and Weber's law of gravitation. The resulting theory is based on simultaneous nonlocal gravitational interactions between particles in the solar system and others in the remote universe beyond the Milky Way galaxy. It explains the precession of the perihelion of Mercury. A most important implication of the Mach–Weber theory of the force of inertia is the necessity for a large amount of uniformly distributed matter in the galactic universe. This matter could be the source of the cosmic background radiation. Nonlocal inertia forces are compatible with a static universe and also with an expanding universe but the latter would demand slow changes in the mass of particles and the gravitational constant.

1. INTRODUCTION

Since the Aharonov–Bohm effect was first mentioned(1) and experimentally confirmed(2) thirty years ago, physics has moved imperceptibly away from field-contact action. This movement gathered strength with Bell's logical arguments,(3) and the confirmatory experiment by Aspect *et al.*,(4) showing that local actions cannot govern remote quantum correlations, although the Aspect *et al.* claims have been criticized by Wesley.(5) To soften the blow, the new term "nonlocal action" was introduced into the physics vocabulary to distinguish the new phenomena from energy impact and recoil concepts of quantum field theory.

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At the same time, a dozen or more decisive electrodynamic experiments were found to be incompatible with relativistic electromagnetism,[6] and could only be explained with the pre-Maxwellian theories of Ampère, F. E. Neumann, Kirchhoff, and Weber which were all based on simultaneous far-actions, less precisely described as instantaneous action at a distance. Only with this type of nonlocal action can the functioning of the common induction motor be understood.[7] The recent experiments which are being quantitatively explained with simultaneous far-actions deal with: jet propulsion in liquids,[8] the exploding wire phenomena,[9,10] the electromagnetic impulse pendulum and the mechanism of railguns,[11-15] Ampère forces in gaseous and liquid conductors,[16-18] etc.

Yet another development drew attention to nonlocal actions. This was the full implementation of Mach's principle[19] with a modified law of Newtonian universal gravitation. This law was first proposed by Weber and Tisserand in the nineteenth century.[20,21] The identity of gravitational and inertial mass, as established by this model[19] on the basis of Weber's law, now appears to be the strongest argument in favor of simultaneous far-actions. According to this every particle in the universe is permanently interconnected to every other particle.[22] The purpose of our paper is to examine some of the cosmological consequences of this interconnectedness. It should be remarked that the main aspects of the Mach–Weber model to be discussed in this paper (application of a Weber's law to gravitation in order to derive Mach's principle) had been derived in an extremely important paper by Erwin Schrödinger[23] of 1925, although he was not aware of the previous investigations of Weber and Tisserand. Schrödinger dealt only with the velocity-dependent potential energy, while in Ref. 19 we worked also with Weber's force. The essential results can be obtained in both-ways.

2. MACH'S PRINCIPLE

Einstein[24] coined the phrase "Mach's principle." It stands for the conjecture that the forces of inertia on local matter are determined by the distribution of matter throughout the universe. According to Mach, there is no meaning in Newton's absolute space, absolute time, and absolute motion. All motion of matter is relative to other matter. Mach therefore referred Newton's first and second laws of motion to the frame of the "fixed stars." In 1883 he said (Ref. 25, p. 336): "I have remained to the present day the only one who insists upon referring the law of inertia to the earth, and in the case of motions of great spatial and temporal extent, to the fixed stars."

Today we know that most visible objects in the sky belong to our own galaxy which rotates relative to the distant galaxies with a period of about 250 million years. It is this set of remote galaxies which we will take as our inertial reference frame and call it the "fixed stars."

Mach identified the reference frame of the fixed stars to be Newton's "absolute space." But Mach's principle stood for more. It had dynamical consequences. For example, according to Mach a spinning spherical shell, or any spinning isotropic distribution of matter, should generate within itself centrifugal forces on every body not lying on the axis of rotation. With regard to Newton's bucket experiment, Mach (Ref. 25, pp. 279 and 284) said: "Try to fix Newton's bucket and rotate the heaven of the fixed Stars and then prove the absence of centrifugal forces.... The principles of mechanics can, indeed, be so conceived that even for relative rotations centrifugal forces arise."

Three important quantitative facts strongly support Mach's philosophy. In Newtonian physics these facts are treated as coincidences and no explanation is supplied. A theory incorporating Mach's principle should justify these facts within a coherent picture of the universe.

