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LONGITUDINAL FORCES IN WEBER'S ELECTRODYNAMICS

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We analyze the existence of longitudinal forces with Weber's electrodynamics. We show that they exist. But they cannot explain the exploding wire phenomena due to their order of magnitude.

1. Introduction

The exploding wire is one of the main experiments showing the existence of longitudinal forces in current carrying circuits.[1–4] This effect cannot be explained by Grassmann's force $Id\mathbf{l} \times \mathbf{B}$ acting on a current element $Id\mathbf{l}$. The reason is that this force is always orthogonal to the element, no matter the value of the magnetic field $\mathbf{B}$. A possible explanation might be Ampère's force between current elements.[5] Nowadays there is a great controversy over this subject and many people, including the authors, believe that even in the case of a single closed circuit the net force on each current element according to Ampère's expression will be orthogonal to the element, as is the case with Grassmann's force. A proof of the non-existence of a net longitudinal force acting on a straight piece of wire asymmetrically located in a circuit has been given recently with an important null experiment by Robson and Sethian.[6] This, however, does not rule out the possible existence of an internal tension in the conductor, as shown in the exploding wire phenomena. Due to this fact, we decided to analyze this problem with Weber's electrodynamics.[7] Accordingly, we calculate the longitudinal force acting on the lattice of an asymmetrical part of a rectangular circuit.

2. The Circuit

The geometry analyzed here is represented in Fig. 1. We have a closed rectangular circuit 1, 2, …, 7, B. The bridge is represented by piece B and the support by pieces 1 to 7. There is a constant dc current I flowing in the circuit. We suppose

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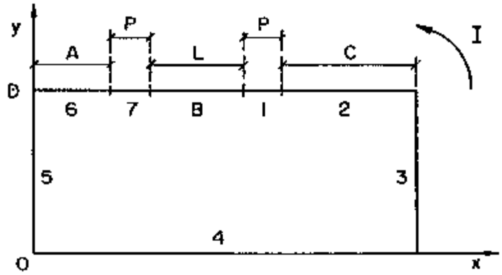

Fig. 1. A rectangular circuit carrying a constant current I. With a counter clockwise current we have a clockwise flow of electrons.

the wire to be made up of a positive lattice fixed relative to the laboratory and of free electrons which move relative to the lattice yielding the electric current I. The source of the current may be a chemical battery (which produces a constant tension) and this electromotive force is balanced by the uniform electrical resistance of the wire.

What we need to calculate is the longitudinal force on the lattice of the bridge B when it is not symmetrically located in the circuit. Pieces 1 and 7 have the same length P. So, we need to calculate the x-component of the force on B when $A \neq C$, where A and C are the lengths of the pieces 6 and 2, respectively. The bridge has a length L and pieces 3 and 5 has length D.

To calculate this force we utilize Weber's electrodynamics. Weber's force exerted by the charge dq_j on dq_i is given by[7]:

$$d^2\mathbf{F}_{ji} = \frac{dq_i dq_j}{4\pi\varepsilon_0}\frac{\hat{r}_{ij}}{r_{ij}^2}\left[1 + \frac{1}{c^2}\left(\mathbf{v}_{ij}\cdot\mathbf{v}_{ij} - \frac{3}{2}(\hat{r}_{ij}\cdot\mathbf{v}_{ij})^2 + \mathbf{r}_{ij}\cdot\mathbf{a}_{ij}\right)\right]. \qquad (1)$$

The charges dq_i and dq_j are located at $\mathbf{r}_i$ and $\mathbf{r}_j$, respectively. In this equation we have $\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$, $\mathbf{v}_{ij} \equiv \mathbf{v}_i - \mathbf{v}_j \equiv d\mathbf{r}_{ij}/dt$, $\mathbf{a}_{ij} \equiv \mathbf{a}_i - \mathbf{a}_j \equiv d\mathbf{v}_{ij}/dt = d^2\mathbf{r}_{ij}/dt^2$, $r_{ij} \equiv |\mathbf{r}_{ij}|$, $\hat{r}_{ij} = \mathbf{r}_{ij}/r_{ij}$ is the unit vector pointing from dq_j to dq_i, $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}$ is the vacuum permittivity and $c = 3 \times 10^8\,\mathrm{m/s}$.

