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The Oscillatory Motion of Charged Particles by Weber's Electrodynamics

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Abstract

We analyse the oscillatory motion of a charged particle outside an ideal capacitor by four different models: classical, relativistic, Weber's electrodynamics and Weber's electrodynamics plus Schrödinger's mechanics. These two last models yield a period of oscillation depending on the voltage of the capacitor.

PACS: 41.20. -q Electric, magnetic and eletromagmetic fields
12.25. +e Models for gravitational interactions
03.20 +i Classical mechanics of discrete systems: general mathematical aspects

Key words: Weber's potential, Weber force law, Weber's electrodynamics, mass variation, Schrödinger's potential energy.

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1 – Introduction

It is a kwown fact that Weber's electrodynamics predicts a force on a test charge moving outside an ideal capacitor, [1] – [2]. This force should not exist according to classical electrodynami

cs based on Lorentz's force law.

In this paper we analyse the oscillatory motion of a charged particle outside an ideal capacitor (Figure 1). Our aim is to verify how the Weberian force mentioned above modify the period of oscillation and the energy of the system. For this reason we are going to analyse four different models.

The capacitor is located parallel to the yz plane with positive (negative) plates of surface charge densities σ_0 ($-\sigma_0$) located at x_0 ($-x_0$). The test charge q of mass m is located at $\vec{r} = x\hat{x}$ with $x < -x_0$ and oscillates along this axis connected to a spring of elastic constant k. The equilibrium position of the spring is at x_E. That is, when $x = x_E$ the test charge does not feel any force from the spring. We choose the zero of the potential at $x = 0$, in the middle of the plates.

In the classical harmonic oscillator (C.H.O.) we have a point charge q outside the ideal capacitor oscillating orthogonally to its plates. Its constant energy is given by the sum of the kinetic ($mv^2/2$), elastic ($k(x - x_E)^2/2$) and electrostatic ($-q\Delta\Phi/2$) energies. Here $\Delta\Phi = 2\sigma_0 x_0/\varepsilon_0$ is the voltage between the plates of the capacitor.

In the relativistic harmonic oscillator (R.H.O.) we only change the kinetic energy to the equivalent relativistic expression.

In Weber's theory we have a charged particle and a difference of potential in the capacitor. The energy of the particle is given by the classical kinetic energy plus the elastic potential energy and the Weberian interaction energy that we are going to present below. We call it the Weberian harmonic oscillator (W.H.O.).

When we consider high velocities and a charged particle interacting with a charged capacitor we utilize the theory named Schrödinger's harmonic oscillator (S.H.O.). In this theory the energy is given in the same way that in W.H.O. where we change the classical kinetic energy by Schrödinger's kinetic energy, [3].

2 – Period of oscillation and energy of the system

We write the Lagrangian of the S.H.O. as

$$L = -mc^2\sqrt{1-\frac{\dot\eta^2}{c^2}} + \frac{q\Delta\Phi}{2} - \frac{q\Delta\Phi}{4}\frac{\dot\eta^2}{c^2} - \frac{k\eta^2}{2} + mc^2\,, \tag{1}$$

where $\eta = x - x_E$. The Lagragian above has three parts. The first part is due to kinetic energy

$$L_k = -mc^2\sqrt{1-\frac{\dot\eta^2}{c^2}} + mc^2\,. \tag{2}$$

Erwin Schrödinger obtained this Lagrangian considering only gravitational interactions. For more details see [3]. It must be remarked that Eq. (2) has the same form than the relativist Lagrangian but its meaning is different. In Schrödinger's approach the kinetic energy has its origins in interactions of the test particle with the distant universe through a generalized Weber's law for gravitation. The second part of the Lagrangian is the Weberian electromagnetic interaction given by

$$L_W = \frac{q\Delta\Phi}{2} - \frac{q\Delta\Phi}{4}\frac{\dot\eta^2}{c^2}\,. \tag{3}$$