The first and most important of the three facts is the identity of inertial and gravitational mass. By inertial mass we mean the mass which appears in linear and angular momentum, in Newton's second law of motion ($m_i\vec{a}$), and in expressions of kinetic energy. The inertial mass of a body determines the inertial force acting on the body, that is, the force which resists acceleration, deceleration, and change of direction relative to the fixed stars. On the other hand, gravitational mass is the mass which appears in Newton's law of universal gravitation, in the weight of a body, and in the gravitational potential energy. As they arise from totally different experiments, there is no reason that the two masses of a body should be the same, unless this reason is provided by Mach's principle.

The identity of inertial and gravitational mass revealed itself for the first time in Galileo's free fall experiments. Newton demonstrated it with carefully designed pendulum experiments, and showed it was correct to at least one part in a thousand. At the turn of the century Eötvos concluded that this agreement was better than one part in 10^8, and nowadays it is known to hold to at least one part in 10^{12}. This mass-equivalence became the basis of Einstein's general theory of relativity. His principle of equivalence unequivocally linked inertial interactions to gravitational interactions.

The second fact which suggests the validity of Mach's point of view can be expressed in two ways. (A) The universe as a whole does not rotate relative to absolute space, or (B) the kinematical rotation of the earth is equal to its dynamical rotation. If the universe as a whole did rotate, most galaxies would be located in a disc normal to the axis of rotation, as is the case with the stars of a galaxy and the planets of the solar system. Although clusters of galaxies have now been observed, their distribution shows no preferred directions as seen from the earth.

With respect to (B), we observe an apparent circular motion of the fixed stars and galaxies about the Pole Star (in the northern hemisphere) with a period of 24 hours. This rotation we attribute to the kinematic spin of the earth. We also have dynamical proof of the spin of the earth. This is provided by the Foucault pendulum and the distortion of the shape of the earth. The equatorial radius of our planet is approximately one part in three hundred greater than the polar radius. The kinematically and the dynamically determined angular velocities of the earth are equal to one part in 10^8 radians per year.[26] In practice this tells us that the best inertial frame is that of the distant galaxies. It may be another coincidence and does not necessarily confirm Mach's theory, but it appears likely that the inertial properties of bodies on earth depend on the remote universe.

The third fact supporting Mach's argument was discovered after his death. Since about 1930 it has been known that

$$H_o^2 \approx G\rho_o \tag{1}$$

where H_o is Hubble's constant, G Newton's gravitational constant, and ρ_o the mean mass density of the universe. Although this quantitative relation has been known for 65 years, the origin of the cosmological redshift is still uncertain (Doppler effect, tired light, redshift based on a gravitational effect, etc.) It could be a chance coincidence that these three quantities are interrelated, but if we accept Mach's principle and nonlocal actions it is not at all surprising that the gravitational constant should depend on the extent of the matter distribution throughout the universe.

3. THE MACH–WEBER MODEL

Weber proposed a modified form of Newton's law of gravitation.[20] With Weber's law it has been possible to fully implement Mach's principle using gravitational forces.[19, 23] The Mach–Weber model is based on two postulates:

(I) The sum of all forces (mechanical contact, gravitational, inertial, electromagnetic, nuclear, etc.) on any material particle, or body, is always zero in all coordinate frames.

(II) The gravitational attraction between two material particles, or bodies, 1 and 2 is given by Weber's force:

$$F_{1,2} = -H_g \frac{m_1 m_2}{r^2}\left[1 - \frac{6}{c^2}\left(\frac{\dot{r}^2}{2} - r\ddot{r}\right)\right] \tag{2}$$

where H_g is a constant (it may be put equal to G to simplify the analysis, but this is not essential as we will see), m_1 and m_2 are the gravitational masses of the particles 1 and 2, and c is a constant with the value of the velocity of light. The distance between the two particles is r, and their relative velocity and acceleration along r are denoted by $\dot{r}$ and $\ddot{r}$. As we see in the sequel, it will be the $\ddot{r}$-contribution which will generate Newton's inertial force $-m\vec{a}$. We can distinguish the $\dot{r}$-contribution from the inertial force $m\vec{a}$ by observing that the second one is fixed and homogeneous, while the first can generate an anisotropy in the effective inertial mass of a body. The $\dot{r}$-contribution can also generate an effective inertial mass which depends on the potential at the location of the body. Possible experimental consequences of these facts (with the $\dot{r}$ present in Weber's electromagnetic force) have been discussed in Refs. 27 and 28.