In order to perform the integrations we replace dq by λdl, where λ is the linear charge density of the wire. We are interested here in longitudinal forces so that we only calculate the x-component of the forces acting on the charges of the bridge from now on. As we are supposing a constant current I and the wire has no curvatures we have $\mathbf{a} = 0$ for all charges, neglecting the corners. This simplifies Eq. (1). Let us calculate, at first place, the force of the charges of piece 2 on the charges of the bridge B. With $dq_\mathrm{B} = \lambda_\mathrm{B} dx_\mathrm{B}$, $\mathbf{r}_\mathrm{B} = x_\mathrm{B}\hat{x} + D\hat{y}$, $\mathbf{v}_\mathrm{B} = v_\mathrm{B}\hat{x}$, $dq_2 = \lambda_2 dx_2$, $\mathbf{r}_2 = x_2\hat{x} + D\hat{y}$ and $\mathbf{v}_2 = v_2\hat{x}$ in Eq. (1) yields (with Fig. 1):

$$\begin{aligned} F_{2\mathrm{B}} &= -\frac{\lambda_\mathrm{B}\lambda_2}{4\pi\varepsilon_0}\int_{A+P}^{A+P+L} dx_\mathrm{B} \int_{A+2P+L}^{A+2P+L+C} dx_2 \frac{1 - (v_\mathrm{B}^2 - 2v_\mathrm{B}v_2 + v_2^2)/2c^2}{(x_2 - x_\mathrm{B})^2} \\ &= \frac{\lambda_\mathrm{B}\lambda_2}{4\pi\varepsilon_0}\left(1 - \frac{v_\mathrm{B}^2 - 2v_\mathrm{B}v_2 + v_2^2}{2c^2}\right)\left(\ln\frac{P+L+C}{P+C} - \ln\frac{P+L}{P}\right). \qquad (2)\end{aligned}$$

With $\lambda_B = \lambda_+$ and $\mathbf{v}_B = 0$ (or $\lambda_B = \lambda_-$ and $\mathbf{v}_B = V_d\hat{x}$) we represent the positive lattice of the bridge (or its free electrons, where V_d is the drifting velocity of the electrons responsible for the current, namely, their velocity relative to the lattice). With $\lambda_2 = \lambda_+$ and $\mathbf{v}_2 = 0$ (or $\lambda_2 = \lambda_-$ and $\mathbf{v}_2 = V_d\hat{x}$) we represent the positive lattice of piece 2 (or its free electrons). With these replacements, Eq. (2) can represent the force of the lattice of piece 2 on the lattice or free electrons of the bridge, or the force of the electrons of piece 2 on the lattice or free electrons of the bridge.

Now the x-component of the force of the charges of piece 3 on those of the bridge. With $dq_3 = \lambda_3 dy_3$, $\mathbf{r}_3 = (A+2P+L+C)\hat{x} + y_3\hat{y}$, $\mathbf{v}_3 = v_3\hat{y}$, see Fig. 1, and the previous expressions for the charges of the bridge in Eq. (1) we arrive at:

$$
\begin{aligned}
F_{3B} = {} & \frac{\lambda_B\lambda_3}{4\pi\varepsilon_0}\int_{A+P}^{A+P+L} dx_B \int_0^D dy_3 \left[\left(1 + \frac{v_B^2 + v_3^2}{c^2}\right)\frac{x_B - (A+2P+L+C)}{r_{B3}^3}\right. \\
& - \frac{3v_B^2}{2c^2}\frac{[x_B - (A+2P+L+C)]^3}{r_{B3}^5} \\
& - \frac{3v_3^2}{2c^2}\frac{[x_B - (A+2P+L+C)](D-y_3)^2}{r_{B3}^5} \\
& \left. + \frac{3v_Bv_3}{c^2}\frac{[x_B - (A+2P+L+C)]^2(D-y_3)}{r_{B3}^5}\right] \\
= {} & \frac{\lambda_B\lambda_3}{4\pi\varepsilon_0}\left[\left(1 + \frac{v_3^2}{2c^2}\right)\left(\ln\frac{\sqrt{(P+L+C)^2 + D^2} + D}{P+L+C}\right.\right. \\
& \left. - \ln\frac{\sqrt{(P+C)^2 + D^2} + D}{P+C}\right) \\
& + \frac{v_B^2 - v_3^2}{2c^2}\left(\frac{D}{\sqrt{(P+C)^2 + D^2}} - \frac{D}{\sqrt{(P+L+C)^2 + D^2}}\right) \\
& + \frac{v_Bv_3}{c^2}\left(\ln\frac{P+L+C}{P+C} - \ln\frac{\sqrt{(P+C)^2 + D^2} - (P+C)}{\sqrt{(P+L+C)^2 + D^2} - (P+L+C)}\right. \\
& \left.\left. + \frac{P+L+C}{\sqrt{(P+L+C)^2 + D^2}} - \frac{P+C}{\sqrt{(P+C)^2 + D^2}}\right)\right], \qquad (3)
\end{aligned}
$$

where $r_{B3} = \sqrt{[x_B - (A+2P+L+C)]^2 + (D-y_3)^2}$. With $\lambda_3 = \lambda_+$ and $\mathbf{v}_3 = 0$ we represent the positive lattice of piece 3. With $\lambda_3 = \lambda_-$ and $\mathbf{v}_3 = -V_d\hat{y}$ (see Fig. 1) we represent its free electrons.