We obtain Eq. (3) when we integrate the expression

$$U_{ij} = \frac{q_i q_j}{4\pi\varepsilon_0}\frac{1}{r_{ij}}\left(1+\frac{1}{2}\frac{\dot{r_{ij}}^2}{c^2}\right)\,, \tag{4}$$

in both plates of capacitor, [1]–[2]. Here r_{ij} is the distance between q_i and q_j and $\dot r_{ij} = dr_{ij}/dt$ is their radial velocity. With $\dot r_{ij} = 0$ we recover Coulomb's potential energy. The last part of the of the Lagrangian is related to the elastic potential energy:

$$L_S = -\frac{k\eta^2}{2}\,. \tag{5}$$

The equation of energy E is obtained from Eq. (1) by the usual procedure, see [4]. It is given by

$$\frac{E}{mc^2} = \frac{1}{\sqrt{1-\dot{\eta}^2/c^2}} - \frac{q\Delta\Phi}{2mc^2}\frac{\dot{\eta}^2}{2c^2} - \frac{q\Delta\Phi}{2mc^2} + \frac{k\eta^2}{2mc^2} - mc^2\,. \tag{6}$$

When $t=0$ we suppose $\eta=0$ and $\dot{\eta}=\dot{\eta}_0$. When $\eta=\pm b$ we have $\dot{\eta}=0$. By conservation of energy we can equate these two situations yielding

$$1 + \frac{kb^2}{2mc^2}\left(1-\frac{\eta^2}{b^2}\right) = \frac{1}{\sqrt{1-\dot{\eta}^2/b^2}} - \frac{\alpha}{4}\frac{\dot{\eta}^2}{c^2}\,, \tag{7}$$

where $\alpha \equiv q\Delta\Phi/mc^2$ is a dimensionless parameter.

We shall examine the first order corrections. In this aproximation the elastic potential energy is always small compared to the rest mass energy, $kb^2/2 \ll mc^2$. Expanding the radical in Eq. (7) up to v^4/c^4 yields, with $\dot{\eta}\equiv v$:

$$\frac{v^4}{c^4} + \frac{4}{3}\left(1-\frac{\alpha}{2}\right)\frac{v^2}{c^2} - \frac{8}{3}\frac{kb^2}{2mc^2}\left(1-\frac{\eta^2}{b^2}\right) = 0\,. \tag{8}$$

From Eq. (8) we can obtain the period of oscillation τ utilizing the standard procedure presented in [4], p. 324 and [5]. The final result is given by

$$\frac{\tau}{\tau_0} = \sqrt{1-\alpha/2}\left\{1 + \frac{3}{16}\frac{kb}{mc^2}\frac{1}{(1-\alpha/2)^2}\right\}\,,\ \alpha < 2\,. \tag{9}$$

Here $\tau_0 = 2\pi\sqrt{m/k}$ is the period of oscillation for the classical harmonic oscillator.

An interesting aspect of the S. H. O. is the dependence of the period of oscillation with the difference of potencial between the plates. It is somewhat similar to a gravitational redshift.

Now we analyse the limits for low velocities ($v/c \ll 1$). If we consider v/c only up to v^2/c^2 in Eq. (1) we obtain

$$L = \frac{m\dot{\eta}^2}{2} + \frac{q\Delta\Phi}{2} - \frac{q\Delta\Phi}{4}\frac{\dot{\eta}^2}{c^2} - \frac{k\eta^2}{2}\,, \tag{10}$$

From Eq. (10) by the usual procedure, see [4], the equation of motion is given by

$$m(1-\alpha/2)\ddot{\eta} + k\eta = 0\,, \tag{11}$$

and the normalized energy by

$$\frac{E}{mc^2} = \left(1 - \frac{\alpha}{2}\right)\frac{\dot{\eta}^2}{2c^2} - \frac{\alpha}{2} + \frac{k\eta^2}{2mc^2}\,. \tag{12}$$

This is the equation of energy of the W. H. O. It must be remarked that Eq. (12) and Eq. (11) are similar to the C. H. O. with m_{ei} instead of m. Here m_{ei} is the effective inertial mass given by

$$m_{ei} = m - \frac{q\Delta\Phi}{2c^2} = m\left(1 - \frac{q\Delta\Phi}{2mc^2}\right) = m\left(1 - \frac{\alpha}{2}\right)\,. \tag{13}$$