The negative sign of Eq. (2) stands for a force of attraction. It is Newton's nonlocal law of universal gravitation with terms added which depend on the relative velocity and acceleration of the interacting particles. As in Newton's cosmology, the Weber law represents interconnectedness of all matter in the universe via simultaneous far-actions.

The virtual work concept ($F = -dU/dr$) may be used to derive the Weber force from the potential energy

$$U = -H_g \frac{m_1 m_2}{r}\left(1 - \frac{3\dot{r}^2}{c^2}\right) \tag{3}$$

It was based on this expression that Schrödinger developed his theory.[23]

To derive (2) from (3) we utilize $F = -dU/dr$ and $\dot{r} = \dot{r}(t)$, so that

$$\frac{d\dot{r}^2}{dr} = 2\dot{r}\frac{d\dot{r}}{dr} = 2\dot{r}\frac{d\dot{r}}{dt}\frac{dt}{dr} = 2\ddot{r}$$

The third term in Weber's force law, Eq. (2), is proportional to $1/r$ and, therefore, stands for a long-range force compared to the other two terms which are proportional to $1/r^2$ and represent short-range forces. It is the long-range third term which implements Mach's principle.[19]

Other authors have tried to explain the origin of inertia with the help of $1/r$ long-range forces proportional to relative acceleration. These investigators include Sciama,[29] Brown,[30] Treder,[31] Edwards,[32] Barbour and Bertott,[33] Eby,[34] Ghosh,[35] and others.

A good analysis of this kind was provided by French.[36] In his words (p. 543):

"We know that the act of giving an object an acceleration **a**, with respect to the inertial frame defined by the fixed stars, calls into play an inertial force, equal to $-m\mathbf{a}$, that expresses the resistance of the object to being accelerated."

On this basis he derives the inertial force on a particle of mass m as being given by

$$\mathbf{F}_i = -m\mathbf{a} \sum_j \left(\frac{GM_j}{c^2 r} \right) \tag{4}$$

In French's formula m is the mass of an object on earth and M_j the mass of a celestial body and the summation extends over all the universe. The distance between the centers of mass is r and G is Newton's gravitational constant. French claims that Eq. (4) should be Newton's second law of motion, and then we would have

$$\sum_j \left(\frac{GM_j}{c^2 r} \right) = 1 \tag{5}$$

French recognized that the (M_j/r) ratios of the earth, the sun, and even the Milky Way were far too small to balance Eq. (5). Only the universe beyond our galaxy contained sufficient mass which, in spite of the great distances, could possibly satisfy the requirement of (5). Should the origin of inertia be due to remote matter interactions, it has to involve the furthest reaches of the universe.

What is missing in French's theory is a force law which reciprocally couples the particle on earth with another particle in the distant universe, complying with Newton's third law. It would have to be a long-range force as indicated by the $(1/r)$ relationship of Eq. (4). This alone rules out Newton's law of gravitation from being responsible for inertia. Furthermore, if the inertia force on the particle on earth is to be the same in all directions of the acceleration, the mass M_j must be distributed homogeneously throughout the universe.

The most startling assumption underlying French's derivation of the origin of inertia is the simultaneity of the matter interactions over distances that are normally measured in light-years. The force $\mathbf{F}_i$ of Eq. (4), arises instantly when the acceleration **a** commences. As Mach is supposed to have put it:[37] "When the subway jerks, it's the fixed stars that throw you down." The remote universe acts on us without time-delay.

To examine the Mach–Weber model of the origin of inertia we study the motion of a particle of mass m_1 inside a homogeneous and stationary spherical matter shell of particles j. It is known that, whatever the state of motion of m_1, the combined forces of Newtonian gravitation on m_1, due to the matter shell, cancel identically to zero. Suppose the stationary shell has a radius r, is of thickness dr, and has uniform density ρ. If the acceleration of this particle m_1 is $\mathbf{a}_1$ relative to the stationary shell, in any direction and position whatsoever, it has been shown in Ref. 19 that, according to Weber's law of gravitation, the force on it will be

$$\mathbf{F}_i = \frac{H_g}{c^2} 8\pi r \cdot dr \cdot \rho(-m_1 \mathbf{a}_1) \tag{6}$$

This is similar to French's result of Eq. (4).