With $dq_4 = \lambda_4 dx_4$, $\mathbf{r}_4 = x_4\hat{x}$, $\mathbf{v}_4 = v_4\hat{x}$, in Eq. (1) yields, by a similar reasoning, the x-component of the force of the charges of piece 4 on the charges of the bridge, namely:

$$F_{4\mathrm{B}} = \frac{\lambda_\mathrm{B}\lambda_4}{4\pi\varepsilon_0}\left[\left(1-\frac{(v_\mathrm{B}-v_4)^2}{2c^2}\right)\left(\ln\frac{\sqrt{(A+P)^2+D^2}+(A+P)}{\sqrt{(P+L+C)^2+D^2}-(P+L+C)}\right.\right.$$
$$\left.-\ln\frac{\sqrt{(A+P+L)^2+D^2}+(A+P+L)}{\sqrt{(P+C)^2+D^2}-(P+C)}\right)$$
$$-\frac{(v_\mathrm{B}-v_4)^2}{2c^2}\left(\frac{A+P+L}{\sqrt{(A+P+L)^2+D^2}}+\frac{P+C}{\sqrt{(P+C)^2+D^2}}\right.$$
$$\left.\left.-\frac{A+P}{\sqrt{(A+P)^2+D^2}}-\frac{P+L+C}{\sqrt{(P+L+C)^2+D^2}}\right)\right]. \tag{4}$$

With $dq_5 = \lambda_5 dy_5$, $\mathbf{r}_5 = y_5\hat{x}$, $\mathbf{v}_5 = v_5\hat{y}$, we get for the x-component of the force:

$$F_{5\mathrm{B}} = \frac{\lambda_\mathrm{B}\lambda_5}{4\pi\varepsilon_0}\left[\left(1+\frac{v_5^2}{2c^2}\right)\left(\ln\frac{\sqrt{(A+P)^2+D^2}+D}{A+P}\right.\right.$$
$$\left.-\ln\frac{\sqrt{(A+P+L)^2+D^2}+D}{A+P+L}\right)$$
$$+\frac{v_\mathrm{B}^2-v_5^2}{2c^2}\left(\frac{D}{\sqrt{(A+P+L)^2+D^2}}-\frac{D}{\sqrt{(A+P)^2+D^2}}\right)$$
$$-\frac{v_\mathrm{B}v_5}{c^2}\left(\frac{A+P}{\sqrt{(A+P)^2+D^2}}-\frac{A+P+L}{\sqrt{(A+P+L)^2+D^2}}\right.$$
$$\left.\left.+\ln\frac{\sqrt{(A+P+L)^2+D^2}+(A+P+L)}{\sqrt{(A+P)^2+D^2}+(A+P)}-\ln\frac{A+P+L}{A+P}\right)\right]. \tag{5}$$

And finally, with $dq_6 = \lambda_6 dx_6$, $\mathbf{r}_6 = x_6\hat{x}+D\hat{y}$ and $\mathbf{v}_6 = v_6\hat{x}$ we get:

$$F_{6\mathrm{B}} = \frac{\lambda_\mathrm{B}\lambda_6}{4\pi\varepsilon_0}\left(1-\frac{v_\mathrm{B}^2-2v_\mathrm{B}v_6+v_6^2}{2c^2}\right)\left(\ln\frac{P+L}{P}-\ln\frac{A+P+L}{A+P}\right). \tag{6}$$

Adding $F_{6\mathrm{B}}$ to $F_{2\mathrm{B}}$ with $\mathbf{v}_2 = \mathbf{v}_6$ and $\lambda_2 = \lambda_6$ yields a zero value if $A = C$, for any value of P. This shows that the sum of the forces of pieces 1 and 7 on the charges of the bridge is equal to zero, as they have the same size and are symmetrically located relative to the bridge:

$$F_{1\mathrm{B}} + F_{7\mathrm{B}} = 0\,, \tag{7}$$

supposing $\lambda_7 = \lambda_1$ and $\mathbf{v}_7 = \mathbf{v}_1$. This is the reason why we do not need to consider these forces separately.