For $\alpha = 0$ Eq. (12) and Eq. (11) reduce to the C. H. O. From Eq. (12) the period of oscillation for the W. H. O. with $\alpha < 2$ is given by

$$\frac{\tau}{\tau_0} = \sqrt{1-\alpha/2}\,. \tag{14}$$

The solution of the equation of motion for the C.H.O. with initial conditions $\eta = b$ and $\dot{\eta} = 0$ is given by

$$\eta = b\cos\omega_0 t\,, \tag{15}$$

$$\dot{\eta} = -\omega_0 b \sin\omega_0 t\,. \tag{16}$$

For the same initial conditions the solution of Eq. (11) with $\alpha < 2$ is given by

$$\eta = b\cos\left[\frac{\omega_0 t}{\sqrt{1-\alpha/2}}\right] , \tag{17}$$

$$\dot{\eta} = -\frac{\omega_0 b}{\sqrt{1-\alpha/2}}\sin\left[\frac{\omega_0 t}{\sqrt{1-\alpha/2}}\right] . \tag{18}$$

When $\alpha = 0$ we recover the solution of C.H.O. We can get the energy and the period of oscillation for the R. H. O. from Eq. (6) and Eq. (9) with $\alpha = 0$, see [4] – [5]. The diagram below resumes the models dicussed here.

$$\begin{array}{ccccc} \text{S. H. O} & \xrightarrow{v/c\ll 1} & \text{W. H. O.} & \xrightarrow{\alpha=0} & \text{C. H. O.} \\ & \alpha = 0 \searrow & & \nearrow v/c \ll 1 & \\ & & \text{R. H. O.} & & \end{array}$$

3 – Discussion and Conclusion

In Figure 2 we plot Eq. (9) and Eq. (14). For the aproximation $kb^2/2 \ll mc^2$, which is equivalent to $\dot{\eta} \ll c$, the period of oscillation for R. H. O. goes to the period of oscillation for C. H. O. In this Figure we utilize $kb^2/2mc^2 = 0.001$.

Eqs. (17) and (18) for the W.H.O. are plotted in Figures 3A to 3D for several values of α. With $\alpha = 0$ we recover the C.H.O. In Figures 3A and 3C we plot the normalized position, η/b, against $\omega_0 t$. In Figures 3B and 3D we plot the normalized velocity, $\dot{\eta}/\omega_0 b$, where $\omega_0 b$ is the initial velocity, against $\omega_0 t$. In Figures 3A to 3D we utilize $\alpha = -5$, 0 and 1.

In the range $-\infty < \alpha \leq 0$ the period of oscillation for the W.H.O. is greater than the period of oscillation for the C.H.O. The maximal velocity in Weber's theory is smaller than in the C.H.O. due to effective inertial mass in W.H.O. In the range $0 \leq \alpha < 2$ the period of the C.H.O. is greater than in the W.H.O. This is due to the effective inertial mass in the W.H.O. which is smaller than m.

When $\alpha > 2$ the effective inertial mass, m_{ie}, of W.H.O. is a negative constant. The solution of the equation of motion in this case is given by

$$\eta = \frac{b(e^{\omega_2 t} + e^{-\omega_2 t})}{2} \tag{19}$$

where $\omega_2 = \omega_0/\sqrt{\alpha/2 - 1}$. In Figure 4A we plot the normalized position, η/b, and in Figure 4B the normalized velocity, $\dot{\eta}/\omega_0 b$, both against $\omega_0 t$ for $\alpha = 5$ and 3.

The velocity increases continuously due to the negative effective inertial mass for $\alpha > 2$. Thus the particle is accelerated contrary to the force.

An interesting aspect of the W.H.O. or of the S. H. O. comes out when we consider $v/c \ll 1$ and $\alpha \ll 1$ in Eq. (9). This yields the normalized frequency of oscillation $\omega/\omega_0 = \tau/\tau_0$ as:

$$\frac{\omega}{\omega_0} = \frac{1}{\sqrt{1-\alpha}} \, . \tag{20}$$

So, for $\alpha \ll 1$ Eq. (20) yields:

$$\frac{\Delta\omega}{\omega_0} = \frac{\omega - \omega_0}{\omega_0} = \frac{1}{2}\frac{q\Delta\Phi}{2mc^2} \, . \tag{21}$$

This is similar to a gravitational redshift. For an electron and $\Delta\Phi = 1\text{V}$ we have $\Delta\omega/\omega_0 = 4.92 \times 10^{-7}$.

It would be important to test this prediction in a laboratory experiment.