If we now extend the calculation to a spherical volume of radius R we have to integrate (6) and obtain (supposing a constant mass density ρ):

$$\mathbf{F}_i = \frac{H_g}{c^2} 4\pi R^2 \rho(-m_1 \mathbf{a}_1) \tag{7}$$

Since the volume of the sphere is $V = (4/3)\,\pi R^3$ and the total mass of the sphere is $M = V\rho$, Eq. (7) may be written

$$\mathbf{F}_i = \frac{3H_g M}{Rc^2}(-m_1 \mathbf{a}_1) \tag{8}$$

The Milky Way is surrounded by matter in all directions. We then consider M as being the homogeneous mass surrounding our galaxy. We will suppose that the body m_1 (for instance, a planet) is interacting with l local bodies (other planets, the sun, etc.) and represent the forces of these local bodies on m_1 by $\sum_l \mathbf{F}_1$. According to Eq. (8) and to postulate (I) of the Mach–Weber model we get

$$\sum_l \mathbf{F}_1 + \left(-\frac{3H_g M}{Rc^2} m_1 \mathbf{a}_1 \right) = 0 \tag{9}$$

And this can be identified with Newton's second law of motion.

In order to see this more clearly we can study the "two-body" planetary problem (with Mach's point of view it is the two bodies—the sun and a planet—plus the fixed stars). The particle m_1 is a planet interacting with the fixed stars through (8), and with the sun through Weber's gravitational force (2). For the typical velocities and accelerations of the planets in the solar system we can usually neglect the $\dot{r}^2/c^2$ and $r\ddot{r}/c^2$ in (2). Substituting $\sum_l \mathbf{F}_l$ in (9) by Eq. (2) without the small corrections in $\dot{r}$ and $\ddot{r}$ yields

$$-H_g \frac{m_1 m_s}{r^2} \hat{r} - \frac{3H_g M}{Rc^2} m_1 \mathbf{a}_1 = 0 \tag{10}$$

where m_s is the mass of the sun, r is the distance between the sun and the planet m_1, and $\hat{r}$ is the unit vector pointing from the sun to the planet m_1. This is exactly the two-body problem in Newtonian mechanics, provided that $H_g/(3H_g M/Rc^2) = G$, Newton's gravitational constant. As French emphasized, this has been known to be true for almost sixty years now [this relation is the same as Eq. (1) remembering that $M = \rho_o 4\pi R^3/3$ and $H_o = c/R$]. It shows that Eqs. (8) and (9) agree with French's formulation of Mach's principle [Eq. (4)]. From the beginning we utilized a force law which complies with Newton's third law (action and reaction).

Equation (10) also shows that the constant H_g is undetermined as it can be cancelled out in both terms. This is a consequence of the first postulate of the Mach–Weber model as we can multiply all forces by the same constant α without altering any result. This is also in compliance with Mach's principle as now only ratios of forces will be relevant, but not the absolute value of any single force.

Here we neglected the terms with $\dot{r}^2/c^2$ and $r\ddot{r}/c^2$ only to show how to arrive at Eq. (1) in the Mach–Weber model. If the orbits of the planets were perfect circles we would have constant r's so that $\dot{r}$ and $\ddot{r}$ would be zero for the planets. In reality the orbits are ellipses with small eccentricities. This means that these terms cannot be completely neglected. As a matter of fact it will be exactly these terms which will generate the precession of the perihelion of the planets in agreement with observations (the algebraic value of this precession in the Mach–Weber model is exactly the same as the expression of Einstein's general relativity, although coming from a different orbit equation). This was shown by Paul Gerber in the last century,[38] although his work was criticized by Seeliger.[39] The precession of the perihelion of the planets with Weber's law has peen calculated independently by Tisserand,[21] Schrödinger,[23] Eby,[34] and Assis.[19]

Wesley believes that the correct gravitational force up to second order in $1/c$ should be given by Eq. (2) with a $1/c^2$ factor, instead of $6/c^2$.[40] His expression is then completely similar to Weber's original force applied to electromagnetism (replacing $-Gm_1 m_2$ by $q_1 q_2/4\pi\varepsilon_o$).