We now add the x-components of the forces of the positive charges of pieces 1 to 7 acting on the positive charges of the bridge (with $\lambda_1 = \cdots = \lambda_7 = \lambda_B = \lambda_+$ and $v_1 = \cdots = v_7 = v_B = 0$) to the x-components of the forces of the electrons of pieces 1 to 7 acting on the positive charges of the bridge (with $\lambda_1 = \cdots = \lambda_7 = \lambda_-$, $\lambda_B = \lambda_+$, $v_1 = v_2 = v_5 = v_6 = v_7 = V_d$, $v_3 = v_4 = -V_d$, see Fig. 1, and $v_B = 0$). The force of the positive lattice of the bridge on itself is obviously zero and the same happens with the force of the electrons of the bridge on its lattice. With the previous results, supposing the charge neutrality of the wire ($\lambda_- = -\lambda_+$) and utilizing the identifications $I = |\lambda_- V_d|$ and $c^2 = 1/\mu_0\varepsilon_0$ ($\mu_0 = 4\pi \times 10^{-7}$ kg m C^{-2} is the vacuum permeability), yields the final result:

$$
\begin{aligned}
&F_{\mathrm{S}_+ \,\mathrm{on}\, \mathrm{B}_+} + F_{\mathrm{S}_- \,\mathrm{on}\, \mathrm{B}_+} \\
&\quad = -\frac{\mu_0}{4\pi} I^2 \left[\ln \frac{A+P+L}{A+P} - \ln \frac{P+C+L}{P+C} \right. \\
&\qquad + \frac{1}{2} \left(\ln \frac{\sqrt{(A+P+L)^2 + D^2} + (A+P+L)}{\sqrt{(A+P)^2 + D^2} + (A+P)} \right. \\
&\qquad - \ln \frac{\sqrt{(P+C+L)^2 + D^2} + (P+C+L)}{\sqrt{(P+C)^2 + D^2} + (P+C)} \\
&\qquad + \ln \frac{\sqrt{(P+C+L)^2 + D^2} + D}{\sqrt{(P+C)^2 + D^2} + D} - \ln \frac{\sqrt{(A+P+L)^2 + D^2} + D}{\sqrt{(A+P)^2 + D^2} + D} \\
&\qquad + \frac{A+P+D}{\sqrt{(A+P)^2 + D^2}} - \frac{A+P+D+L}{\sqrt{(A+P+L)^2 + D^2}} \\
&\qquad \left.\left. + \frac{P+C+D+L}{\sqrt{(P+C+L)^2 + D^2}} - \frac{P+C+D}{\sqrt{(P+C)^2 + D^2}} \right) \right] . \qquad (8)
\end{aligned}
$$

This is the net longitudinal force (x-component) exerted by the positive and negative charges of the support (S_+ and S_-) on the positive lattice of the bridge, B_+.

Calculating the force of the positive and negative charges of the support on the negative charges of the bridge (B_-) yields the negative of this result, namely:

$$
F_{\mathrm{S}_+ \,\mathrm{on}\, \mathrm{B}_-} + F_{\mathrm{S}- \,\mathrm{on}\, \mathrm{B}-} = -(F_{\mathrm{S}_+ \,\mathrm{on}\, \mathrm{B}_+} + F_{S_- \,\mathrm{on}\, \mathrm{B}_+}) . \qquad (9)
$$

Adding these two together, $F_{\mathrm{SB}_+} + F_{\mathrm{SB}_-}$, with $\mathrm{S} \equiv \mathrm{S}_+ + \mathrm{S}_-$, yields a zero value. That is, although the neutral bridge is not symmetrically located relative to the circuit when $A \neq C$, the net longitudinal force on it is zero.

This zero resultant force has been shown conclusively in Robson and Sethian's important experiment.[6]

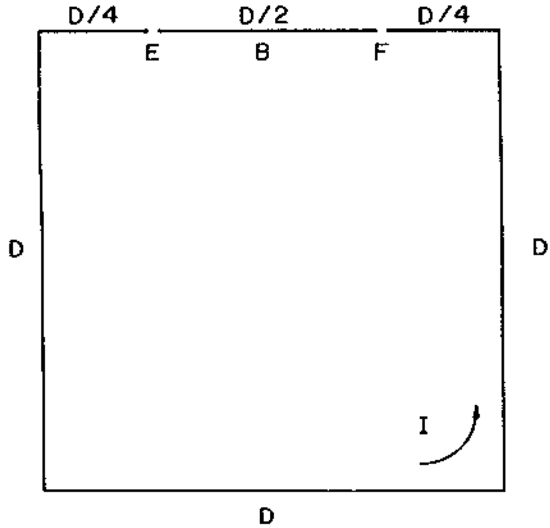

Fig. 2. A square circuit of side D. The bridge of length $D/2$ is at the middle of the upper side and is symmetrically located in the circuit.