4 – Acknowledgments

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Figure Captions

Figure 1: Geometry of the problem.

Figure 2: Behaviour of the normalized period against $\alpha \equiv q\Delta\Phi/mc^2$, where $\tau/\tau_0 = 2\pi/\omega_0$. We utilize $kb^2/2mc^2 = 0.001$.

Figure 3: Normalized position and velocity against $\omega_0 t$ for different values of α. The range for α is $\alpha < 2$.

Figure 4: Normalized position and velocity against $\omega_0 t$ for different values of α. The range for α is $\alpha > 2$.

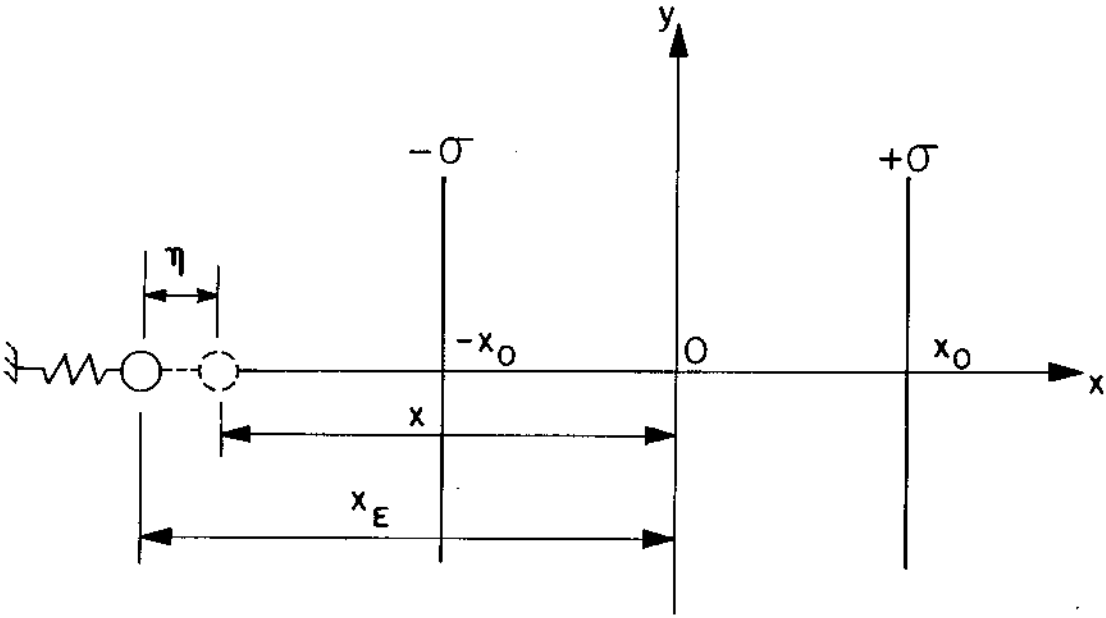

Figure 1

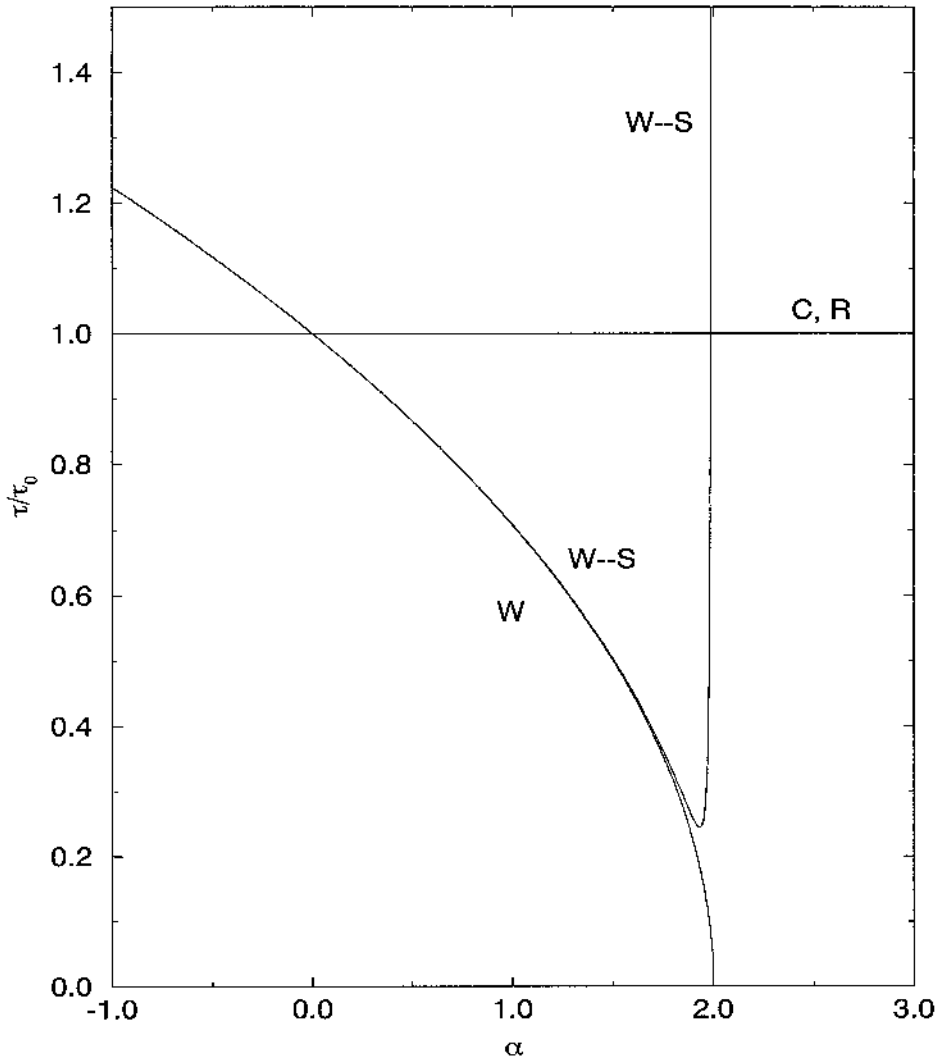

Figure 2

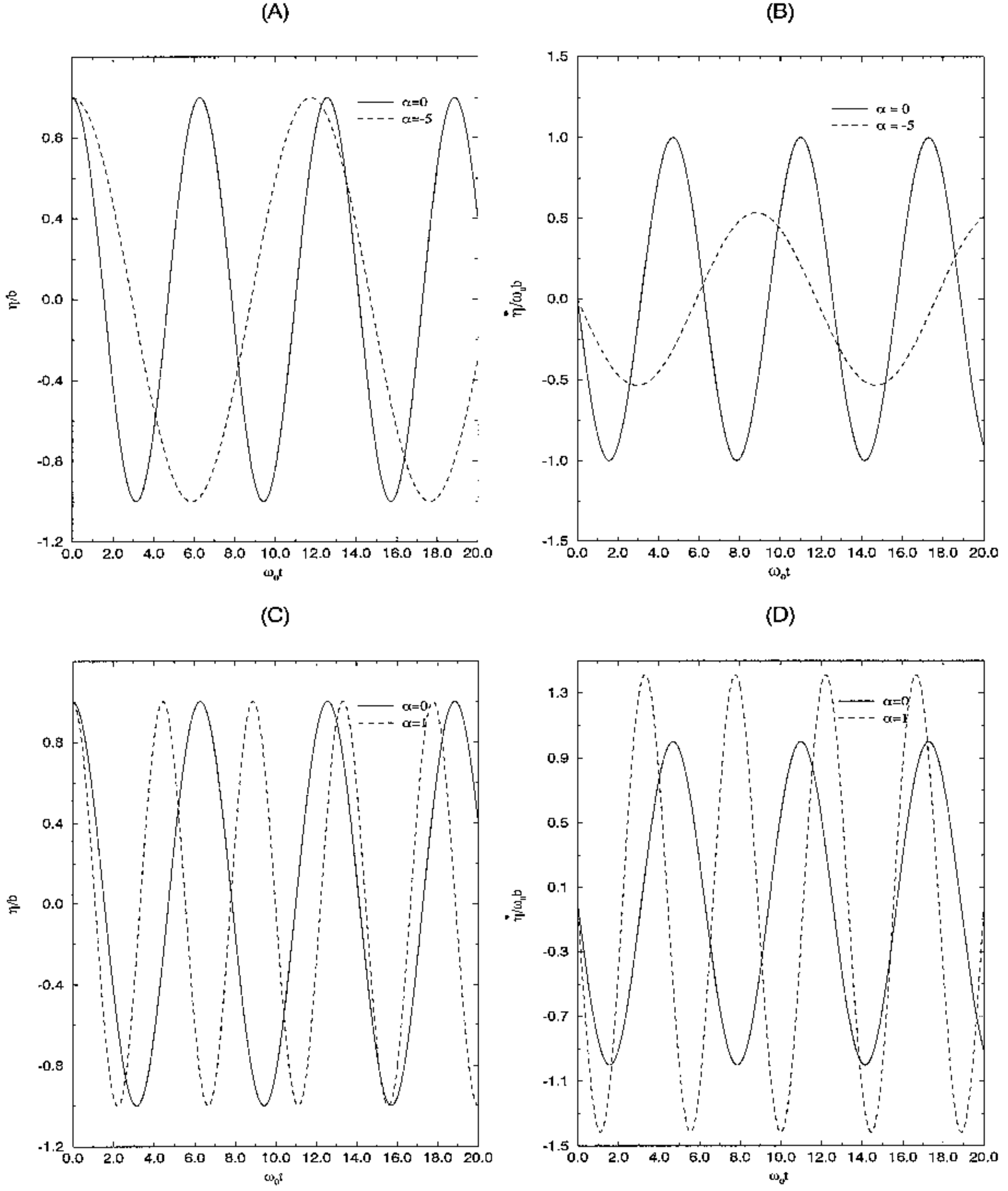

Figure 3

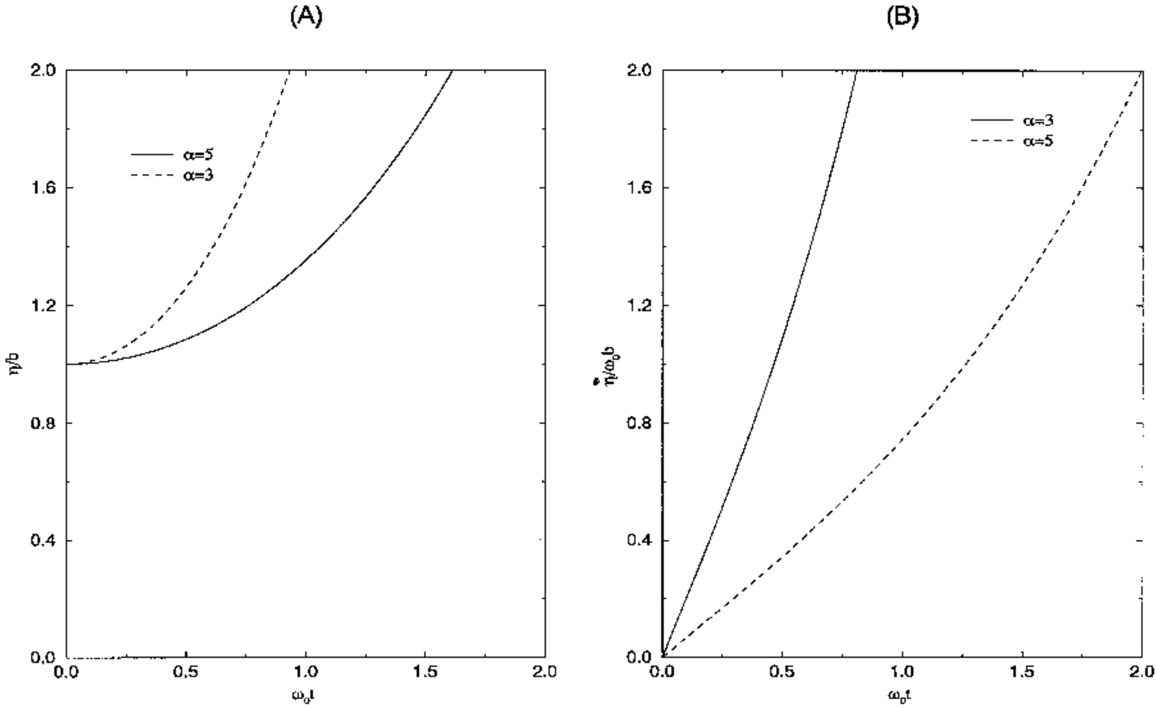

Figure 4