Once Newton's second law has been derived from Mach's principle, the other forces of inertia (e.g., centrifugal force, Coriolis force, and the forces on gyroscopes) follow from the normal Newtonian mechanics. The only important fact to keep in mind is that instead of absolute space the inertial frame has been shown to be the frame of the "fixed stars."

This model explains the three quantitative facts which support Mach's principle, as mentioned previously. Equation (8) arises from a gravitational interaction [through Weber's force, Eq. (2)] between m_1 and the remaining universe. As the concept of inertial mass was never introduced in the model, all the masses which appear in both sides of (10) are gravitational masses. Equation (10) proves the identity of inertial and gravitational mass. The second fact (absolute space is the frame of the fixed stars, or the kinematical rotation of the earth is the same as its dynamical rotation) follows directly from this model, because we derived Newton's second law [our Eq. (10)] in the frame of the fixed stars. The centrifugal and Coriolis forces will appear in systems which rotate relative to the fixed stars, as was emphasized by Mach. And the third fact [Eq. (1)] is a necessary consequence of this model, as Eq. (10) and the numerical relationship between G, H_o and ρ_o indicate. As these three facts are determined quantitatively by the Mach–Weber model, we now discuss other related cosmological issues.

4. COSMOLOGICAL CONSEQUENCES OF MACH–WEBER INERTIA

Gravitation theory is more than three hundred years old. Newton's universal law has accounted for almost all gravitational observations made in three centuries. Apart from inertia, which Newton did not treat as a gravitational phenomenon, the only remaining mystery was the anomalous precession of the perihelion of the planet Mercury (we will not consider here the interaction between gravitation and electromagnetism, as in the gravitational deflection of light and the gravitational redshift). General relativity finally provided a reason for the odd behavior of Mercury.

With an expanded law of gravitation, as the one proposed by Weber, there arose the prospect that the precession of the elliptic orbit of Mercury around the sun could be explained without invoking the gravitation and local actions of general relativity. As we observed previously, this was accomplished and represents the first astronomical success of Mach–Weber inertia.

A truly cosmological consequence of the Mach–Weber theory is that, as shown by Eq. (7), the universe in this model is a sphere of finite radius R.[41] With Hubble's constant H_o, this radius would be $R = c/H_o$. The size of the universe has been in the forefront of all cosmological speculations. As Mach's philosophy is closely allied to Newton's point of view, we might examine what Newton had to say about the size of the universe.

Early in his life Newton believed in a finite-size cosmos immersed in an infinite void, as suggested by the Stoics in Greece. Later he reasoned that a finite universe would fall inward and congregate in a large spherical mass.[42] To avoid gravitational collapse, Newton thought the Creator must have made the universe infinitely large. Then every star could be pulled equally strongly in all directions. The cancellation of multi-directional pulls, however, rested precariously on a uniform matter distribution throughout the universe. Newton spoke of each star being poised on the point of a needle, ready to fall in any direction.

If we invoke inertia forces to stabilize the universe, by way of centrifugal action, it seems that in this model we can return to the finite universe of the Stoics. The reason is that if we have a constant and uniform matter density ρ and make $R \to \infty$ in Eq. (7), we obtain that the inertial force diverges to infinity. To avoid this we can say that the matter in the universe is limited to a finite volume. Whether this is situated in a larger void, or not, is unimportant. If the universe is infinite and contains an infinite amount of matter, the present model would fail and must be modified. A proposal in this direction, utilizing the exponential decay for gravitation proposed by Seeliger and C. Neumann in a Weber law applied to an infinite and homogeneous universe, has been given recently.[43, 44]

Another consequence of the Mach–Weber model is the necessity for a homogeneous distribution of matter in the remote regions of the cosmos.[41] Not all the cosmic and nearby matter need be distributed uniformly. The inhomogeneous part does give rise to Newtonian gravitational attraction represented by the first term of Eq. (2). It is responsible for the weight of all bodies on the surface of the earth. Space travel relies on the concentrated masses of the sun and all its planets. The clusters of galaxies cause no noticeable gravitational effect in the solar system. The distribution of all visible matter in the sky may be nonuniform. If this is the case, it cannot be the cause of the Mach–Weber inertia forces, unless the forces of inertia (or the inertial masses) are found to have different values in different directions.