3. Tension in the Bridge

In order to analyze the possible existence of tension in this circuit, despite the zero net force, and to present some orders of magnitude for these expressions we consider the square circuit of Fig. 2. We now have $A+2P+L+C = D$, $A+P = P+C = D/4$, $L = D/2$, so that the bridge is symmetrically located in this square circuit. The bridge is composed of two equal parts $B_{\rm L}$ and $B_{\rm R}$, the left half and the right half respectively. Each half has a length of $D/4$. With these values in Eq. (8) we obtain that the longitudinal force exerted on the positive lattice of the left half of the bridge by the remainder of the circuit (positive and negative charges of the support and of the right half) is given by

$$F_{\rm S\,on\,B_{L+}} + F_{\rm B_R\,on\,B_{L+}} = -0.27\frac{\mu_0}{4\pi}I^2\,. \tag{10}$$

This force is independent of the value of D. With a current of 6×10^3 A, this yields a force to the left of approximately 0.7 N. The force on the positive lattice of the right half exerted by the positive and negative charges of the left half and the support is also given by this value, but now pointing to the right, see Fig. 3.

In an experiment with a geometry of this kind (although the support has not been clearly specified) and currents of this order of magnitude, an aluminum wire 1 meter long and with 1.2 mm diameter was broken in many pieces.[1] The analysis of the fragments indicated that the wire was broken in solid state due to a mechanical tension and not, for instance, due to melting or Joule heating. This aluminum wire was mechanically disconnected to the remainder of the circuit and we represent it

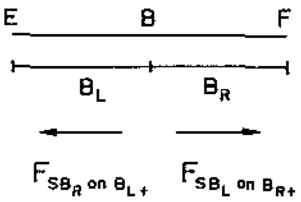

Fig. 3. The net force on the positive lattice of the left (right) half of the bridge exerted by the positive and negative charges of the support and of the right (left) half points to the left (right), indicating a tension.

by the bridge B in Fig. 2. At E and F of this figure there are electric arcs closing the current but allowing the bridge to be mechanically disconnected from the support. As the bridge is mechanically disconnected from the remainder of the circuit it does not feel any force or tension exerted on the support. The net force on the bridge is obviously zero due to its symmetrical location. But it might be under tension and this would be represented by the force on its left half, which was given by Eq. (10). But the longitudinal force acting on the lattice which we obtained with Weber's expression was only of the order of 1 N. And the tensile strength for aluminum wires is of the order 50–500 N/mm^2.[8] This shows that in this experiment it was not Weber's force responsible for breaking the wire.

4. Discussion

We obtained a zero net force acting on the lattice plus electrons according to Weber's electrodynamics, in agreement with the experimental findings of Robson and Sethian.[6] However, when considering only the lattice we have found a net longitudinal force which might put the wire under tension. When comparing these forces with a real exploding wire experiment we have found this tension to be two orders of magnitude smaller than the required tensile strength to break the wire. Despite this fact let us analyze a little more the consequences of this force.

It acts on the positive lattice and we have seen previously that there is an opposite force acting on the electrons. Considering that we are in a steady state with constant current, this force on the electrons need to be balanced by something else. The electromotive force of the battery is already balanced by the resistance of the wire. The consequence is that this extra force on the electrons will cause an accumulation of electrons at the middle of each side, as they will be under compression while the positive charges composing the lattice are under tension. In equilibrium this distribution of charges should balance this compression.

We do not attempt to calculate this new distribution of electrons here. But one consequence is that there will be a new Coulombian repulsion between them, which did not exist previously as we were considering a neutral current. The effects of this Coulombian repulsion will not be analyzed here, as they are beyond the scope of this work.

Another aspect should be mentioned. In the exploding wire experiments cited above, we have usually a current generated by the discharging of a capacitor bank, instead of a *dc* current due to a chemical battery. This means that the explosion of the wire happens with an alternate current, while the analysis of the present work has been restricted to a *dc* current.

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