Equation (7), the inertia force law, stands and falls by the existence of a considerable amount of uniformly distributed matter outside our own galaxy. At present we feel this may be some form of cosmic dust for matter dispersed homogeneously in atomic or molecular form, or the uniform distribution of galaxies in the sky, or cosmic plasmas.

The finely divided dark cosmic matter may not only be required to imbue matter on earth with inertia, but as Arp *et al.*[45] suggested, it could also be the source of the cosmic background radiation which impinges on earth from all directions. Therefore, the requirement of a uniform dark matter distribution and the measured isotropy of the cosmic background radiation[46] go hand-in-hand. It must be stressed that nonlocal inertia forces and the local emission and reception of electromagnetic radiation are not mutually exclusive phenomena. The cosmic dust, it seems, continuously emits radiation, representing a temperature of 2.7 K, and we receive it in our part of the universe much later.

The Mach–Weber origin of inertia is not in obvious conflict with the expanding universe and the "big-bang" concept of creation, but it is also consistent with a static universe. If the total amount of matter in the cosmos is being conserved, the expansion implies, however, a very slow decrease in the density of cosmic dust. Matter conservation demands $\rho R^3 = \text{constant}$. This is not guaranteed by the inertia force law, Eq. (7). Hence an expanding universe may be accompanied by slow changes in particle mass and the gravitational constant. In a nonexpanding universe, on the other hand, the gravitational constant and the particle masses would stay constant in time.[43, 44]

Further discussion on this subject can be found in Ref. 47, Chapters 7 and 8.

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REFERENCES

1. Y. Aharonov and D. Bohm, "Significance of electromagnetic potentials in quantum theory," *Phys. Rev.* **115**, 485 (1959).
2. G. Moellenstedt and W. Bayh, "Messung der kontinuierlichen Phasenschiebung im kraftfreiem Raum durch das magnetische Vektor Potential," *Naturwissenschaften* **49**, 81 (1962).
3. J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, New York, 1987).
4. A. Aspect, J. Dalibard, and G. Roger, "Experimental test of Bell's inequalities using time-varying analyzers," *Phys. Rev. Lett.* **49**, 1804 (1982).
5. J. P. Wesley, *Phys. Essays* **7**, 240 (1994).
6. P. Graneau, *Ampère–Neumann Electrodynamics of Metals*, 2nd edn. (Hadronic Press, Palm Harbor, 1994).
7. P. Graneau, "Nonlocal action in the induction motor," *Found. Phys. Lett.* **4**, 499 (1991).

8. P. Graneau, "Electromagnetic jet-propulsion in the direction of current flow," *Nature* **295**, 311 (1982).
9. P. Graneau, "First indication of Ampère tension in solid electric conductors," *Phys. Lett. A* **97**, 253 (1993).
10. P. Graneau, "Wire explosions," *Phys. Lett A* **120**, 77 (1987).
11. P. T. Pappas, "The original Ampère force and Biot–Savart and Lorentz forces," *Nuovo Cimento B* **76**, 189 (1983).
12. P. G. Moyssides and P. T. Pappas, "Rigorous quantitative test of Biot–Savart–Lorentz forces," *J. Appl. Phys.* **59**, 19 (1986).
13. P. Graneau, "Railgun Recoil and relativity," *J. Phys. D* **20**, 391 (1987).
14. P. G. Moyssides, "Experimental verification of the Biot–Savart–Lorentz and Ampère force laws in a closed circuit, revisited," *IEEE Trans. Magn.* **25**, 4313 (1989).
15. A. K. T. Assis, "On the mechanism of railguns," *Galilean Electrodynam.* 3, 93 (1992).
16. J. Nasilowski, "A note on longitudinal Ampère forces in gaseous conductors," *Phys. Lett. A* **111**, 315 (1985).
17. T. E. Phipps, Jr., "New evidence for Ampère longitudinal forces," *Phys. Essays* 3, 198 (1990).
18. T. E. Phipps and T. E. Phipps, Jr., "Observation of Ampère forces in mercury," *Phys. Lett. A* **146**, 6 (1990).
19. A. K. T. Assis, "On Mach's principle," *Found. Phys. Lett.* **2**, 301 (1989).
20. W. Weber, *Wilhelm Weber's Werke*, 6 volumes (Springer, Berlin, 1892-4); see especially Vol. 4, p. 479.
21. M. F. Tisserand, "Sur le mouvement des planètes autour du soleil, d'après la loi électrodynamique de Weber," *C. R. Acad. Sci. (Paris)* **75**, 760 (1872).
22. P. Graneau, "Interconnecting action-at-a-distance," *Phys. Essays* **3**, 340 (1990).
23. E. Schrödinger, "Die Erfüllbarkeit der Relativitätsforderung in der klassischen Mechanik," *Ann. Phys. (Leipzig)* **77**, 325 (1925).
24. A. Einstein, "Prinzipielles zur allgemeinen Relativitatstheorie," *Ann. Phys. (Leipzig)* **55**, 241 (1918).
25. E. Mach, *The Science of Mechanics* (Open Court, La Salle, 1960).
26. L. I. Schiff, "Observational basis of Mach's principle," *Rev. Mod. Phys.* **36**, 510 (1964).
27. A. K. T. Assis, "Centrifugal electrical force," *Commun. Theor. Phys.* **18**, 475 (1992).
28. A. K. T. Assis, "Changing the inertial mass of a charged body," *J. Phys. Soc. Japan* **62**, 1418 (1993).
29. D. W. Sciama, "On the origin of inertia," *Mon. Not. R. Astron. Soc.* **113**, 34 (1953).
30. G. B. Brown, *Retarded Action-at-a-distance* (Cortney Publications, Luton, 1982).
31. H. J. Treder, *Die Relativituet der Tragheit* (Akademie-Verlag, Berlin, 1972).
32. W. F. Edwards, "Inertia and an alternative approach to the theory of interactions," *Proc. Utah Acad. Sci., Arts, Lett.* **51** (2), 1 (1974).
33. J. B. Barbour and B. Bertotti, "Gravity and intertia in a Machian framework," *Nuovo Cimento B* **38**, 1 (1977).
34. P. B. Eby, "Perihelion precession as a Machian effect," *Lett. Nuovo Cimento* **18**, 93 (1977).
35. A. Ghosh, "Velocity-dependent intertial induction: a possible tired-light mechanism," *Apeiron* **9–10**, 35 (1991).
36. A. P. French, *Newtonian Mechanics* (Norton, New York, 1971).
37. T. E. Phipps, "Should Mach's principle be taken seriously?" *Spec. Sci. Technol.* **1**, 499 (1978).
38. P. Gerber, "Die räumliche und zietliche ausbreitung der gravitation," *Z. Math. II* **43**, 93 (1898).

39. H. Seeliger, "Bemerkung zu P. Gerbers Aufsatz: 'Die Fortplanzungsgeschwindligkeit der Gravitation,'" *Ann. Phys.* (*Leipzig*) **53**, 31 (1917).
40. J. P. Wesley, *Selected Topics in Advanced Fundamental Physics* (Benjamin Wesly, Blumberg, 1991) pp. 185–189.
41. P. Graneau, "Some cosmological consequences of Mach's principle," *Hadronic J. Suppl.* **5**, 335 (1990).
42. E. Harrison, "Newton and the infinite universe," *Phys. Today* **39**, 24 (1986).
43. A. K. T. Assis, "On the absortion of gravity," *Apeiron* **13**, 3 (1992).
44. A. K. T. Assis, "A steady-state cosmology," in *Progress in New Cosmologies: Beyond the Big Bang*, H. C. Arp, C. R. Keys, and K. Rudnicki, eds. (Plenum, New York, 1993), pp. 153–167.
45. H. Arp, G. R. Burbidge, F. Hoyle, J. Narlikar, and C. Wickramasinghe, "The extragalactic universe—an alternative view," *nature* **346**, 807 (1990).
46. A. A. Penzias and R. W. Wilson, "A measurement of excess antenna temperature at 4080 mc/s," *Astrophys. J.* **142**, 419 (1965).
47. A. K. T. Assis, *Weber's Electrodynamics* (Kluwer Academic, Dordrecht, 1994).